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Abstract: A class of discrete-time nonlinear system and measurement equations having incrementally conic nonlinearities and finite energy disturbances is considered. A linear matrix inequality based resilient observer design approach is presented to guarantee the satisfaction of a variety of performance criteria ranging from simple estimation error boundedness to dissipativity in the presence of bounded perturbations on the gain. Some simulation examples are included to illustrate the proposed design methodology.

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SECTION I.

Introduction

In recent years, many new nonlinear state observer design techniques have been developed: feedback linearization, variable structure, extended linearization, high gain observers and Lyapunov-based techniques, among others. In references,[1,2,3] several feedback linearization techniques for a class of nonlinear systems are proposed. A variable structure technique is proposed in reference.[4] Performance of several nonlinear state observation techniques are compared in.[5] A design methodology for state estimation of nonlinear stochastic systems and measurement models with colored noise process is presented in reference.[6] In,[7] an extension is given of the variable structure observers to unbounded noise and measurement uncertainties. In,[8] an adaptive extension of the sliding-mode observer to state reconstruction of nonlinear systems with uncertainty having unknown bounds is presented. An extended linearization technique, a design method based on the family of linearizations of the system, parameterized by constant operating points for a single input and multiple output nonlinear system model is considered in[9] High gain observers are introduced for nonlinear systems in.[10,11] The Lyapunov-based observer design introduced in[12] for a class of nonlinear systems is extended and improved further by several researchers.[13,14,15,16,17,18,19,20] These are only some of the major approaches to nonlinear observer design included due to space limitations.

In this paper, a novel design of resilient observers is introduced for discrete-time nonlinear systems with incrementally conic nonlinearities and finite energy type disturbances. An observer for which the closed-loop system is destabilized by a small perturbation in the observer gains is referred to as a "fragile" or "non-resilient" observer. Although, this problem was addressed in the gain margin studies in classical control, the topic has regained attention recently.[21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36] Since more and more implementations of controllers and observers are done digitally, there are numerical round off errors in computation. Also, some implementations need manual tuning with obtaining the preferred performance of the observer system. For that reason, it is desired to design an observer that has tolerance to the readjustment of the gain coefficients.

In this work, an observer design method is presented to accommodate such perturbations in the gain where nonlinearities are allowed in both the state and the measurement equations and are more general than the Lipschitz type nonlinearity used in.[12,13,14,15,16,17] Linear matrix inequality (LMI) techniques[30] are used as the main mathematical tool. This result is a natural follow up to the LMI-based robust observer design method presented in[31] for continuous-time uncertain nonlinear systems with integral quadratic constraints and its control counterpart of,[32] which is the design of linear state feedback controllers for a class of continuous-time nonlinear systems with uncertain nonlinear dissipative dynamics in the feedback loop. This result is also generalization of the results in.[33] In the next section, the problem of nonlinear observer design according to various performance criteria is formulated. Then the LMI solutions are introduced in Section 3. Simulation examples presented in Section 4 provide validation for the theoretical results.

The following notation is utilized in this work: $x \in R^n$ denotes an n-dimensional vector with real elements and with the associated norm $\| x \| = (x^T x)^{\frac{1}{2}}$ where $(\cdot)^T$ represents the transpose. $A \in R^{m \times n}$ denotes an $m \times n$ matrix with real elements. A^{-1} is the inverse of matrix A,$A>0(A<0)$ means A is a positive (negative) definite matrix, and I_m is an identity matrix of dimension $m\lambda_{max}(A)\left(\lambda_{min}(A)\right)$ denotes the maximum(minimum) eigenvalue of a symmetric matrix $A. \ell_2$ is the space of vector valued signals with finite energy. Rayleigh's inequalities for a symmetric matrix A will be used in this work. Also the following Schur complement results:

$$[\begin{matrix} A & B \\ B_T & C \end{matrix}] \geq 0 \Leftrightarrow (A - BC^{-1}\ B^T\ > 0\ and$$

$$C > 0) \Leftrightarrow (C - B^T\ A^{-1}\ B > 0\ and\ A > 0)$$

for suitably defined matrices will be used.

SECTION II.

Problem Formulation

Consider a state space representation of a non-linear system of the general form:

$$X_{k+1} = f(X_k, u_k, W_k), y_k = h(X_k, u_k, W_k) \quad (1)$$

where $x_k \in R^n$ is the state to be estimated from knowledge of the control input $u_k \in R^m$ and the measurement output $y_k R^p$. w_k is an ℓ_2 disturbance input. The nonlinear functions f and h are assumed to be measurable functions of their arguments.

We assume the following incrementally conic condition on the nonlinearites:

$$\left\| \begin{bmatrix} F(e_k, w_k) \\ H(e_k, w_k) \end{bmatrix} \right\| \doteq \left\| \begin{bmatrix} f(x_k, u_k, w_k) - f(\hat{x}_k, u_k, 0) - (Ae_k + Bw_k) \\ h(x_k, u_k, w_k) - h(\hat{x}_k, u_k, 0) - (Ce_k + Dw_k) \end{bmatrix} \right\| \le \| A_f e_k + B_f w_k \| \quad (2)$$

for $e_k = x_k - \hat{x}_k$ for any two vectors $x_k, \hat{x}_k \in R^n$ and for some matrices $A, B, C, D, A_f,$ and B_f. This describes in incremental terms the maximum deviation given by the right side of (2) of the nonlinearities F and H from the central linear system describing the error evolution

$$e_{k+1} = Ae_k + Bw_k, y_k = Ce_k + Dw_k$$

where (A,C) is a detectable pair. Note that vector functions f and h which are globally Lipschitz in their arguments used in [12]"[17] are special cases of incremental conicity defined in (2) with A=0,B=0,C=0 and D=0, where (A,C) is trivially detectable.

Let $\hat{x}_k$, the estimate of the true state, obey the following nonlinear Luenberger observer equation

$$\hat{x}_{k+1} = f(\hat{x}_k, u_k, 0) + (K + \Delta_k)\big(y_k - h(\hat{x}_k, u_k, 0)\big) \quad (3)$$

where Δ_k represents the additive perturbation (due to computational or tuning errors) in the observer gain which is bounded as follows:

$\Delta_k^T \Delta_k \le rI$ for r>0 and for all k≥0.

The dimension of Δ is identical to the dimension of the gain. Substituting from equations (1) – (3), adding and subtracting the same terms and then rearranging, we find that the error dynamics obey

$$\begin{aligned} e_{k+1} &= f(x_k,u_k,w_k) - f(\hat{x}_k,u_k,0) \pm (Ae_k + Bw_k) \\ &-(K+\Delta_k)\{h(x_k,u_k,w_k) - h(\hat{x}_k,u_k,0) \pm (Ce_k + Dw_k)\} \\ &= (A-(K+\Delta_k)C)e_k + (B-KD)w_k \\ &+F(e_k,w_k) - (K+\Delta_k)H(e_k,w_k) \\ &= (A-KC)e_k + [I, -(K+\Delta_k)]\begin{bmatrix} Bw_k + F(e_k,w_k) \\ Dw_k + H(e_k,w_k) \end{bmatrix} \end{aligned} \quad (4)$$

Let Z_k denote the performance output where

$$z_k = C_z e_k + D_z w_k \quad (5)$$

and consider the general performance objective

$$V_{k+1} - V_k + \delta \parallel z_k \parallel^2 + \in \parallel w_k \parallel^2 - \beta {z_k}^T w_k \leq 0 \quad (6)$$

for an energy function $V_k = {e_k}^T P e_k$ where $P>0$.

Notice that upon summation, inequality (6) yields

$${e_N}^T P e_N \leq {e_0}^T P e_0 - \sum_{k=0}^{N} (\delta | z_k \parallel^2 + \in \parallel w_k \parallel^2 - \beta {z_k}^T w_k) \quad (7)$$

or by using Rayleigh's inequalities, we obtain

$$\begin{aligned} \lambda_{min}(P) \parallel e_N \parallel^2 &\leq \lambda_{max}(P) \parallel e_0 \parallel^2 \\ &- \sum_{k=0}^{N} (\delta \parallel z_k \parallel^2 + \in \parallel w_k \parallel^2 - \beta {z_k}^T w_k) \end{aligned} \quad (8)$$

that allows several optimization possibilities in a unified eigenvalue problem[30] framework. We can design different observers for a variety of performance criteria for this class of systems.

First of all, in the absence of noise $w_k \equiv 0, k \geq 0$ if we take $\delta = 0, \beta = 0$, and $\in=0$, (8) yields

$$\parallel e_N \parallel^2 \leq \frac{\lambda_{max}(P)}{\lambda_{min}(P)} \parallel e_0 \parallel^2$$

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This means that by minimizing $\lambda_{\max}(P)$ and maximizing $\lambda_{\min}(P)$, we can lower the bound on the norm of the estimation error, which will guarantee a faster response for the observer. Note that this implies boundedness of the estimation error (stability).

By taking $\delta>0$, $\beta=0$, and $\in=0$, (8) will yield a bound on the energy of the performance output in terms of the initial estimation error e_0

$$\sum_{k=0}^{N} \| z_k \|^2 \leq \frac{1}{\delta} \lambda_{max}(P) \| e_0 \|^2$$

Minimizing $\lambda_{\max}(P)$ and maximizing δ will give us a smaller bound on the energy of the performance output. This is suboptimal H_2 observer.[30]

In the noisy case, by setting $\delta=1$, $\beta=0$, and, $\in<0$ for $e_0=0$, gives the result

$$\sum_{k=0}^{N} \| z_k \|^2 \leq -\in \sum_{k=0}^{N} \| w_k \|^2$$

which means a bound on the ℓ_2 to ℓ_2 gain of the estimator (Suboptimal H_∞ observer). Maximizing $\in$ will minimize the energy of the performance output.

When $e_0=0$, if we use this formulation, we can design several dissipative controllers by using different values of δ,β, and $\in$.

If we take $\delta=0,\beta=1$, and, $\in>0$, it yields the input strict passivity result:

$$\sum_{k=0}^{N} {z_k}^T w_k \geq \in \sum_{k=0}^{N} \| w_k \|^2$$

Maximizing $\in$ will maximize the lossy nature of this observer.

Similarly, other dissipativity results can be obtained by changing δ,β, and $\in$ values. For example, taking $e_0=0$ $\delta=0,\beta=1$, and $\in=0$ gives passivity

$$\sum_{k=0}^{N} {z_k}^T w_k \geq 0$$

If we set δ>0,β=1, and ∈=0, we get output strict passivity:

$$\sum_{k=0}^{N} {z_k}^T w_k \geq \delta \sum_{k=0}^{N} \parallel z_k \parallel^2$$

Very strict passivity, which is the strict passivity both in the terms of the input and the output, can be obtained if we set δ>0,β=1, and ∈> 0:

$$\sum_{k=0}^{N} {z_k}^T w_k \geq \in \sum_{k=0}^{N} \parallel w_k \parallel^2 + \delta \sum_{k=0}^{N} \parallel Z_k \parallel^2$$

Again maximizing ∈ and δ will maximize the dissipative nature of the observer.

Therefore, this LMI formulation enables us to design different observers according to a variety of performance criteria in a common framework.

SECTION III.

LMI Solution

Let us first consider the case where there is no noise with B=0, D=0, B_f=0, D_Z=0, ∈=0 and β=0 . Substituting for the terms in inequality (6), we obtain

$$\left\{(A-(K+\Delta_k)C)e_k+[I,-(K+\Delta_k)]\begin{bmatrix}F(e_k)\\H(e_k)\end{bmatrix}\right\}^T$$
$$\times P\left\{(A-(K+\Delta_k)C)e_k+[I,-(K+\Delta_k)]\begin{bmatrix}F(e_k)\\H(e_k)\end{bmatrix}\right\}$$
$$-e_k^T P e_k+\delta e_k^T C_Z^T C_z e_k \le 0$$

(9)

The following is true for any α>0

$$e_k{}^T P[I,(K+\Delta_k)]\begin{bmatrix}F\\H\end{bmatrix}+[F^T,H^T]\begin{bmatrix}I\\-(K+\Delta_k)^T\end{bmatrix}Pe_k$$
$$\le \alpha e_k{}^T P[I,(K+\Delta_k)]\begin{bmatrix}I\\-(K+\Delta_k)^T\end{bmatrix}Pe_k+\alpha^{-1}[F^T,H^T]\begin{bmatrix}F\\H\end{bmatrix}$$
$$\le \alpha e_k^T P(I+(K+\Delta_k)(K+\Delta_k)^T)Pe_k+\alpha^{-1}(A_f e+B_f w)^T(A_f e+B_f w)$$

(10)

where we have used (2).

Using (10), a sufficient condition for (9) is

$$e_k^T[(P-\delta C_Z^T C_Z-\alpha A_f^T A_f)-(A-(K+\Delta_k)C)^T$$
$$\times(P-\alpha^{-1}P(I+(K+\Delta_k)(K+\Delta_k)^T)P)^{-1}P(A-(K+\Delta_k)C)]e_k\ge 0$$

(11)

Using the Schur complement result given in the introduction twice for the quadratic terms in (11), we obtain the following sufficient condition for (9):

$$Q=\begin{bmatrix}q_{11} & q_{12} & q_{13} & q_{14}\\ * & q_{22} & q_{23} & q_{24}\\ * & * & q_{33} & q_{34}\\ * & * & * & q_{44}\end{bmatrix}\ge 0$$

(12)

for

$$q_{11} = P - \delta C_z^T C_z - \alpha {A_f}^T A_f, q_{12} = A^T P - C^T Y^T - C^T {\Delta_k}^T P$$
$$q_{13} = 0, q_{14} = 0, q_{22} = P, q_{23} = P, q_{24} = -Y - P\Delta_k,$$
$$q_{33} = q_{44} = \alpha I, q_{34} = 0$$

for $Y=PK$ By arranging (12)

$$\begin{bmatrix} P - \delta {C_z}^T C_z - \alpha {A_f}^T A_f & A^T P - C^T Y^T & 0 & 0 \\ * & P & P & -Y \\ * & * & \alpha I & 0 \\ * & & * & \alpha I \end{bmatrix} \geq \begin{bmatrix} 0 & C^T {\Delta_k}^T P & 0 & 0 \\ * & 0 & 0 & P\Delta_k \\ * & * & 0 & 0 \\ * & * & * & 0 \end{bmatrix} \quad (13)$$

and from

$$[0\Delta_k^T P00]^T [C00I] + [C00I]^T [0\Delta_k^T P00] \leq [0\Delta_k^T P00]^T [0\Delta_k^T P00] + b^{-1} [C00I]^T [C00I]$$

for any $b>0$, we can derive the upper bound of the right hand side of (13). By replacing the right hand side of (13) with the upper bound, and by substituting $b=r^{-1}$, and using Schur's complement on the resulting matrices, we obtain

$$\begin{bmatrix} P - \delta C_z^T C_z - \alpha {A_f}^T A_f - rC^T C & A^T P - C^T Y^T & 0 & -rC^T & 0 \\ * & P & P & -Y & P \\ * & * & \alpha I & 0 & 0 \\ * & * & * & (\alpha - r)I & 0 \\ * & * & * & 0 & I \end{bmatrix} \geq 0 \quad (14)$$

The LMI (14) needs to be solved for $P>0, Y$ and $r>0$ in the non-noisy case and K is found from $K=P^{-1} Y$

In the presence of noise, (6) yields

$$\begin{aligned}&\left\{(A-KC)e_k+[I,-(K+\Delta_K)]\begin{bmatrix}Bw_k+F(e_k,w_k)\\Dw_k+H(e_k,w_k)\end{bmatrix}\right\}^T\\&\times P\left\{(A-KC)e_k+[I,-(K+\Delta_k)]\begin{bmatrix}Bw_k+F(e_k,w_k)\\Dw_k+H(e_k,w_k)\end{bmatrix}\right\}\\&-{e_k}^TPe_k+\delta(C_ze_k+D_zw_k)^T(C_ze_k+D_zw_k)+\in w_k^Tw_k-\beta(C_ze_k+D_zw_k)^Tw_k\le 0\end{aligned}\quad(15)$$

Using inequality (10) in a similar manner, a sufficient condition for (13) to hold is given by

$$\begin{bmatrix}{e_k}^T & {w_k}^T\end{bmatrix}\begin{bmatrix}r_{11} & r_{12}\\ * & r_{22}\end{bmatrix}\begin{bmatrix}e_k\\w_k\end{bmatrix}\ge 0\quad(16)$$

for $\alpha>0$, where

$$\begin{aligned}r_{11}&=P-\delta{C_z}^TC_Z-\alpha{A_f}^TA_f-(A-(K+\Delta_k)C)^T\\&\times P(P-\alpha^{-1}P(I+(K+\Delta_k)(K+\Delta_k)^T)P)^{-1}P(A-(K+\Delta_k)C)\\r_{12}&=-\delta{C_z}^TD_z+\frac{\beta}{2}{C_z}^T-\alpha{A_f}^TB_f-(A-(K+\Delta_k)C)^T\\&\times P(P-\alpha^{-1}P(1+(K+\Delta_k)(K+\Delta_k)^T)P)^{-1}P(B-(K+\Delta_k)D)\\r_{22}&=-\delta{D_z}^TD_z-\in I+\frac{\beta}{2}({D_z}^T+D_z)-\alpha{B_f}^TB_f-(B-(K+\Delta_k)D)^T\\&\times P(P-\alpha^{-1}P(1+(K+\Delta_k)(K+\Delta_k)^T)P)^{-1}P(B-(K+\Delta_k)D)\end{aligned}$$

By using the Schur complement twice, we obtain

$$S=\begin{bmatrix}s_{11} & s_{12} & s_{13} & s_{14} & s_{15}\\ * & s_{22} & s_{23} & s_{24} & s_{25}\\ * & * & s_{33} & s_{34} & s_{35}\\ * & * & * & s_{44} & s_{45}\\ * & * & * & * & s_{55}\end{bmatrix}\ge 0\quad(17)$$

for

$$s_{11} = P - \delta {C_z}^T C_z - \alpha {A_f}^T A_f,$$
$$s_{12} = -\delta {C_z}^T D_z + \frac{\beta}{2} {C_z}^T - \alpha {A_f}^T B_f,$$
$$s_{13} = A^T P - C^T Y^T - C^T {\Delta_k}^T P, s_{33} = s_{34} = P,$$
$$s_{14} = s_{15} = s_{24} = s_{25} = s_{45} = 0, s_{44} = s_{55} = \alpha I$$
$$s_{22} = -\delta {D_z}^T D_z - \in I + \frac{\beta}{2}\left(D_z + {D_z}^T\right) - \alpha {B_f}^T B_f$$
$$s_{23} = B^T P - D^T Y^T - D^T {\Delta_k}^T P, s_{35} = Y - P\Delta_k$$

where $Y=PK$.

Then, by proceeding in a similar manner to the non-noisy case, leads to

$$SN = \begin{bmatrix} sn_{11} & sn_{12} & sn_{13} & sn_{14} & sn_{15} & sn_{16} & sn_{17} \\ * & sn_{22} & sn_{23} & sn_{24} & sn_{25} & sn_{26} & sn_{27} \\ * & * & sn_{33} & sn_{34} & sn_{35} & sn_{36} & sn_{37} \\ * & * & * & sn_{44} & sn_{45} & sn_{46} & sn_{47} \\ * & * & * & * & sn_{55} & sn_{56} & sn_{57} \\ * & * & * & * & * & sn_{66} & sn_{67} \\ * & * & * & * & * & sn_{76} & sn_{77} \end{bmatrix} \geq 0$$

for

$$\begin{gathered} sn_{11} = P - \delta {C_z}^T C_z - \alpha {A_f}^T A_f - rC^T C, \\ sn_{12} = -\delta {C_z}^T D_z + \frac{\beta}{2} {C_z}^T - \alpha {A_f}^T B_f - C^T D, \\ s_{13} = A^T P - C^T Y^T, s_{15} = -rC^T \\ sn_{14} = sn_{16} = sn_{17} = sn_{24} = sn_{25} = sn_{27} \\ = sn_{45} = sn_{46} = sn_{47} = sn_{56} = sn_{57} = sn_{67} = 0, \\ sn_{22} = -\delta {D_z}^T D_z - \in I + \frac{\beta}{2}\left(D_z + {D_z}^T\right) - \alpha {B_f}^T B_f \\ sn_{23} = B^T P - D^T Y^T, sn_{26} = P, sn_{33} = P - rD^T D, \\ sn_{34} = P, sn_{35} = Y, sn_{36} = 0, sn_{37} = P, sn_{44} = \alpha I \\ sn_{55} = (\alpha - r)I, sn_{66} = sn_{77} = I, \end{gathered} \tag{18}$$

The LMI (18) needs to be solved for $P>0$, Y and $r>0$ in the non-noisy case and K is found from $K=P^{-1} Y$

The above development is summarized in the following theorem:

Theorem:

Given the nonlinear system and measurement scheme in (1) and (2) where $w_k \in \ell_2$, the use of the observer (3) leads to the satisfaction of the general performance objective (6) for z_k given by (5) if LMIs (14) and (18) are feasible, respectively for the non-noisy ($w_k \equiv 0, k \geq 0$) and noisy cases, for P>0, Y and r>0. The necessary gain is found from $K = P^{-1} Y$

Remark:

The magnitude of maximum perturbation that the designed observer can tolerate for any directions can be calculated from (14) for non-noisy case, and (18) for additive noise case. However, the actual magnitude of the perturbation as a function of the direction can be calculated from (12) and (17) for the noisy-free and noisy cases, respectively.

SECTION IV.

Illustrative Examples

Chaotic synchronization is chosen to demonstrate one of the possible applications of the proposed observer design. Chua's circuit[37] has become almost a benchmark for design involving chaotic systems because of its strong nonlinear dynamical behavior. The discretized (with sampling time T=0.01 sec) version of the example in[38] is chosen for this demonstration. The simulation is done for the case of boundedness of the estimation error. The state and measurement equation of this model in[37] is written as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -\alpha_c & \alpha_c & 0 \\ 1 & -1 & 1 \\ 0 & -\beta_c & -\mu \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} \alpha_c f(x_1) \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

$$y = [111] \cdot [x_1 x_2 x_3]^T$$

$$\text{where} f(x_1) = b x_1 + 0.5(a_n - b_n)(|x_1 + 1| - |x_1 - 1|).$$

and we use the following parameters in the simulation with a randomly chosen initial state:

$$\alpha_c = 9.1, \beta_c = 16.5811, \mu = 0.138083, a_n = -1.39386, b_n = -0.75590$$
$$'A_f = 0.0094, C_z = [100], \text{and } \alpha = 0.3.$$

For the given system and the performance criterion, the observer gain from LMI (14) is found to be: $K = [0.7798,1.4067,4.5295]^{\mathrm{T}}$ with $\sqrt{\gamma} = 0.2062$ which is the maximum bound on the perturbation for which (14) holds. With this obtained gain, the LMI (12) is solved for individual $\Delta = \Omega^{*}[\cos\varphi\cos\theta, \sin\varphi\cos\theta, \sin\theta]^{T}$ to calculate the magnitude Ω of the actual (constant) perturbation. This value varies according to the changes in the direction of perturbation. ϕ and θ are the angles from the horizontal and vertical axes, respectively. To obtain a more detailed picture of the magnitude of the perturbation with respect to the direction, the computations of Ω in LMI (12) were conducted for $0 \leq \phi, \theta \leq 360°$ with an increment of 2°. Magnitudes of allowable perturbations in the observer gain for various ϕ and θ values are depicted in Fig. 1. From this figure, the minimum over these ranges of angles of the maximum allowable perturbations is found to be 0.206235 which is very close to the value of $\sqrt{\gamma} = 0.2062$ found from (14), therefore the conservatism introduced when going from (12) to (14) in the derivation has been kept to a minimum and the sufficient conditions are close to being necessary also.

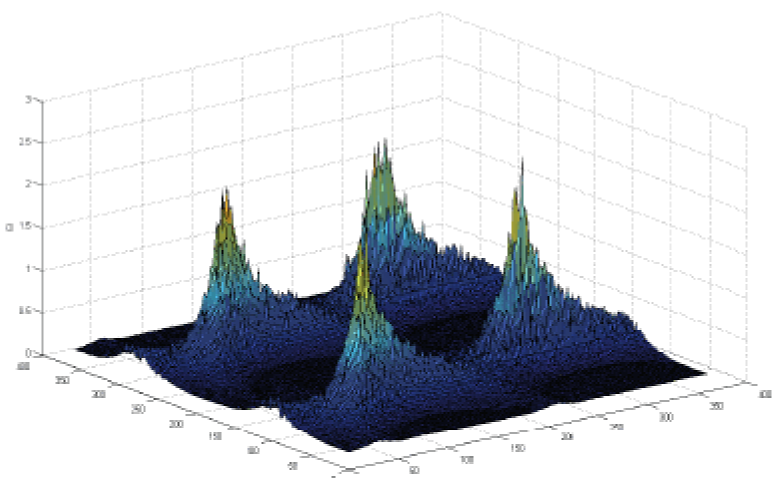

Fig. 1. Magnitude of perturbations for different Φ and θ.

The maximum constant gain perturbation magnitude found from (12) for system (19) is: $\Delta = 2.6679^{*}[\cos 74°\cos 74°, \sin 74°\cos 74°, \sin 74°]^{T}$ and this perturbation is applied for the simulation.

The simulation results involving co-plots of state variables together with their estimates (Fig.s 2(a)–(c)) and the norm of the error vector (Fig. 2(d)) show that the proposed observer is able to estimate the state successfully.

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The calculated gain and perturbation are applied to the system in (17), with the observer equation (3).

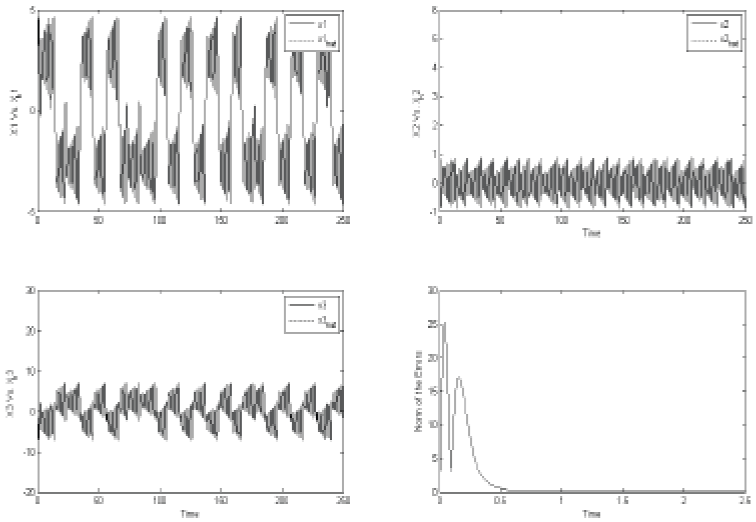

Fig. 2. Plots of state variables, their estimates and the norm of the error vector vs. time (sec).

Fig.s 3(a)–(c) are included to clearly depict the transient response of the observer state variable estimates. Fig. 4 not only indicates that the original system shows chaotic behavior, but also that the suggested observer successfully estimates the state.

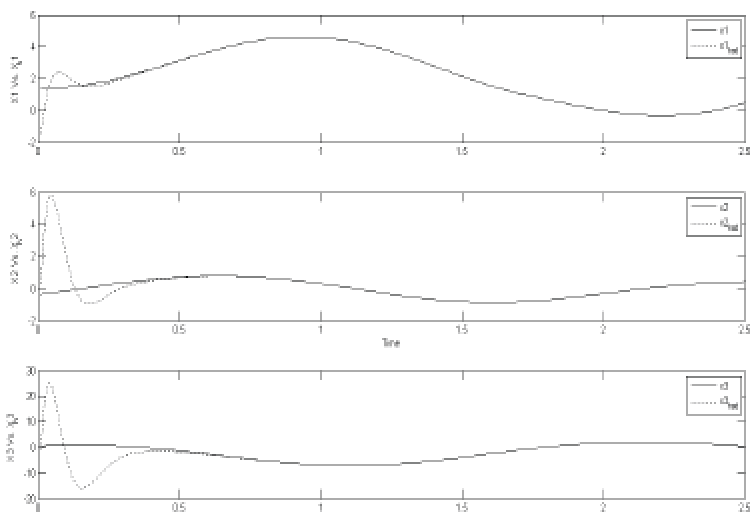

Fig. 3. Plots of state variables, their estimates and the norm of the error vector vs. time (sec)

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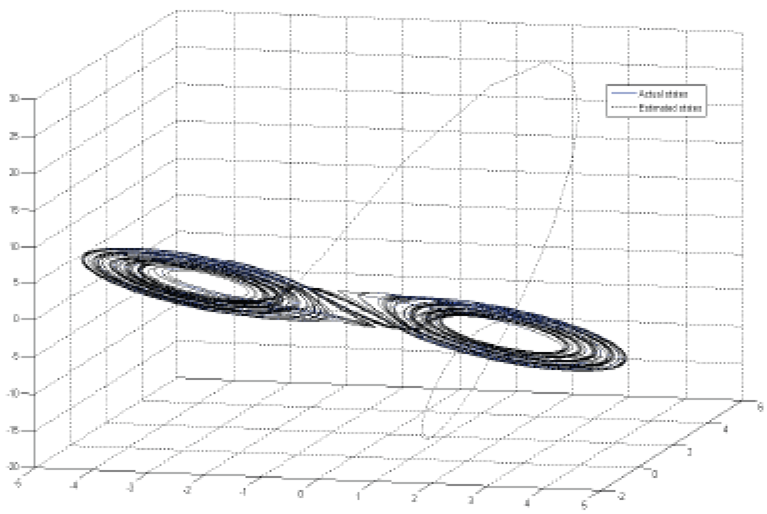

Fig. 4. Three-Dimensional plots of state variables and their estimates.

SECTION V.

Conclusions

A resilient discrete-time observer design procedure based on linear matrix inequalities has been presented for a class of nonlinear system and measurement models. A common framework is provided to design observers according to a variety of performance criteria. The results of a chaotic synchronization simulation illustrate the effectiveness of the proposed methodology.

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