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TIMOSHENKO BEAM EFFECTS IN LATERAL-MODE MICROCANTILEVER-BASED SENSORS IN LIQUIDS

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Introduction and Motivation

Dynamic-mode microcantilever-based devices are well suited to biological and chemical sensing applications. However, these applications often necessitate liquid-phase sensing, introducing significant fluid-induced inertial and dissipative forces which reduce resonant frequencies (f_{res}) and quality factors (Q) and, thus, adversely affect sensitivity and limit of detection. In an effort to mitigate some of these fluid effects, the use of unconventional resonant modes of microcantilever-based sensors has been investigated [1-4]. Recent analytical [2,3] and experimental [4] research has shown that higher in-fluid Q is achieved by exciting microcantilevers in lateral flexural (Fig. 1), which reduces the viscous energy dissipation in the fluid as compared to the transverse (out-of-plane) mode. In particular, both theoretical and experimental results show that the lateral-mode designs offering the most promise in liquid-phase sensing applications are those for which the micro-beams are relatively short and wide. However, such geometries may violate the assumptions employed in Euler-Bernoulli (EB) beam theory due to the large width-to-length ratio. This may be observed, for example, in the deviation between EB predictions and experimental data for f_{res} and Q for short, wide cantilevers, for which EB theory overestimates the results [4]. To understand the behavior of such sensor geometries, a beam-fluid interaction model that accounts for “Timoshenko beam” (TB) effects (shear deformation and rotatory inertia) is warranted and is the focus of this study.

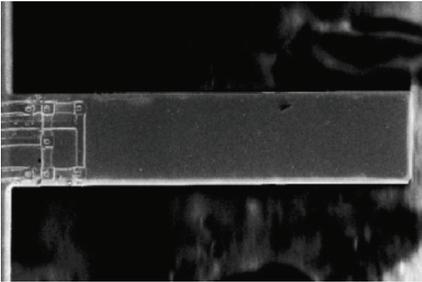


Fig. 1: Microcantilever-based Chemical Sensor: Thermal Resistors near Support to Excite Lateral (In-plane) Bending [4]

Modeling Assumptions

The major assumptions employed in the new model are (1) viscous dissipation in the fluid is the dominant loss mechanism; (2) the cross section is relatively thin (thickness $h \ll$ width b), so the fluid resistance associated with the pressure on the smaller faces is negligible compared with that due to the fluid’s shear resistance on the larger faces; (3) the shear stress exerted by the fluid on the beam is approximated by local application of the classical solution of Stokes’s second problem for harmonic motion of an infinite rigid plate in a viscous fluid.

Boundary Value Problem (BVP)

By modeling the microcantilever as a Timoshenko beam with distributed Stokes-type fluid resistance, two 4th-order PDEs which govern the total deflection, \bar{v} , and the rotation angle of the cross section, $\bar{\phi}$, may be derived. (Overbars denote dimensionless quantities.) Employing separation of variables leads to two 4th-order ODEs for the spatially dependent deflection and rotation fields, $\bar{V}(\xi)$ and $\Phi(\xi)$, where $\xi = x/L$ is a normalized coordinate:

$$\bar{V}'''' + \lambda^3 (r^2 + s^2) [\lambda + (1-i)\zeta] \bar{V}'' - \lambda^3 [\lambda + (1-i)\zeta] \{1 - r^2 s^2 \lambda^3 [\lambda + (1-i)\zeta]\} \bar{V} = 0, \quad (1a,b)$$

(Equation (1b) for $\Phi(\xi)$ is identical in form.) Quantities \bar{V} and Φ are the amplitudes of the *total* beam deflection (bending plus shear) and the rotation angle (associated with bending only). The TB parameters are defined as the rotational inertia parameter, $r^2 \equiv I/AL^2$, and the shear deformation parameter, $s^2 \equiv EI/kAGL^2$, where A and I are the cross section’s area and second moment of area, E and G are the Young’s modulus and shear modulus, $k = 5/6$ is the shear coefficient, and L is beam length. Parameters λ and ζ are the frequency and fluid resistance parameters, which are related to the fundamental system parameters by

$$\lambda \equiv \left(\frac{12\rho_b L^4 \omega^2}{Eb^2} \right)^{1/4}, \quad \zeta \equiv \frac{L}{hb^{1/2}} \left(\frac{48\rho_f^2 \eta^2}{E\rho_b^3} \right)^{1/4}, \quad (2a,b)$$

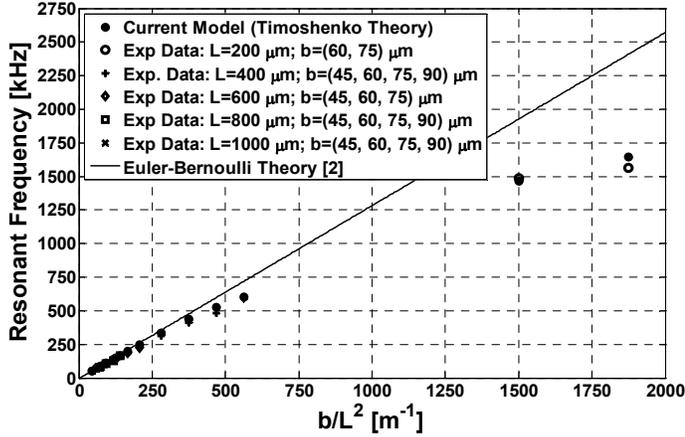


Fig. 2: Resonant Frequency Comparison (Lateral Mode in Water, $h=7.02 \mu\text{m}$, $E=151 \text{ GPa}$): Current Model, Euler-Bernoulli Model, and Experimental Data.

where ρ_b is the density of the beam material, ρ_f and η are the density and viscosity of the fluid, and ω is the driving (and thus the response) frequency (rad/s). Imposed boundary conditions correspond to electrothermal harmonic excitation via integrated heating resistors near the base of the cantilever (Fig. 1) and are given by

$$\bar{V}(0)=0, \Phi(0)=\theta_0, \Phi'(1)=0, \bar{V}'(1)-\Phi(1)=0, \quad (3a-d)$$

where θ_0 represents the amplitude of the “effective support rotation” imparted by the heating resistors [3].

Results

The BVP defined by (1a,b) and (3a-d) was solved analytically and the results expressed in terms of two “output signals”: total tip displacement and bending tip displacement, corresponding respectively to optical and piezoresistive detection methods. Resonant frequency (f_{res}) and Q were extracted from the theoretical beam response and were insensitive to the type of output signal for the fluid resistance values considered. Results (Figs. 2 and 3) show that the new model reproduces experimental trends in f_{res} and Q for lateral-mode microcantilevers at higher b/L ratios (i.e., for the high- Q devices for which EB models prove inadequate). Over the practical ranges of system parameters considered, the results indicate that TB effects can account for a reduction in f_{res} and Q of up to $\sim 40\%$ and $\sim 25\%$, respectively, but have effects of less than 2% when $L/b > 10$. The more accurate frequency estimates are smaller than the EB results because the Timoshenko beam model has lower stiffness (due to shear deformation) and greater mass (due to rotatory inertia). These effects therefore lead to the departure from the linear EB frequency results (Fig. 2). Similar conclusions apply to the Q comparisons among the experimental data

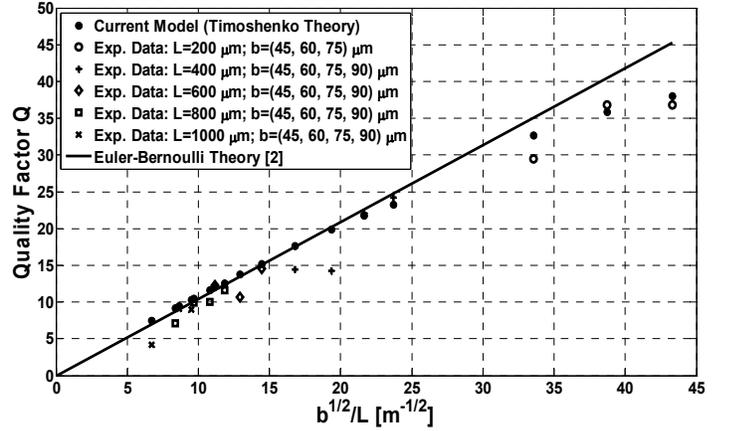


Fig. 3: Quality Factor Comparison (Lateral Mode in Water, $h=7.02 \mu\text{m}$, $E=151 \text{ GPa}$): Current Model, Euler-Bernoulli Model, and Experimental Data [4].

and the TB and EB models (Fig. 3).

To show more explicitly the effect of system parameters on Q , a new analytical equation has been established:

$$Q \approx 0.7182 \frac{hb^{1/2}}{L} \left(\frac{E\rho_b^3}{\rho_f^2 \eta^2} \right)^{1/4} \left[1 - 0.0794 \left(\frac{b}{L} \right)^{1.902} - 0.0670 \left(\frac{b}{L} \right)^{1.422} \left(\frac{E}{G} \right)^{0.811} \right]. \quad (4)$$

The bracketed expression, obtained by fitting the results of the new model, represents a correction factor associated with shear deformation and rotatory inertia effects, which reduces the EB result [2] appearing in front of the correction factor. The results of (4) are within 2.6% of those generated by the analytical model over the following parameter ranges: $\zeta = (0, 0.05)$, $r = (0, 0.2)$, $\sqrt{E/kG} = (0, 3)$.

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