Corrigendum to "Taxonomies of Model-theoretically Defined Topological Properties"

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Abstract. An error has been found in the cited paper; namely, Theorem 3.1 is false.

1. I would like to correct a simple, but serious, error in [1]; namely Theorem 3.1 therein is quite false: It can happen that there are compact Hausdorff spaces $X$ and $Y$ with $X \equiv Y$ (indeed $X \equiv Y$) but $X \not\equiv_{T} Y$. I am most grateful to Lutz Heindorf for communicating [3] the following straightforward example: Let $X$ and $Y$ be any two Boolean spaces with infinite dense sets of isolated points. Then $B(X)$ and $B(Y)$ are Wallman bases for $X$ and $Y$ respectively, are infinite atomic Boolean algebras, and hence, by the Tarski invariants theorem, are elementarily equivalent. Thus $X \equiv Y$. However, one can easily pick $X$ and $Y$ as above so that $X \not\equiv_{T} Y$; e.g., let $X$ and $Y$ be the ordinal spaces $\omega + 1$ and $\omega^2 + 1$ respectively. Then $Y$ has a point of Cantor–Bendixson derivative 2, while $X$ does not. This fact can be expressed in a sentence of $\Phi$. The faulty inference in the proof of Theorem 3.1 of [1] occurs in the penultimate sentence: If $W$ and $Z$ are two Tichonov spaces with Wallman bases that are lattice-isomorphic, it does not generally follow that $W$ and $Z$ are homeomorphic. (We could make the inference if either $W$ and $Z$ were both compact or the Wallman bases contained all the singletons, but in our case $W$ and $Z$ are topological ultrapowers and neither condition holds.)

2. In Professor Heindorf’s communication [3], there were some further interesting facts that enrich the content of [1].

2.1. The 3-cell $\mathcal{S}^3$ is characterized by $T_F$ in \{metrizable\} [2]. (This augments Theorem 1.2 in [1].)

2.2. There is a complete description of the spaces that are (finitely) characterized by certain taxonomies in \{metrizable Boolean\}. Let $\mathcal{R}$ be the class of R. S. Pierce’s “compact 0-dimensional metric spaces of finite type” [7].

Theorem [5]. For any metrizable Boolean space $X$, the following are equivalent:
(i) $X \in \mathcal{R}$.

Received September 17, 1990.

1980 Mathematics Subject Classifications. (1985 Revision) Primary: 03C15, 03C20, 06D99, 54D30, 54F15, 54F25

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(ii) \( X \) is finitely characterized by \( T_F \) in \{metrizable Boolean\}.

(iii) \( X \) is finitely characterized by \( T_r \) in \{metrizable Boolean\}.

(This result addresses issue (I2) in [1].)

2.3. **Theorem** [6]. There are \( c \) \( T_r \)-taxa (hence \( c \) \( T_F \)-taxa) in \{metrizable Boolean\}. 

(This result addresses issue (I3) in [1], and answers a question raised in the penultimate paragraph on p. 592 therein. See also the paragraphs following the proof of Theorem 2.10.)

2.4. **Theorem** [4]. \{metrizable Boolean\} is dense in \{Boolean\}, relative to \( T_r \).

(This result addresses issue (I6) in [1].)

**REFERENCES**


