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The contingent claims model has been used to value a variety of risky debt securities since the seminal work of Black and Scholes [1973]. The model is also called the option-theoretic model or the structural model. It treats a debt security as a contingent claim against the value of an underlying asset.

In 1974, Merton first applied this methodology to estimate the value of a defaultable zero-coupon bond; since then, many authors have applied it to value corporate debt. Extensions of Merton [1974] include Black and Cox [1976], who incorporate classes of senior and junior debt; Geske [1977], who considers bonds that make coupon payments; and Ho and Singer [1984], who value bonds with sinking fund provisions. Other authors modify Merton's assumption of a flat term structure.[1]

Despite the substantial literature, the research has been of limited empirical success in explaining the price behavior of corporate

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debt instruments and their credit risk spreads. Even though the contingent claims model is not particularly good at valuing corporate debt securities, this approach has become the norm for valuing mortgages, as mortgage loan characteristics more readily permit its direct application.

Real estate is usually financed with one debt source, and not the frequently complex capital structure of corporations. Mortgages also typically use a single asset as collateral and have relatively homogeneous contract terms. Because of this simpler capital structure and one collateral source, borrower default decisions and the foreclosure procedures for real estate are easier to model using the contingent claims approach.

Real estate researchers first used the contingent claims model to assess prepayment and default risk in residential mortgages. The commercial mortgage literature follows the history of the residential mortgage literature but lags it by approximately a decade, mainly because commercial mortgage data are limited. Applying the framework established in the residential mortgage literature, most commercial mortgage pricing studies assume a rational borrower defaults when the property value drops below the market value of the mortgage.[2]

These studies tend to ignore some key differences between residential mortgages and commercial mortgages. First, commercial mortgages are used to finance income-producing properties. Therefore, a borrower's default decision depends on not only the asset value (i.e., investor equity) but also the property liquidity (i.e., property income). A rational borrower would not default when property net cash flow is positive, even if the owner's equity position is negative. To properly reflect a rational borrower's default decision, a pricing model for commercial mortgages needs to include both property value and property income as default triggers.[3]

Second, unlike residential mortgages that are typically fully amortizing, most commercial mortgages are partially amortizing; that is, a balloon payment is due when the mortgage matures. Typical commercial mortgages have a 7-to 12-year term and a 25- to 30-year amortization schedule. Borrowers usually fund the balloon payment by refinancing the current mortgage. Even a borrower in good standing during the term of the mortgage might be unable to refinance at maturity due to higher interest rates or tighter underwriting standards, among other factors. Pricing models that ignore the impact of balloon risk may thus underestimate the overall credit risk in commercial mortgages.[4]

We use the contingent claims model to assess credit risk in commercial mortgages. Rather than assume a single default trigger based on property value (measured by contemporaneous loan-to-value, LTV), our model incorporates a second trigger based on property income (represented by contemporaneous debt service coverage, DSC). We also explicitly consider balloon risk as a second source of credit risk in commercial mortgages.

Our findings reveal that the effect of a property income trigger is significant, and, depending on the size of the property reserve account if required, the property income trigger can dramatically reduce the estimated credit risk premiums. While the inclusion of a second default trigger helps improve estimates of term default risk, it is necessary to expand the model to include balloon risk in order to assess total credit risk adequately.

We find that while weaker properties with sizable reserve requirements may not default during the term of the loan, thus reducing term default risk, they are often unable to meet stricter underwriting standards at mortgage maturity, resulting in higher balloon risk. Therefore, the current low commercial mortgage term default rates may be merely illusion if balloon risk is not appropriately considered. This may lead to significant mispricing of both investment-grade and subordinate classes of commercial mortgage-backed securities (CMBS).

I. Recent Developments in Commercial Mortgage Default

Most studies that examine the impact of default risk on the valuation of mortgages, both commercial and residential, focus on asset value as the sole default trigger, assuming that borrowers will default when the property value falls below the mortgage value. Recent theoretical and empirical research, however, suggests that asset value is not the sole default trigger, and in some cases not even an important trigger.

Archer et al. [2002], for example, argue that loan-to-value at origination is an endogenous risk measure, and suggest there is no empirical relationship between LTV and mortgage default. In their investigation of 495 multifamily loans, they find no significant relationship between LTV at origination and mortgage default. On the other hand, their results reveal that initial property income (measured by debt service coverage at origination) is a strong predictor of default.

Ciochetti et al. [2002] further extend the literature by including contemporaneous measures of LTV and DSC in their empirical analysis. They estimate default and prepayment functions for commercial mortgages using a competing risks proportional hazards model and loan-level data, and find that an asset value-based model alone cannot fully explain default incidence. The authors also reveal that both contemporaneous DSC and a binary variable representing balloon year show a strong impact on default incidence, indicating the importance of including property income and balloon risk in commercial mortgage pricing.

Goldberg and Capone [2002] propose a theoretical default model that incorporates both property value and property cash flow to predict multifamily mortgage default. They test the model empirically using a data set of 13,482 multifamily loans. The results show that a double-trigger joint-probability model is better than models with a single default trigger (either LTV or DSC). Their findings also reveal a sizable increase in default risk in the balloon year, confirming the results of Ciochetti et al. and the need to explicitly include balloon risk in commercial mortgage pricing models.

When a borrower is unable to make the balloon payment, the lender may either foreclose on the property or renegotiate the loan contract. While academic research has shown that negotiating a discounted loan payoff eliminates the default costs associated with property liquidation or transfer (see Riddiough and Wyatt [1994]), evidence in practice suggests that extending the mortgage maturity is a more common form of workout (Harding and Sirmans [2002]).

Harding and Sirmans argue that maturity extension better aligns the incentives of borrowers and lenders than principal renegotiation. They find that borrowers who expect lenders to renegotiate loan maturity to avoid default generally have less incentive to extract cash flow from the property during the term of the mortgage, and are less likely to take on additional risk, resulting in reduced agency costs.[5]

Leveraging these recent developments in the literature, we propose a model that considers the interaction of term default and balloon risk in commercial mortgage pricing. In our model, the borrower default decision is based on both contemporaneous property value and property income, and balloon risk is incorporated in the form of maturity extension rather than discounted principal payoff.

II. Mortgage Pricing Methodology

The most popular methodology for pricing mortgages in the academic literature is the contingent claims model, where the partial differential equation is solved using a backward numeric method. The beauty of this pricing approach is that it recognizes the nature of compound default options and explicitly considers the value of these options. Under highly restrictive circumstances—e.g., the borrower considers only its equity position; the lender forecloses the property immediately when the borrower defaults; and there is no lag time between default and investment recovery-this is the best mortgage pricing method. It becomes very difficult, if not impossible, however, to apply this approach when property cash flow, balloon risk, and other factors are also incorporated.

One limitation of the backward numeric approach is that computation time increases exponentially as the number of state variables rises. Studies applying this approach were limited to two state variables until recently (see Brunson, Kau, and Keenan [2001] for a mortgage valuation model with three state variables). When two default triggers are considered in the pricing model, at least one more state variable must be added to account for the volatility of property income, making it complicated to solve using the backward approach.

Another limitation in pricing mortgages using the backward numeric method is that terminal conditions must be specified in order to work backward in time. Consequently, loan workout, maturity extension, delay of investment recovery, and other possible situations cannot be properly addressed.[6]

Overall, a pricing model that incorporates double default triggers and balloon risk is difficult if not impossible to solve using the backward approach, so we use a forward Monte Carlo simulation approach. Other studies that have adopted the forward mortgage pricing approach include Schwartz and Torous [1989a and 1989b] and Riddiough and Thompson [1993].

With the forward pricing model, we use both contemporaneous LTV and DSC as default triggers, and consider the possibility of loan extension if minimum mortgage refinance terms are not met at mortgage maturity. The Monte Carlo approach is also more flexible than the backward pricing method, as it allows factors such as time to foreclosure, foreclosure costs, and property income payout rates, among other factors, to vary, instead of maintaining a constant value.

The main criticism of Monte Carlo simulation for pricing commercial mortgages is that it cannot explicitly measure the value of the borrower's default options. To address this issue, we use the default probability function developed by Riddiough and Thompson [1993], which replaces sharp borrower default boundaries in rigid default models with fuzzy default boundaries. The Riddiough-Thompson model recognizes the influence of default transaction costs on the borrower's default decision and considers the value of default options implicitly.

In this model, default probability is a function of time to maturity and net equity level, E, which is the inverse of the contemporaneous loan-to-value ratio:

$$E_t = \frac{P_t}{M_t(r)} \quad (1)$$

where P_t is the property value at time t, and M_t is the mortgage value, which is stated as a function of the current mortgage rate, r.

Riddiough and Thompson [1993] establish default probability rate bounds at mortgage origination, $f(E_0, 0)$, and at mortgage maturity, $f(E_T, T)$, given different equity levels. These bounds are then used to determine a default probability function during the term of the loan. As a result, the lower the property's net equity level or the closer to mortgage maturity, the higher the probability of default.[7]

Exhibit 1 graphs the relationship between default probabilities and a property's net equity level at 1) origination, 2) halfway through the loan term, and 3) maturity.

III. Simulation Model

To investigate the effect of credit risk on commercial mortgage values, we first specify the state variables employed in the simulation model. Most contingent claims studies focus on two state variables: interest rate and property value. Using the Cox, Ingersoll, and Ross [1985] mean-reverting interest rate process, the dynamics of interest rate variation in our model are specified as:

$$dr = \kappa(\theta - r)\,dt + \sigma_r\sqrt{r}\,dz_r \quad (2)$$

Where κ is the speed of reversion parameter; θ is the long-term reversion rate; $\sigma_r\sqrt{r}$ is the standard deviation of changes in the current spot rate; dz_r and is a standard Wiener process. A variety of shapes of the yield curve can be described by using a different initial interest rate, r_0..

Property values are assumed to follow a lognormal diffusion process:

$$dP = (\alpha_P - \beta_P)P\,dt + \sigma_P P\,dz_P$$

where P is property value; α_P is the expected total return on the property; β_P is the continuous property income payout rate; σ_P is a volatility parameter of property returns; and dz_P is a standard Wiener process. To estimate the credit risk premium of commercial mortgages, we apply the risk-neutral valuation principle, where the risk-neutral property price process is specified as:

$$dP = (r - \beta_P)P\,dt + \sigma_P P\,dz_P \quad (4)$$

and r is the riskless spot rate. It is assumed there is an instantaneous correlation between changes in property prices and interest rates, ρ_{Pr}

A third stochastic variable that must be specified in the mortgage pricing model is property cash flow. Monthly property income is determined by multiplying the property value by the property income payout rate. Since interest rate and payout rate are correlated, we specify the changes in payout rate as a function of the contemporaneous interest rate where:[8]

$$d\beta_P = \lambda\,dr + \sigma_\beta\,dz_\beta \quad (5)$$

where β_P is the property income payout rate; λ is an estimated parameter; r is the interest rate; σ_β is a volatility parameter of the payout rate; and dz_β is a standard Wiener process.

In each simulation iteration, random seeds are generated for each month over the term of the mortgage. Given the parameters of the stochastic processes, spot interest rates, property values, and property income are calculated for each month. Values of these variables are then used to project borrower default behavior.

Borrower default during the mortgage term is modeled using a modified Riddiough-Thompson default function, where we include a second default trigger, property income. If property income is adequate to cover debt service, we assume a rational borrower will not default and forgo the positive cash flow and the time value of the default options. Therefore, in our double-trigger default model, the borrower must incur a negative cash flow position in addition to an adverse net equity level to trigger default. In other words, a property income trigger event (i.e., a DSC of less than 1.0) is a necessary condition of default.

Furthermore, a borrower is unlikely to default immediately when the DSC initially drops below parity. The borrower may fund a debt service shortfall through a property reserve account or other equity sources until it either becomes illiquid, uses all property reserves, or perceives that the negative cash flow is likely to persist and the property value is unlikely to recover. Unfortunately, no empirical research has examined the extent or the length of property cash flow deficiencies before default occurs. In the simulation analysis, we consider a series of borrower default criteria related to the contemporaneous DSC while also including the contemporaneous LTV-based default trigger.

The model also accounts for balloon risk by examining the possibility that the borrower cannot make the balloon payment even though the mortgage is not in default during the term of the loan. At maturity, we estimate the loan amount the borrower is able to refinance (i.e., the justified loan amount) based on the contemporaneous property value, property income, interest rate, and underwriting standards (LTV and DSC). If the justified loan amount is lower than the balloon balance, the borrower is presumed to be unable to payoff the current mortgage. In this case, the lender and the borrower are likely to negotiate a workout.

We assume the lender will agree to extend the loan maturity, while the borrower continues to make periodic payments. At the end of each extended month, the mortgage may be: paid off (if the justified loan amount exceeds the balloon payment), in default (if both LTV and DSC default triggers are satisfied), or extended again (otherwise). It is assumed that the mortgage can be extended for up to two years, and the borrower will be forced to liquidate the property and terminate the mortgage if neither default nor payoff occurs during the two-year extension period.

Having established the criteria of default and extension, we can project the entire cash flow stream throughout the life of the mortgage for each simulation path. These cash flows are then discounted on a risk-neutral basis to determine the initial mortgage value. A large number of Monte Carlo simulation paths are generated until the mortgage value converges. The credit risk premium is determined by solving for the mortgage contract rate that makes the mortgage sell at par.

IV. Model Parameters and Simulation Results

We use the simulation model to examine how term default risk and balloon risk affect the value of a 10-year commercial mortgage with a 30-year amortization schedule. To isolate the impact of credit risk on mortgage pricing, we assume a non-callable mortgage.[9]

Exhibit 2 describes the mortgage terms, interest rate parameters, property value processes, property income payout parameters, and other variables used in the I simulations. The mortgage is assumed to have an initial amount of $1 million, with LTV of 70% and DSC of 1.30. In the base case, we consider an upward-sloping yield curve with = 2.0%, = 25.0%, = 5.0%, and = 8.0%.[10]

While the literature generally assumes that property prices follow a lognormal diffusion process, there is no consensus on property value volatility (σ_P). We test a series of property volatility rates ranging from 12% to 22%, and find that the risk premiums estimated using an 18% volatility are most comparable to those observed in the market. This range is consistent with the literature. For example, Titman and Torous [1989] use a series of volatility measures from 15.0% to 22.5%, Riddiough and Thompson [1993] use 12% and 16%; and Childs, Ott, and Riddiough [1996] use 15% and 20%. Therefore, we discuss only the simulation results using the 18% volatility.

The property income payout rate is defined as a function of the long-term interest rate, with an initial rate of 6.5% and a standard deviation (σ_β) of 0.3%.[11] Additional assumptions made in the base case simulation analysis include: 1) Average lender loss rate of 15% of the mortgage value in foreclosure, with a standard deviation of 5% and minimum of 5%; 2) a 12-month investment recovery lag; and 3) investor carrying costs between default and foreclosure of 0.5% per month of the outstanding loan balance.

To separate the impact of term default and balloon risk on mortgage values and, more important, to highlight the interaction between these two types of risk, we present the simulation results in two sections. First, term default risk premiums are estimated using the double-trigger default model without considering balloon risk. These results reveal the effect of including the property income as a default trigger on mortgage pricing. Next, loan extension at maturity is considered in the model to determine the impact of balloon risk on the overall credit risk of commercial mortgages.

Simulation Results with Term Default Only

Exhibit 3 presents the simulated mortgage values and default risk premiums using the double-trigger default model and 10,000 Monte Carlo state variable paths without considering balloon risk. To highlight how the addition of a second property income default trigger (DSC) influences pricing results, we first estimate the mortgage value and default risk premium using the single-trigger LTV-based default criterion (Model 1 in Exhibit 3). The single-trigger risk premium of189 basis points in Model 1 provides a baseline for comparing the results of adding the property cash flow trigger to the default model.

The Riddiough-Thompson [1993] default function recognizes unobservable borrower default transaction costs, and thus reflects what we know to be non-optimal borrower default decisions. To test whether the Riddiough-Thompson model adequately reflects borrower default behavior indicated by a double-trigger model, we start with a less restrictive DSC-based criterion. In Model 2 of Exhibit 3, a second necessary condition of default is a negative net cash flow, or DSC < 1. The results reveal no significant difference in the mortgage values and default risk premiums.

While the default behavior in Model 2 is plausible, we find it unlikely, as most borrowers are able to cover small or temporary cash flow deficits to keep the options alive. Generally speaking, when the DSC is slightly lower than 1.0, the probability that property net cash flow will again become positive is high, and the cost of keeping the default options open is relatively low.[12] Therefore, the borrower is unlikely to default immediately when the DSC drops below 1.0.

Thus, we consider more restrictive property income default triggering conditions. Model 3 in Exhibit 3 considers three consecutive months of cash flow deficits, which reveals a slightly lower credit risk premium under Model 1 (6 basis points). Although the difference is small, it is statistically significant. The results indicate that explicitly considering the DSC trigger is necessary to properly measure the credit risk of commercial mortgages.

A recent trend in commercial mortgage underwriting is the requirement of cash reserves or escrows as a cash flow volatility buffer.[13] With capital improvement, tenant buildout, or property expense reserves, borrowers can fund small-to-moderate cash flow deficits with these reserve accounts.

Models 4-6 assume the borrower has sufficient reserves to fund a one-month, three-month, and six-month cumulative debt service shortfall over a 12-month period, where a one-month shortfall is equal to one month's debt service, and so on. The possibility of the borrower funding debt service out of a reserve account or out of pocket for the cumulative amount of one month or three or even six months' debt service is entirely reasonable. Under these conditions, default risk premiums drop, and in Models 5 and 6 drop sharply.[14]

In Model 4, the one-month debt service deficiency case, the estimated risk premium drops to 165 basis points, representing a 24 basis point reduction from Model 1 (the asset value-only model). When the borrower has the ability to fund a three-month cumulative debt service shortfall (Model 5), the risk premium shrinks to 112 basis points. In Model 6, when the borrower can fund a six-month debt service shortfall over the 12-month period, the term default risk premium plummets to 17 basis points.

While these simulated default risk premiums appear low, they are consistent with the default rates experienced for mortgages originated over the past decade. See Esaki [2002] and the delinquency rates compiled by the ACLI.[15]

In additional simulations, we also adjust the shape of the yield curve, the property payout rate, and loan origination terms, among other parameters. We find a similar pattern of default risk premium reductions between single-trigger and the double-trigger models.

Simulation Results with Both Term Default and Balloon Risk

It is quite plausible that a borrower will be unable to make the balloon payment if the current mortgage cannot be refinanced due to higher interest rates or tighter underwriting standards, among other reasons. In this case, the lender and the borrower are likely to negotiate a loan extension.

At mortgage maturity, a justified refinance loan amount is calculated based on the contemporaneous property value, property cash flow, interest rate, and underwriting standards (LTV and DSC). If the justified refinance loan amount is lower than the balloon amount and the loan is not in default, we assume the mortgage is extended.

Exhibit 4 presents the probabilities of term default, payoff at maturity, and extension across the six models in Exhibit 3. As expected, default probabilities decline monotonically across default models. Interestingly, the percentage of mortgages that are paid off at maturity is relatively stable, but balloon risk (the risk of loan extension) increases with the borrower's ability to fund property income shortfalls. In other words, loans that would otherwise have defaulted prior to maturity now reach the balloon payment date; then, these weaker properties are less likely to meet contemporaneous underwriting standards necessary to obtain a refinancing loan at maturity.[16]

We further investigate the effect of balloon risk on commercial mortgage pricing in Exhibit 5, which presents the simulated mortgage values and risk premiums considering a double-default trigger model that includes balloon risk, where borrower default behavior during the term of the mortgage is the same as in Exhibit 3.

During loan extension, the borrower is assumed to make the periodic debt service payment. At the end of each extension month, the mortgage may be paid off, in default, or extended again. Loans that default during the extension period have the same loss and foreclosure parameters as term defaults. Additionally, it is assumed that the mortgage can be extended for up to two years, at which point the borrower will be forced to liquidate the property and terminate the mortgage if neither default nor payoff occurs during the extension period.

The credit risk premium estimated in Model 1 in Exhibit 5 is 198 basis points, indicating a 9-basis point balloon risk premium (compared to Model 1 in Exhibit 3). When the second default trigger is added in Models 2-6, the balloon risk premium increases, while the total credit risk premium rises slightly at first and then drops dramatically. As the borrower is able to fund a three-month and six-month debt service shortfall (see Models 5 and 6), the estimated balloon risk premiums are is 51 and 93 basis points, respectively.

These results reveal an interesting interaction between term default and balloon risk. As more restrictive default criteria reduce the probability of term default and thus the resulting risk premium (see the impact of including a second default trigger across models in Exhibit 6), more properties are unable to meet the refinance requirements, increasing the probability of extension and the balloon risk premium. Consequently, although the double-trigger models (Models 2-6) are superior to the single-trigger model (Model 1) in the sense that they better simulate borrower default behavior and thus improve the estimation of default risk premiums, simply adding the property income-based default trigger without considering its interaction with balloon risk may introduce a different kind of bias in mortgage pricing, one that underestimates the total credit risk premium in commercial mortgages.

V. Summary and Conclusions

We have assessed credit risk in commercial mortgages using a double-trigger default model that includes balloon risk. Like other researchers, we find property cash flow is an important predictor of default, in addition to the property value. We therefore develop a commercial mortgage pricing model that uses both asset value and property income as default triggers, and apply Monte Carlo simulation to estimate the risk premium associated with borrower default during the term of the mortgage. The results reveal that failure to consider property cash flow significantly changes the probability of default and the credit risk premium.

As most commercial mortgages are not fully amortizing, a balloon payment is often required to pay off the loan at maturity. While most research on commercial mortgage pricing assumes a performing mortgage is immediately paid off at maturity, it is possible the borrower will be unable to make the balloon payment if the property does not meet contemporary underwriting standards. We therefore examine the effect of loan extension and possible default during the extension period on mortgage values and credit risk premiums.

The analysis reveals an interaction between term default and balloon risk. As more restrictive default criteria reduce the probability of term default and the resulting risk premium, more weak properties that would otherwise have defaulted survive to maturity but cannot satisfy the refinance requirements, increasing the balloon risk. Applying the double-trigger default criteria results in lower to significantly lower term default risk premiums, but the effect on total credit risk premiums is not as dramatic due to higher balloon risk premiums.

As commercial mortgage originators and underwriters require property reserve accounts for capital improvements, tenant buildouts, and building expenses as cash flow volatility buffers, term default risk is expected to be low. This expected outcome is confirmed by studies of commercial mortgage performance by Corcoran [2000] and Esaki [2002]. Yet a property with a depressed value and weak income-producing ability makes refinancing at maturity tenuous. As a result, the reduction in term default is likely to lead to an increase in balloon risk.

These results should be of particular interest to CMBS investors. As mortgage default may be deferred to maturity, the non-rated and B tranche buyers are likely to remain in the pool for longer periods, thus enhancing their returns. As term defaults are delayed until maturity, however, the risk of loss or delayed payments could work its way up the subordination levels to investment-grade tranches. As a result, a pricing model that incorporates double default triggers and considers balloon risk is critical to accurately assess the credit risk in commercial mortgages, particularly those included in CMBS pools.

Endnotes

1. Shimko, Tejima, and Van Deventer [1993] and Longstaff and Schwartz [1995] assume that short-term risk-free rates follow a Vasicek process; and Kim, Ramaswamy, and Sundaresan [1993] use a Cox, Ingersoll, and Ross [1985] model for short-term rates.

2. These studies include Kau et al. [1987], Titman and Torous [1989], and Childs, Ott, and Riddiough [1996], among others.

3. If property value and property income are perfectly correlated, separate modeling of property income is unnecessary, but experience suggests

this is not the case. The correlation between the National Council of Real Estate Investment Fiduciaries (NCREIF) property net operating income (NOI) growth rate and capital return in the period 1978 to 2002 was less than 0.5. Also, if property NOI and value are highly correlated, capitalization rates should remain stable over time. American Council of Life Insurers (ACLI) data on capitalization rates over the same time period ranged from 8.3% to 13.7% with a standard deviation of 1.24%.

4. Tu and Eppli [2002] estimate the probability of balloon risk and its associated losses to the lender. They find balloon risk is sensitive to property cash flow volatility and changes in underwriting standards between loan origination and maturity.

5. These two types of agency problem are referred to as underinvestment and overinvestment by Gertner and Scharfstein [1991]. The agency issues are based on the work by Jensen and Meckling [1976] and Myers [1977].

6. Studies using the backward approach assume that a certain proportion of the property value will be recovered immediately in the event of default. In reality, both the amount and the timing of investment recovery are uncertain.

7. Riddiough and Thompson use a quadratic weighting system to determine the default probability function for a commercial mortgage. For example, when a loan is halfway through its term, the lower bound is weighted 75% (1 – 0.52) while a 25% weight is placed on the upper bound (0.5^2).

8. Since data on commercial property income payout rates are not available, we estimate the relationship between payout rates and interest rates using property capitalization rates as a proxy. A regression of capitalization rates on mortgage contract rates is estimated using ACLI data, similar to the approach employed by Goldberg and Capone [2002].

9. Commercial mortgage pricing studies have generally presumed non-callable mortgages (for example, see Titman and Torous [1989]; Riddiough and Thompson [1993]; and Childs, Ott, and Riddiough [1996]). Most commercial mortgages have lockout periods and strict prepayment penalties in the form of defeasance and yield maintenance prepayment penalties.

10. These parameters are consistent with studies on commercial mortgage pricing, and the resulting yield curve resembles the Treasury yield curve observed in early 2003.

11. The initial payout rate is a function of the long-term interest rate where: $\beta_P = \alpha + \lambda r$. Using ACLI data on mortgage interest rates and property capitalization rates, we estimate that α = 4.8% and λ = 0.45. According to the Cox-Ingersoll Ross [1985] process and r_0, the initial ten-year interest rate is approximately 3.8%.

12. For example, the monthly cost of maintaining the option of future cash flows and property appreciation is 0.054% of the loan amount, when the DSC drops to 0.90 for a loan with 6.50% mortgage constant. In other words, multiplying the monthly mortgage constant of 0.54% by 0.10 (e.g., 1 – DSC) returns a monthly debt service shortfall of 0.054% of the loan amount, which is the cost of keeping the options alive each month.

13. For example, see the *Fitch Commercial Mortgage Presale Report, GE Capital Commercial Mortgage Corp., Series 2003-C2.* The summary statistics on page 2 reveal that 82% of all mortgages in the pool have capital reserve requirements, and 87% have up-front or ongoing expense reserve requirements.

14. With a 6.50% mortgage constant, the 12-month cumulative cost of keeping the option open is 3.25% of the loan amount in the most restrictive 6-month cash flow shortfall case.

15. Esaki [2002] reports that average annual default rates (not risk premiums) for recently originated commercial mortgages were between 0.07% and 0.59% for loans originated between 1991 and 1995 (i.e., in the fifth to ninth year of the loan term).

    Over the past three years, ACLI commercial mortgage delinquency rates have hovered around 30 basis points, with a December 2001 delinquency rate of12 basis points. If approximately 30% of delinquent loans default, and the loss rate on default is approximately 35% of the outstanding loan balance, commercial mortgage loss rates should average less than 10 basis points over the last three years for ACLI loans.

16. Note that while the mortgage values and risk premiums are estimated in a risk-neutral framework, default and extension probabilities must be stated in real terms. Hence, an expected property total return (α_P) is necessary. In the simulation, we assume α_P = 11.0%.

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Appendix

Exhibit 1: Default Probability as Function of Property Net Equity Level

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Tu, C. C., and M. J. Eppli. (2003.) "Term Default, Balloon Risk, and Credit Risk in Commercial Mortgages." *Journal of Fixed Income,* 13(3): 42-52.

Exhibit 2: Parameter Values in Monte Carlo Simulations

Parameter	Value
Contract Terms	
Mortgage Amount	\$1,000,000
Mortgage Term (years)	10
Amortization Schedule (years)	30
LTV [a]	70%
DSC [a]	1.30
Property Parameters	
Initial Property Value (P_0) [b]	\$1,428,571
Standard Deviation (σ_P)	18.0%
Initial Payout Rate (β_P) [c]	6.5%
Interest Rate Parameters	
Initial Spot Rate (r_0)	2.0%
Reversion Speed (κ)	25.0%
Long-Term Reversion Rate (θ)	5.0%
Standard Deviation (σ_r)	8.0%
Correlation Between r and P (ρ_{Pr})	0
Number of Iterations	10,000

[a] Mortgage underwriting standards at loan origination and at maturity.

[b] Property value is calculated based on mortgage amount and original LTV.

[c] The initial payout rate is a function of the long-term interest rate where:

$\beta_P = \alpha + \lambda r$

Using ACLI data on mortgage interest rates and property capitalization rates, we estimate that α = 4.8% and λ = 0.45. Based on the Cox-Ingersoll-Ross process and r_0, the ten-year interest rate is approximately 3.8%.

Exhibit 3: Monte Carlo Simulation Results—Mortgage Values and Risk Premiums Assuming Term Default Only

	Mortgage Value ($)	Risk Premium (bp)	Change in RP from Model 1 (bp)
Model 1	869,161	189	
LTV Only	(2,399)	(4)	
Model 2	868,768	189	0
LTV and DSC < 1	(2,450)	(4)	
Model 3	872,836	183	-6
LTV and DSC < 1 for 3 Consecutive Months	(2,683)	(4)	
Model 4	884,123	165	-24
LTV and One-Month Cash Flow Deficiency	(2,742)	(4)	
Model 5	919,525	112	-77
LTV and Three-Month Cash Flow Deficiency	(2,384)	(3)	
Model 6	987,563	17	-172
LTV and Six-Month Cash Flow Deficiency	(1,151)	(2)	

Numbers in parentheses are standard deviations.

Exhibit 4: Probabilities of Term Default, Payoff, and Extension

	Probability		
	Term Default	Payoff	Extension
Model 1			
LTV Only	24.62%	68.81%	6.57%
Model 2			
LTV and DSC < 1	20.36%	71.11%	8.53%
Model 3			
LTV and DSC < 1 for 3 Consecutive Months	17.78%	72.14%	10.08%
Model 4			
LTV and One-Month Cash Flow Deficiency	13.61%	73.47%	12.92%
Model 5			
LTV and Three-Month Cash Flow Deficiency	6.58%	74.69%	18.73%
Model 6			
LTV and Six-Month Cash Flow Deficiency	0.94%	74.94%	24.12%

Probabilities are estimated in real terms, assuming an expected property total return, α_P, of 11%. Other parameters are the same as the risk-neutral simulation presented in Exhibit 3.

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Exhibit 5: Monte Carlo Simulation Results—Mortgage Values and Risk Premiums Assuming Term Default and Balloon Risk

	Mortgage Value ($)	Credit Risk Premium (bp)	Balloon RP (Change from Exhibit 3)	Change in RP from Model 1
Model 1	863,347	198	9	
LTV only	(2,389)	(4)		
Model 2	859,203	204	15	6
LTV and DSC < 1	(2,471)	(4)		
Model 3	860,514	202	19	4
LTV and DSC < 1 for 3 Consecutive Months	(2,743)	(4)		
Model 4	866,144	193	28	-5
LTV and One-Month Cash Flow Deficiency	(2,911)	(5)		
Model 5	885,769	163	51	-35
LTV and Three-Month Cash Flow Deficiency	(2,872)	(4)		
Model 6	921,297	110	93	-88
LTV and Six-Month Cash Flow Deficiency	(1,881)	(3)		

Numbers in parentheses are standard deviations.

Exhibit 6: Interaction of Term Default and Balloon Risk