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The Limits of Central Bank Forward Guidance under Learning

By

Stephen J. Cole

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Stephen J. Cole[♦]
Department of Economics
Marquette University

Abstract.

Central bank forward guidance emerged as a pertinent tool for monetary policymakers since the Great Recession. Nevertheless, the effects of forward guidance remain unclear. This paper investigates the effectiveness of forward guidance while relaxing two standard macroeconomic assumptions: rational expectations and frictionless financial markets. Agents forecast future macroeconomic variables via either the rational expectations hypothesis or a more plausible theory of expectations formation called adaptive learning. A standard Dynamic Stochastic General Equilibrium (DSGE) model is extended to include the financial accelerator mechanism. The results show that the addition of financial frictions amplifies the differences between rational expectations and adaptive learning to forward guidance. The macroeconomic variables are overall more responsive to forward guidance under rational expectations than under adaptive learning. During a period of economic crisis (e.g. a recession), output under rational expectations displays more favorable responses to forward guidance than under adaptive learning. These differences are exacerbated when compared to a similar analysis without financial frictions. Thus, monetary policymakers should consider the way in which expectations and credit frictions are modeled when examining the effects of forward guidance.

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[♦] Corresponding author
Email: stephen.cole@marquette.edu

1 Introduction

The conventional monetary policy tool of lowering overnight interest rates was exhausted when the zero lower bound (ZLB) on U.S. short-term nominal interest rates was effectively reached during the 2007-2009 global financial crisis. Central banks around the world responded by pursuing "unconventional" monetary policies to stimulate their economies. One of these alternatives tools was forward guidance where the central bank communicates to the public information about the future path of the policy rate. For instance, the Federal Reserve issued forward guidance in the September 2012 Federal Open Market Committee (FOMC) statement: "the Committee also . . . anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." In addition, Eggertsson and Woodford (2003) and Woodford (2012) argue that communication on the future path of interest rates can have stimulative economic effects. Standard New Keynesian models (e.g. Woodford [2003]) predict agents being forward looking when making current period decisions. If households and firms expect higher interest rates in response to future expansions, economic activity today may be limited. However, if a central bank communicates a low policy rate through part of the expansion, output today will not be as constrained.

The effectiveness of forward guidance hinges on two key channels–financial markets and expectations–that are largely overlooked in standard macroeconomic models. The addition of credit frictions in macroeconomic models is not a standard assumption. Frictionless financial markets are largely assumed for simplicity and not to model realistic features of an economy. However, this absence removes the prominent role of credit frictions in the macroeconomy and a key medium through which forward guidance influences the economy. In addition, the way in which private sector expectations about macroeconomic variables (e.g. output and inflation) respond to forward guidance defines a key channel through which this unconventional monetary policy operates. The standard way to model expectations in macroeconomic models is the rational expectations hypothesis. However, this expectations formation scheme makes strong assumptions about the amount of knowledge agents possess when constructing forecasts. Therefore, it is natural to investigate the effectiveness of forward guidance when agents construct forecasts through a more realistic theory of expectations formation.

This paper studies the effectiveness of forward guidance in an environment where credit market frictions persist and rational expectations has been replaced by an adaptive learning rule similar to one proposed by Marcet and Sargent (1989) and Evans and Honkapohja (2001). In particular, the economic environment is based on the Federal Reserve Bank of New York-Dynamic Stochastic General Equilibrium (FRBNY-DSGE) model presented in both Del Negro, Giannoni, and Patterson (2012) and Del Negro et al. (2013). The model adds to a standard DSGE model both financial frictions and central bank communication about the future path of interest rates. A standard monetary policy rule is augmented with anticipated shocks as in Laséen and Svensson (2011). The anticipated shocks represent future changes from a normal interest rate rule that the central bank communicates to agents today. The shocks are also included to model time-contingent forward guidance in which the central bank communicates to the public a forward guidance completion date.

Agents are assumed to form expectations of future macroeconomic variables via two options: the rational expectations hypothesis or a popular alternative called adaptive learning. Rational expectations is a strong assumption. Agents form expectations based on the true model of the economy as they know the model's deep parameters, structure of the model, beliefs of other agents, and distribution of the error terms. A popular alternative to rational expectations is adaptive learning in which agents behave as real-life economists (see, for instance, Evans and Honkapohja [2013]). Adaptive learning agents formulate forecasts of future endogenous variables by creating an econometric model using variables based on the solution found under rational expectations. They estimate the parameters of the model using ordinary least squares and appropriately adjust their forecasts to new data each period.[1]

The inclusion of financial frictions follows Bernanke, Gertler, and Gilchrist (1999) and Christiano, Motto, and Rostagno (2009) and adds a realistic feature. The new components model the borrowing and lending of funds seen in the real economy by adding two types of agents to a standard medium scale DSGE model: banks and entrepreneurs. Banks take in deposits from households and lend to entrepreneurs. The latter type of agents use these funds to purchase capital and rent it to intermediate goods producers. Banks charge entrepreneurs a premium over the riskless interest rate as there is a possibility they default. This "spread" fluctuates based on entrepreneurs' leverage and an idiosyncratic shock that affects the perceived riskiness of entrepreneurs by banks. If riskiness increases, entrepreneurs have a harder time receiving funds, and thus, are constrained in the amount of capital they can funnel to the production side of the economy. The spread or riskiness shock captures how the financial sector contributed to the Great Recession. Del Negro et al. (2013) explain that spread shocks caused about half the decrease in U.S. output during the Great Recession.

The results show that the addition of financial frictions amplifies the differences between

[1]When they construct forecasts each period, adaptive learning agents do not take into account they will be updating their expectations in the future. They believe their forecasts to be optimal in each period. The reader can refer to Kreps (1998) for additional description on anticipated utility.

rational expectations and adaptive learning to forward guidance statements. These outcomes are first shown under impulse responses of the macroeconomic variables to forward guidance shocks. For instance, output's response to forward guidance is stronger under rational expectations than adaptive learning. Output displays a larger reaction to forward guidance under the former than latter expectations formation scheme. The results are also presented during a period of economic crisis (e.g. a recession). The central bank responds to the recession by communicating to the public that the nominal interest rate will equal zero throughout the forward guidance horizon. This exercise shows that the effects of forward guidance are overstated under rational expectations relative to adaptive learning. Specifically, the value of output is higher under rational expectations than adaptive learning throughout the forward guidance horizon. In addition, when the effect of financial frictions in model is reduced, the "wedge" that existed between the responses of rational expectations and adaptive learning to forward guidance diminishes. When compared to an analysis without a financial sector included in the model (e.g. Cole [2015]), the differences between rational expectations and adaptive learning in this present paper are also exacerbated. For instance, in response to forward guidance announcements, the difference in output during an economic crisis is larger between rational expectations and adaptive learning in this current paper than in a similar analysis without financial frictions in Cole (2015).

The reasons for the differences arise from the amount of knowledge that agents are assumed to hold and the additional financial variables to forecast. Under rational expectations, agents base their expectations of future macroeconomic variables on the true model of the economy. Thus, rational expectations agents compute precise expectations about how forward guidance statements affect future macroeconomic variables. However, adaptive learning agents are not endowed with this level of knowledge. Instead, they estimate the effects of forward guidance using their econometric model of the economy. In addition, the inclusion of a financial sector magnifies the differences between rational expectations and adaptive learning agents. There exists more inertia in the adaptive learning forecasting model and more variables to forecast than in a model without financial frictions. These previous reasons and the fact that adaptive learning agents are estimating the effects of forward guidance create bigger differences between the two types of expectations assumptions.

Overall, the results of the paper suggest a main finding for monetary policymakers: the effects of forward guidance depend on the manner in which expectations and financial frictions are modeled. Under the assumption of rational expectations, forward guidance produces more favorable values of output than under adaptive learning. When including credit frictions, these differences are magnified.

1.1 Previous Literature

This paper contributes to the growing literature on forward guidance. The seminal work by Eggertsson and Woodford (2003) explains the importance of the expectations channel on the economy. The path of short-term interest rates affect long-term interest rates and asset prices, and thus, the management of interest rate expectations is pertinent for a central bank. McKay, Nakamura, and Steinsson (2015) explain that the extreme responses of macroeconomic variables to forward guidance found in standard macroeconomic models depend on the assumption of complete markets. The addition of precautionary savings into a macroeconomic model limits the effectiveness of forward guidance at the ZLB. The results from Del Negro et al. (2012) and Carlstrom, Fuerst, and Paustian (2012) display unusually large responses of the macroeconomic variables to forward guidance relative to the data. Del Negro et al. (2012) label this outcome "the forward guidance puzzle." This present paper suggests that the unusually large responses may be due to the rational expectations assumption employed in Del Negro et al. (2012), and a more realistic expectation formation assumption (e.g. adaptive learning) produces results that better match the data.

The current paper follows recent literature examining the effectiveness of forward guidance. De Graeve, Ilbas, and Wouters (2014) study the effects of forward guidance through a different lens than the expectations channel. They find that the effects of this unconventional monetary policy tool vary depending on the type of forward guidance and the underlying reasons for implementing it (e.g. monetary stimulus or sign of future economic crisis). Levin, López-Salido, Nelson, and Yun (2010) explain that the power of forward guidance is sensitive to the type of structural shock affecting the economy. Swanson and Williams (2014) show that Federal Reserve forward guidance statements influence the economy by affecting medium- and longer-term interest rates. Kool and Thornton (2012) test the effectiveness of forward guidance across four countries: New Zealand, Norway, Sweden, and the United States. They find forward guidance helped market participants forecast future short-term yields. In addition, Cole (2015) examines the effects of forward guidance across rational expectations and adaptive learning assumptions, but utilizes a DSGE model without financial frictions. The present paper shows that the differences between rational expectations and adaptive learning to forward guidance are amplified when a DSGE model is expanded to include a financial sector.

This paper fits into the literature on expectation formation and policy. Caputo, Medina, and Soto (2010) use a DSGE model with adaptive learning and a financial accelerator as in Bernanke et al. (1999). They find that the financial accelerator model with adaptive learning leads to large business cycle fluctuations, but a central bank that aggressively responds to inflation can limit the volatility in output and inflation. When the fiscal authority communicates information about the future path of government purchases and taxes, Mitra, Evans, and Honkapohja (2012) show that output multipliers match the data more under adaptive learning than rational expectations. Eusepi and Preston (2010) examine the benefits of central bank communication in which agents have an incomplete model of the economy when constructing expectations. When a central bank provides more information to the public about policy (e.g. the variables within a monetary policy rule), the macroeconomic variables exhibit greater stability as agents are able to construct more precise forecasts. Woodford (2010) studies optimal monetary policy in which the central bank understands agents may not form forecasts via the rational expectations hypothesis. He stresses the importance of policy commitment (e.g. guaranteeing stable inflation) regardless of agents' expectations not being model consistent. Slobodyan and Wouters (2011) examine the medium-scale DSGE model of Smets and Wouters (2007) under the assumption of adaptive learning. The DSGE model's match to the data is on par or exceeds the results under adaptive learning than rational expectations.

The remaining sections are organized as follows. Section two presents the DSGE model with financial frictions. Section three discusses expectations formation under both rational expectations and adaptive learning. Section four contains the results of forward guidance under both types of expectations formations. Impulse response functions, forward guidance during an economic crisis, and the importance of financial frictions for forward guidance are examined. The results are also studied when the degree in which adaptive learning agents discount previous observations is varied. Section five concludes.

2 Model

The aggregate dynamics of the economy are described by a medium scale DSGE model with financial frictions following Del Negro et al. (2012) and Del Negro et al. (2013). It contains a large number of frictions found in standard DSGE models (e.g. Smets and Wouters [2007]). These include price and wage stickiness, price and wage indexation, habit formation in consumption, capital utilization, and investment adjustment costs. The model also includes credit frictions following Bernanke et al. (1999) and Christiano et al. (2009). The remainder of this section presents a brief description of the model followed by the log-linearized equations.

2.1 Description

Households and Labor Packers: Each household $j \in [0, 1]$ maximizes the sum of its expected discounted utility. They receive utility from consumption and disutility from providing work to firms. A household supplies its work to labor packers (e.g. employment agencies). The latter group bundles labor to sell to intermediate goods producers in a perfectly competitive market. In addition, a household can put its wealth in government issued bonds, deposits held at banks, and money. As will be discussed later, the deposits held at banks are important as entrepreneurs use it to purchase capital. The entrepreneurs funnel capital to the production side of the economy.

The frictions in the household sector take the form of habit formation in consumption and wage stickiness. Households have market power in the labor market and choose their nominal wage subject to the amount of work demanded by intermediate goods producers. Following Calvo (1983), a household has probability $(1 - \zeta_w)$ of choosing its wage each period, and a probability ζ_w of not being able to choose its wage. Under the latter scenario, wages are indexed to either previous period's inflation times last period's productivity with probability ι_w, or steady state inflation times the economy's growth rate with probability $(1 - \iota_w)$.

Firms: There exist two types of firms: intermediate and final goods producers. Intermediate goods producers operate in a monopolistically competitive market and use labor and capital to create differentiated products to sell to final goods producers. The source of their labor and capital comes from households (via employment agencies) and entrepreneurs, respectively. The intermediate goods producers are subject to nominal price rigidities in the form of a Calvo (1983) pricing scheme. In each period, firms have a probability $(1 - \zeta_p)$ of freely changing their price. The remaining fraction ζ_p of firms index their price to either previous period's inflation with probability ι_p or the steady state rate of inflation with probability $(1 - \iota_p)$. The final goods producers conduct business in a competitive market and bundle the intermediate goods into one composite good.

Financial Sector and Capital Producers: The modeling of credit frictions starts with two agents: banks and entrepreneurs. Banks pay interest on deposits received from households and use the funds to issue loans to entrepreneurs. The entrepreneurs use the funds to purchase capital from capital producers and rent it to intermediate goods firms. Banks also charge entrepreneurs a premium over the risk-free interest rate as there is a risk of default. This "spread" varies with entrepreneurs' leverage, that is, the ratio of the value of capital to net worth. The spread widens as the value of entrepreneurs' capital, which is positively related to the amount it borrows from banks, increases relative to its own net worth. In every period, an idiosyncratic shock also affects the amount of capital that entrepreneurs manage. An adverse shock shrinks the amount of capital they can lend, and thus, the proceeds they earn from lending to intermediate goods firms. Consequently, this negative shock decreases the ability of entrepreneurs to repay their loans to the bank. In addition, there exist spread shocks which affect the volatility of the idiosyncratic shock. This event can reflect entrepreneurs' perceived riskiness by banks to repay loans. If the riskiness increases, banks will increase the amount it charges entrepreneurs for loans. As the cost of borrowing rises, the ability of entrepreneurs to buy capital to rent to intermediate goods producers diminishes.

Capital producers operate in a perfectly competitive market and are responsible for the creation of the stock of capital. They purchase a part of output from final goods producers and transform it into capital subject to adjustment costs. They also purchase a fraction of capital from entrepreneurs. These two sources of capital comprise the amount of capital for use next period. Capital producers sell capital back to entrepreneurs who then rent it to intermediate goods producers.

Government Policy: The model includes both monetary and fiscal policies. The monetary authorities follow a Taylor-type rule and adjust the short-term nominal interest rate to changes in output, inflation, monetary policy shock, and anticipated or forward guidance shocks. The fiscal authorities collect lump-sum taxes and satisfy a government budget constraint. There also exists a government spending shock which captures exogenous fluctuations in aggregate demand.

2.2 Log-linearized Equations

The following are the set of log-linearized equations that describe the DSGE model with financial frictions. The " ˆ " and " * " symbols represent log deviations from steady state and steady state values, respectively. The $\hat{E}_t$ indicates (potentially) non-rational expectations, while E_t denotes the model-consistent rational expectations operator. From the household's first-order conditions, one can get the consumption Euler equation:

$$\hat{\xi}_t = \hat{R}_t + \hat{E}_t\hat{\xi}_{t+1} - \hat{E}_t\hat{\pi}_{t+1} \tag{1}$$

where $\hat{\xi}_t$ is the marginal utility of consumption, $\hat{R}_t$ is the nominal interest rate paid on government issued bonds and controlled by the central bank, and $\hat{\pi}_t$ is the inflation rate. Consumption is defined according to the following equation:

$$\begin{aligned}(e^{\gamma} - h\beta)(e^{\gamma} - h)\hat{\xi}_t = e^{\gamma}(e^{\gamma} - h)\hat{b}_t - (e^{2\gamma} + \beta h^2)\hat{c}_t + he^{\gamma}\hat{c}_{t-1} \\ - \beta h(e^{\gamma} - h)\hat{E}_t\hat{b}_{t+1} + \beta he^{\gamma}\hat{E}_t\hat{c}_{t+1}\end{aligned} \tag{2}$$

where $\hat{c}_t$ is consumption, $\hat{b}_t$ is a stochastic shock to household utility, β is the discount factor, h represents habit formation in consumption, and e^{γ} is the steady-state (gross) growth rate of the economy. The demand for money by households is given by

$$v_m\hat{m}_t = -\frac{1}{R^* - 1}\hat{R}_t - \hat{\xi}_t \tag{3}$$

where $\hat{m}_t$ is money.

Households have market power in the labor market. Wages are chosen by households according to a Calvo (1983) scheme. In each period, a fraction $1-\zeta_w$ of households can choose their wage. The remaining ζ_w of households index wages to either previous period's inflation times last period's productivity with probability ι_w, or steady state inflation times the economy's growth rate with probability $(1-\iota_w)$. The optimal reset wage equation is given by

$$\begin{aligned}(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{\tilde{w}}_t + (1+\zeta_w\beta\nu_l(\frac{1+\lambda_w}{\lambda_w}))\hat{w}_t &= \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{E}_t(\hat{\tilde{w}}_{t+1}+\hat{w}_{t+1}) \\ &+ (1-\zeta_w\beta)(e^{2\gamma}+h^2\beta)\frac{e^{-\gamma}}{e^{\gamma}-h}\hat{b}_t + \hat{\varphi}_t + (1-\zeta_w\beta)(\nu_l\hat{L}_t-\hat{\xi}_t) \\ &- \zeta_w\beta\iota_w(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{\pi}_t + \zeta_w\beta(1+\nu_l\frac{1+\lambda_w}{\lambda_w})\hat{E}_t\hat{\pi}_{t+1}\end{aligned} \tag{4}$$

$\hat{\tilde{w}}_t$ represents the freely chosen wage by households, $\hat{w}_t$ is the aggregate wage, $\hat{L}_t$ is aggregate labor, and $\hat{\varphi}_t$ is a stochastic shock that affects the marginal utility of labor. λ_w defines the elasticity of substitution between differentiated labor services, and ν_l represents the inverse Frisch elasticity of labor supply. In addition, the aggregate wage equation is given by

$$\hat{w}_t = \hat{w}_{t-1} + \iota_w\hat{\pi}_{t-1} - \hat{\pi}_t + \frac{1-\zeta_w}{\zeta_w}\hat{\tilde{w}}_t \tag{5}$$

The production side of the economy is populated by intermediate and final goods producing firms. The intermediate goods firms operate in a monopolistically competitive market while final goods producers conduct business in a competitive market. Prices do not freely adjust in the former market. Specifically, a fraction $(1-\zeta_p)$ of firms can freely adjust its price every period. The remaining ζ_p of firms either index prices to previous period's inflation with probability ι_p or steady state rate of inflation with probability $(1-\iota_p)$. Consequently, the Phillips Curve is given by

$$\hat{\pi}_t = \frac{\iota_p\zeta_p}{1+\iota_p\beta}\hat{\pi}_{t-1} + \frac{\beta}{1+\iota_p\beta}\hat{E}_t\hat{\pi}_{t+1} + \frac{(1-\zeta_p\beta)(1-\zeta_p)}{(1+\iota_p\beta)\zeta_p}\hat{mc}_t + \frac{1}{(1+\iota_p\beta)\zeta_p}\hat{\lambda}_{f,t} \tag{6}$$

where $\hat{\lambda}_{f,t}$ represents a cost-push shock. $\hat{mc}_t$ is marginal cost and is defined by

$$\hat{mc}_t = (1-\alpha)\hat{w}_t + \alpha\hat{r}^k_t \tag{7}$$

where $\hat{r}^k_t$ is the rental rate of capital and α captures capital's share of output. Intermediate goods firms utilize both labor and capital in a Cobb-Douglas production function given by

$$\hat{y}_t = \frac{\alpha(y^*+\Phi)}{y^*}\hat{k}_t + \frac{(1-\alpha)(y^*+\Phi)}{y^*}\hat{L}_t \tag{8}$$

where $\hat{k}_t$ represents effective capital in the economy and Φ is fixed costs in production.

The model's resource constraint satisfies

$$\hat{y}_t = \hat{g}_t + \frac{c*}{c* + i*}\hat{c}_t + \frac{i*}{c* + i*}\hat{i}_t + \frac{r^{k*}k*}{c* + i*}\hat{u}_t \tag{9}$$

where $\hat{i}_t$ is investment, $\hat{u}_t$ is capital utilization, and $\hat{g}_t$ is a government spending shock capturing exogenous aggregate demand fluctuations in the economy.

The capital-to-labor ratio is given by

$$\hat{k}_t = \hat{w}_t - \hat{r}_t^k + \hat{L}_t \tag{10}$$

The financial side of the economy is populated by banks and entrepreneurs. Entrepreneurs borrow funds from banks to purchase capital from capital producers and rent it to intermediate goods producers. The amount of funds entrepreneurs can borrow is a function of their net worth, which evolves according to the following equation:

$$\begin{aligned} \hat{n}_t = \zeta_{n,\tilde{R}^k}(\hat{\tilde{R}}_t^k - \hat{\pi}_t) - \zeta_{n,R}(\hat{R}_{t-1} - \hat{\pi}_t) + \zeta_{n,qK}(\hat{q}_{t-1}^k + \hat{\bar{k}}_{t-1}) \\ + \zeta_{n,n}\hat{n}_{t-1} + \hat{\gamma}_t - \frac{\zeta_{n,\mu^e}}{\zeta_{sp,\mu^e}}\hat{\mu}_{t-1}^e - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}}\hat{\sigma}_{\omega,t-1} \end{aligned} \tag{11}$$

where $\hat{q}_t^k$ is the price of capital, $\hat{\bar{k}}_t$ measures the amount of installed capital, $\hat{\gamma}_t$ defines the time-varying exogenous fraction of entrepreneurs that survive each period shock, $\hat{\mu}_t^e$ is a bankruptcy cost shock, and $\hat{\sigma}_{\omega,t}$ is a spread shock. $\zeta_{n,\tilde{R}^k}, \zeta_{n,R}, \zeta_{n,qK}, \zeta_{n,n}, \zeta_{n,\mu^e}$, and ζ_{n,σ_ω} are the elasticities of net worth with respect to the return on capital, nominal interest rate, price of capital, net worth itself, bankruptcy cost shock, and the spread shock, respectively. ζ_{sp,σ_ω} represents the elasticity of the spread with respect to the volatility of the spread shock. ζ_{sp,μ_e} is the elasticity of the spread with respect to the bankruptcy cost shock. $\hat{\tilde{R}}_t^k$ is the gross return on capital entrepreneurs receive from renting capital to intermediate goods producers and is defined by

$$\hat{\tilde{R}}_t^k - \hat{\pi}_t = \frac{r_*^k}{r_*^k + (1-\delta)}\hat{r}_t^k + \frac{1-\delta}{r_*^k + (1-\delta)}\hat{q}_t^k - \hat{q}_{t-1}^k \tag{12}$$

where δ is the depreciation rate. The expected excess return on capital or spread is defined by the following equation

$$\hat{E}_t(\hat{\tilde{R}}_{t+1}^k - \hat{R}_t) = \zeta_{sp,b}(\hat{q}_t^k + \hat{\bar{k}}_t - \hat{n}_t) + \hat{\mu}_t^e + \hat{\sigma}_{\omega,t} \tag{13}$$

where $\hat{\sigma}_{\omega,t}$ is defined as a spread shock. It characterizes banks' perception of the riskiness of entrepreneurs. For example, if this shock increases, banks perceive entrepreneurs to be risky and thus, bank loans are harder to receive. This decrease in funds hampers the ability of entrepreneurs to funnel capital to the intermediate goods sector. $\zeta_{sp,b}$ characterizes the elasticity of the spread to entrepreneurs' leverage, which is defined as the ratio of the value of capital to nominal net worth. The amount of installed capital in the model is given by

$$\hat{\bar{k}}_t = (1 - \frac{i*}{k*})\hat{\bar{k}}_{t-1} + \frac{i*}{k*}\hat{\mu}_t + \frac{i*}{k*}\hat{i}_t \tag{14}$$

where $\hat{\mu}_t$ is a shock to the amount of capital. The amount of investment $\hat{i}_t$ is defined by

$$\hat{i}_t = \frac{1}{1+\beta}\hat{i}_{t-1} + \frac{\beta}{1+\beta}\hat{E}_t\hat{i}_{t+1} + \frac{1}{(1+\beta)S''e^{2\gamma}}\hat{q}_t^k + \hat{\mu}_t \tag{15}$$

where $S(\bullet)$ captures the cost of adjusting capital and $S' > 0$ and $S'' > 0$.

The amount of capital is described by the following equation:

$$\hat{k}_t = \hat{u}_t + \hat{\bar{k}}_{t-1} \tag{16}$$

$\hat{u}_t$ defines the capital utilization rate and the corresponding equation is given by

$$r_*^k \hat{r}_t^k = a'' \hat{u}_t \tag{17}$$

where a'' captures capital utilization costs.

Monetary Policy: The model's central bank adjusts the short-term nominal interest rate using the following monetary policy rule:

$$\hat{R}_t = \psi_\pi \hat{\pi}_t + \psi_y \hat{y}_t + \varepsilon_t^{MP} + \sum_{l=1}^{L} \varepsilon_{l,t-l}^R \tag{18}$$

where ε_t^{MP} defines an unanticipated monetary policy shock. Forward guidance is added into the model by augmenting the monetary policy rule with anticipated shocks similar to Del Negro et al. (2012), Laséen and Svensson (2011), and Cole (2015). Each anticipated or forward guidance shock ($\varepsilon_{l,t-l}^R$) is contained in the term $\sum_{l=1}^{L} \varepsilon_{l,t-l}^R$ found in equation (18) and is *i.i.d.*[2] A forward guidance shock defines a central bank announcement in period $t-l$ that the interest rate will change l periods later, that is, in period t. In addition, L represents the length of the central bank's time-contingent forward guidance horizon. Thus, there are L forward guidance shocks in equation (18) that affect the monetary policy rule in period t. Following Del Negro et al. (2012) and Laséen and Svensson (2011), the system is also augmented with L state variables $v_{1,t}, v_{2,t}, ..., v_{L,t}$ whose laws of motion

[2]These shocks are also described as news shocks in Schmitt-Grohé and Uribe (2012). They study the contribution of anticipated shocks to U.S. business cycles.

is given by

$$\begin{aligned} v_{1,t} &= v_{2,t-1} + \varepsilon^R_{1,t} && (19)\\ v_{2,t} &= v_{3,t-1} + \varepsilon^R_{2,t} && (20)\\ v_{3,t} &= v_{4,t-1} + \varepsilon^R_{3,t} && (21)\\ &\vdots \\ v_{L,t} &= \varepsilon^R_{L,t} && (22) \end{aligned}$$

Each term in $v_t = [v_{1,t}, v_{2,t}, ..., v_{L,t}]'$ contains the sum of all forward guidance statements known to agents in period t that change the interest rate $1, 2, ...,$ and L periods later, respectively. Equations $(19) - (22)$ can also be simplified to find that $v_{1,t-1} = \sum_{l=1}^{L} \varepsilon^R_{l,t-l}$, which is the sum of all forward guidance commitments announced by the central bank $1, 2, ...,$ and L periods ago that affect the interest rate in period t. In addition, the main reason to model forward guidance in this manner regards indeterminacy. If forward guidance is alternatively modeled as pegging the interest rate to a certain value, indeterminacy can arise as described in Honkapohja and Mitra (2005) and Woodford (2005). For instance, without a monetary policy that responds to economic activity, real disturbances to the economy can produce multiple equilibria.[3] However, forward guidance modeled by equations $(19) - (22)$ can still attain a constant interest rate path. As will be described in Section 4.3, the forward guidance shocks can be chosen such that the $\hat{R}_t = \bar{R}$ throughout the forward guidance horizon.

The following example is provided to gain intuition on equations (19)-(22). Consider the case in which the central bank's forward guidance horizon is 2 periods ahead, i.e. $L = 2$. The model's system of equations includes $v_{1,t}$ and $v_{2,t}$ whose laws of motion are defined as

$$\begin{aligned} v_{1,t} &= v_{2,t-1} + \varepsilon^R_{1,t} = \varepsilon^R_{2,t-1} + \varepsilon^R_{1,t} && (23)\\ v_{2,t} &= \varepsilon^R_{2,t} && (24) \end{aligned}$$

The variable $v^R_{1,t}$ contains all central bank forward guidance known in period t that affects the interest rate one period later, that is, $\varepsilon^R_{2,t-1}$ and $\varepsilon^R_{1,t}$. Forward guidance known in period t that affects the interest rate two periods later is defined by $v_{2,t}$. This variable consists of $\varepsilon^R_{2,t}$, which is defined as current period forward guidance that affects the interest rate two periods later. Furthermore, the " ˆ " symbol over the variables is removed for the remainder of the paper to simplify notation.

[3] Determinacy can occur from modeling forward guidance as an interest rate peg as described in Carlstrom, Fuerst, and Paustian (2012). However, terminal conditions need to be known, a standard monetary policy rule needs to be followed after the interest rate peg, and exceedingly large responses of output and inflation to forward guidance occur.

2.3 Exogenous Shocks

The model's exogenous shocks consist of a spread shock ($\sigma_{w,t}$), price mark-up shock ($\lambda_{f,t}$), labor shock (φ_t), stochastic preference shock (b_t), government spending shock (g_t), marginal efficiency of investment shock (μ_t), bankruptcy cost shock (μ_t^e), time-varying exogenous survival rate of entrepreneurs shock (γ_t), monetary policy shock (ε_t^{MP}), and forward guidance shocks ($\varepsilon_{1,t}^R, \varepsilon_{2,t}^R, \ldots, \varepsilon_{L,t}^R$). Except for the unanticipated monetary policy and forward guidance shocks, the structural shocks follow an AR(1) process with autocorrelation parameters ($\rho_{\sigma_w}, \rho_{\lambda_f}, \rho_\varphi, \rho_b, \rho_g, \rho_\mu$, ρ_{μ^e}, and ρ_γ).

3 Expectations Formation

This paper assumes agents evaluate the expectations in equations (1) − (17) following either the rational expectations hypothesis or adaptive learning. Rational expectations agents form expectations based on the true model of the economy. They know the structure of the model, parameters of the model (e.g. ζ_p, h, etc.), distribution of the error terms, and beliefs of other agents. Adaptive learning agents do not know the true model of the economy. Instead, they operate as real-life economists (e.g. econometricians) by creating an econometric model of the economy to produce forecasts of future economic variables. They estimate the parameters using standard econometric techniques and revise their forecasts as new data arrives.[4]

Rational Expectations–The model under rational expectations is solved using standard techniques (e.g. Sims [2002]). The model is written in general state-space form:

$$\widetilde{\Gamma}_0 \widetilde{Y}_t = C + \widetilde{\Gamma}_1 \widetilde{Y}_{t-1} + \widetilde{\Gamma}_2 \widetilde{\varepsilon}_t + \widetilde{\Gamma}_3 \zeta_t \tag{25}$$

where

$$\widetilde{Y}_t = [Y_t, \epsilon_t, v_t, \Xi_t]' \tag{26}$$

$$Y_t = [\xi_t, R_t, c_t, \bar{k}_t, i_t, k_t, u_t, r_t^k, q_t, \tilde{R}_t^K, \pi_t, n_t, w_t, \tilde{w}_t, L_t, mc_t, y_t, m_t]' \tag{27}$$

$$\epsilon_t = [\lambda_{f,t}, \mu_t, \varphi_t, g_t, \sigma_{w,t}, b_t, \mu_t^e, \gamma_t]' \tag{28}$$

$$v_t = [v_{1,t}, v_{2,t}, \ldots, v_{L,t}]' \tag{29}$$

$$\Xi_t = [E_t\xi_{t+1}, E_t\pi_{t+1}, E_t c_{t+1}, E_t i_{t+1}, E_t \tilde{R}_{t+1}^k, E_t \tilde{w}_{t+1}, E_t w_{t+1}]' \tag{30}$$

$$\widetilde{\varepsilon}_t = [\varepsilon_t^\lambda, \varepsilon_t^\mu, \varepsilon_t^\varphi, \varepsilon_t^g, \varepsilon_t^{\sigma_w}, \varepsilon_t^b, \varepsilon_t^{\mu^e}, \varepsilon_t^\gamma, \varepsilon_t^{MP}, \varepsilon_{1,t}^R, \varepsilon_{2,t}^R, \ldots, \varepsilon_{L,t}^R]' \tag{31}$$

[4] For a more in-depth discussion, see Marcet and Sargent (1989), Evans and Honkapohja (2001), and Evans, Honkapohja, and Mitra (2009).

C denotes a vector of constants of required dimensions, Y_t defines a vector containing the model's endogenous variables, ϵ_t is a vector of the model's exogenous processes, and Ξ_t denotes the vector of expectations. The *i.i.d.* structural disturbances and forward guidance shocks are contained in the vector $\widetilde{\varepsilon}_t$. The expectational errors (e.g. $\zeta_t^\pi = \pi_t - E_{t-1}\pi_t$) are contained in the vector ζ_t of required dimensions. When using the technique proposed by Sims (2002) and the parameter values in Table 1 in Appendix A, the solution under rational expectations is

$$\widetilde{Y}_t = \widetilde{C} + \xi_1 \widetilde{Y}_{t-1} + \xi_2 \widetilde{\varepsilon}_t \tag{32}$$

where the matrices $\widetilde{C}$, ξ_1, and ξ_2 are nonlinear functions of the model's parameters.[5]

Adaptive Learning–Adaptive learning agents evaluate the expectations in equations (1) – (17) by forming an econometric model and estimating the coefficients. This model is called the "Perceived Law of Motion" (PLM) and contains the variables that appear in the minimum state variable (MSV) solution that exists under rational expectations.[6] The PLM is given by

$$Y_t = a + bY_{t-1} + cv_t + d\tilde{\bar{\epsilon}}_t + ev_{1,t-1} + \varepsilon_t \tag{33}$$

where $\tilde{\bar{\epsilon}}_t = [\epsilon_t, \varepsilon_t^{MP}]'$. Y_t, v_t, and ϵ_t are defined as in the rational expectations model. In addition, the reader should note that v_t and $\widetilde{\bar{\epsilon}}_t$ can be expressed as

$$v_t = \Phi v_{t-1} + \eta_t \tag{34}$$

$$\tilde{\bar{\epsilon}}_t = \widetilde{\phi}\tilde{\bar{\epsilon}}_{t-1} + \bar{\epsilon}_t \tag{35}$$

where Φ is an L x L matrix given by

$$\Phi = \begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & 0 & 1 & \dots & 0 & 0 \\ \vdots & & & & \ddots & \vdots & \\ 0 & 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & 1 \\ 0 & 0 & 0 & 0 & \dots & 0 & 0 \end{bmatrix} \tag{36}$$

[5] Section 4.1 contains a discussion of the model's parameter values.

[6] This paper utilizes a PLM that is based on the unique non-explosive rational expectations equilibrium.

and

$$\eta_t = [\varepsilon^R_{1,t}, \varepsilon^R_{1,t}, \ldots, \varepsilon^R_{L,t}]' \tag{37}$$

$$\widetilde{\phi} = \begin{bmatrix} \rho_{\lambda_f} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \rho_\mu & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \rho_\varphi & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \rho_g & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \rho_{\sigma_w} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \rho_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_{\mu^e} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_\gamma & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{38}$$

$$\bar{\epsilon}_t = [\varepsilon^\lambda_t, \varepsilon^\mu_t, \varepsilon^\varphi_t, \varepsilon^g_t, \varepsilon^w_t, \varepsilon^b_t, \varepsilon^{\mu^e}_t, \varepsilon^\gamma_t, \varepsilon^{MP}_t]' \tag{39}$$

a, b, c, d, and e are unknown coefficient matrices of appropriate dimensions that adaptive learning agents estimate each period. The time subscript is not added to the PLM coefficients to highlight that adaptive learning agents believe their expectations to be optimal every period. They do not take into account they will be updating their beliefs in the future. However, the coefficients in the PLM will evolve each period as will be described later. Furthermore, the addition of $v_{1,t-1}$ in the PLM is necessary. $v_{1,t-1}$ is contained in the rational expectations solution but not found in the vector v_t as seen in equations (34) and (36).

This paper assumes the following time line of events for the learning expectations formation process:

1. At the beginning of period t, adaptive learning agents observe v_t, and $\tilde{\bar{\epsilon}}_t$ and add these variables to their information set.

2. Agents use previous period's estimates (i.e. a_{t-1}, b_{t-1}, c_{t-1}, d_{t-1}, and e_{t-1}) and Y_{t-1}, v_t, $\tilde{\bar{\epsilon}}_t$, and $v_{1,t-1}$ to construct forecasts of future endogenous variables.

3. The values of the endogenous variables contained in Y_t are realized.

4. Adaptive learning agents update their parameter estimates by computing a least squares regression of Y_t on 1, Y_{t-1}, v_t, $\tilde{\bar{\epsilon}}_t$, and $v_{1,t-1}$.

Agents update their parameter estimates of the PLM by following the recursive least squares (RLS) formula

$$\phi_t = \phi_{t-1} + \tau_t R_t^{-1} z_t (Y_t - \phi'_{t-1} z_t)' \tag{40}$$

$$R_t = R_{t-1} + \tau_t (z_t z'_t - R_{t-1}) \tag{41}$$

where $\phi = (a, b, c, d, e)'$ contains the PLM coefficients to be estimated and $z_t \equiv [1, Y_{t-1}, v_t, \tilde{\bar{\epsilon}}_t, v_{1,t-1}]'$ defines the regressors in the PLM. R_t is the precision matrix of the regressors in the PLM. Adaptive learning agents' recent prediction error is given by the last expression in (40). The "gain" parameter τ_t governs the degree in which ϕ_t responds to new information.

This current paper examines the discounted or constant gain learning (CGL) case in which the gain parameter is fixed to a certain value, that is, $\tau_t = \bar{\tau}$. As described in Evans and Honkapohja (2001), the coefficients will converge in distribution to their rational expectations counterparts with a variance proportional to $\tau_t = \bar{\tau}$. Under this scheme, recent observations also play a larger role when adaptive learning agents are updating their coefficients. This assumption allows agents to update their beliefs every period to new information (e.g. forward guidance) as is similar to real-life economists updating their forecasts as new data arrive.[7]

Adaptive learning agents solve for $\hat{E}_t Y_{t+1}$ by using equation (33). Specifically, expectations are given by

$$\begin{aligned} \hat{E}_t Y_{t+1} &= (I_{18} + b_{t-1})a_{t-1} + b_{t-1}^2 Y_{t-1} + (b_{t-1}c_{t-1} + c_{t-1}\Phi)v_t \\ &+ (b_{t-1}d_{t-1} + d_{t-1}\widetilde{\phi})\tilde{\bar{\epsilon}}_t + b_{t-1}e_{t-1}v_{1,t-1} + e_{t-1}v_{1,t} \end{aligned} \tag{42}$$

Equation (42) is substituted into equations (1) – (17) to give the "Actual Law of Motion" (ALM):

$$Y_t = \Gamma_0(\phi_{t-1}) + \Gamma_1(\phi_{t-1})Y_{t-1} + \Gamma_2(\phi_{t-1})v_t + \Gamma_3(\phi_{t-1})v_{t-1} + \Gamma_4(\phi_{t-1})\tilde{\bar{\epsilon}}_t + \Gamma_5(\phi_{t-1})\tilde{\bar{\epsilon}}_{t-1} \tag{43}$$

The previous equation describes the evolution of the model's endogenous variables implied by the PLM in (33).

4 Results

4.1 Parameterization

Table 1 in Appendix A displays the values of the parameters used in simulation. The values largely follow from empirical work by Del Negro et al. (2012) and Del Negro et al. (2013). There exists a high degree of habit formation in consumption with $h = 0.71$. $a'' = 0.2$ indicates a smaller reaction of the rental rate of capital to changes in the capital utilization rate. The value of the price stickiness parameter implies that prices change once a year, which also corresponds to empirical work by Klenow and Malin (2011). The inclusion of a financial sector also adds additional credit

[7] This approach is in contrast to the decreasing gain or RLS case in which $\tau_t = t^{-1}$. As described by Evans and Honkapohja (2001), this latter case implies past observations are equally weighted and the coefficients converge to their rational expectations counterparts with probability one as $t \to \infty$.

market parameters. The survival rate of entrepreneurs is set to 0.99. $\zeta_{sp,b}$ defines the elasticity of the spread $(E_t(\tilde{R}^K_{t+1} - R_t))$ with respect to leverage $(q^k_t + \bar{k}_t - n_t)$ and equals to 0.05. For simplicity, the structural shocks are assumed to be *i.i.d.* The distribution of the noise shocks is not assumed to be highly dispersed. There also is no covariance between the structural shocks.

This current paper uses the CGL model as described in Section 3. The CGL parameter, $\bar{\tau}$, is chosen to be 0.02. This value closely follows Orphanides and Williams (2005), Milani (2007), and Branch and Evans (2006).

The values of the monetary policy parameters in Table 1 closely match the existing literature. Monetary policy positively responds to output and positively adjusts at more than a one-to-one rate to inflation. The value of χ_x closely follows Gilchrist, Ortiz, and Zakrajšek (2009) who estimated a medium-scale DSGE model with financial frictions. The value of the inflation feedback parameter (i.e. χ_π) closely follows empirical adaptive learning work by Milani (2007). In addition, L represents the length of central bank time-contingent forward guidance and is set equal to 12. This number is based off the FOMC September 2012 statement, which was one of its last announcements to exclusively use time-contingent forward guidance language. In this statement, the FOMC said "the Committee also decided today to keep the target range for the federal funds rate at 0 to 1/4 percent and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015." The number of quarters from September 2012 to "mid-2015" is twelve when taking "mid-2015" to be at most the end of the third quarter of 2015.

4.2 Normal Economics Times

I first examine the differences between rational expectations and adaptive learning to forward guidance under the DSGE model with financial frictions. K-period impulse responses of output, investment, and inflation to negative one standard deviation forward guidance shocks under different expectations assumptions are examined in Figures 1 and 2.[8] In addition, adaptive learning impulse response functions cannot be computed using standard linear techniques as equation (43) exhibits a nonlinear structure. Thus, the following approach from Eusepi and Preston (2011) is utilized. The model is simulated twice for $T+1+K$ periods where K is the impulse response horizon and is chosen to be twenty periods.[9] One simulation contains a negative one standard deviation forward guidance shock in period $T+1$. The impulse responses are given by the difference between the two simulations over the final $K = 20$ periods. This process is repeated a large number of times and

[8] The forward guidance shocks are found in equations (19) - (22).

[9] To ensure that the adaptive learning coefficients converge to its stationary distribution, T is chosen to be a large number.

the average is taken to arrive at the reported impulse response function. Furthermore, the solid lines in Figures 1 and 2 represent rational expectations impulse response functions. The dashed lines denote the adaptive learning impulse response functions with 95% confidence bands given by the dotted lines.[10]

Figures 1 and 2 show that the macroeconomic variables overall display a stronger reaction to forward guidance under rational expectations than adaptive learning. Even though forward guidance has stimulative effects on both expectations assumptions, the adaptive learning output path exhibits a smaller reaction to forward guidance shocks than rational expectations. Rational expectations agents' forecasts are based on the true model of the economy. Consequently, rational expectations agents understand the effects that statements about the future interest rate have on future macroeconomic variables. However, adaptive learning agents are unable to base their expectations on the true model of the economy as they are not endowed with that knowledge. Instead, they estimate the effects of forward guidance utilizing an econometric model of the economy. Adaptive learning agents are continually adjusting their forecasts each period causing a smaller reaction to forward guidance. In addition, the inclusion of a financial sector contributes to the differences between adaptive learning and rational expectations. The financial sector produces additional variables to forecast and more inertial behavior (lagged variables) in the PLM relative to a model without financial frictions (e.g. Cole [2015]). Therefore, adaptive learning agents are slower to understand the positive effects of forward guidance on the economy.

Overall, the message from this section is that rational expectations exhibits a stronger reaction to forward guidance and a financial sector compounds the differences between the two types of agents. When the central bank communicates forward guidance to agents, the adaptive learning path of output is different than rational expectations. Rational expectations agents precisely understand the effects forward guidance has on macroeconomic variables as they base their beliefs on the true model of the economy. However, the expectations of adaptive learning agents are slower to adjust to forward guidance statements as they base their forecasts on an estimated model of the economy. The presence of financial frictions also creates a slower response of adaptive learning to forward guidance than rational expectations.

4.3 Economic Crisis

Central bank forward guidance was implemented in response to the 2007-2009 financial crisis. With that event in mind, this section examines the effects of forward guidance during a period of eco-

[10] To make sure that adaptive learning agents' beliefs are not explosive, a projection facility is utilized.

nomic crisis (e.g. a recession) under both rational expectations and adaptive learning assumptions. Specifically, the central bank communicates forward guidance information such that the interest rate $\bar{R} = 0$ throughout the recession and forward guidance horizon. The policy simulation is described next and is motivated by similar exercises in Cole (2015) and Del Negro et al. (2012).

The model is first simulated until period $T+1$. This time frame reflects a period of economic stability (e.g. the period before the Great Recession). In period $T+1$, the economy experiences a recession that lasts six periods.[11] A large negative spread shock impacts the model in period $T+1$ followed by a sequence of five more adverse spread shocks.[12] To counter the adverse effects in the economy, the central bank implements forward guidance. It communicates to the public that the interest rate will equal $\bar{R} = 0$ in period $T+1$ and L periods into the future. This forward guidance announcement corresponds to an unanticipated change in the interest rate in period $T+1$ and anticipated changes in the interest rate in periods $T+2$ through $T+L+1$. Specifically, the central bank chooses the unanticipated monetary policy shock, ε^{MP}_{T+1}, and the anticipated forward guidance shocks $\eta_{T+1} = [\varepsilon^{R}_{1,T+1}, \varepsilon^{R}_{2,T+1}, \ldots, \varepsilon^{R}_{L,T+1}]$ such that the nominal interest rate equals 0 from the time period $T+1$ through $T+L+1$. In addition, the length of the central bank's forward guidance spans a recession and normal times since $L = 12$. If agents expect the interest rate to be lower than usual even during economic expansions, that is, normal times, forward guidance can have additional stimulative effects. The central bank also assumes that agents form their expectations via the rational expectations hypothesis. This expectations formation scheme is the standard assumption in macroeconomic models. The same forward guidance is then given to adaptive learning agents in order to examine the differences between the two types of expectations formation assumptions.

The previously described exercise assumes that the central bank is committed to keeping the interest rate at zero throughout the forward guidance horizon. Rational expectations agents precisely understand how the central bank's forward guidance statements affect the economy. Thus, the interest rate equals $\bar{R} = 0$ throughout the forward guidance horizon. However, adaptive learning agents have an incomplete model of the economy when forming expectations. By giving adaptive learning agents the same forward guidance information that was given to rational expectations agents, the interest rate will not achieve a model implied $\bar{R} = 0$ throughout the forward guidance horizon. To model the central bank promising to keep $\bar{R} = 0$ over the forward guidance horizon and ensure the interest rate is the same value in both rational expectations and adaptive learning, this policy exercise follows Cole (2015) such that the central bank chooses ε^{MP}_{t} each period to

[11]The recession's length is chosen in accordance with the National Bureau of Economic Research's definition of the 2007-2009 Great Recession.

[12]After the recession, the shocks are drawn from a normal distribution.

guarantee that $\bar{R} = 0$.

The spread shock operates through the financial sector to cause a downturn in the economy. A higher spread implies banks perceive entrepreneurs to be riskier, and thus, borrowing costs and cost of capital for firms increase. This result hinders firms from receiving capital from entrepreneurs. Lower economic activity results from less capital being channeled to the production side of the economy. Furthermore, the modeling of a recession via a spread shock closely matches the data. Del Negro et al. (2013) show that spread shocks accounted for about half the decline in output growth during the Great Recession in the U.S.

Figure 3 displays the macroeconomic effects of forward guidance during an economic recession. The difference between rational expectations and adaptive learning of different macroeconomic variables is plotted. A positive value indicates the macroeconomic variable's value is higher under rational expectations than adaptive learning. If the value is negative, the value of the macroeconomic variable is lower under rational expectations than adaptive learning. The figure shows that the stimulative economic effects of forward guidance are overstated under rational expectations than adaptive learning. Specifically, the value of output in the top panel of Figure 3 is higher under rational expectations than adaptive learning across the entire forward guidance horizon.

What accounts for the higher response of output to forward guidance under rational expectations than adaptive learning? The first source comes from the financial sector of the model. In the bottom three panels of Figure 3, differences occur between the responses of rational expectations and adaptive learning to forward guidance. However, the disparity is greater under investment than consumption and inflation indicating that financial elements are driving the disparity in output between rational expectations and adaptive learning. The differences between the amount of the knowledge rational expectations and adaptive learning agents have about the economy also influence the results seen in Figure 3. Since they construct forecasts using the true model of the economy, rational expectations agents precisely understand how central bank forward guidance will stimulate the economy. However, adaptive learning agents do not know the true model of the economy when constructing their expectations. Since they use an econometric model to build their forecasts, adaptive learning agents *estimate* the effects of forward guidance on the economy. Thus, they fail to understand all of the positive benefits of forward guidance.

The results in this section also relate to the "the forward guidance puzzle" found in Del Negro et al. (2012). Their paper showed that central bank forward guidance produced an exceedingly large reaction of the macroeconomic variables in relation to the data. Del Negro et al. (2012) also solved expectations via the rational expectations hypothesis. In addition, the model in this present paper is based on the model in Del Negro et al. (2012), but is solved under both the assumptions of rational expectations and adaptive learning. As shown in the top panel of Figure 3, the value of output exhibits a much larger and more favorable reaction to forward guidance under rational expectations than adaptive learning. Thus, this paper suggests that the extreme responses of the macroeconomic variables to forward guidance found in Del Negro et al. (2012) could be due to the expectations assumption.

Overall, the effect of forward guidance is overstated when agents form beliefs via the rational expectations hypothesis rather than adaptive learning. Since they construct forecasts of future endogenous variables using the true model of the economy, rational expectations agents precisely understand the positive effects of forward guidance. However, adaptive learning agents have partial knowledge about the true model of the economy, and must *estimate* the effects of forward guidance using an econometric model. In addition, financial factors play an important role in explaining the more favorable response of output to forward guidance under rational expectations than adaptive learning. Specifically, the differences in investment between rational expectations and adaptive learning drive the disparity in output. The results of adaptive learning to forward guidance also seem to match the data better than rational expectations.

4.4 Importance of Financial Frictions

While the previous section commented on the importance of credit frictions, this current section investigates in depth how the addition of financial frictions to a standard DSGE model affects the differences between rational expectations and adaptive learning to forward guidance statements. This examination is important for two reasons. Financial frictions play an integral part of an economy. Del Negro et al. (2013) show that spread shocks emanating from the financial sector contributed to about half the decrease in U.S. output during the 2007-2009 financial crisis. In addition, the inclusion of a financial component in modern macroeconomic models is not standard practice. This exclusion may leave out an important channel through which forward guidance operates.

The impulse responses of macroeconomic variables to forward guidance shocks across rational expectations and adaptive learning assumptions are computed under different values of ζ_{spb}. This parameter defines the elasticity of the spread with respect to leverage of entrepreneurs and governs the strength of the financial sector's influence on the economy. When ζ_{spb} decreases, the influence of entrepreneurs' leverage (i.e. the ratio of the value of capital to net worth) on the economy diminishes, that is, the effects of financial conditions on the economy decrease. Therefore, the results of Section 4.2 are examined under the baseline case of $\zeta_{spb} = 0.05$ as well as $\zeta_{spb} = 0.001$ to show the contribution of the financial sector to the differences between rational expectations and adaptive learning to forward guidance.

Figures 4 and 5 show that the addition of a financial sector into a standard New Keynesian model amplifies the disparity between rational expectations and adaptive learning to forward guidance. The top rows in the figures display the difference in output between the two expectations formation schemes to forward guidance shocks under different values of ζ_{spb}. When the effect of financial conditions on the economy diminishes, that is, $\zeta_{spb} = 0.001$, the differences between rational expectations and adaptive learning to forward guidance reduce. However, when financial factors are allowed to exist, that is, $\zeta_{spb} = 0.05$, the disparity between the two increases. As financial conditions play a bigger role in the economy, a bigger "wedge" exists between the output responses of rational expectations and adaptive learning to forward guidance. Thus, the removal of financial frictions from standard DSGE models leaves out an important channel through which forward guidance operates.

The impulse responses of investment in the bottom rows of Figures 4 and 5 also show how the addition of a financial sector can exacerbate the differences between rational expectations and adaptive learning to forward guidance statements. The same type of large wedge that exists between rational expectations and adaptive learning under output is apparent under investment. When credit frictions play a larger role on the economy (e.g. $\zeta_{spb} = 0.05$), adaptive learning agents' forecasting model is more influenced by the financial sector. Thus, bigger differences between rational expectations and adaptive learning exist.

This exacerbated difference can also be seen when examining the results of this paper in comparison to a DSGE model in which the financial sector is removed. In Cole (2015), the model of the economy was based on a smaller scale DSGE model *without* financial frictions. During an economic crisis and in response to forward guidance, output was lower under adaptive learning than rational expectations. With the inclusion of a financial sector in the present paper, the differences in output between rational expectations and adaptive learning during the economic crisis exercise in Section 4.3 are larger than in Cole (2015). The addition of the financial sector includes more inertia in the PLM and more variables to forecast (e.g. $E_t\tilde{R}^k_{t+1}$), and thus, more overall differences between rational expectations and adaptive learning. The adaptive learning agents must estimate how forward guidance will alleviate the recession by forecasting not only future variables concerning households and firms, but also the financial sector. This creates more errors by the adaptive learning agents relative to their rational expectations counterparts causing the effects of forward guidance to be greatly overstated under rational expectations relative to adaptive learning.

Overall, the findings in this section suggest a key takeaway for policymakers. If monetary policymakers want to understand the effects of forward guidance and utilize macroeconomic models with the standard assumptions of rational expectations and frictionless financial markets, the results may be potentially misleading. This section shows that the addition of financial frictions into a standard macroeconomic model exacerbates the differences between the responses of rational expectations and adaptive learning to forward guidance.

4.5 Alternative Constant Gains

This section examines the importance of financial frictions for forward guidance effectiveness when the degree in which adaptive learning agents discount previous observations is changed. Specifically, the financial friction "wedge" that exists between the responses of rational expectations and adaptive learning to forward guidance is examined to see how sensitive it is to different values of $\bar{\tau}$. The exercise in Section 4.2 is rerun under the baseline value of $\zeta_{spb} = 0.05$. Lower and higher values of $\bar{\tau}$ are also chosen. To capture adaptive learning agents placing less weight on new information, the results are examined under $\bar{\tau} = 0.001$ represented by the dotted line in Figure 6. To capture adaptive learning agents placing more weight on new information, the results are examined under $\bar{\tau} = 0.03$ represented by the dashed line. The benchmark rational expectations impulse response functions are also displayed and denoted by the solid line in Figure 6.

Figure 6 shows that the degree in which financial frictions amplify the differences between rational expectations and adaptive learning depends on the value of $\bar{\tau}$. When adaptive learning agents place more weight on previous observations, that is, as $\bar{\tau}$ increases, financial conditions in the economy have a bigger impact on their forecasts. Thus, output does not exhibit as strong of a response to forward guidance as under a lower value of $\bar{\tau}$. Figure 6 shows a larger wedge between rational expectations and adaptive learning to forward guidance under $\bar{\tau} = 0.03$ than $\bar{\tau} = 0.001$ from the time of the announcement of the forward guidance shock to its realization. As adaptive learning agents weight previous observations less, that is, as $\bar{\tau}$ decreases, their beliefs and forecasts should not vary as much from the previous period. Consequently, current financial conditions in the economy do not play as big of a role in their forecasts. Thus, the financial friction wedge between rational expectations and adaptive learning diminishes. Figure 6 shows the smaller difference between rational expectations and CGL with $\bar{\tau} = 0.001$ from the time of the announcement of the forward guidance shock to its realization.

5 Conclusion

The 2007-2009 global financial crisis caused central banks around the world to implement the unconventional monetary policy of forward guidance to stimulate their economies. The effectiveness of forward guidance hinges on two key channels–expectations and financial markets–that are largely overlooked in standard macroeconomic models. The standard expectations formation assumption is the rational expectations hypothesis, while frictionless financial markets are largely assumed for convenience. Thus, it is of interest to investigate the effectiveness of forward guidance when the rational expectations assumption has been relaxed and credit frictions are included.

This paper utilizes a medium scale DSGE model with financial frictions to compare the effects of forward guidance under both rational expectations and adaptive learning. The results show that the addition of financial markets into a DSGE model amplifies the differences between rational expectations and adaptive learning to forward guidance statements. Adaptive learning agents do not respond as strongly to a forward guidance shock relative to their rational expectations counterparts. During a period of economic crisis (e.g. a recession), output under rational expectations also displays more favorable responses to forward guidance than under adaptive learning. Rational expectations agents form their forecasts based on the true model of the economy, and thus, can understand how forward guidance will precisely help the economy. However, adaptive learning agents must estimate the effects of forward guidance on the economy as their forecasts are based on an econometric model of the economy. In addition, the differences between the responses of rational expectations and adaptive learning to forward guidance reduce as the effect of financial frictions in the model diminishes. These differences between the two expectations formations are also magnified when compared to an analysis without financial frictions (e.g. Cole [2015]). The additional inertia in the PLM, more financial sector variables to forecast, and the fact that adaptive learning agents estimate the effects of forward guidance create bigger differences between the two types of expectation assumptions. Furthermore, these results are especially important to policymakers. If they want to understand the effects of forward guidance on the economy, monetary policymakers should consider the way in which expectations and financial frictions are modeled.

There are other modifications to the model presented in this paper that are worth noting. For example, the credibility of central bank forward guidance announcements could be examined as in Dong (2014). In the model presented above, agents believe the forward guidance statements, and the central bank implements its forward guidance promises. However, the results could be examined when agents do not completely believe the central bank will follow through with its forward guidance statements. The type of forward guidance could also be changed. This current paper examines time-contingent forward guidance in which the central bank communicates the end date of forward guidance. Forward guidance could be state-contingent in which the completion date of central bank forward guidance is linked to economic conditions (e.g. unemployment rate and output). The RLS formula could also be modified to allow agents to better track structural changes in the economy as described in Marcet and Nicolini (2003) and Milani (2014). Specifically, the gain parameter would be a constant if the recent prediction errors were large and decreasing if the recent prediction errors were small. Overall, the roles of expectations and financial frictions are important to understand when examining the effects of forward guidance on the economy.

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Appendix

A Parameter Values

Table 1: Parameter Values

	Description	Value
α	Capital's Share of Output	0.33
ζ_p	Price Stickiness	0.75
ι_p	Price Indexation	0.54
δ	Depreciation	0.025
Φ	Share of Fixed Costs	0.3
S''	Investment Adjustment Cost	4
h	Habit Formation	0.71
a''	Capital Utilization Cost	0.2
ν_l	Elasticity of Labor Supply	2
ν_m	Money Demand	2
β	Discount Factor	0.99
ζ_w	Wage Stickiness	0.75
ι_w	Wage Indexation	0.5
λ_w	Elast. of Sub. Diff. Labor Services	0.3
ψ_π	Feedback Inflation	1.40
ψ_y	Feedback Output	0.10
ζ_{spb}	Elast. of Spread w.r.t. Leverage	0.05
γ	Steady-State Growth Rate of Economy	2.75
λ_f	Steady-State Price Mark-Up	0.15
g_*	Steady-State Government	0.3
$F(\bar{\omega})$	Steady-State Default Rate	0.03
L	FG Horizon	12
$\bar{\tau}$	CGL	0.02

Note: The standard deviations of the structural shocks are set to 0.0001. FG stands for forward guidance. The autoregressive parameters for the structural shocks are set to equal 0.

B Figures

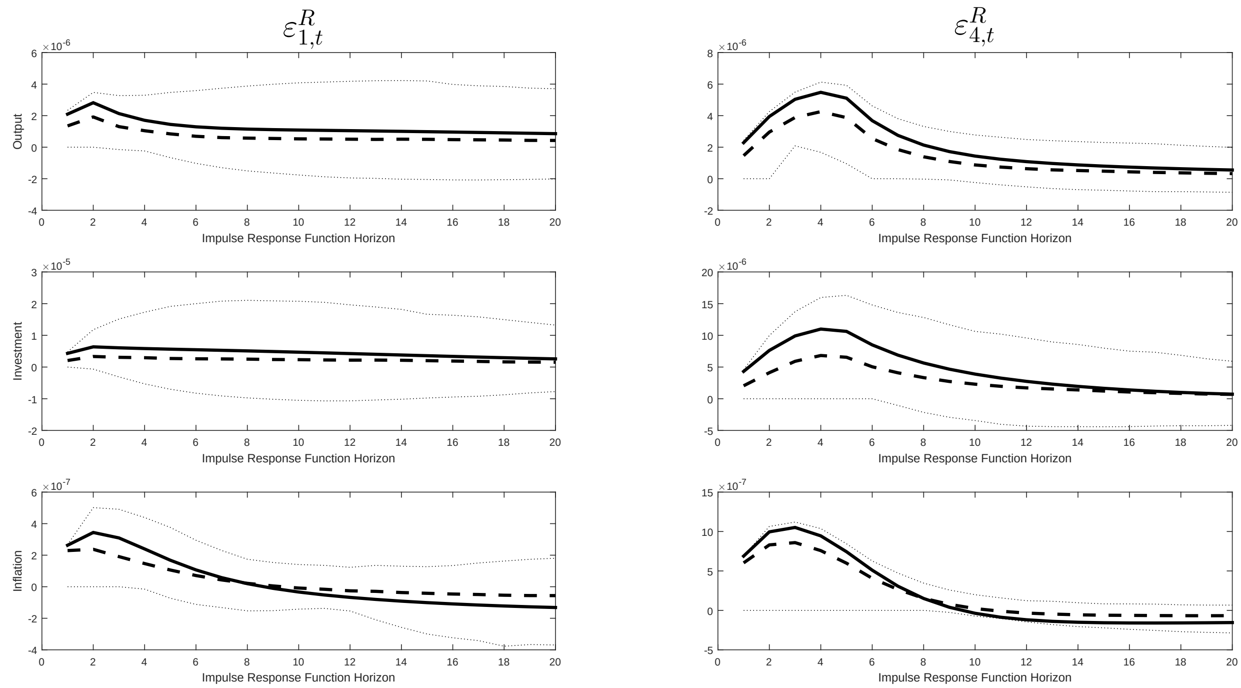

Figure 1: Impulse Responses of Macroeconomic Variables to Forward Guidance Shocks. Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Lines: 95% Confidence Bands.

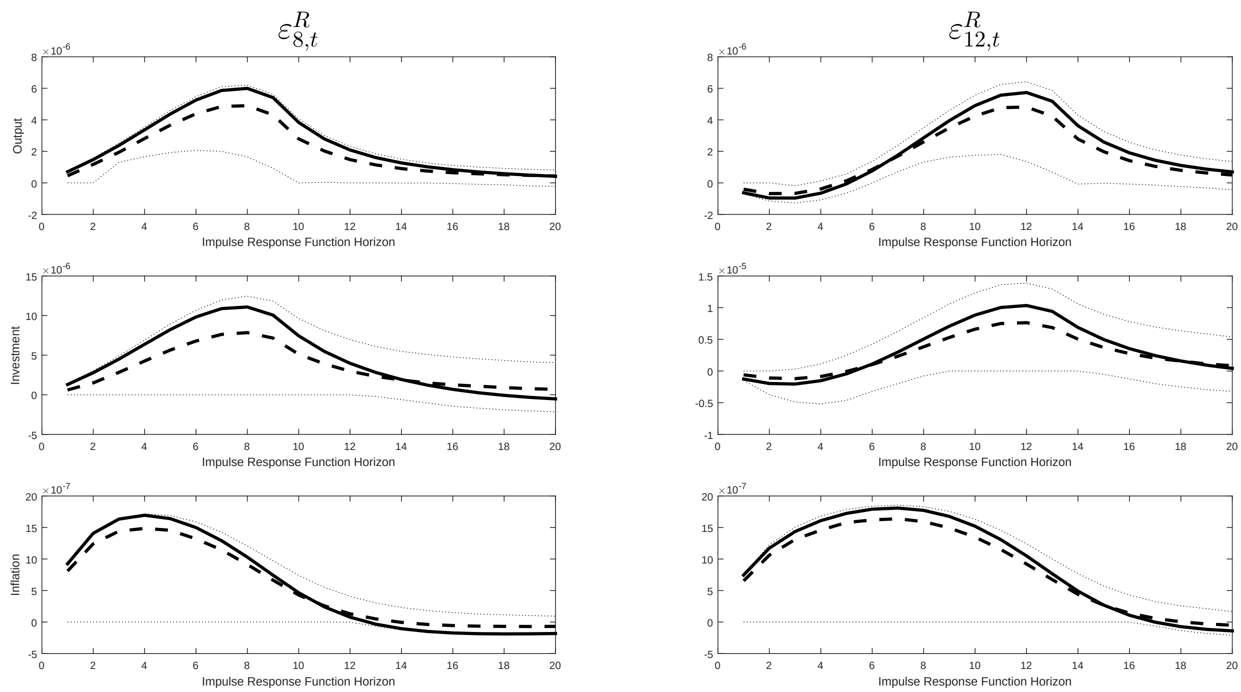

Figure 2: Impulse Responses of Macroeconomic Variables to Forward Guidance Shocks. Solid Line: Rational Expectations; Dashed Line: CGL; Dotted Lines: 95% Confidence Bands.

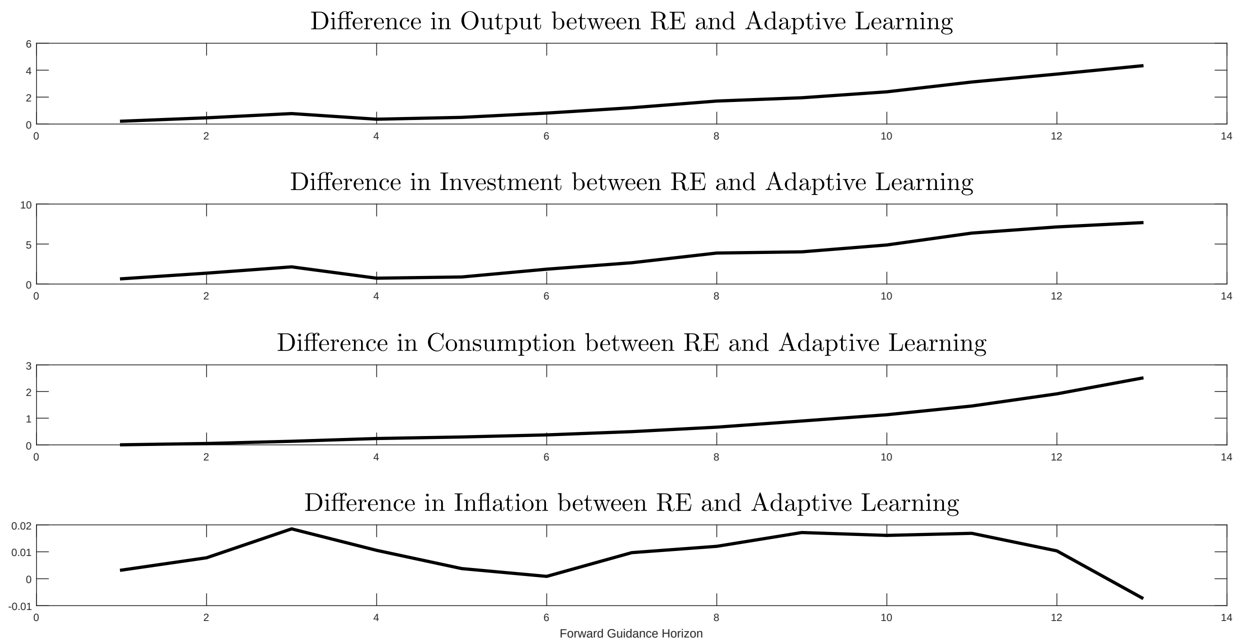

Figure 3: Macroeconomic Effects of Forward Guidance during an Economic Crisis

Note: The graphs show the difference in the macroeconomic variables between rational expectations and adaptive learning agents. A positive value indicates the value under rational expectations is higher than under adaptive learning. A negative value indicates the variable's value under rational expectations is lower than under adaptive learning.

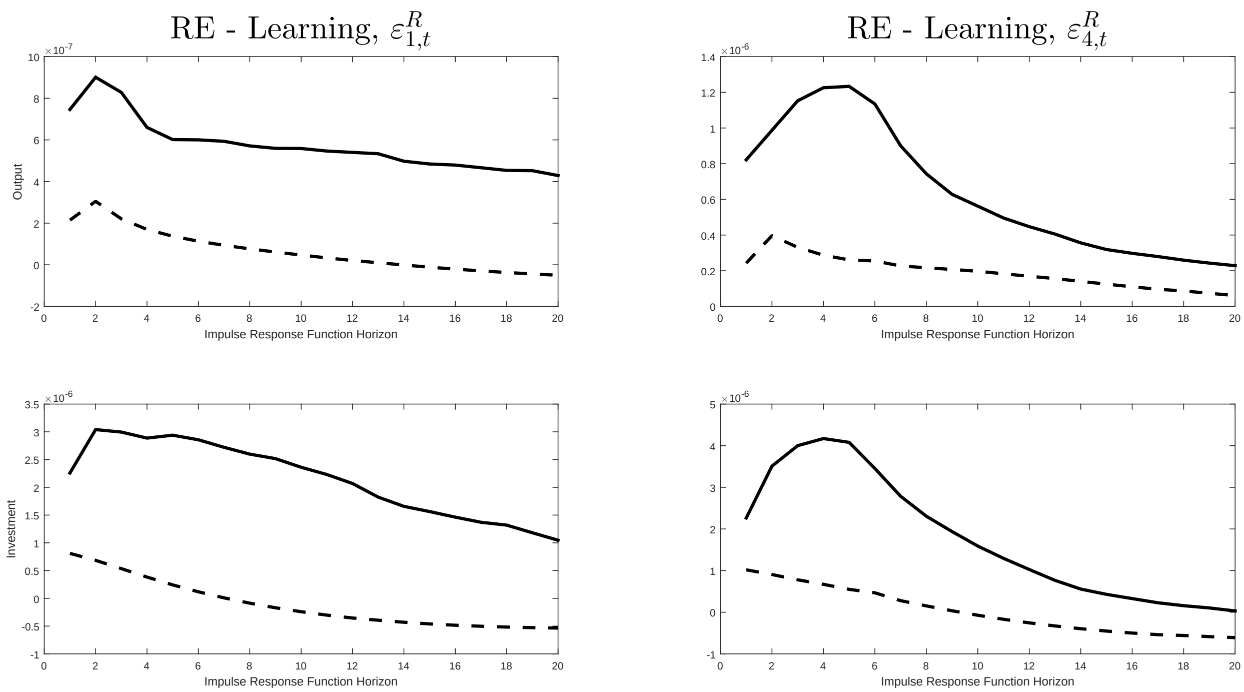

Figure 4: Difference between Impulse Response Functions of Rational Expectations and Adaptive Learning to Forward Guidance Shocks under Different Values of ζ_{spb}.

Note: Solid Line: $\zeta_{spb} = 0.05$. Dashed Line: $\zeta_{spb} = 0.001$.

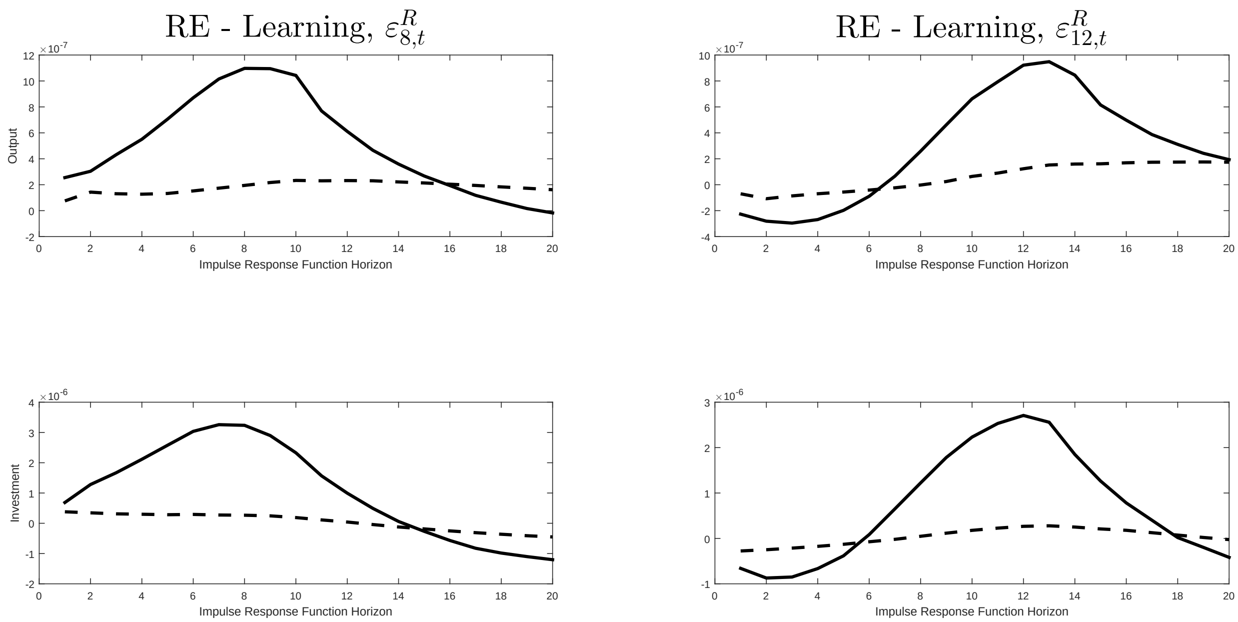

Figure 5: Difference between Impulse Response Functions of Rational Expectations and Adaptive Learning to Forward Guidance Shocks under Different Values of ζ_{spb}.

Note: Solid Line: $\zeta_{spb} = 0.05$. Dashed Line: $\zeta_{spb} = 0.001$.

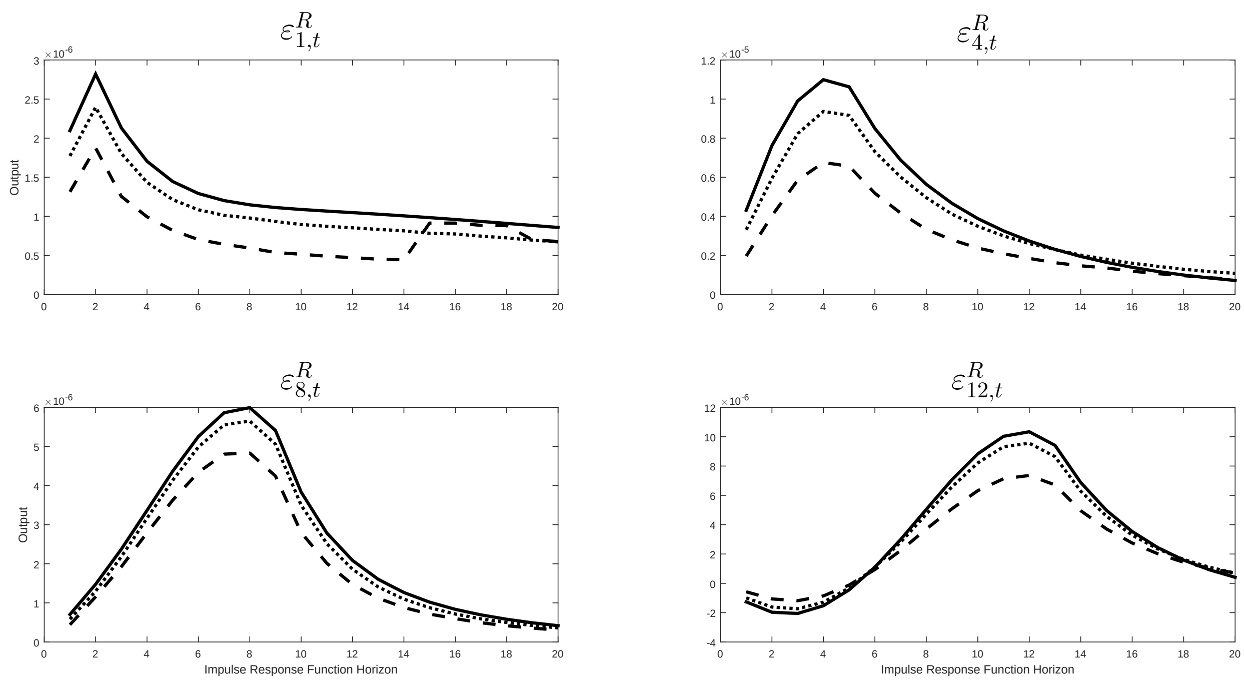

Figure 6: Impulse Response Functions to Forward Guidance Shocks under Different Values of $\bar{\tau}$. Solid Line: Rational Expectations. Dotted Line: CGL with $\bar{\tau} = 0.001$. Dashed Line: CGL with $\bar{\tau} = 0.03$.