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Signal and Noise in Complex-Valued SENSE MR Image Reconstruction

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Signal and Noise in Complex-Valued SENSE MR Image Reconstruction

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OUTLINE

1. Motivation

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Motivation

In MRI it is not voxel values that are measured.

The actual measurements are spatial frequencies (k -space).

The k -space measurements are not acquired instantaneously.

In parallel imaging, k -space is subsampled and measured in parallel then combined to form a single image.

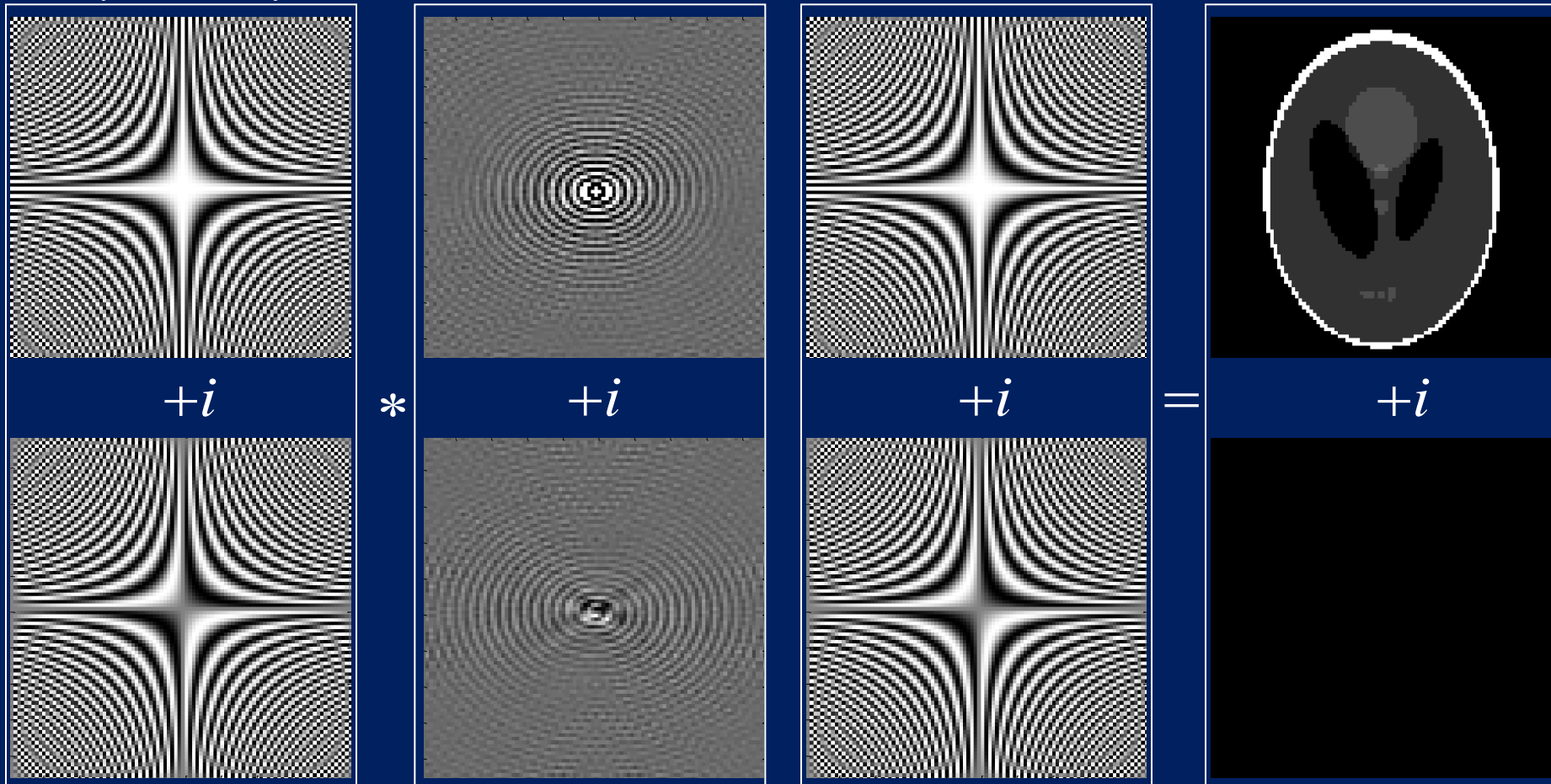
Image and volume measurement time is decreased at the expense of increased image reconstruction difficulty and time.

One popular parallel imaging method is SENSE.

Background

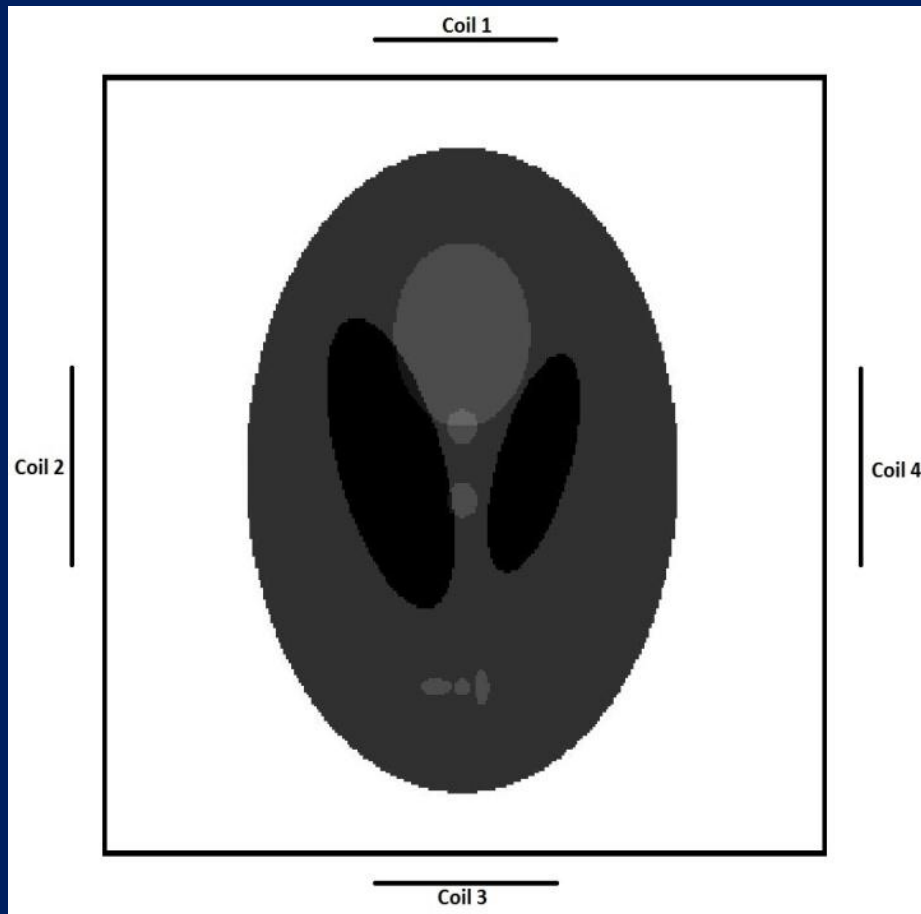
Image inverse Fourier Reconstruction for single coil.

$$(\Omega_{yR} + i\Omega_{yI}) * (F_R + iF_I) * (\Omega_{xR} + i\Omega_{xI})^T = (V_R + iV_I)$$

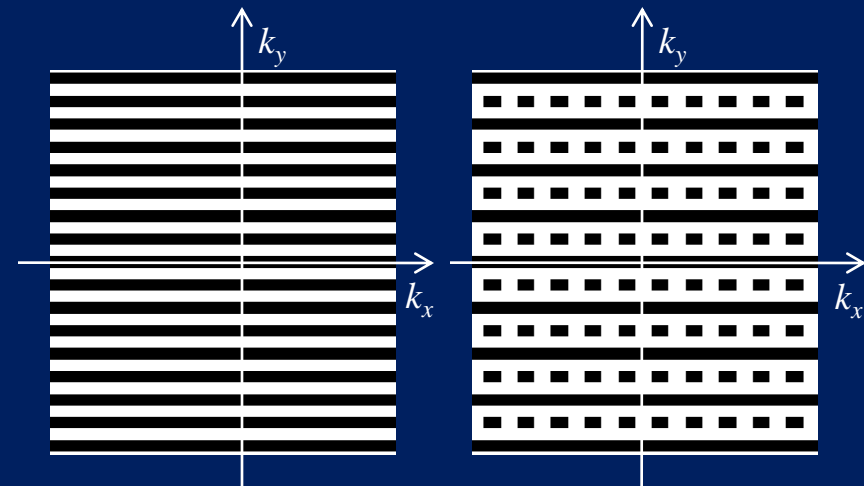


Background

In parallel imaging there is more than one receive coil.



Each coil measures a k -space array where every A^{th} line is skipped.

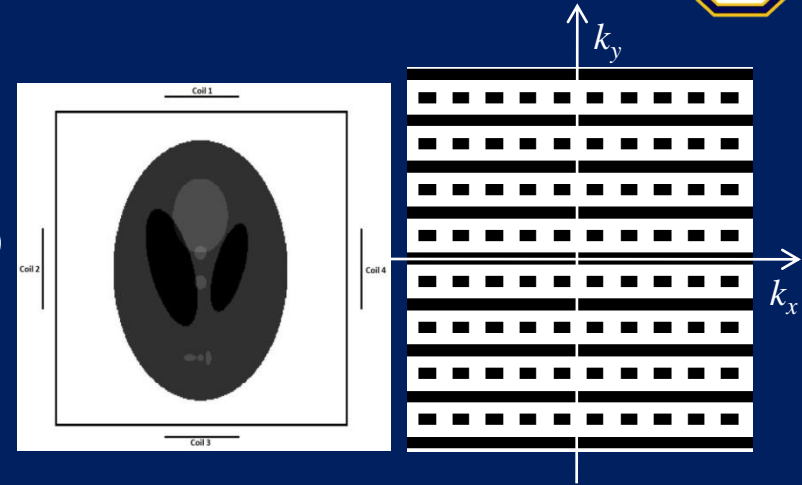


Full k -space.

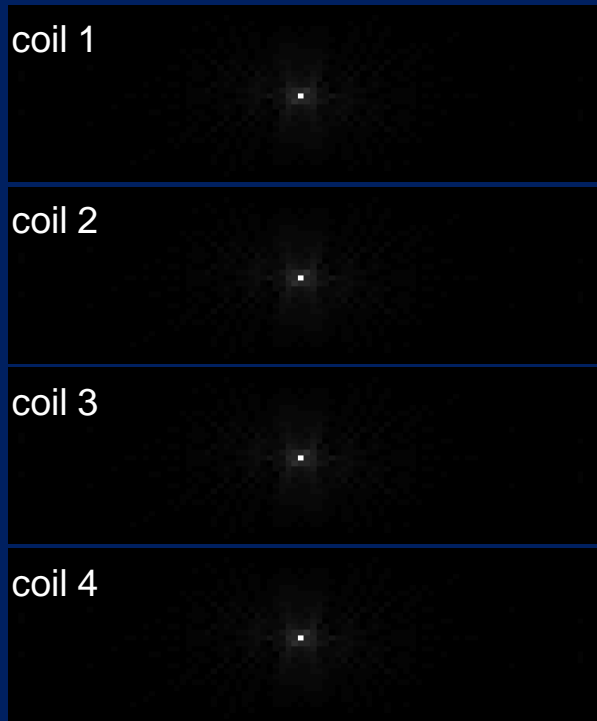
Skipped k -space.

Background

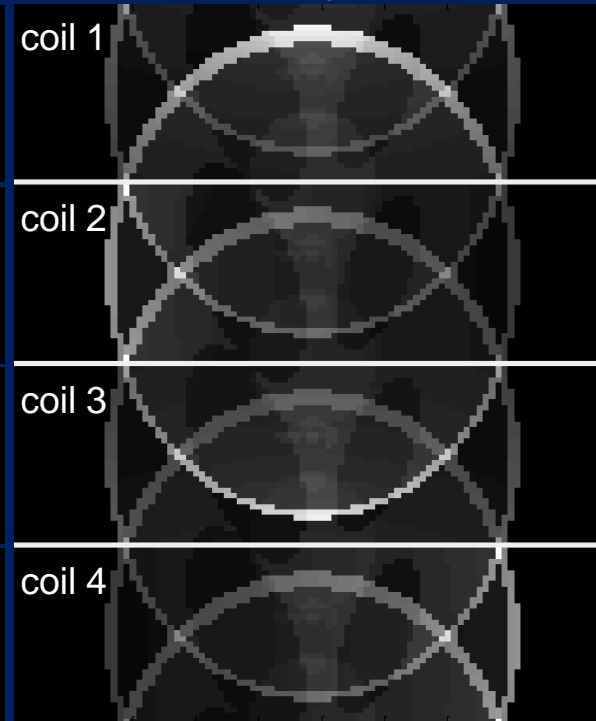
The k -space arrays where every A^{th} line is skipped are reconstructed into an aliased image to be combined to form a single image.



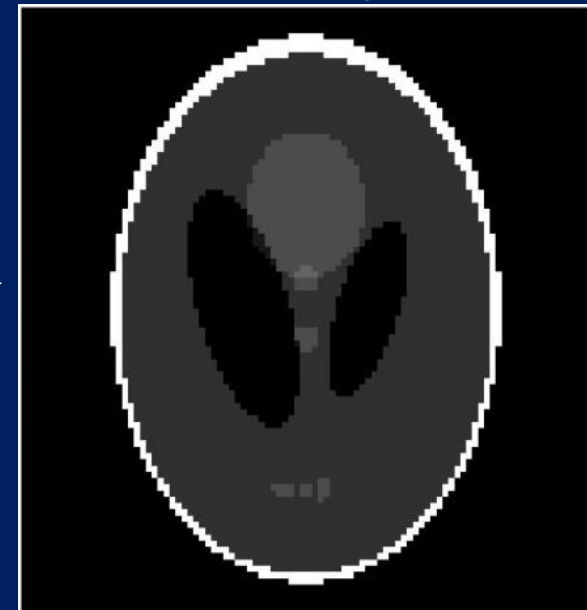
Skipped k -space.



Aliased images.

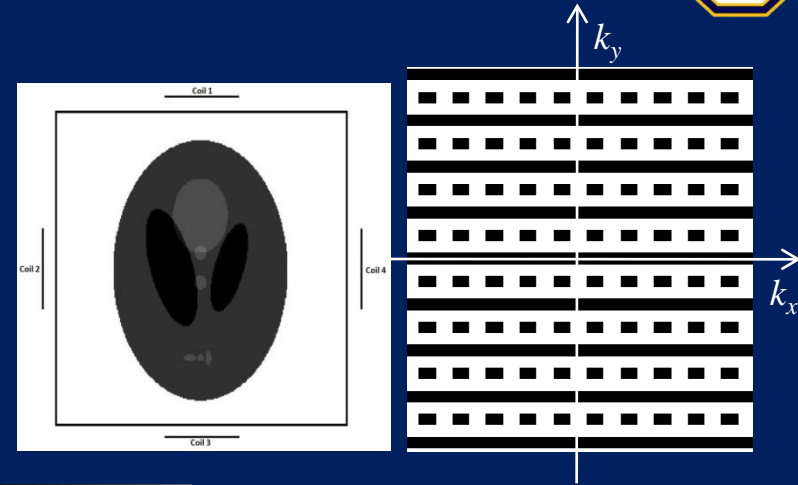


Combined image.



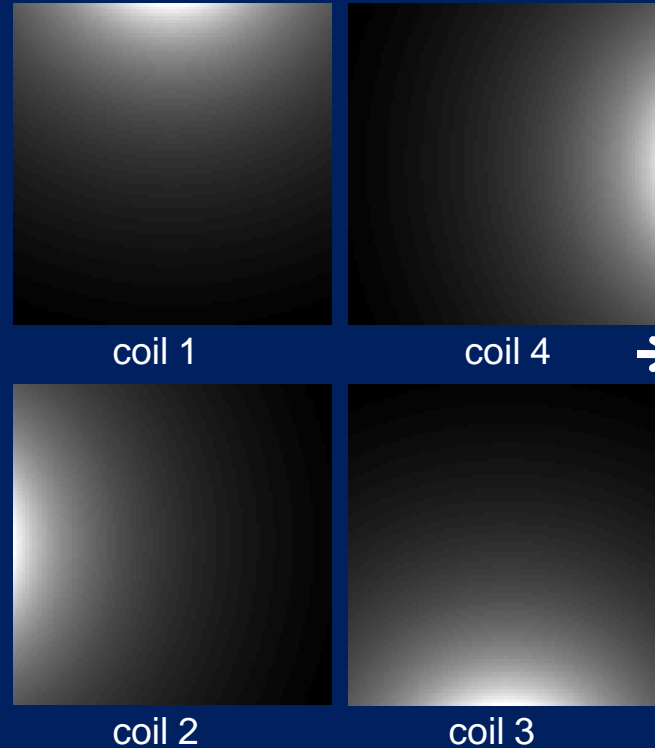
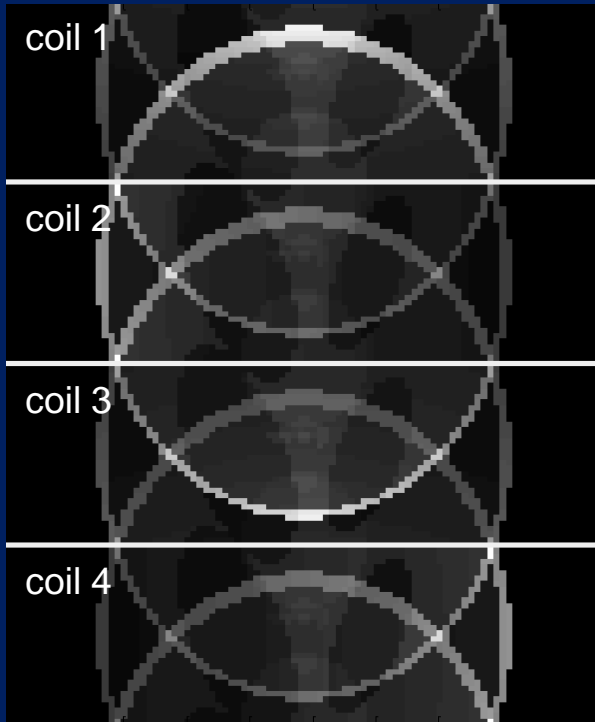
Background

The combination of aliased images to form a single image utilizes coil sensitivities.

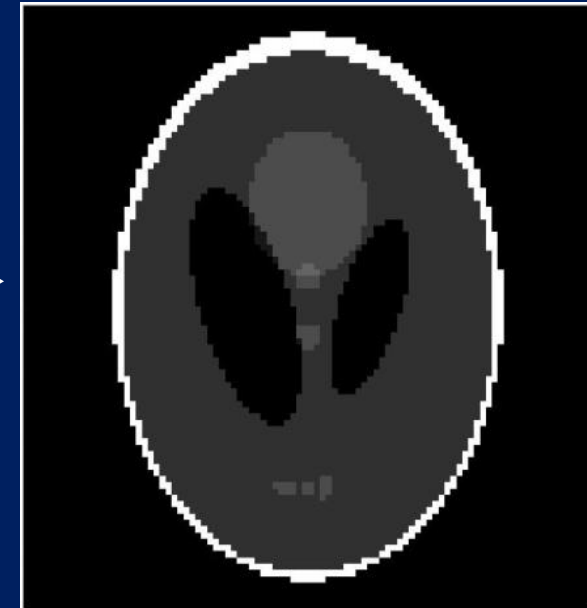


Aliased images. a_C

Coil sensitivities. S_C



Combined image. v_C



Methods

The SENSE model for aliased voxel values from n coils is

$$\underset{n \times 1}{a_C} = \underset{n \times A}{S_C} \underset{A \times 1}{v_C} + \underset{n \times 1}{\varepsilon_C}, \quad \varepsilon_C \sim CN(0, \Psi_C)$$

where for each voxel

$$\Psi_C = \Psi_R + i\Psi_I$$

a_C is a vector of the n complex-valued aliased voxel values

$$a_C = a_R + ia_I$$

v_C is a vector of the A unaliased voxel value

$$v_C = v_R + iv_I$$

S_C is an $n \times A$ matrix of complex-valued coil sensitivities

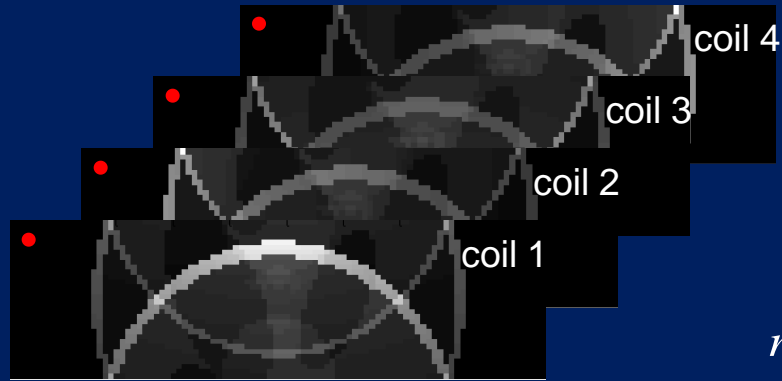
$$S_C = S_R + iS_I$$

ε_C is a vector of the n complex-valued error values

$$\varepsilon_C = \varepsilon_R + i\varepsilon_I$$

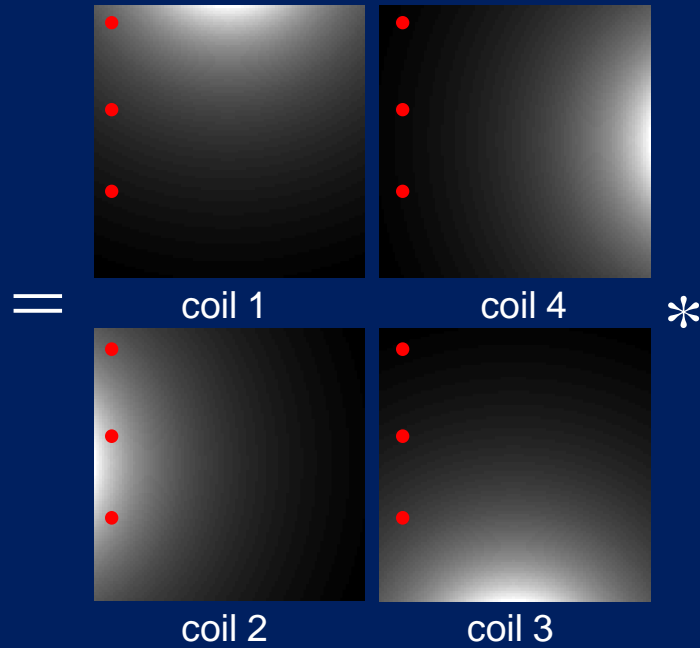
Methods

$n \times 1$

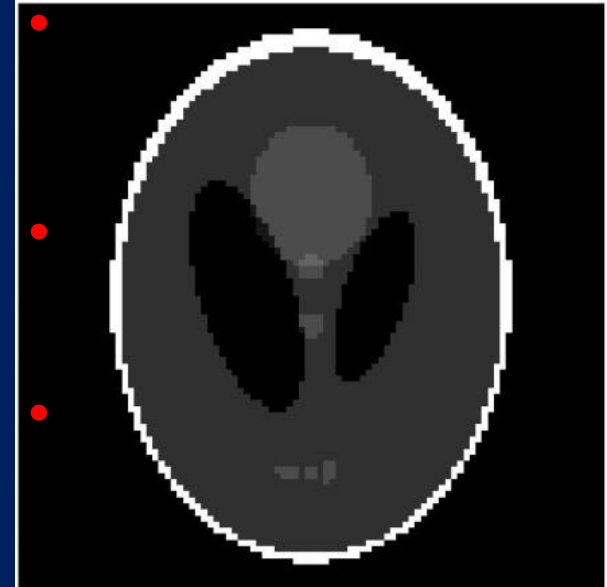


$$a_C \begin{matrix} = \\ n \times 1 \end{matrix} = \begin{matrix} S_C \\ n \times A \end{matrix} \begin{matrix} v_C \\ A \times 1 \end{matrix} + \begin{matrix} \epsilon_C \\ n \times 1 \end{matrix}$$

$n \times A$

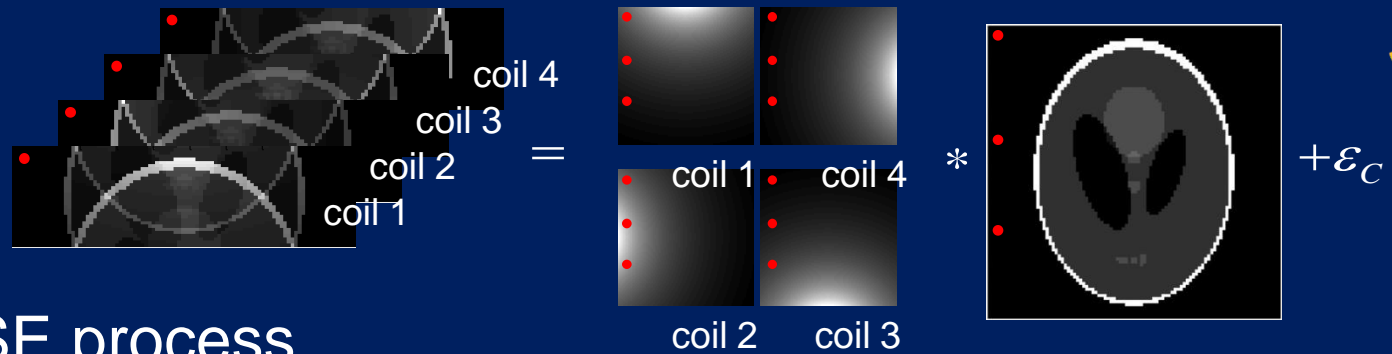


$A \times 1$



$n \times 1$
 $+\epsilon_C$

Methods



The SENSE process

$$a_C = S_C v_C + \varepsilon_C, \quad \varepsilon_C \sim CN(0, \Psi_C)$$

$$\Psi_C = \Psi_R + i\Psi_I$$

uses the complex-valued normal distribution

$$f(\varepsilon_C) = (2\pi)^{-n} |\Psi_C|^{-1} e^{-1/2 \varepsilon_C^H \Psi_C^{-1} \varepsilon_C},$$

H is the conjugate transpose (Hermetian)

and for n coil measurements

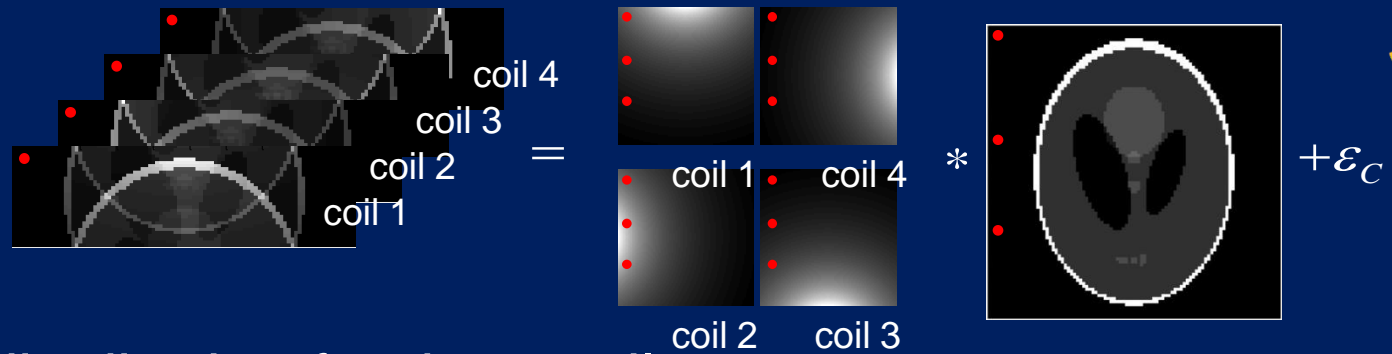
$$f(a_C) = (2\pi)^{-n} |\Psi_C|^{-1} e^{-1/2 (a_C - S_C v_C)^H \Psi_C^{-1} (a_C - S_C v_C)}$$

Pruessmann et al.: SENSE: Sensitivity Encoding for Fast MRI. MRM 42:952–962, 1999.

Wooding The multivariate distribution of complex normal variables. Biometrika 43:212–215, 1956.

Bruce, Karaman, and Rowe: In Submission, 2011.

Methods



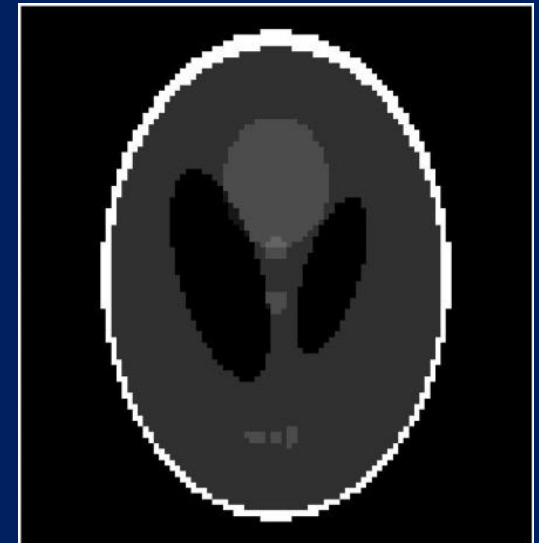
From the distribution for the n coil measurements

$$f(a_C) = (2\pi)^{-n} |\Psi_C|^{-1} e^{-1/2 (a_C - S_C v_C)^H \Psi_C^{-1} (a_C - S_C v_C)}$$

the voxel values can be estimated as

$$v_C = (S_C^H \Psi_C^{-1} S_C)^{-1} S_C^H \Psi_C^{-1} a_C$$

with knowledge of S_C and Ψ_C .



Methods

Instead of writing the model with complex numbers as

$$\mathbf{a}_C = \mathbf{S}_C \mathbf{v}_C + \boldsymbol{\varepsilon}_C,$$

$n \times 1$ $n \times A$ $A \times 1$ $n \times 1$

$$\mathbf{a}_C = \mathbf{a}_R + i\mathbf{a}_I \quad \mathbf{S}_C = \mathbf{S}_R + i\mathbf{S}_I \quad \mathbf{v}_C = \mathbf{v}_R + i\mathbf{v}_I \quad \boldsymbol{\varepsilon}_C = \boldsymbol{\varepsilon}_R + i\boldsymbol{\varepsilon}_I$$

we can write the model using an isomorphism as

$$\mathbf{a} = \mathbf{S} \mathbf{v} + \boldsymbol{\varepsilon}$$

$2n \times 1$ $2n \times 2A$ $2A \times 1$ $2n \times 1$

$$\mathbf{a} = \begin{pmatrix} \mathbf{a}_R \\ \mathbf{a}_I \end{pmatrix} \quad \mathbf{S} = \begin{pmatrix} \mathbf{S}_R & -\mathbf{S}_I \\ \mathbf{S}_I & \mathbf{S}_R \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} \mathbf{v}_R \\ \mathbf{v}_I \end{pmatrix} \quad \boldsymbol{\varepsilon} = \begin{pmatrix} \boldsymbol{\varepsilon}_R \\ \boldsymbol{\varepsilon}_I \end{pmatrix}.$$

Methods

Then the distribution for n coil measurements is

$$f(a) = (2\pi)^{-n} |\Psi|^{-1/2} e^{-1/2(a-Sv)' \Psi^{-1} (a-Sv)} ,$$

with

$$a = \begin{pmatrix} a_R \\ a_I \end{pmatrix}_{2n \times 1} \quad S = \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix}_{2n \times 2A} \quad v = \begin{pmatrix} v_R \\ v_I \end{pmatrix}_{2A \times 1} \quad \mathcal{E} = \begin{pmatrix} \mathcal{E}_R \\ \mathcal{E}_I \end{pmatrix}_{2n \times 1} ,$$

and the imposed skew-symmetric covariance structure

$$\Psi = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix} .$$

Methods

The SENSE voxel values can be estimated by

$$v_C = (S_C^H \Psi_C^{-1} S_C)^{-1} S_C^H \Psi_C^{-1} a_C$$

or in terms of an isomorphism

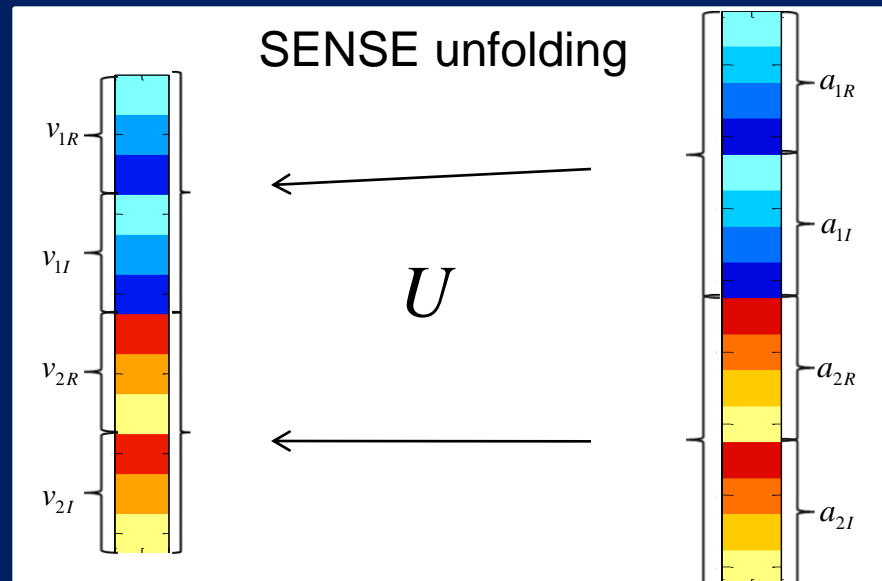
$$\begin{pmatrix} v_R \\ v_I \end{pmatrix} = \begin{bmatrix} \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix}^T & \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix}^{-1} \\ \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix} & \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix} \end{bmatrix}^{-1} \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix}^T \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix}^{-1} \begin{pmatrix} a_R \\ a_I \end{pmatrix}$$

$2A \times 1$ $2A \times 2n$ $2n \times 2n$ $2n \times 2A$ $2A \times 2n$ $2n \times 2n$ $2n \times 1$

$$\begin{pmatrix} v_R \\ v_I \end{pmatrix} = U * \begin{pmatrix} a_R \\ a_I \end{pmatrix}$$

$2A \times 1$ $2A \times 2n$ $2n \times 1$

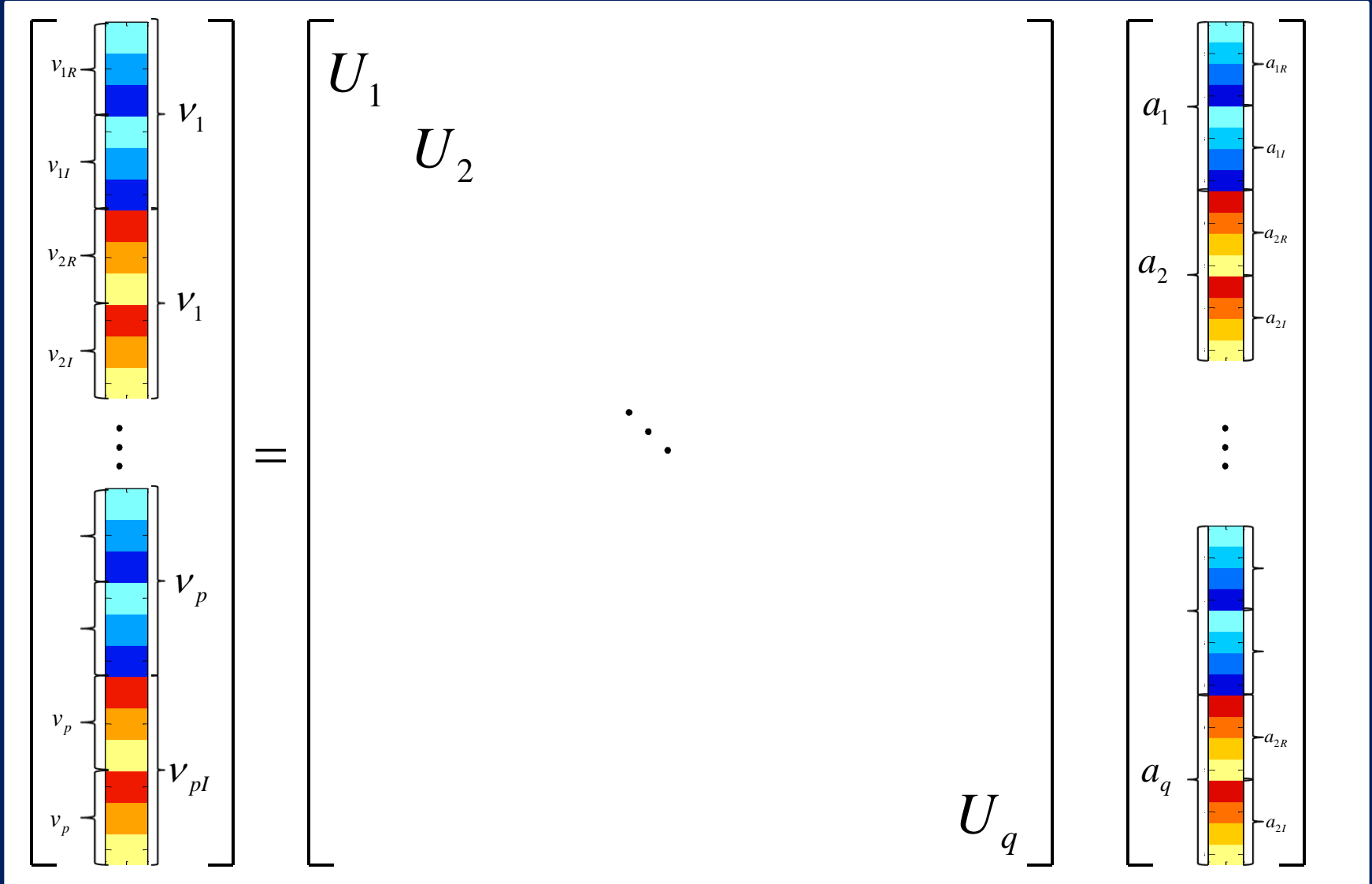
$n=4$ real / $n=4$
imaginary
true aliased
voxel values



Acceleration A
 $A=3$ real / $A=3$
imaginary
un-aliased
fold values

$$\text{SENSE unfolding} \begin{pmatrix} v_R \\ v_I \end{pmatrix} = U * \begin{pmatrix} a_R \\ a_I \end{pmatrix}$$

Methods

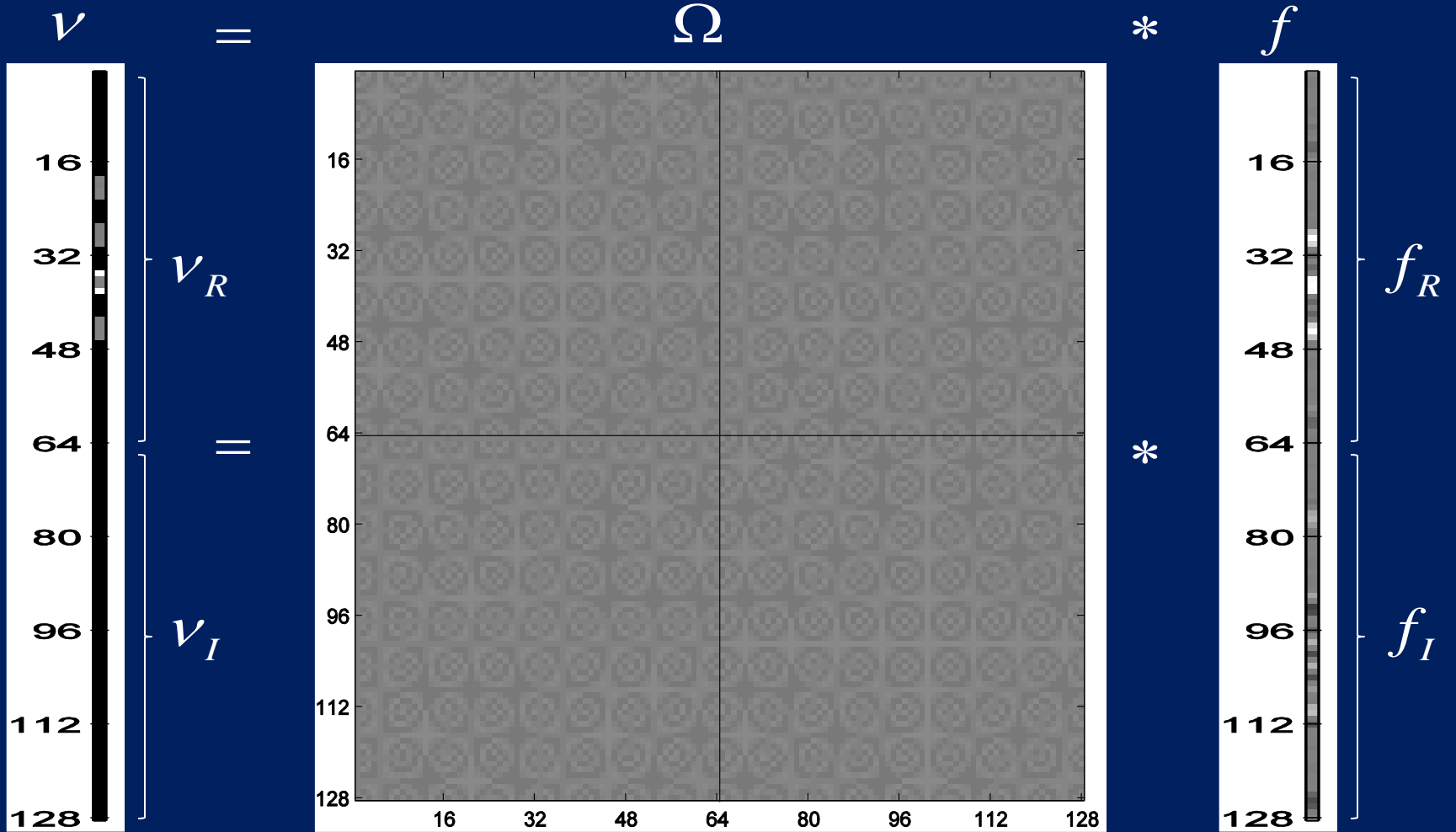
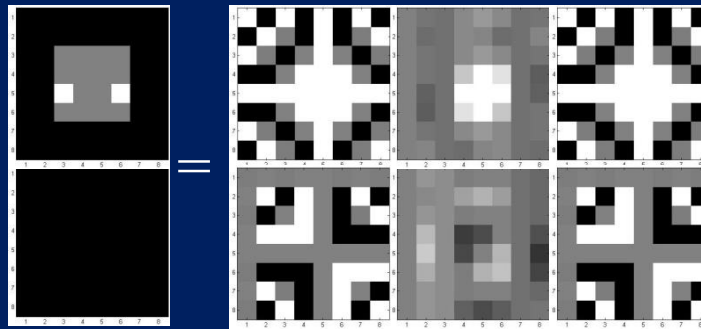


Methods

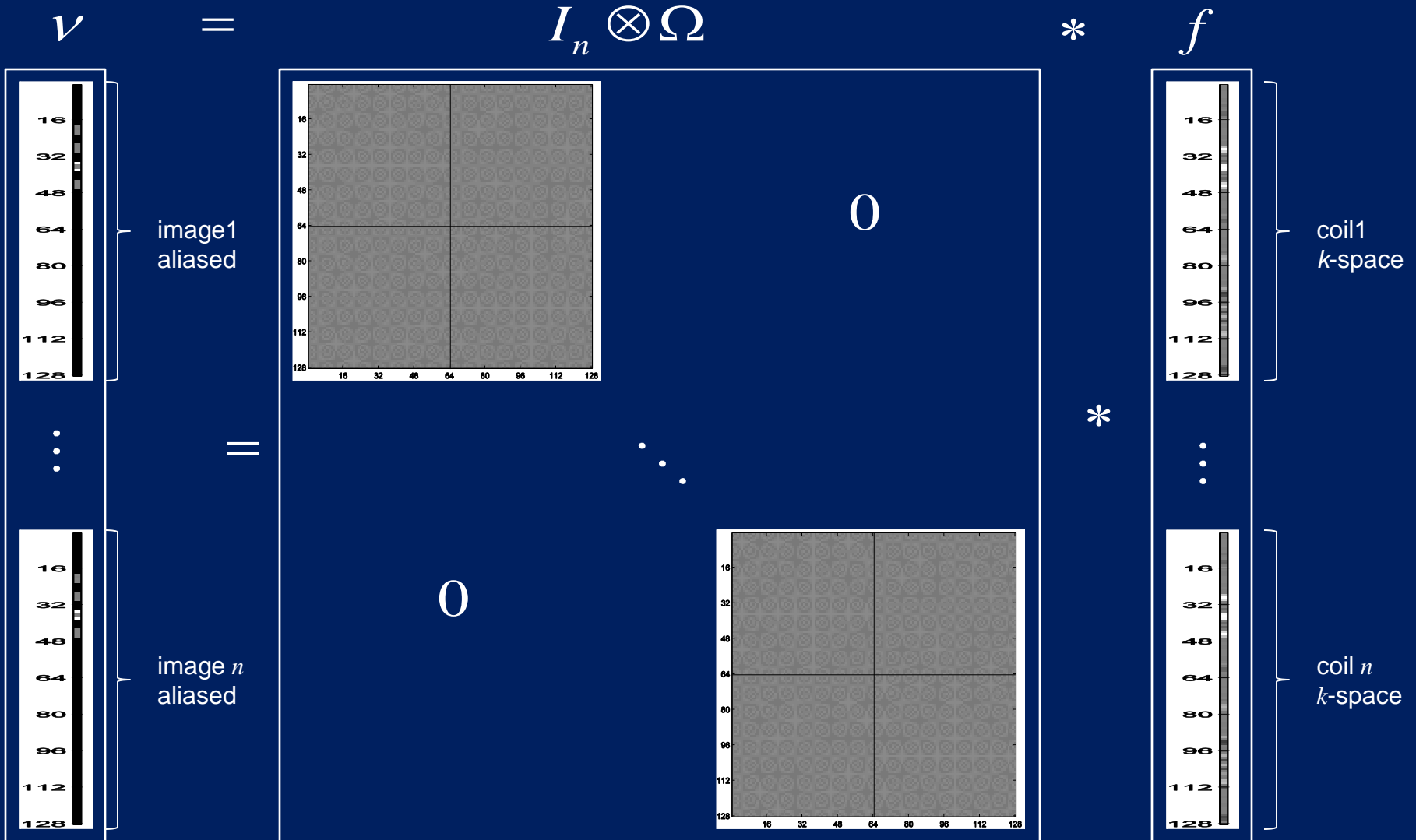
Real-valued isomorphism

real

imaginary



Methods



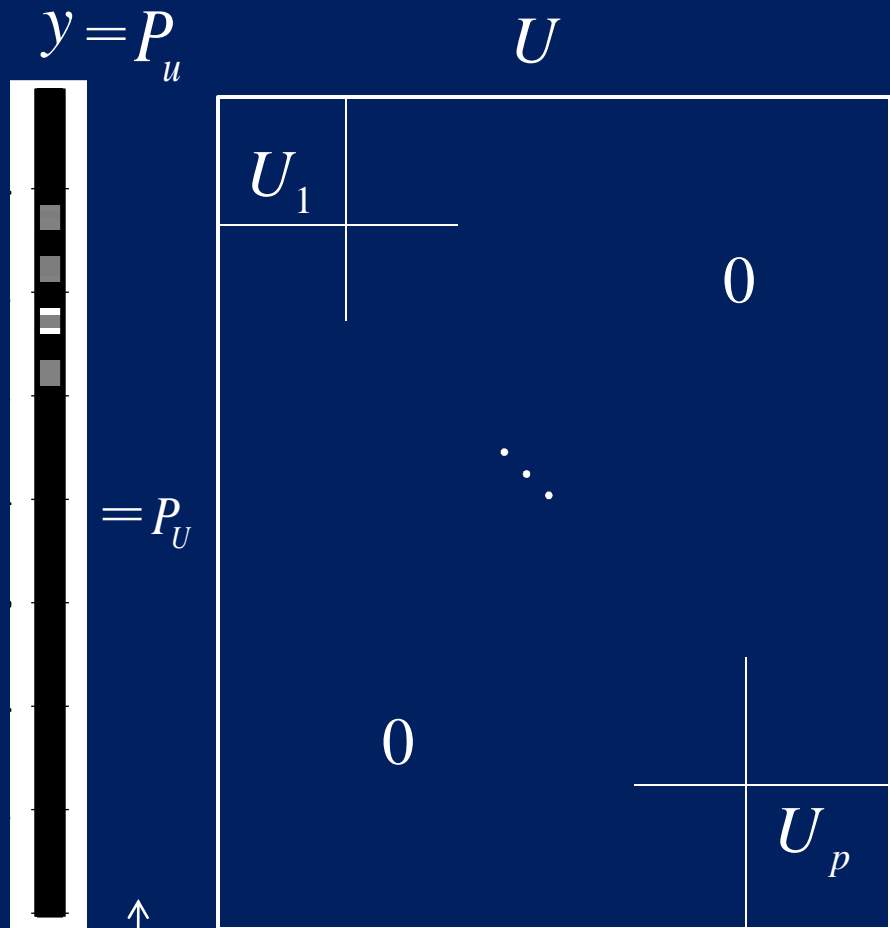
Methods

here is a for each voxel



$$P_S P_C$$

v



$$P_S P_C$$



permute to by folded voxel

unfold matrix U 's have S and Ψ

Single image vector

Methods

processing on unfolded image vector

here is a for each voxel

processing on each coil image vector

insert processing on each coil k -space vector

$$y = P_u$$

U

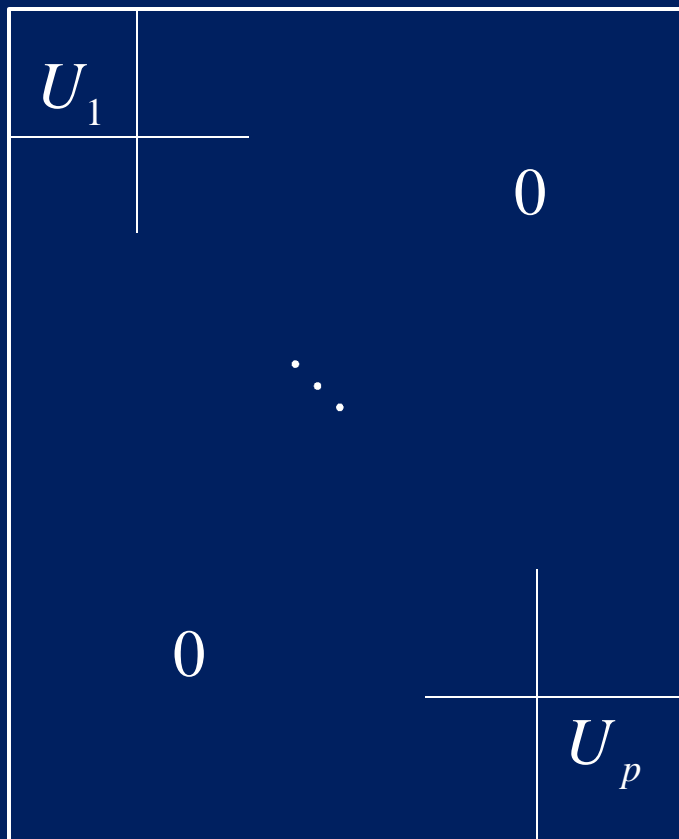
$$P_S P_C$$

$$(I_n \otimes \Omega)$$

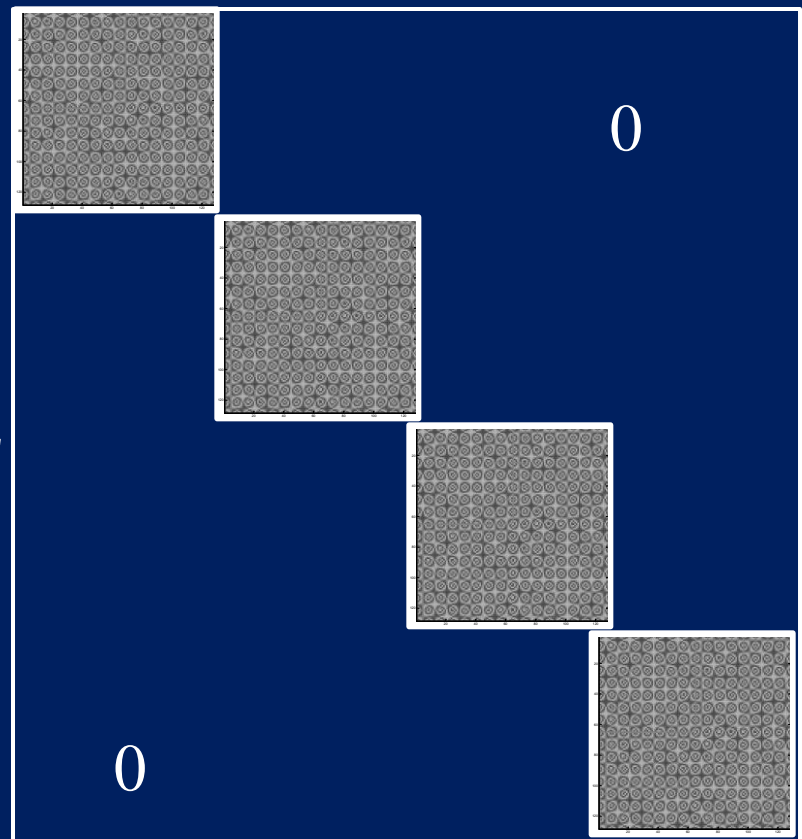
f



$$= P_U$$



$$P_S P_C$$



Single image vector

unfold matrix U 's have S and Ψ

permute to by folded voxel

reconstruct $n=4$ images

k -space vector of n images

19

Methods

$$y = \underbrace{O_I P_u U P_S P_C}_{O} (I_n \otimes \Omega_a O_k) f$$

where

$f = (f_1, \dots, f_n)'$ are coil k -space

O_k is k -space preprocessing

Ω_a is adj. inverse Fourier matrix

P_u, P_S, P_C , permutation matrices

U SENSE unfolding matrix

O_I is image space preprocessing

$$f = P_C \mathcal{R} C \mathcal{F}$$

$$O_k = \mathcal{A} \mathcal{Z} \mathcal{H} \underbrace{P_R^{-1} \Omega_{row}^{-1} \Phi \Omega_{row} P_R}_{\text{row reverse permute}}$$

$$\Omega_a = \Omega \quad \begin{array}{l} \text{adjusted for } \Delta B \\ \text{and for } T_2^* \end{array}$$

$$O_I = S_m \quad \leftarrow \text{Image smoothing}$$

←
k-space vector
censor blip points
row reverse
permute

Methods

Statistical Expectation and Covariance.

If $E(f)=f_0$, then for Mf , $E(Mf)=Mf_0$.

If $\text{cov}(f)=\Gamma$, then for Mf , $\text{cov}(Mf)=M\Gamma M'$.

This means that with $y = Of$,

$$E(y) = Of_0 \quad \text{and} \quad \text{cov}(y) = O\Gamma O' = \Sigma_{2p \times 2p}$$

$$\Rightarrow \text{cor}(v) = D_{\Sigma}^{-1/2} \Sigma D_{\Sigma}^{-1/2}$$

So even if $\Gamma = \sigma^2 I$, it is not necessarily true that $\Sigma = \sigma^2 I$!

This has H_0 fMRI noise and fcMRI connectivity implications!

Results

Since

$$y = Of,$$

we inverted and made the n coil spatial frequencies from

$$(O^T O)^{-1} O^T v = f$$

where O and v are known

v is true/noiseless Shepp-Logan phantom (scaled by 50)

$$O = S_m P_U U P_S P_C (I_n \otimes \Omega)$$

The number of coils, n , and the reduction factor, A , are specified in the dimensions of operators, O .

$$y = Of$$

Results

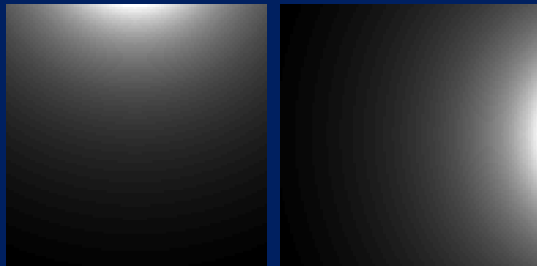
Noiseless data $f = (O^T O)^{-1} O^T v$ generated for $N_X = N_Y = 96$,
 $n=4$, $A=3$

$$O = S_m P_U U P_S P_C (I_n \otimes \Omega)$$

O had diagonal blocks $U_j = (S_j^T \Psi^{-1} S_j)^{-1} S_j^T \Psi^{-1}$

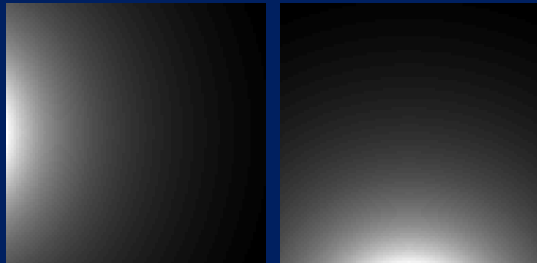
Sensitivities, S

Markovian coil covariance, Ψ



coil 1

coil 4



coil 2

coil 3

$$\Psi = \begin{pmatrix} \Psi_R & \Psi_{RI} \\ \Psi'_{RI} & \Psi_I \end{pmatrix}$$

Not skew-symmetric

$$\Psi_R = \begin{pmatrix} 1 & .33 & .11 & .33 \\ .33 & 1 & .33 & .11 \\ .11 & .33 & 1 & .33 \\ .33 & .11 & .33 & 1 \end{pmatrix}$$

$$\Psi_I = \Psi_R$$

$$\Psi_{RI} = \begin{pmatrix} 0 & -.11 & -.07 & -.11 \\ .26 & 0 & -.11 & -.07 \\ .42 & .26 & 0 & -.11 \\ .26 & .42 & .26 & 0 \end{pmatrix}$$

Results

Gaussian Smoothing applied in image-space

- FWHM = 3 voxels,
- Normalized to leave variance unaffected (Scales mean by 4.516)

By definition, smoothing induces a covariance and correlation between voxels and their neighbors.

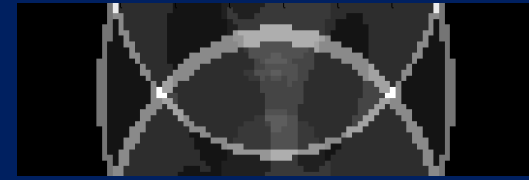
This effect is in turn transferred to the correlated voxels from each fold in SENSE.

Gaussian smoothing kernel, S_m , was applied in image-space to reconstructed images.

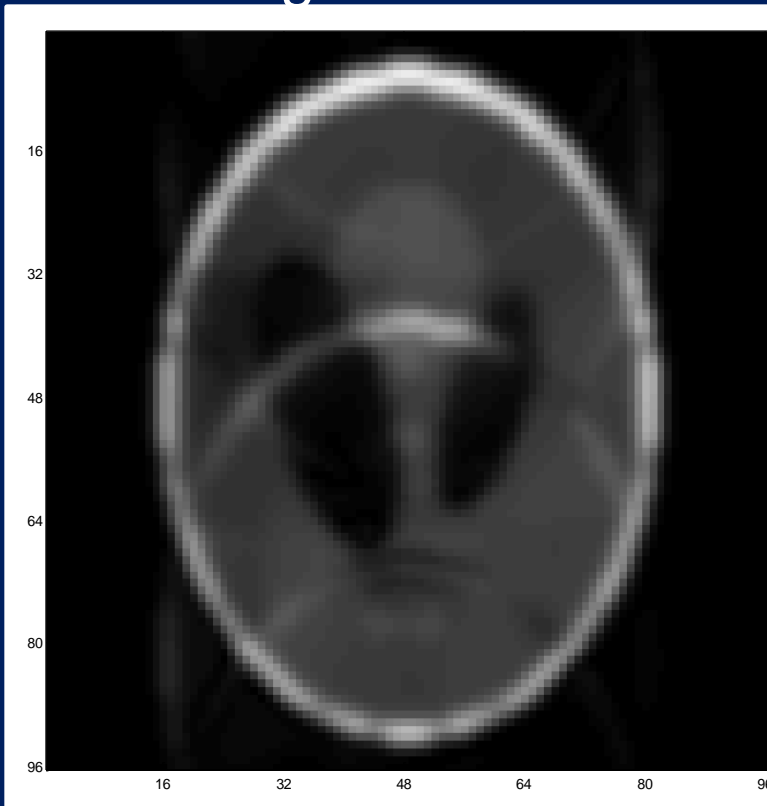
Bruce, Karaman, and Rowe: In Submission, 2011.

Nencka et al.: A Mathematical Model for Understanding the Statistical effects of k-space (AMMUST-k) Preprocessing Operators on Observed Voxel Measurements in fcMRI and fMRI. Journal of Neuroscience Methods 181:268-282, 2009.

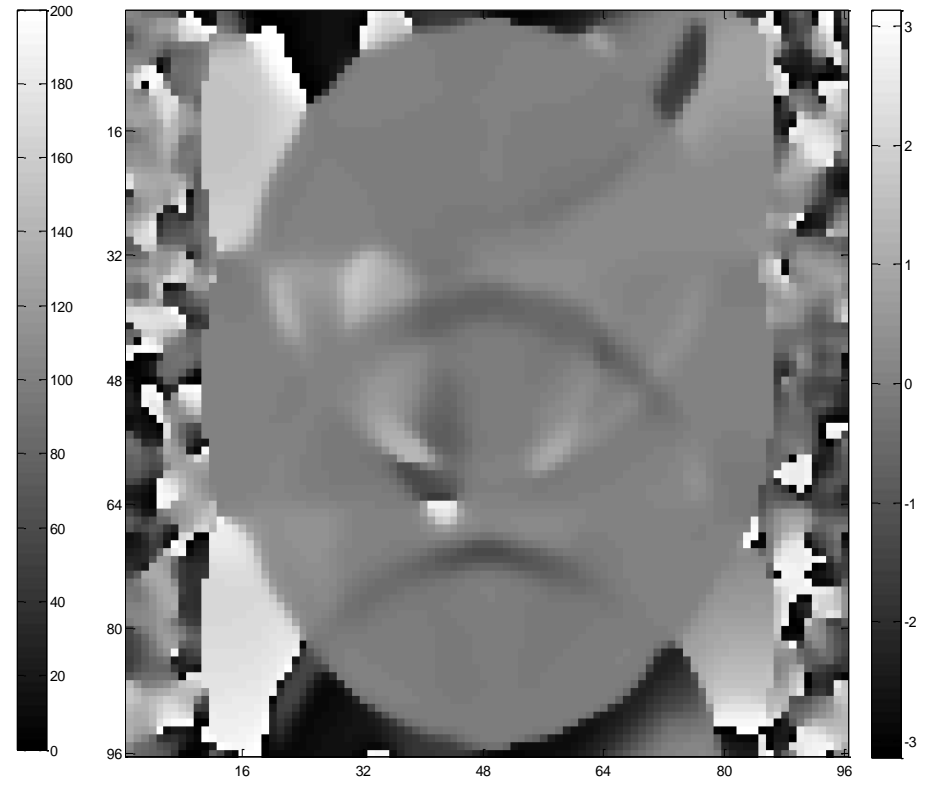
Results



Magnitude



Phase



Ghosting because symmetric coil cov Ψ used $\rightarrow \Psi = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix}$

Alternative symmetric coil cov Ψ proposed.

Phase is important in complex-valued fMRI!

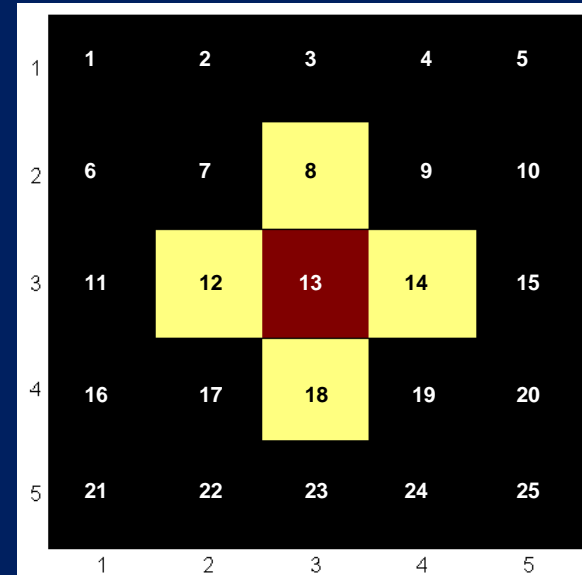
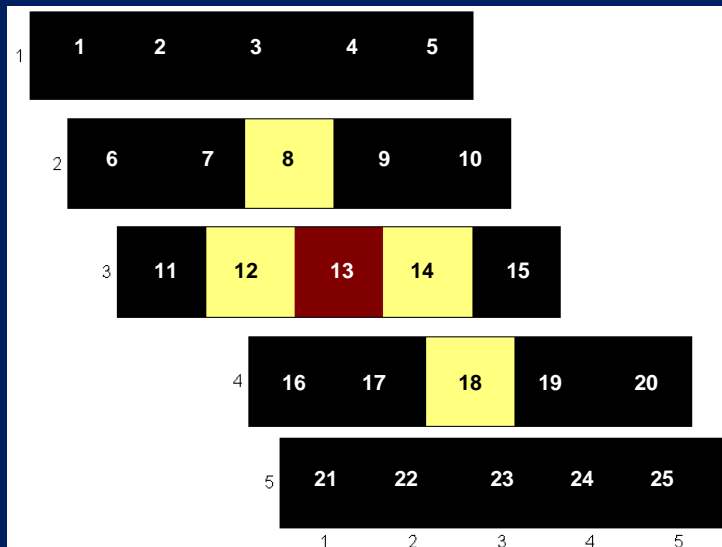
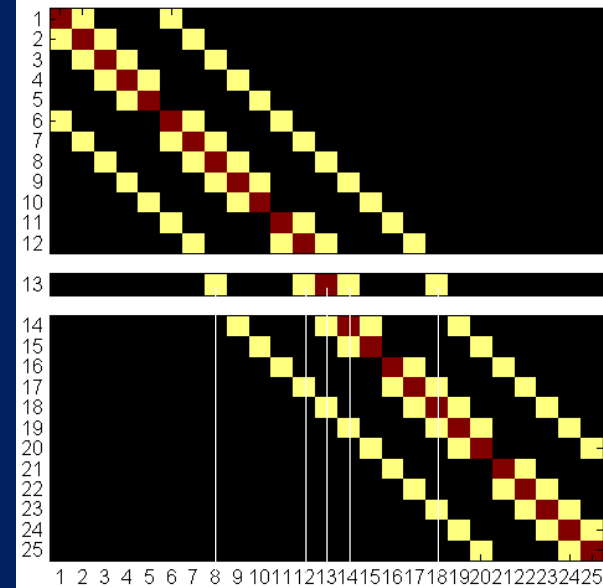
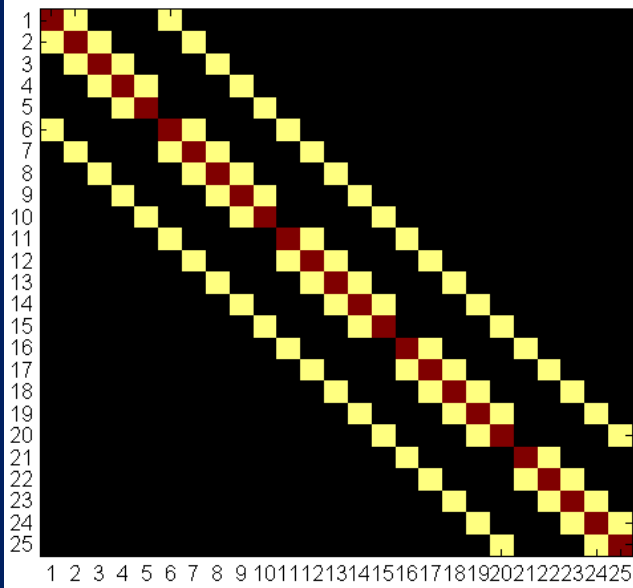
$$\Psi = \begin{pmatrix} \Psi_R & \Psi_{RI} \\ \Psi'_{RI} & \Psi_I \end{pmatrix}$$

Results

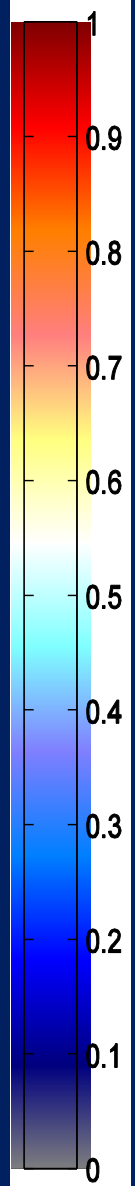
1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

5x5 image

COR =
25x25
correlation
matrix



5x5 correlation image



Correlations induced about the center voxel.

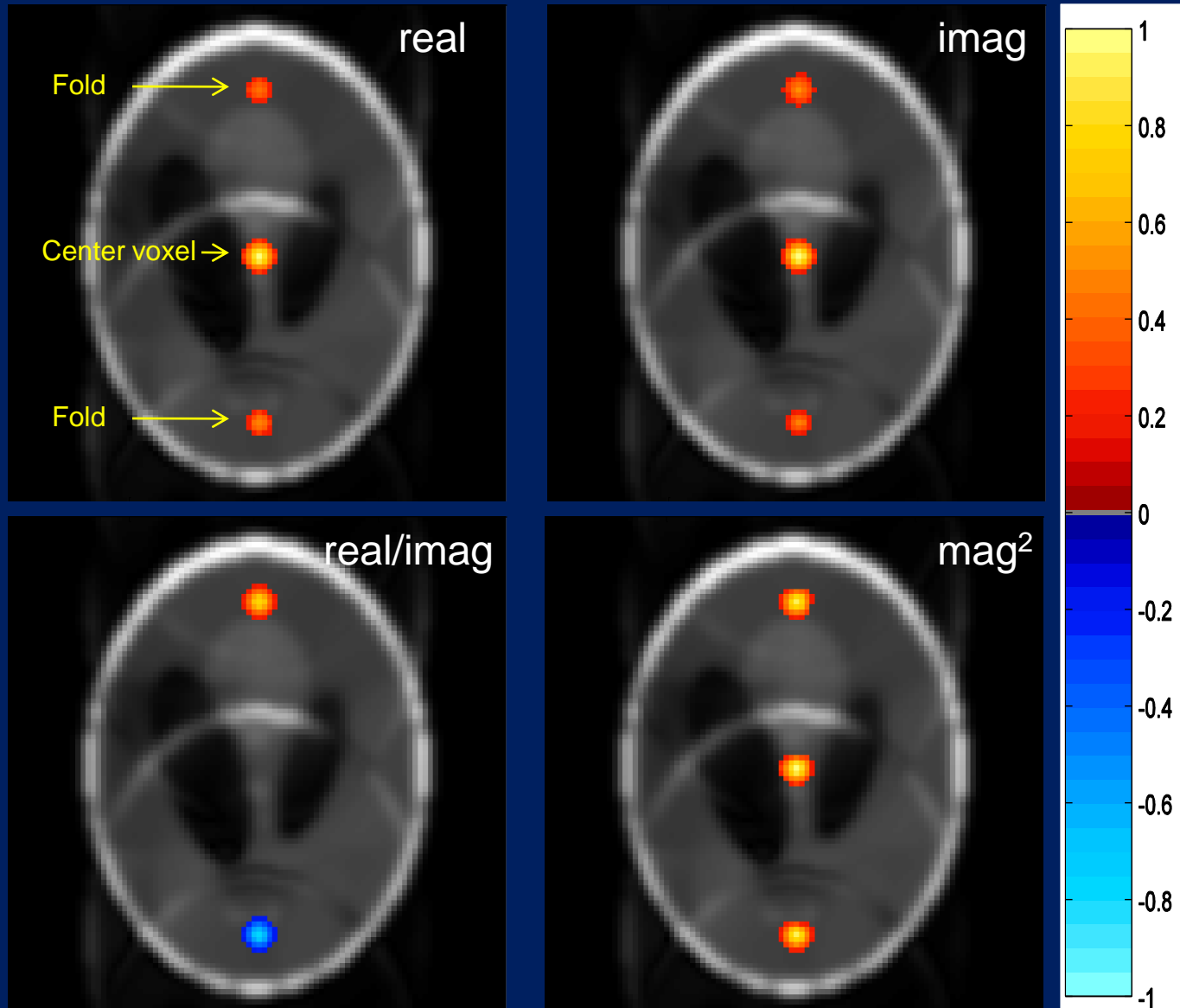
Results

$$N_x=96$$

$$N_y=96$$

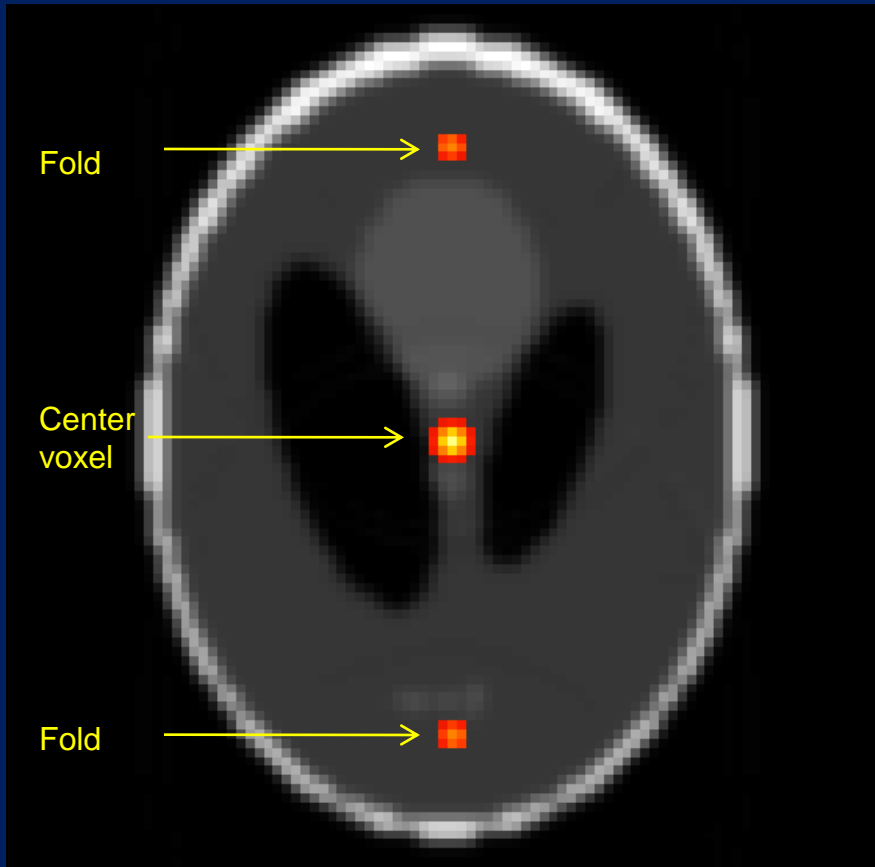
$$n = 4$$

$$A = 3$$

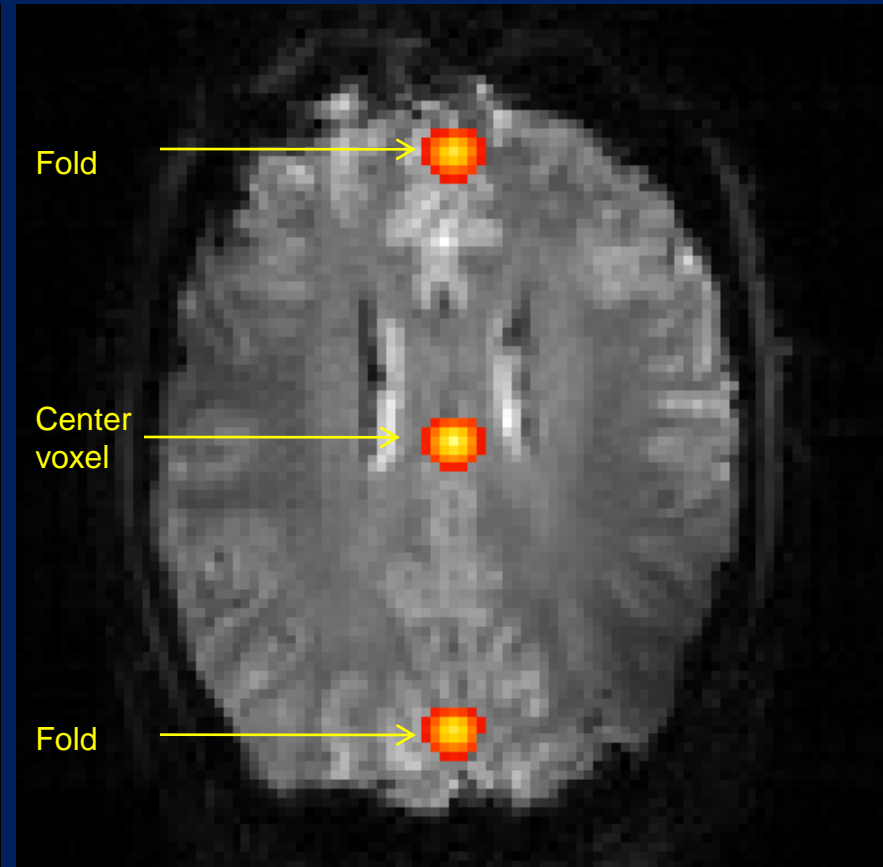
Functional
connectivity
implications

Results

Phantom



Human



underlay artificially expanded

Extrapolate to human, mistakenly conclude regions correlated!

Discussion

The SENSE image reconstruction method was described.

Wrote SENSE reconstruction with an isomorphism

$$y = O_I P_U U P_S P_C (I_n \otimes \Omega O_k) f = O f .$$

The new mean $E(y) = O f_0$ and covariance $\Sigma = O \Gamma O'$ of complex-valued SENSE described.

Theoretical results of SENSE reconstruction presented.

Ghosting present in SENSE magnitude and phase images.

Induced correlation between folds of no biological origin.

Thank You

Acknowledgements:

Iain Bruce, Marquette University

Muge Karaman, Marquette University

Sweet 16!



vs.



Go Marquette!