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# Time Series Models for Computing Activation in fMRI

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# Time Series Models for Computing Activation in fMRI

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## [1]AR(p) Rician model

From model (1) to the right, we model the magnitude such that, in the context of the **EM algorithm**,

- complete data: complex data as in (1)
- observed data: magnitude data
- "missing" data: phase data. *Note*: the phase data is not really missing, just modeled as such.

The model is **AR(p) Rician** because

- In (1), real and imaginary errors $\eta_R$ and $\eta_I$ are independent AR(p) time series.
- Marginal distribution of each magnitude observation is Rician, resulting from independent normality of real and imaginary components.

Other magnitude AR(p) models assume normality of magnitude observations, an assumption only valid for **high SNR**. Rician AR(p) model is valid for all SNR.

## [3]AR(p) normal model

- Den Dekker et al (2009) model the magnitude time series as a normal linear model with AR(p) errors.
- The AR(p) dependence is incorporated into the likelihood function and **likelihood-based activation statistics** include likelihood ratio test, Wald test, and Rao (score) test, which avoid the approximations inherent in prewhitening procedures.
- **Disadvantage:** Magnitude is modeled with normal distribution, which is only a good approximation of magnitude's true distribution (Rice) at high SNR.

## [4]Independent Rician model

- Because real and imaginary time series and independent and normally distributed, magnitude data is Rice-distributed.
- Rician distribution approaches normal distribution at high SNR.
- Den Dekker & Sijbers (2005) showed that likelihood ratio tests (LRTs) based on the Rician distribution has **constant false alarm rate (CFAR)** for all SNR; normal LRTs do not have CFAR property.
- Also, Rician LRTs show higher detection probability than normal LRTs at low SNR, same detection probability at high SNR.
- **Disadvantage:** Independent Rician model does not incorporate AR(p) dependence because **AR(p) errors are predicated upon the normal distribution**.

## [8]Independent normal model

This was the first model for computing activation, as in Bandettini et al (1993).

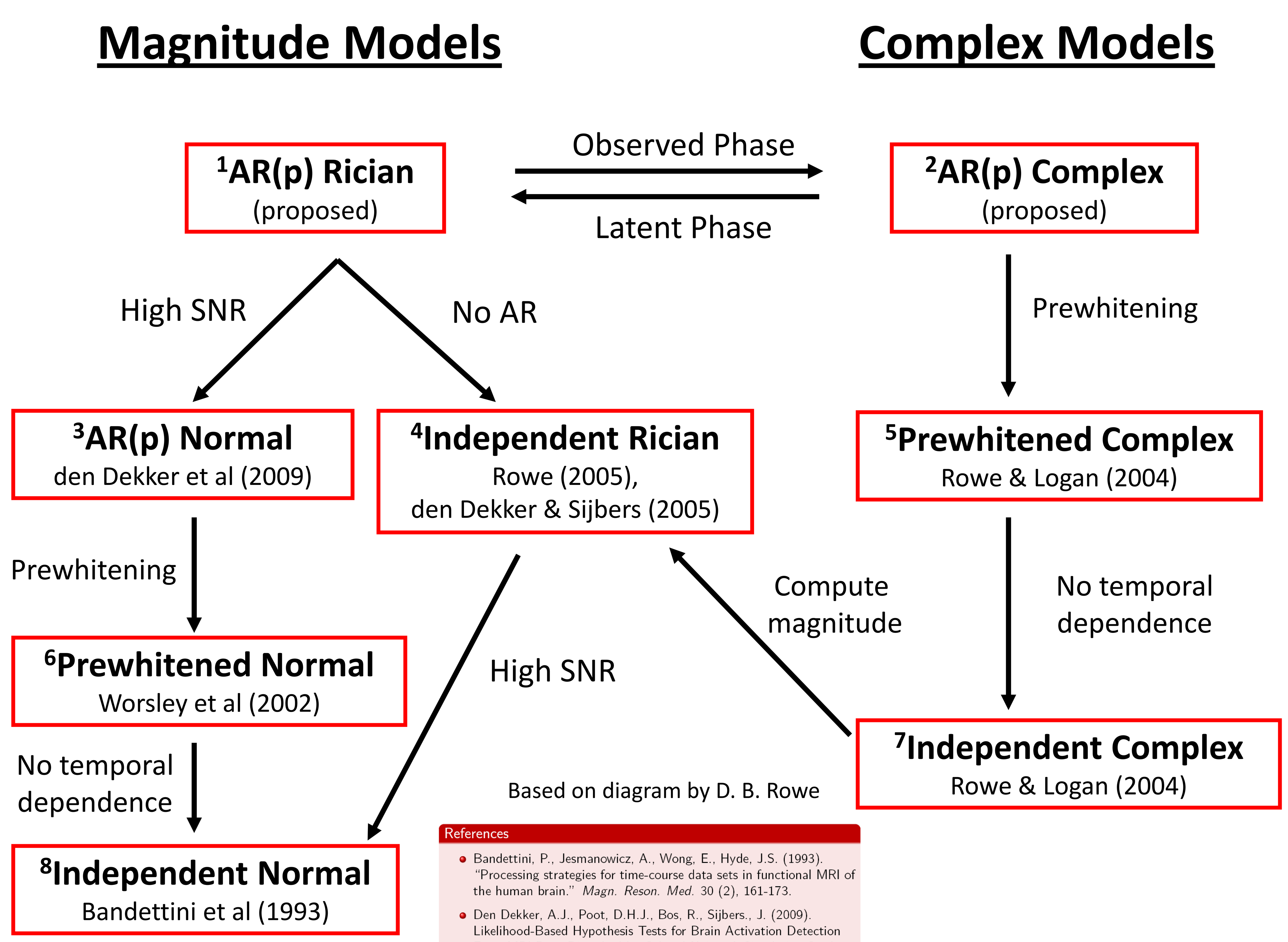
## General Complex Model in Rowe & Logan (2004)

For a single voxel, the complex model is

$$\begin{pmatrix} y_R \\ y_I \end{pmatrix} = \begin{pmatrix} X & 0 \\ 0 & X \end{pmatrix} \begin{pmatrix} \beta\cos\theta \\ \beta\sin\theta \end{pmatrix} + \begin{pmatrix} \eta_R \\ \eta_I \end{pmatrix}, \qquad (1)$$

where

- $y_R$ and $y_I$ are the real and imaginary time series, respectively.
- $X$ is a design matrix that models the baseline, drift, and HRF.
- $\beta$ is the corresponding parameter vector.
- $\theta$ is the phase imperfection, assumed constant.
- $\eta_R$ and $\eta_I$ are the real and imaginary error vectors, respectively.
- $(\eta_R', \eta_I')' \sim N(0, \Sigma \otimes \Phi)$, where $\Sigma = \sigma^2 I_2$, i.e. real and imaginary components are independent with same temporal dependence structure.




Based on diagram by D. B. Rowe

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## [2]AR(p) complex model

- Uses all observed **complex data**, not just the magnitude data.
- In model (1) to the left, $\eta_R$ and $\eta_I$ are independent AR(p) time series.
- Through incorporating AR(p) dependence into the likelihood function, likelihood-based activation statistics are developed, which do not rely on **prewhitening** approximations. Note that den Dekker et al (2009) also make this distinction for magnitude data (see **AR(p) normal model**).
- Magnitude AR(p) models assume normality of magnitude observations, an assumption only valid for **high SNR**.
- Complex AR(p) model is valid for all SNR.

## [5 and 6]Prewhitened Models

Rowe & Logan (2004) describe a procedure for prewhitening complex time series

- In (1), estimate $\Phi$ with $\hat{\Phi}$ based on AR model.
- For factorization $\hat{\Phi} = PP'$, multiply real and imaginary components on both sides by $P^{-1}$.
- This produces (approximately) independent real and imaginary error terms, for use in the **independent complex model**.

Prewhitening is widely used in the magnitude model, as in Worsley et al (2002).

**Disadvantage:** Prewhitening (and the resulting assumption of independence) is based on an *estimate* of the true covariance structure. Models that incorporate dependence directly into the likelihood function, such as **AR(p) complex**, **AR(p) Rician**, and **AR(p) normal**, do not rely on such an estimate.

## SNR

- SNR (signal-to-noise ratio) is defined as the ratio of baseline level to noise standard deviation.
- Lower SNR is a consequence of fMRI studies with **increased resolution**.

## [7]Independent complex model

- In (1), Rowe & Logan (2004) make the assumption that $\Phi = I$ (possibly after the prewhitening described above)
- Complex likelihood ratio test (LRT) is compared to LRTs under independent normal model and independent Rician model (in Rowe, 2005).
- Rowe & Logan (2004) demonstrate higher power of complex LRT at low SNR.