1-1-2010

A Comparative Analysis of the Bootstrap versus Traditional Statistical Procedures Applied to Digital Analysis Based on Benford's Law

Ikseon Suh
Marquette University, ikseon.suh@marquette.edu

T. Christopher Headrick
Southern Illinois University Carbondale

Journal of Forensic & Investigative Accounting
Vol. 2, Issue 2

A Comparative Analysis of the Bootstrap versus Traditional Statistical Procedures Applied to Digital Analysis Based on Benford's Law

Ik Seon Suh
T. Christopher Headrick*

Many fraud detection problems involve large numbers of financial transactions such as those associated with credit cards, accounts receivables, payments to vendors, payrolls, or other expense accounts (Panigrahi 2006; Bolton and Hand 2002). Computer Assisted Auditing Tools and Techniques (CAATTs) e.g., ACL (2006) allow auditors to perform digital analysis based on Benford's Law (Benford 1938) for scrutinizing high volumes of complex financial data and detecting unintentional errors or fraud (AICPA 2008; Panigrahi 2006; Coderre 1999). However, the advantages that these software packages offer for assessing data conformity to Benford's Law are limited due to problems associated with the underlying traditional statistical procedures.

Specifically, previous studies (e.g., Cho and Gaines 2007; Geyer and Williamson 2004; Nigrini 2000) have issued caveats on the use of traditional statistical tests such as chi-square goodness-of-fit or *Z*-tests in the context of Benford's Law because applications of these tests to large data sets may falsely lead auditors to believe that evidence of fraud exists when in fact there is none. Nigrini (2000) defines this situation as the problem of "excessive power" (p.75). Similarly, Geyer and Williamson (2004) note that "…one has to be careful in such situations…where the sample size may be very large, for this [*Z*] test is almost certain to reject the null hypothesis for a given significance level" (p. 234). Further, Cho and Gaines (2007) also indicate that "…chi-square goodness-of-fit tests are very sensitive to sample size, having

* The authors are, respectively, at Marquette University and Southern Illinois University Carbondale.

enormous power for large N, so that even quite small differences will be statistically significant" (p. 220).

In view of the above, we introduce a bootstrap procedure to assess data conformity to Benford's Law that addresses the problem of excessive power (false alarms). The proposed procedure is based on developing bootstrap confidence intervals for the mean, variance, skew, and kurtosis associated with the first two digits of Benford data (e.g. non-fraudulent data) and actual data that are subject to investigation. In addition, bootstrap confidence intervals for the Pearson correlation between the first digits and the second digits can also be used as a supportive (or decisive) analysis. Unlike the conventional analysis based on Benford's Law that examines financial data on a digit-by-digit basis, the proposed procedure allows auditors to diagnose financial transactions on an overall basis. Thus, this procedure takes an approach that is similar to what Cleary and Thibodeau (2005) suggested "Perhaps the most prudent approach would be to begin the analysis stage with an overall analysis…" (p. 80).

Applications of the proposed procedure to reported annual earnings of S&P 1500 companies, Federal Election Commission data, and extremely fraudulent data demonstrate the robustness of our procedure over different periods of time and across small or large financial data sets. For example, the proposed bootstrap procedure and the traditional statistical tests were applied to data sets that closely follow Benford's Law such as reported annual earnings from S&P 1500 companies. The results associated with the bootstrap procedure consistently confirmed that the S&P 1500 companies are not likely to be engaged in earnings rounding-up behaviors. In contradistinction, the results corresponding with the traditional statistical tests were inconsistent across various sample sizes and time periods. Further, the proposed procedure was applied to allegedly fraudulent data from the Federal Election Commission (Cho and Gaines

2007) and extremely fraudulent data (Geyer and Williamson 2004; Hill 1998). The overall results indicate that our procedure accurately detects fraud.

In the next section we provide the essential requisite information associated with the bootstrap in the context of digital analysis and Benford's Law. In Section III, we present applications of the bootstrap procedure to reported annual earnings of S&P 1500 companies, the allegedly fraudulent data from the FEC, and extremely fraudulent data. In Section IV, we discuss the applications of the procedure and make comments and suggestions for users of the methodology.

II. METHODOLOGY: THE BOOTSTRAP AND BENFORD'S LAW

The Bootrap

The bootstrap is a statistical procedure that has the advantage of being able to make inferences about a fixed parameter, such as a population mean, without having to make assumptions about the shape of the sampling distribution associated with the parameter's estimate. In the context of a population mean (θ), its sampling distribution can be thought of as the distribution of sample means ($\hat{\theta}_i$) from all possible samples of a given sample size. In contrast, traditional statistical tests such as *Z* or *t*-tests assume that the shape of the sampling distribution for the mean has a normal distribution (i.e., a bell-shaped curve).

The bootstrap method consists of randomly sampling *N* observations from a data set of size *N*. The random sampling is conducted with replacement and where each observation is selected with equal probability. For each bootstrap sample, a sample statistic (e.g. $\hat{\theta}_i$) is computed. This process is repeated T times to obtain T sample statistics $\hat{\theta}_i$ where $i = 1, \dots, T$. The sample estimates are subsequently ordered from minimum to maximum i.e., $\min(\hat{\theta}_i) = \hat{\theta}^{(1)}$,…,$\max(\hat{\theta}_i) = \hat{\theta}^{(T)}$ and are used to construct a $(1 - \alpha)100\%$ bootstrap confidence interval

$\left[\hat{\theta}^{(T*\alpha/2)}, \hat{\theta}^{(T*(1-\alpha)/2)}\right]$ where α is the false alarm or Type I error rate (Efron and Tibshirani 1998).

For example, suppose we have a data set that consists of N =5,000 payroll transactions. Our first bootstrap sample is generated by sampling with replacement from the data 5,000 transactions and then the mean ($\hat{\theta}_1$) is computed for this sample. We repeat this procedure until we have generated T =25,000 bootstrap samples and, thus, have computed 25,000 sample means ($\hat{\theta}_{i=1,T}$) based on the original 5,000 payroll transactions. The sample means ($\hat{\theta}_{i=1,T}$) are subsequently sorted from minimum to maximum. If one is willing to tolerate false alarms 1% of the time (i.e., a Type I error rate of 0.01), then one would select the 125th and 24,875th values from the ordered sample means of payroll transactions to be the lower and upper limits of the bootstrap confidence interval (C.I.), $\left[\hat{\theta}^{(125)}, \hat{\theta}^{(24875)}\right]$. In short, a 99% bootstrap C.I. has been constructed for the sample mean of the payroll transactions. See Figure 1 for an illustration of this process.

Given this introduction to the bootstrap, we subsequently discuss how bootstrapping can be used in the context of Benford's Law and for determining whether or not data are fraudulent or contain unintentional errors. For additional details on the bootstrap see Hogg and Tanis (2001), Efron and Tibshirani (1998), and Mooney and Duval (1993).

Benford's Law

Benford (1938) observed numerous cases where the probabilities associated with the first nine digits follow a logarithmic distribution. For example, digit 1 occurs more often than digit 2, in turn, digit 2 occurs more often than digit 3, and successively up to digit 8 which occurs more often than digit 9. See Table A.1 in Appendix A for the empirical probabilities associated with the Benford (1938) digits. Benford (1938) formalized his observations in formulae to determine the exact probabilities (P) for the first digits (d_1), the second digits (d_2), and the joint probabilities for the first two digits (d_1d_2) as: $P_{d_1} = \log_{10}(1 + 1 \,/\, d_1)$ where d_1 =1,…9; $P_{d_2} = \sum_{d_1=1}^{9} \log_{10}(1 + 1/d_1d_2)$ where d_2 =0,…9; and $P_{d_1d_2} = \log_{10}(1 + 1 \,/\, d_1d_2)$ where $d_1d_2 =$ 10,…,99. Tables A.1, A.2, and A.3 give the exact (Benford) probabilities associated with the first, second, and first two digits, respectively. It is noted that although Benford (1938) did not use financial data, research in accounting and auditing has demonstrated that the probabilities of the leading (first, second, or first two) digits of these types of data can also follow Benford's Law (e.g., Caneghem 2004; Durschi et al. 2004; Kinnunnen and Koskela 2003; Nigrini 1996). More specifically, and for the purposes considered in this study, if the leading digits associated with a set of data follow Benford's Law then these digits will have the exact probabilities listed in Tables A.1, A.2, and A.3. And, that these data are presumed not to contain fraudulent transactions or unintentional errors.

As such, if data are assumed to follow Benford's Law, then the probabilities listed in Tables A.1, A.2, and A.3 can be used to determine the population parameters of the mean (μ), variance (σ^2), skew (γ), and kurtosis (δ) for the distributions of the first digits, the second digits, and the first two digits from the data set[1]. More specifically, if we assume that data follow Benford's Law then the following four null hypotheses (H_0^i) can be formulated using Table A.3 for the first two digits (d_1d_2) as H_0^1: μ_{12} =38.5898, H_0^2: σ_{12}^2 =621.8317, H_0^3: γ_{12} =0.771864, and H_0^4: $\delta_{12} = -0.546544$ (See Appendix B for more specific details on computing these parameters). That is, the first two digits (d_1d_2) of a Benford data set must have these parameters for the mean, variance, skew, and kurtosis. These parameters are referred to as "fraud-free" because they are derived under the assumption that data follow Benford's Law. Similarly, using

[1] Suppose we have the following payroll transactions: \$5846, \$2508, \$8046, and \$1174. The first digits related to these transactions are 5, 2, 8, and 1. The second digits are 8, 5, 0, and 1. The first two digits are 58, 25, 80, and 11.

Tables A.1 and A.2, null hypotheses could also be separately formulated for the first digits (d_1) with fraud-free parameters of μ_1 =3.440237, σ_1^2 =6.056513, γ_1 =0.795604, and $\delta_1 = -0.548225$; and for the second digits (d_2) with fraud-free parameters of μ_2 =4.18730, σ_2^2 =8.25381, γ_2 =0.133114, and $\delta_2 = -1.208390$.

Given the specified fraud-free parameters above, the proposed bootstrapping procedure has essentially three basic steps: (1) obtain the leading first two digits from the actual data set under investigation and the exact (or Benford) first two digits which are generated by the user, (2) construct bootstrap C.I.s for the mean, variance, skew, and kurtosis based on the actual and exact digits, and (3) use the decision criteria provided in Figure 2 to infer whether the actual data conform to Benford' Law (see Figure 2).

Specifically, and in terms of step (1), the exact digits are generated based on the specific *N* size of the actual data and the exact probabilities for the first, second, and first two digits listed in Tables A.1, A.2, and A.3. Algorithms and instructions for generating exact digits that have the probabilities listed in these tables are available at the following website: http://www.siuc.edu/~epse1/headrick/Benford/ExactDigits.doc.

In terms of step (2), the bootstrap C.I.s for the exact and actual digits can be easily constructed using Spotfire *S*+ (2008) since there are no programming skills required. The notations in *S+*, under the command *resample*→*bootstrap*, for the sample statistics are: *mean*(*X*), *var*(*X*), *skewness*(*X*), and *kurtosis*(*X*). We recommend that 99% C.I.s be generated for both the exact digits and actual digits using T =25,000 bootstrap samples with the *Bias-Corrected accelerated* (*BCa*) default option. The 99% C.I. is based on a Boneferroni adjustment to the usual false alarm rate (or Type I error rate) of 0.05. Thus, because there are four hypotheses, we have a false alarm rate of $0.05/4 = 0.0125$ and then rounding to the more commonly used error rate of 0.01. We recommend this conservative level of tolerance for false alarms to prevent auditors from making false conclusions when testing all four hypotheses. We note that additional step-by-step instructions are also provided for constructing bootstrap C.I.s using $S+$ at the website indicated above.

The 99% C.I.s generated in step (2) for the exact digits and actual digits are then used in conjunction with the decision criteria in Figure 2 to assess whether the actual data contain any anomalies such as fraud or unintentional errors. As indicated in Figure 2, if the sample size of the actual data is $N \geq 1{,}000$ then the first two digits ($d_1 d_2$) are considered first. Specifically, if all four 99% C.I.s based on the actual digits either (a) contain their corresponding population parameter (μ_{12} =38.5898, σ_{12}^2 =621.8317, γ_{12} =0.771864, $\delta_{12} = -0.546544$) or (b) overlap with their corresponding exact digit C.I., then we would fail to reject (or retain) the four null hypotheses that are associated with the first two digits ($d_1 d_2$). For example, consider the population mean for the first two digits. Suppose that we have a 99% C.I. for the exact digits of [38.122 ≤ μ_{12} ≤39.013] and a 99% C.I. for the actual digits of [38.557≤ μ_{12} ≤39.591]. Inspection of these C.I.s indicate that the latter C.I. contains the population mean (μ_{12}).

Alternatively, assume that the actual digit C.I. is [37.555≤ μ_{12} ≤**38.529***]. This C.I. does not contain the population mean, but overlaps with the exact (Benford) digit C.I. [**38.122*** ≤ μ_{12} ≤39.013] because the lower limit of the exact digit C.I. (38.122) is less than the upper limit of the actual digit C.I. (38.529). Thus, to reiterate, if the C.I.s for the mean, variance, skew, *and* kurtosis either contain their associated parameter or overlap with their corresponding exact digit C.I. then the conclusion would be made that the actual data (e.g., payroll transactions) conform to Benford's Law and, thus, do not have any anomalies (i.e., fraud or unintentional errors). Conversely, if we were to reject all four null hypotheses because the actual digit C.I.s neither contain the parameters nor overlap with the exact digit C.I.s, then we would conclude that the actual data are likely to contain unintentional errors or fraud and that additional investigation is necessary.

For smaller sample sizes of data ($N < 1{,}000$) or for cases where the four hypotheses associated with the first two digits ($d_1 d_2$) are inconsistent i.e. the four hypotheses were not all rejected (retained), separate single digit analyses are appropriate. That is, check whether the first digits and the second digits bootstrap C.I.s contain their corresponding population parameter or overlap with their respective Benford C.I.s. If either of these criteria is met, then infer that the actual data do not contain unintentional errors or fraud. Otherwise, use the bootstrap C.I. for the Pearson correlation between the first digits (d_1) and the second digits (d_2). In a situation where the population correlation ($\rho_{d_1 d_2}$) falls within the bootstrap C.I., then conclude that the actual data (e.g., payroll transactions) do not contain fraudulent transactions. Otherwise, further investigation is necessary. Applications of single digit analysis and Pearson correlation bootstrap C.I.s are discussed in more detail using actual data sets in the next section.

III. APPLICATIONS

Data

The proposed bootstrap procedure was applied to three different kinds of data sets. The first data set is the reported annual earnings of S&P 1500 companies (1990-2008) from Compustat. Specifically, we used positive net income and income before extraordinary items (IBEIs) to assess whether the U.S. publicly trading companies engage in cosmetic earnings management. Our approach consisted of examining leading (first, second or first two) digits on an overall basis rather than on a digit-by-digit basis.

Kinnunnen and Koskela (2003) defined a cosmetic earnings management when upward (downward) rounding of positive (negative) earnings cause leading digit (first or second digit) frequencies to deviate significantly from their expected probabilities according to Benford's Law. For example, Carslaw (1988) found that the frequencies of second digit zeros and nines associated with positive reported earnings of New Zealand companies significantly differ from their expected probabilities. Further, Thomas (1989) indicated that U.S. companies also engaged in cosmetic earnings managements by reporting more second digit zeros and less second digit nines for positive earnings. However, the pattern of the actual probabilities for second digits zeros and nines is reversed for negative earnings.

Subsequent studies provide evidence of European and Japanese listed companies also exhibited cosmetic earnings management behaviors (Kinnunen and Koskela 2003, Caneghem 2004, Skousen et al. 2004, Caneghem 2002, Niskanen and Keloharju 2000). Nevertheless, study results suggest that external auditors (Guan et al. 2006) or the Sarbanes-Oxley Act [SOX] (Aono and Guan 2008) likely deter companies from rounding up second digits of earnings towards the nearest reference points. All of the above studies examined reported earnings on a digit-by-digit basis. However, Cleary and Thibodeau (2005) indicated that conducting nine (or ten) separate tests to examine first (second) digits likely increases false alarms (or Type I error). For example, a digit-by-digit analysis of the first nine digits likely increases false alarms seven times more often than an overall basis analysis of first nine digits for a Type I error rate of 0.05 (Cleary and Thibodeau 2005, 80). In view of this problem, it is important to examine reported annual earnings on an overall basis. The bootstrap procedure can be used to address this issue.

In addition, Aono and Guan (2008) used a conservative approach, i.e., a two-year window period before and after SOX, to control the problem of excessive power. The authors commented that "…larger sample size due to the longer pre-SOX windows would increase the *Z*-statistic used to measure the significance of changes in the observed proportion of digits between the two [pre- and post-SOX] period[s]…" (p. 218). We consider the use of the proposed procedure to be appropriate in the context of larger sample sizes. As a result, we selected S&P 1500 reported earnings for pre- (1990-2002) and post-SOX (2003-2008) periods to examine the effects of SOX on U.S. listed companies rounding-up behaviors.

The second data set relates to the Federal Election Commission (FEC) committee-to-committee in-kind contributions. The FEC in-kind contributions represent a cash value for services donated or bills paid by a third party on behalf of a campaign committee. Unlike regular funds raised for candidates that must meet the maximum level requirements (e.g., individual contributions up to $2,000 per federal candidates), in-kind contributions (i.e., soft money for party committees) are exceptions for this limitation. As a result, the data qualifies for digital analysis based on Benford's Law.

The third data set concerns known fraudulent data (i.e., cash disbursements and payroll information) taken from a 1995 King's County, New York, District Attorney's Office study (Geyer and Williamson 2004, Hill 1998). We replicated first digits of the fraudulent data based on the first digit probabilities listed in Table A. 1 provided by Hill (1998, 363).

Results

Tables 1 through 3 give the 99% bootstrap confidence intervals (C.I.s) associated with the first two digits of Benford data and reported earnings. The S&P 1500 reported earnings relate to the pre- (1990-2002) and post-SOX (2003-2008) periods as well as the combined period (1990-2008). Inspection of these tables indicates that evidence of cosmetic earnings management is unlikely. In fact, we find no evidence of U.S. listed companies to engage in rounding-up behaviors because the bootstrap C.I.s for the first two digits of net income (or income before extraordinary items) either contain population parameters or overlap with the Benford C.I.s in each period (see Tables 1, 2, and 3).

For example, the results in Table 1 show that the lower and upper limits of the 99% bootstrap C.I.s associated with the mean of the first two digits of net income (1990–2002) are 37.743 and 38.658, respectively, and contain the population mean ($\mu_{12} = 38.58976$)—i.e., the fraud free parameter. To indicate this result visually, the lower and upper limits are bold and double asterisks **(****) are included at the end of the upper limit of the actual digits C.I. in each of the Tables. On the other hand, the population parameter for the kurtosis $\delta_{12} = -0.54654$ does not fall within the 99% bootstrap C.I. associated with the first two digits of net income (1990–2002). However, its lower limit (-0.544) overlaps with the upper limit of exact (or Benford) C.I. In this case, the lower limit of the bootstrap C.I. for the actual digits (e.g., net income) and the upper limit of the Benford C.I. are bold and one asterisk (*) at the end of each lower or upper limits is included.

Likewise, the results in Table 1 related to income before extraordinary items (IBEIs) during the pre-SOX period (1990-2002) indicate that the population mean does not fall within the actual digits' C.I. Nevertheless, its upper limit (38.419) overlaps with the lower limit of the Benford C.I. The other bootstrap C.I.s associated with the variance, skew, and kurtosis also display overlapping C.I.s. In addition, we find either overlapping C.I.s or the population parameter falling within the actual digit C.I.s in Tables 2 and 3.

The overall results in Tables 1 through 3 indicate that separate digit analyses are not required. This is because all four first two digits bootstrap C.I.s for reported earnings either contain their corresponding population parameter or overlap with the Benford C.I.s. To demonstrate this point, for example, the results in Tables 4 and 5 show that all four first (or second) digits 99% bootstrap C.I.s for IBEIs either contain their corresponding population parameter or overlap with their respective Benford C.I.s (see Tables 4 and 5).

However, inspection of Table 6 indicates that the results associated with the Nigrini (1996) distortion factor indices (Z-tests) and chi-square goodness-of-fit tests differ from those reported under the bootstrap procedure and suggest evidence of cosmetic earnings management during the pre- and post-SOX periods. These differences indicate that traditional statistical procedures (e.g., Z-tests or chi-square goodness-of-fit tests) may exhibit the problem of excessive power. That is, the results of traditional statistical procedures may provide evidence of cosmetic earnings management as the number of transactions gets larger when, in fact, there is none. To demonstrate this problem, we used the Euclidean distance (ED) on the first digits and bootstrap C.I.s for the Pearson correlation ($\rho_{d_1 d_2}$) between the first digits and second digits.

The application of the Euclidean distance (ED) in the context of Benford' Law consists of finding an index of distance between the actual first digit probabilities (p_i) and the first digit exact (Benford) probabilities (b_i), where i=1,…9. Given these probabilities, the Euclidean distance in the context of Benford's Law is computed as $ED = \sqrt{\sum_{i=1}^{9}(p_i - b_i)^2}$. Intuitively, the larger the distance, the worse the fit of a data set to Benford's Law (i.e., a data set with potential fraud). Despite its simple application in the context of Benford's Law, the Euclidean distance is not an inferential statistic. Thus, it cannot be used to summarize the main characteristic of the actual digits (e.g., amount of variations in payroll transactions) and infer whether the actual data contain unintentional errors or fraudulent transactions for a given level of confidence.

As such, the approach taken here will be that of Cho and Gaines (2007) who compared the Euclidean distance for each one of the FEC data sets (1998-2006) to the cut-off value of

ED^* =0.024. This value of ED^* is computed by using the exact (b_i) and empirical Benford (1938) probabilities (p_i) listed in Table A.1. Cho and Gaines (2007) submitted that this distance measure of ED^* represents "a rough sense for what constitutes a realistic, small value [associated with empirical Benford data]" (p.221). Euclidean distances less than ED^* and closer to zero imply that data more closely conform to Benford's Law than the Euclidean distance greater than ED^* and nearer to one. Thus, the lower and upper limits of the Euclidean distance are zero and one, respectively (i.e., $ED \in [0,1]$), where a value of ED =0 implies an exact correspondence to the first digit probabilities of Benford's Law. The Euclidean distances reported in Table 6 are much smaller than ED^* =0.024, suggesting that the reported earnings data more closely follow Benford's exact probabilities than Benford's (1938) empirical probabilities (see Table 6).

In addition, the Pearson correlation coefficient indicates the strength and direction of the linear relationship between the first digits and second digits. In the context of hypothesis testing, we state that data free of fraud or unintentional errors will have a population correlation coefficient between the first digits (d_1) and the second digits (d_2) of $\rho_{d_1 d_2} = 0.0561$. That is, the null hypothesis associated with the fraud-free parameter is $H_0\colon \rho_{d_1 d_2} = 0.0561$. See at the end of Appendix B for derivation of the population correlation. The results in Table 7 indicate that, for pre- and post-SOX periods, each of the bootstrap C.I.s for the Pearson correlation coefficient contains the population correlation ($\rho_{d_1 d_2}$) and, thus, lends to the bootstrap analyses that the annual reported earnings (net income and IBEIs) conform to Benford's Law. Note that the bootstrap correlation C.I.s were based on 25,000 random samples for a given 99% confidence level that is consistent with the construction of leading digits bootstrap C.I.s (see Table 7).

In summary, both the Euclidean distances and the Pearson correlation bootstrap C.I.s provide evidence that the bootstrap procedure does not exhibit the problem of excessive power.

Therefore, these two indices suggest that the Nigrini (1996) distortion factor index (*Z*-test) and the chi-square goodness-of-fit test have this problem.

In addition to the S&P 1500 reported earnings, we applied the proposed procedure to the Federal Election Commission (FEC) committee to committee in-kind contributions for years 1998, 2000, and 2002. The results in Table 8 indicate that the 1998 FEC data contain allegedly fraudulent transactions. Specifically, all of the four C.I.s associated with the first two digits of the 1998 FEC data do not contain the corresponding population parameter and, thus, suggest that single digit analyses are not necessary. In fact, the results in Tables 9 and 10 provide support for this conclusion (see Tables 8, 9, and 10).

On the other hand, the results in Table 11 show that only one C.I. associated with the first two digits of the 2002 FEC data contain the population parameter (i.e., mean). As a result, single digit analyses are required. The results in Tables 12 and 13 indicate that the decision criteria for separate single digit analyses described in Figure 2 are not met. In other words, all C.I.s associated with the first digits and the second digits of the 2002 FEC data neither contain the corresponding population parameter nor overlap with the exact (Benford) digit C.I.s. Therefore, we conducted the Pearson correlation analysis and concluded that the 2002 FEC data are fraudulent. That is, the 99% bootstrap C.I. for the Pearson correlation between the first digits and the second digits of the 2002 FEC data do not contain the population correlation ($\rho_{d_1 d_2}$). See Table 7 for further details (see Tables 11, 12, and 13).

The bootstrap C.I.s for the Pearson correlation can be used as a decisive factor when the results of the bootstrap procedure do not meet the decision criteria for the digit analyses described in Figure 2. In particular, the results associated with the 2000 FEC data in Tables 14, 15, and 16 indicate that only two out of four C.I.s for the first two digits contain the population parameters. The follow-up separate analyses also show that only two out of four C.I.s for the first digits and one out of four C.I.s for the second digits either contain the corresponding population parameter or overlap with the Benford C.I.s. Up to this point, the evidence is not clear enough to infer whether any potential anomalies (i.e., suspicious transactions from committees to committees) exist in the 2000 FEC data (see Tables 14, 15, and 16).

However, the results in Table 7 show that the Pearson correlation between the first digits and the second digits of the 2000 FEC data is 0.144 which differs significantly from the population correlation ($\rho_{d_1 d_2} = 0.0561$). Altogether the first two digits or single digit analyses for the 1998, 2000, and 2002 FEC data present evidence that support the observations made by Cho and Gaines (2007). That is, allocations of "soft" money from committee to committee in each one of these election cycles (i.e., 1998, 2000, and 2002) were likely manipulated.

Finally, the proposed methodology was applied to simulated first digits of extremely fraudulent data based on the probabilities provided by Hill (1998). We note that Hill's (1998) study is limited to first digits analysis. Despite this limitation, we generated two different sample sizes of data (N=20,229 and N=500) to demonstrate that our proposed procedure accurately detects fraudulent transactions regardless of sample size. The results listed in Table 17 indicate that the bootstrap procedure consistently rejects the four null hypotheses for both large N=20,229 and small N=500 sample sizes and provides the same conclusions as those made by Hill (1998) (see Table 17).

IV. DISCUSSION AND COMMENTS

The bootstrap procedure provides accurate and consistent results over different time periods and across different volumes of transactions as opposed to the Nigrini (1996) distortion factor index (Z-test) or chi-square goodness-of-fit tests. These results show that the bootstrap procedure ameliorates the problem of excessive power described by Nigrini (2000). In addition, our procedure has advantages compared to other approaches such as the mean absolute deviation (MAD), the Euclidean distance, or Bayesian methods that do not exhibit the problem of excessive power. Specifically, the mean absolute deviation and the Euclidean distance do not allow practicing auditors to make probabilistic statements of whether data conform to Benford's Law (Cho and Gaines 2007; Nigrini 2000). Geyer and Williamson (2004) restricted their Bayesian method to first digit analysis. This restriction may prevent auditors from detecting fabricated data where inspection of second, third, or later digits increases the likelihood of discovering suspicious fraudulent entries; and when, in some instances, first digit probability distribution of fabricated data does result in a Benford-like pattern (Diekmann 2007; 328).

It is also noteworthy to point out that the bootstrap procedure allows auditors to examine financial data sets on an overall basis. This approach differs from the statistical procedure used by Aono and Guan (2008) et al. who conducted multiple *Z*-tests on a digit-by-digit basis. To assess data conformity to Benford's Law, we recommend that an overall analysis be used rather than a digit-by-digit analysis because the latter approach inflates false alarms (Type I error) as the number of digits to be examined increases (Cleary and Thibodeau 2005). Further, we suggest that bootstrap confidence intervals (C.I.s) of the Pearson correlation between the first digits and the second digits be used as a decisive factor in situations where the evidence does not clearly indicate whether data follow Benford's Law. Furthermore, we note that the minimum sample size required to conduct the first and second digit analysis is $N=1{,}000$ observations. For samples sizes of less than 1000, it is recommended that only separate single digit analyses be performed. The primary reason is that the proposed bootstrap procedure does not provide stable upper or lower limits for the first two digits exact (Benford) C.I.s for sample sizes less than 1000.

As a final point, we would recommend that practicing auditors be aware of certain conditions where applications of the proposed procedure to large data sets (e.g., transaction-level or large data sets) may not be appropriate. Some of these situations involve assigned numbers, numbers influenced by human thoughts, accounts set up to record firm specific numbers, or numbers with maximum or minimum thresholds that comprise financial data subject to investigations. Also our procedure may not be useful to detect fraud in the case where financial data do not have records of suspicious transactions such as “thefts, kickbacks, bribes or contract rigging” (Durtschi, Hillison and Pacini 2004, p. 24).

REFERENCES

Aono, J.Y., and Guan, L. 2008. The Impact of Sarbanes-Oxley Act on Cosmetic Earnings Management. *Research in Accounting Regulation* 20: 211-221.

ACL for Windows. 2006. Version 9, Workbook, Vancouver: ACL: Services, Ltd.

AICPA. 2008. CAATTs frequently asked questions. *The American Institute of Certified Accountants (AICPA)* Accessed at: http://www.aicpa.org/download/infotech/2008_Top_Tech/CAATTs_FAQ_Document.pdf

Benford, F. 1938. The Law of anomalous numbers. *Proceedings of the American Philosophical Society* 78 (4): 551-572.

Bolton, R.J. and Hand, D.J. 2002. Statistical Fraud Detection: A Review. *Statistical Science* 17(3): 235-249.

Caneghem, T.V. 2004. The impact of audit quality on earnings rounding-up behavior: Some UK evidence. *European Accounting Review* 13 (4): 771-786.

___________ 2002. Earnings management induced by cognitive reference points. *British Accounting Review* 34: 167-178.

Carslaw C.A.P.N. 1988. Anomalies in income numbers: Evidence of goal oriented behavior. *The Accounting Review* LXIII(2): 321-327.

Cho, W. K. and B. J. Gaines. 2007. Breaking the (Benford) Law: Statistical fraud detection in campaign finance. *The American Statistician* 61 (3): 218 - 223.

Cleary, R. and J. C. Thibodeau. 2005. Applying digital analysis using Benford's Law to detect fraud: The dangers of type I error rate. *Auditing: A Journal of Practice & Theory* 24 (1): 77-81.

Coderre .1999. Computer-assisted techniques for fraud detection. *The CPA Journal* 69(8): 57-59.

Diekmann, A. 2007. Not the first digit! Using Benford's Law to detect fraudulent scientific data. *Journal of Applied Statistics* 34(3): 321-329.

Durtschi, C. et al. 2004. The effective use of Benford's Law to assist in detecting fraud in accounting data. *Journal of Forensic Accounting* V: 17-34.

Efron, B. and Tibshirani, R. J. 1998. An introduction to the bootstrap. Monographs on Statistics and Applied Probability 57, Chapman and Hall/CRC, Boca Raton, Florida.

Geyer, C. L., and Williamson, P. P. 2004. Detecting fraud in data sets using Benford's Law. *Communications in Statistics: Simulation and Computation* 33 (1): 229-246.

Guan L. et al. 2006. Auditing, integral approach to quarterly reporting, and cosmetic earnings management. *Managerial Auditing Journal*. 21(6): 569-581.

Hill, T.P. 1998. The first digit phenomenon. *American Scientist* 86: 358-363.

Hogg, R.V., and Tanis, E.A. 2001. *Probability and statistical inference,* 6th Ed. New Jersey, Prentice Hall.

Kendall, M. G. and Stuart, A. 1977. *The advanced theory of statistics*, 4th Ed. New York: MacMillan.

Kinnunnen, J. and Koskela, M. 2003. Who is miss world in cosmetic earnings management? A cross-national comparison of small upward rounding of net income numbers among eighteen countries. *Journal of International Accounting Research* 2: 39-68

Mooney, C. Z. and Duval, R. 1993. *Bootstrapping a nonparametric approach to statistical inference*. Sage Publications, California.

Nigrini, M. J. 2000. *Digital Analysis using Benford's Law: Tests and statistics for auditors*. Global Audit Publications, Canada.

___________ 1996. A taxpayer compliance application of Benford's Law. *The Journal of the American Taxation Association* 18 (Spring): 72-91.

Niskanen, J. and Keloharju, M. 2000. Earnings cosmetics in a tax-driven accounting environment: Evidence from Finnish public firms. *The European Accounting Review* 9(3): 443-452.

Panigrahi, P.K. 2006. Discovering fraud in forensic accounting using data mining techniques. *The Chartered Accountant* (April): 1426-1430.

Skousen C.J. et al. 2004. Anomalies and unusual patterns in reported earnings: Japanese managers round earnings. *Journal of International Financial Management and Accounting* 15(3): 212-234.

Spotfire S+ 8.1 for Windows. 2008. Palo Alto, CA: TIBCO Software.

Thomas, J.K. 1989. Unusual patterns in reported annual earnings. *The Accounting Review* LXIV(4): 773-787.

APPENDIX A

Table A.1. Exact, Benford (1938), and Fraudulent data empirical first digit probabilities.

	First Digits (d_1)								
	1	2	3	4	5	6	7	8	9
Exact Benford	0.301	0.176	0.125	0.097	0.079	0.067	0.058	0.051	0.046
Empirical Benford	0.289	0.195	0.127	0.091	0.075	0.064	0.054	0.055	0.051
Fraudulent Data	0.000	0.019	0.000	0.097	0.612	0.233	0.010	0.029	0.000

Exact = theoretical probabilities are computed as $P_{d_1} = log_{10}(1 + 1 / d_1)$ *where* d_1 *=1,…9.*
Benford = empirical first digit probabilities estimated by Benford (1938) who used a sample of 20,229 observations.
Fraudulent data = fraudulent data relate to cash disbursement and payroll information that was taken from a 1995 King's County, New York, District Attorney's Office study (Hill 1998, p.363).

Table A.2. Exact second digit probabilities.

	Second Digits (d_2)									
	0	1	2	3	4	5	6	7	8	9
Exact Benford	0.120	0.114	0.109	0.104	0.100	0.097	0.093	0.090	0.088	0.085

ExactBenford = theoretical probabilities are computed as $P_{d_2} = \sum_{d_1=1}^{9} log_{10}(1 + 1/d_1 d_2)$ *where* d_2 *=0,…9.*

Table A.3. Exact (Benford) probabilities (*p*) for the joint occurrence of the first two digits ($\boldsymbol{d_1 d_2}$).

$d_1 d_2$	p	$d_1 d_2$	p	$d_1 d_2$	P	$d_1 d_2$	p	$d_1 d_2$	p
10	0.041392	28	0.015240	46	0.009340	64	0.006733	82	0.005264
11	0.037788	29	0.014723	47	0.009143	65	0.006630	83	0.005201
12	0.034762	30	0.014240	48	0.008955	66	0.006531	84	0.005140
13	0.032184	31	0.013788	49	0.008774	67	0.006434	85	0.005080
14	0.029963	32	0.013364	50	0.008600	68	0.006340	86	0.005021
15	0.028029	33	0.012965	51	0.008433	69	0.006249	87	0.004963
16	0.026329	34	0.012589	52	0.008272	70	0.006160	88	0.004907
17	0.024824	35	0.012234	53	0.008118	71	0.006074	89	0.004852
18	0.023481	36	0.011899	54	0.007969	72	0.005990	90	0.004799
19	0.022277	37	0.011582	55	0.007825	73	0.005909	91	0.004746
20	0.021189	38	0.011281	56	0.007687	74	0.005830	92	0.004695
21	0.020203	39	0.010995	57	0.007553	75	0.005752	93	0.004645
22	0.019305	40	0.010724	58	0.007424	76	0.005677	94	0.004596
23	0.018483	41	0.010465	59	0.007299	77	0.005604	95	0.004548
24	0.017729	42	0.010219	60	0.007178	78	0.005532	96	0.004500
25	0.017033	43	0.009984	61	0.007062	79	0.005463	97	0.004454
26	0.016390	44	0.009760	62	0.006949	80	0.005395	98	0.004409
27	0.015794	45	0.009545	63	0.006839	81	0.005329	99	0.004365

Exact Benford = theoretical probabilities are computed as $P_{d_1 d_2} = log_{10}(1 + 1 / d_1 d_2)$ *where* $d_1 d_2 =$ *10,…,99.*

APPENDIX B

The formulae for the population mean (μ), variance (σ^2), skew (γ), and kurtosis (δ) for the probability distributions in Tables A.1, A.2, and A.3 in Appendix A are as follows (Kendall & Stuart, 1977, Eq's 41, 89, 90):

$$\mu = \mu_1 \quad \text{(B.1)}$$

$$\sigma^2 = \mu_2 - \mu_1^2 \quad \text{(B.2)}$$

$$\gamma = (\mu_3 - 3\mu_2\mu_1 + 2\mu_1^3) \ / \ \sigma^3 \quad \text{(B.3)}$$

$$\delta = (\mu_4 - 4\mu_3\mu_1 - 3\mu_2^2 + 12\mu_2\mu_1^2 - 6\mu_1^4) \ / \ \sigma^4 \quad \text{(B.4)}$$

The moments (μ_r) for the probability distributions in Tables A.1, A.2, and A.3 are determined as $\mu_r = \sum_{d_1=1}^{9} d_1^r P_{d_1}$, $\mu_r = \sum_{d_2=0}^{9} d_2^r P_{d_2}$, and $\mu_r = \sum_{d_1 d_2=10}^{99} (d_1 d_2)^r P_{d_1 d_2}$ where r =1,…,4. Substituting the moments into Eq's (B.1), (B.2), (B.3), and (B.4) we obtain the population parameters for the leading digits (the first, second, or first two digits).

For example, the first four moments ($\mu_1, \mu_2, \mu_3,$ and μ_4) would be 3.44024, 17.89174, 115.08205 and 823.27310, respectively. Given these values, the population mean (μ_1), variance (σ_1^2), skew (γ_1) and kurtosis (δ_1) would be $\mu_1 = \sum_{d_1=1}^{9} d_1 P_{d_1} = 3.44024$, $\sigma_1^2 = \mu_2 - (\mu_1^2) = 17.89174 - (3.44024^2) = 6.05651$, $\gamma_1 = (\mu_3 - 3\mu_2\mu_1 + 2\mu^3)/\sigma^3 = \langle 115.08205 - 3(17.89174 \times 3.44014) + 2(3.44024^3)\rangle / 6.05651^{3/2} = 0.79560$ and $\delta_1 = (\mu_4 - 4\mu_3\mu_1 - 3\mu_2^2 + 12\mu_2\mu_1^2 - 6\mu_1^4)/\sigma^4 = \langle 823.2731 - 4(115.08205 \times 3.440234) - 3(17.89174)^2 + 12(17.891743 \times 3.44024^2) - 6(3.44024)^4\rangle / 6.05651^{(4/2)} = -0.54823$, respectively.

Following the same steps as for the first digits, the population mean, variance, skew and kurtosis for the second digits would be $\mu_2 = \sum_{d_2=0}^{9} d_2 P_{d_2} = 4.18739$, $\sigma_2^2 = 8.25381,$, $\gamma_2 =$

$0.133114,$ and $\delta_2 = -1.208390$, respectively. Similarly, the population mean, variance, skew and kurtosis for the first two digits would be $\mu_{12} = \sum_{d_1d_2=10}^{99} (d_1d_2)P_{d_1d_2} = 38.5898,$ $\sigma_{12}^2 = 621.8317,$ $\gamma_{12} = 0.771864,$ $\delta_{12} = -0.546544,$ respectively.

The population correlation between the first digits and the second digits is determined as $\rho_{d_1d_2} = (E[d_1d_2] - \mu_1\mu_2)/(\sigma_1^2\sigma_2^2)^{\frac{1}{2}}$ =0.05605574. The $E[d_1d_2]$ =14.801940 and is based on the probabilities given in Table A.3 and computed as

$E[d_1d_2] = \sum_{d_1=1}^{9}\sum_{d_2=0}^{9}(d_1)(d_2)\log_{10}(1 + 1 / d_1d_2)$.

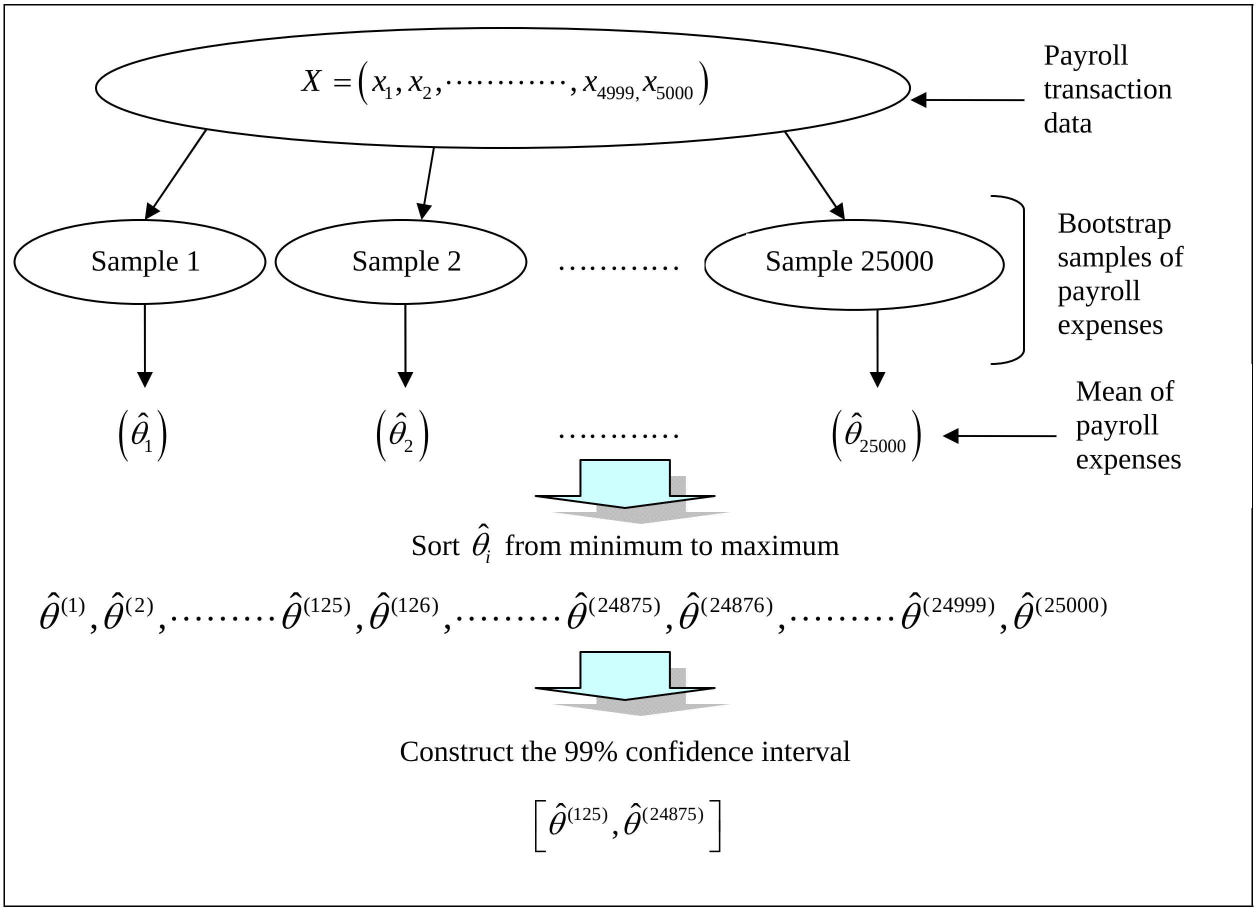

Figure 1. Basic bootstrap process for estimating sample statistics and constructing a confidence interval.

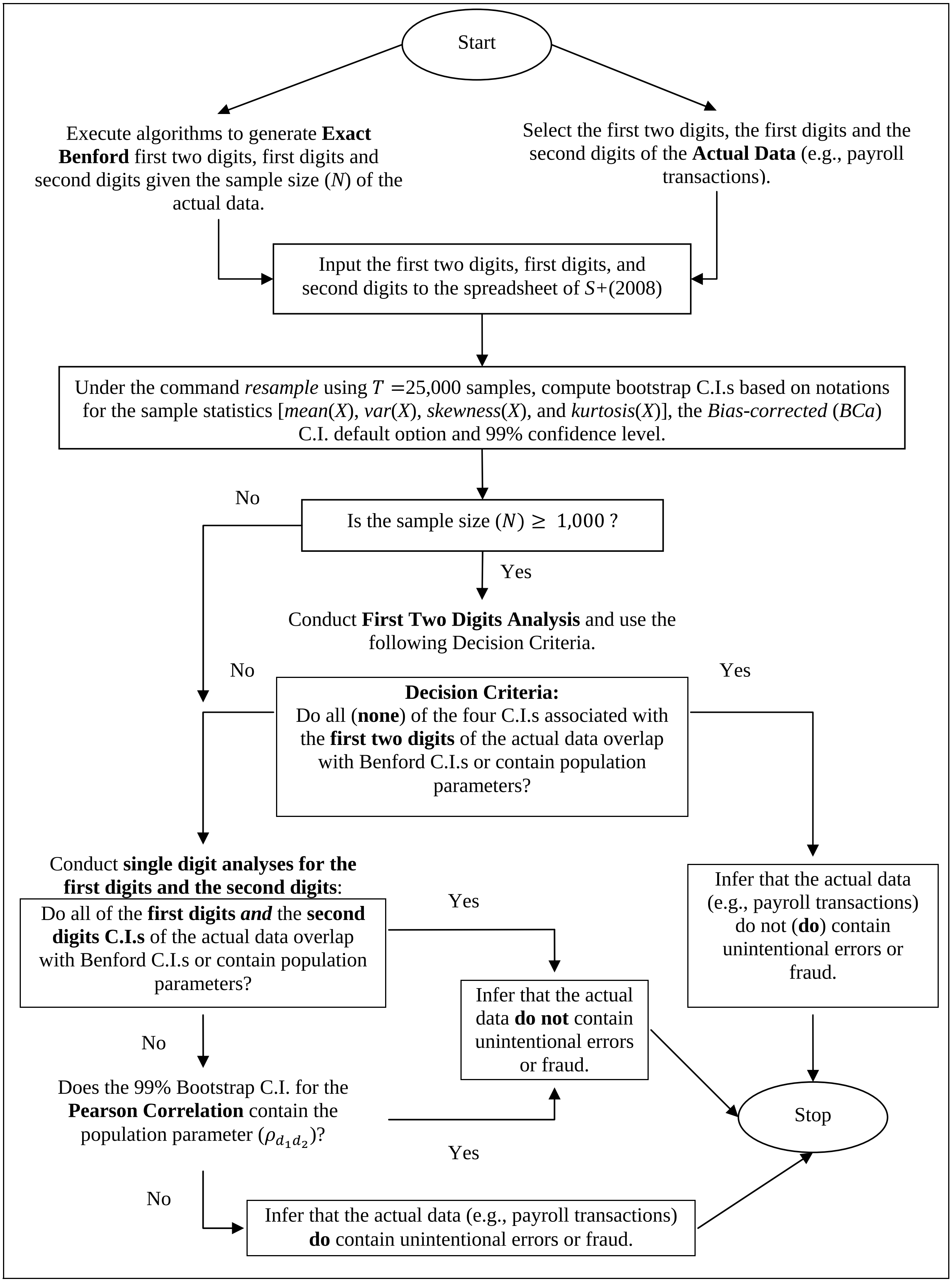

Figure 2. Bootstrap procedure application to digital analysis based on Benford's Law

Table 1. The 99% Bootstrap Confidence Intervals for First Two Digits of S&P 1500 Companies Earnings (1990 - 2002) [a/].

	Population Parameters for the Mean, Variance, Skew, and Kurtosis				
		$\mu_{12} = 38.58976$	$\sigma_{12}^2 = 621.83174$	$\gamma_{12} = 0.77186$	$\delta_{12} = -0.54654$
	Exact Benford	**37.951*** $\le \mu_{12} \le$ 39.170	**608.472*** $\le \sigma_{12}^2 \le$ 636.277	**0.729*** $\le \gamma_{12} \le$ 0.811	-0.603 $\le \delta_{12} \le$ **-0.472***
$N = 19,076$	Net Income (Thousands)	**37.743** $\le \mu_{12} \le$ **38.658****	590.304 $\le \sigma_{12}^2 \le$ **618.271***	**0.765** $\le \gamma_{12} \le$ **0.828****	**-0.544*** $\le \delta_{12} \le$ -0.404
$N = 20,939$	IBEIs*** (Thousands)	37.555 $\le \mu_{12} \le$ **38.419***	588.599 $\le \sigma_{12}^2 \le$ **614.973***	0.774 $\le \gamma_{12} \le$ **0.836***	**-0.526*** $\le \delta_{12} \le$ -0.392

Table 2. The 99% Bootstrap Confidence Intervals for First Two Digits of S&P 1500 Companies Earnings (2003 - 2008) [a/].

	Population Parameters for the Mean, Variance, Skew, and Kurtosis				
		$\mu_{12} = 38.58976$	$\sigma_{12}^2 = 621.83174$	$\gamma_{12} = 0.77186$	$\delta_{12} = -0.54654$
	Exact Benford	**38.114*** $\le \mu_{12} \le$ 39.113	605.754 $\le \sigma_{12}^2 \le$ 636.380	**0.738*** $\le \gamma_{12} \le$ 0.807	**-0.619*** $\le \delta_{12} \le$ -0.474
$N = 16,455$	Net Income (Thousands)	**38.577** $\le \mu_{12} \le$ **39.591****	**610.674** $\le \sigma_{12}^2 \le$ **640.437****	**0.706** $\le \gamma_{12} \le$ **0.773****	**-0.654** $\le \delta_{12} \le$ **-0.517****
$N = 18,391$	IBEIs *** (Thousands)	38.870 $\le \mu_{12} \le$ **39.821***	**611.999** $\le \sigma_{12}^2 \le$ **640.115****	0.697 $\le \gamma_{12} \le$ **0.760***	-0.677 $\le \delta_{12} \le$ **-0.548***

Table 3. The 99% Bootstrap Confidence Intervals for First Two Digits of S&P 1500 Companies Earnings (1900 - 2008) [a/].

	Population Parameters for the Mean, Variance, Skew, and Kurtosis				
		$\mu_{12} = 38.58976$	$\sigma_{12}^2 = 621.83174$	$\gamma_{12} = 0.77186$	$\delta_{12} = -0.54654$
	Exact Benford	38.211 $\le \mu_{12} \le$ 38.907	611.915 $\le \sigma_{12}^2 \le$ 632.071	0.748 $\le \gamma_{12} \le$ 0.795	-0.594 $\le \delta_{12} \le$ -0.496
$N = 35,531$	Net Income (Thousands)	**38.250** $\le \mu_{12} \le$ **38.931****	**604.647** $\le \sigma_{12}^2 \le$ **624.709****	**0.748** $\le \gamma_{12} \le$ **0.794****	**-0.578** $\le \delta_{12} \le$ **-0.478****
$N = 39,330$	IBEIs *** (Thousands)	**38.282** $\le \mu_{12} \le$ **38.930****	**602.968** $\le \sigma_{12}^2 \le$ **622.665****	**0.745** $\le \gamma_{12} \le$ **0.788****	**-0.577** $\le \delta_{12} \le$ **-0.485****

* *Bold numbers mean bootstrap intervals of the actual data overlap with Benford intervals, and thus retain the corresponding null hypothesis.*

** *Bold numbers mean bootstrap intervals of the actual data contain population parameter, and thus retain the corresponding null hypothesis.*

*** *IBEIs=Income before extraordinary items.*

[a/] *Source of data is Compustat.*

Table 4. The 99% Bootstrap Confidence Intervals for First Digits of S&P 1500 Companies Earnings (2003-2008) [a/].

		Population Parameters for the Mean, Variance, Skew, and Kurtosis			
		$\mu_1 = 3.440237$	$\sigma_1^2 = 6.056513$	$\gamma_1 = 0.795604$	$\delta_1 = -0.548225$
	Exact Benford	$3.391 \le \mu_1 \le$ **3.486***	$5.920 \le \sigma_1^2 \le 6.196$	**0.765*** $\le \gamma_1 \le 0.831$	$-0.619 \le \delta_1 \le -0.478$
$N = 18,391$	IBEIs ******* (Thousands)	**3.472*** $\le \mu_1 \le 3.564$	**5.959** $\le \sigma_1^2 \le$ **6.234****	$0.724 \le \gamma_1 \le$ **0.789***	**-0.678** $\le \delta_1 \le$ **-0.547****

Table 5. The 99% Bootstrap Confidence Intervals for Second Digits of S&P 1500 Companies Earnings (2003-2008) [a/].

		Population Parameters for the Mean, Variance, Skew, and Kurtosis			
		$\mu_2 = 4.18739$	$\sigma_2^2 = 8.25381$	$\gamma_2 = 0.133114$	$\delta_2 = -1.208390$
	Exact Benford	$4.131 \le \mu_2 \le 4.240$	$8.124 \le \sigma_2^2 \le 8.403$	$0.108 \le \gamma_2 \le 0.162$	$-1.230 \le \delta_2 \le -1.184$
$N = 18,391$	IBEIs ******* (Thousands)	**4.108** $\le \mu_2 \le$ **4.217****	**8.119** $\le \sigma_2^2 \le$ **8.396****	**0.124** $\le \gamma_2 \le$ **0.179****	**-1.235** $\le \delta_2 \le$ **-1.188****

***** *Bold numbers mean bootstrap intervals of the actual data overlap with Benford intervals, and thus retain the corresponding null hypothesis.*

****** *Bold numbers mean bootstrap intervals of the actual data contain population parameter, and thus retain the corresponding null hypothesis.*

******* *IBEIs=Income before extraordinary items.*

[a/] *Source of data is Compustat.*

Table 6. Nigrini (1996) Distortion Factor Z-Test (Z), Euclidean Distance (ED_{d_1}), and Chi-Square Goodness of Fit Test (χ^2) for Earnings of S&P 1500 Companies.

Period	Reported Earnings	N	Z	ED_{d_1}	χ^2
1990 - 2002	Net Income [a/]	19,076	-2.22949 [b/]	0.008503 [c/]	19.592 [b/] for d_1 16.926 [b/] for d_2
1990 - 2002	Income before Extraordinary Items (IBEIs) [a/]	20,939	3.61030 [b/]	0.007760 [c/]	17.367 [b/] for d_1 27.939 [d/] for d_2
2003 - 2008	Net Income [a/]	16,455	2.41083 [b/]	0.014034 [c/]	19.592 [b/] for d_1 21.958 [d/] for d_2
2003 - 2008	Income before Extraordinary Items (IBEIs) [a/]	18,391	3.92758 [b/]	0.018900 [c/]	38.391 [d/] for d_1 24.001 [d/] for d_2
1990 - 2008	Net Income [a/]	35,531	-0.00261	0.009116 [c/]	23.001 [d/] for d_1 28.213 [d/] for d_2
1990 - 2008	Income before Extraordinary Items (IBEIs) [a/]	39,330	0.04239	0.008579 [c/]	11.056 [b/] for d_1 16.140 [b/] for d_2

[a/] *Source of data is Compustat.*
[b/] *p-value < 0.05*
[c/] *ED < Empirical Benford (1938) Euclidean Distance ED*=0.024 (Cho and Gaines, 2007).*
[d/] *p-value < 0.01*

where

d_1 = first digits and d_2 = second digits
$Z = DF \ / \ SD$ *where* $DF = (AM - EM) \ / \ EM$, *AM is the actual mean of N collapsed numbers scaled to the interval* $[10{,}100)$ *and EM is the expected mean equal to* $EM = 90 \ / \ (N(10^{1/N} - 1))$, *and* $SD = 0.63825342 \ / \ \sqrt{N}$ *(Nigrini, 1996).*
$ED = \sqrt{\sum_{i=1}^{9}(p_i - b_i)^2}$ *where p_i and b_i are the probabilities associated with a data set and exact Benford probabilities, respectively.*
$\chi^2 = \sum_i (O_i - E_i)^2 \ / \ E_i$ *where O_i and E_i are the observed and expected frequencies for the i-th digit.*

Table 7. The 99% Confidence Intervals for the* Pearson *Correlation Coefficients.

Population Correlation ($\rho_{d_1d_2}$= 0.05605574)				
	Sample Size	Time Period	Pearson Correlation Coefficient $\left(\hat{\rho}_{d_1d_2}\right)$	99% Bootstrap Confidence Intervals (C.I.s) for Pearson Correlation Coefficient (ρ)
Net Income [a/]	$N = 19,076$	1990-2002	0.048 [c/]	**0.028018** $\leq \rho_{d_1d_2} \leq$ **0.065154****
	$N = 16,455$	2003-2008	0.057 [c/]	**0.037630** $\leq \rho_{d_1d_2} \leq$ **0.077312****
	$N = 35,531$	1990-2008	0.052 [c/]	**0.038199** $\leq \rho_{d_1d_2} \leq$ **0.065419****
Income before Extraordinary Items (IBEIs) [a/]	$N = 20,939$	1990-2002	0.049 [c/]	**0.032228** $\leq \rho_{d_1d_2} \leq$ **0.067095****
	$N = 18,391$	2003-2008	0.053 [c/]	**0.033172** $\leq \rho_{d_1d_2} \leq$ **0.070844****
	$N = 39,330$	1990-2008	0.051 [c/]	**0.038450** $\leq \rho_{d_1d_2} \leq$ **0.063920****
Committee-to-Committee In-Kind Contribution [b/]	$N = 9,878$	1998	-0.010	-0.037859 $\leq \rho_{d_1d_2} \leq$ 0.014954
	$N = 10,759$	2000	0.144 [c/]	0.118483 $\leq \rho_{d_1d_2} \leq$ 0.167231
	$N = 10,745$	2002	0.178 [c/]	0.152893 $\leq \rho_{d_1d_2} \leq$ 0.201943

a/ *Source of data is Compustat.*

b/ *Data is accessed at the Federal Election Commission website http://www.fec.gov/finance/disclosure/ftpdet.shtml.*

c/ *p-value < 0.01*

****** *Bold numbers mean bootstrap intervals of the actual data contain population parameter, and thus retain the corresponding null hypothesis.*

Table 8. The 99% Bootstrap Confidence Intervals for First Two Digits of All Committee-To-Committee, In-Kind Contributions 1998 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 9,878$	$\mu_{12} = 38.58976$	$\sigma_{12}^2 = 621.83174$	$\gamma_{12} = 0.77186$	$\delta_{12} = -0.54654$
Exact Benford	$37.968 \le \mu_{12} \le 39.255$	$602.032 \le \sigma_{12}^2 \le 641.456$	$0.729 \le \gamma_{12} \le 0.817$	$-0.636 \le \delta_{12} \le -0.452$
Net Income (Thousands)	$35.528 \le \mu_{12} \le 36.686$	$466.633 \le \sigma_{12}^2 \le 503.718$	$0.880 \le \gamma_{12} \le 0.969$	$0.051 \le \delta_{12} \le 0.292$

Table 9. The 99% Bootstrap Confidence Intervals for First Digits of All Committee-To-Committee, In-Kind Contributions 1998 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 9,878$	$\mu_1 = 3.440237$	$\sigma_1^2 = 6.056513$	$\gamma_1 = 0.795604$	$\delta_1 = -0.548225$
Exact Benford	$3.373 \le \mu_1 \le 3.500$	$5.867 \le \sigma_1^2 \le 6.248$	$0.751 \le \gamma_1 \le 0.841$	$-0.636 \le \delta_1 \le -0.446$
Net Income (Thousands)	$3.185 \le \mu_1 \le 3.298$	$4.604 \le \sigma_1^2 \le 4.956$	$0.861 \le \gamma_1 \le 0.948$	$-0.103 \le \delta_1 \le 0.123$

Table 10. The 99% Bootstrap Confidence Intervals for Second Digits of All Committee-To-Committee, In-Kind Contributions 1998 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 9,878$	$\mu_2 = 4.18739$	$\sigma_2^2 = 8.25381$	$\gamma_2 = 0.133114$	$\delta_2 = -1.208390$
Exact Benford	$4.113 \le \mu_2 \le 4.261$	$8.072 \le \sigma_2^2 \le 8.456$	$0.095 \le \gamma_2 \le 0.170$	$-1.237 \le \delta_2 \le -1.174$
Net Income (Thousands)	$3.635 \le \mu_2 \le 3.789$	$8.653 \le \sigma_2^2 \le 9.057$	$0.215 \le \gamma_2 \le 0.292$	**$-1.241 \le \delta_2 \le -1.168$****

Table 11. The 99% Bootstrap Confidence Intervals for First Two Digits of All Committee-To-Committee, In-Kind Contributions 2002 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 10,745$	$\mu_{12} = 38.58976$	$\sigma_{12}^2 = 621.83174$	$\gamma_{12} = 0.77186$	$\delta_{12} = -0.54654$
Exact Benford	$37.943 \le \mu_{12} \le 39.183$	$603.429 \le \sigma_{12}^2 \le 640.802$	$0.733 \le \gamma_{12} \le 0.815$	$-0.634 \le \delta_{12} \le -0.457$
Net Income (Thousands)	**$38.251 \le \mu_{12} \le 39.528$****	$633.833 \le \sigma_{12}^2 \le 680.387$	$0.953 \le \gamma_{12} \le 1.037$	$-0.117 \le \delta_{12} \le 0.127$

***** *Bold numbers mean bootstrap intervals of the actual data overlap with Benford intervals, and thus retain the corresponding null hypothesis.*
****** *Bold numbers mean bootstrap intervals of the actual data contain population parameter, and thus retain the corresponding null hypothesis.*
[b/] *Data is accessed at the Federal Election Commission website http://www.fec.gov/finance/disclosure/ftpdet.shtml.*

Table 12. The 99% Bootstrap Confidence Intervals for First Digits of All Committee-To-Committee, In-Kind Contributions 2002 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 10,745$	$\mu_1 = 3.440237$	$\sigma_1^2 = 6.056513$	$\gamma_1 = 0.795604$	$\delta_1 = -0.548225$
Exact Benford	$3.378 \le \mu_1 \le 3.500$	$5.862 \le \sigma_1^2 \le 6.229$	$0.753 \le \gamma_1 \le 0.840$	$-0.634 \le \delta_1 \le -0.451$
Net Income (Thousands)	$\mathbf{3.401} \le \mu_1 \le \mathbf{3.528**}$	$\mathbf{6.011} \le \sigma_1^2 \le \mathbf{6.422**}$	$0.908 \le \gamma_1 \le 0.994$	$-0.283 \le \delta_1 \le -0.055$

Table 13. The 99% Bootstrap Confidence Intervals for Second Digits of All Committee-To-Committee, In-Kind Contributions 2002 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 10,745$	$\mu_2 = 4.18739$	$\sigma_2^2 = 8.25381$	$\gamma_2 = 0.133114$	$\delta_2 = -1.208390$
Exact Benford	$4.113 \le \mu_2 \le 4.255$	$8.060 \le \sigma_2^2 \le 8.427$	$0.097 \le \gamma_2 \le 0.168$	$-1.241 \le \delta_2 \le -1.180$
Net Income (Thousands)	$\mathbf{4.126} \le \mu_2 \le \mathbf{4.276**}$	$8.908 \le \sigma_2^2 \le 9.284$	$0.017 \le \gamma_2 \le 0.090$	$-1.322 \le \delta_2 \le -1.268$

Table 14. The 99% Bootstrap Confidence Intervals for First Two Digits of All Committee-To-Committee, In-Kind Contributions 2000 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 10,759$	$\mu_{12} = 38.58976$	$\sigma_{12}^2 = 621.83174$	$\gamma_{12} = 0.77186$	$\delta_{12} = -0.54654$
Exact Benford	$37.949 \le \mu_{12} \le 39.194$	$603.981 \le \sigma_{12}^2 \le 640.660$	$0.733 \le \gamma_{12} \le 0.817$	$-0.631 \le \delta_{12} \le -0.451$
Amount	$\mathbf{38.258} \le \mu_{12} \le \mathbf{39.543**}$	$\mathbf{619.184} \le \sigma_{12}^2 \le \mathbf{661.486**}$	$0.866 \le \gamma_{12} \le 0.951$	$-0.324 \le \delta_{12} \le -0.107$

Table 15. The 99% Bootstrap Confidence Intervals for First Digits of All Committee-To-Committee, In-Kind Contributions 2000 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
Exact Benford	$\mu_1 = 3.440237$	$\sigma_1^2 = 6.056513$	$\gamma_1 = 0.795604$	$\delta_1 = -0.548225$
Benford	$3.377 \le \mu_1 \le \mathbf{3.500*}$	$5.880 \le \sigma_1^2 \le 6.243$	$0.751 \le \gamma_1 \le 0.836$	$-0.642 \le \delta_1 \le -0.459$
Amount	$\mathbf{3.441*} \le \mu_1 \le 3.563$	$\mathbf{5.894} \le \sigma_1^2 \le \mathbf{6.283**}$	$0.826 \le \gamma_1 \le 0.909$	$-0.433 \le \delta_1 \le -0.232$

***** *Bold numbers mean bootstrap intervals of the actual data overlap with Benford intervals, and thus retain the corresponding null hypothesis.*

****** *Bold numbers mean bootstrap intervals of the actual data contain population parameter, and thus retain the corresponding null hypothesis.*

[b/] *Data is accessed at the Federal Election Commission website http://www.fec.gov/finance/disclosure/ftpdet.shtml.*

Table 16. The 99% Bootstrap Confidence Intervals for Second Digits of All Committee-To-Committee, In-Kind Contributions 2000 [b/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 10,759$	$\mu_2 = 4.18739$	$\sigma_2^2 = 8.25381$	$\gamma_2 = 0.133114$	$\delta_2 = -1.208390$
Exact Benford	$4.115 \le \mu_2 \le 4.260$	$8.081 \le \sigma_2^2 \le 8.447$	$0.097 \le \gamma_2 \le 0.168$	$-1.238 \le \delta_2 \le -1.177$
Amount	$3.911 \le \mu_2 \le 4.067$	$9.550 \le \sigma_2^2 \le 9.953$	**0.132** $\le \gamma_2 \le$ **0.206****	$-1.343 \le \delta_2 \le -1.284$

Table 17. The 99% Bootstrap Confidence Intervals for First Digits of an Extreme Fraud Data [c/].

Population Parameters for the Mean, Variance, Skew, and Kurtosis				
$N = 20,229$	$\mu_1 = 3.440237$	$\sigma_1^2 = 6.056513$	$\gamma_1 = 0.795604$	$\delta_1 = -0.548225$
Exact Benford	$3.400 \le \mu_1 \le 3.491$	$5.925 \le \sigma_1^2 \le 6.193$	$0.767 \le \gamma_1 \le 0.828$	$-0.612 \le \delta_1 \le -0.477$
Extreme Fraud Data	$5.171 \le \mu_1 \le 5.203$	$0.735 \le \sigma_1^2 \le 0.803$	$-0.0382 \le \gamma_1 \le 0.200$	$3.910 \le \delta_1 \le 4.275$
$N = 500$	$\mu_1 = 3.440237$	$\sigma_1^2 = 6.056513$	$\gamma_1 = 0.795604$	$\delta_1 = -0.548225$
Exact Benford	$3.142 \le \mu_1 \le 3.714$	$5.265 \le \sigma_1^2 \le 6.953$	$0.602 \le \gamma_1 \le 0.996$	$-0.920 \le \delta_1 \le -0.078$
Extreme Fraud Data	$5.084 \le \mu_1 \le 5.286$	$0.570 \le \sigma_1^2 \le 1.004$	$-0.693 \le \gamma_1 \le 0.897$	$3.067 \le \delta_1 \le 5.491$

***** *Bold numbers mean bootstrap intervals of the actual data overlap with Benford intervals, and thus retain the corresponding null hypothesis.*
****** *Bold numbers mean bootstrap intervals of the actual data contain population parameter, and thus retain the corresponding null hypothesis.*
[b/] *Data is accessed at the Federal Election Commission website http://www.fec.gov/finance/disclosure/ftpdet.shtml.*
[c/] *First digits are generated in MINITAB based on the fraud data provided by Hill (1998, p. 363)*

The opinions of the authors are not necessarily those of Louisiana State University, the E.J. Ourso College of business, the LSU Accounting Department, Roosevelt University, the Senior Editor, or the Editor.