Factor Based Statistical Arbitrage in the U.S. Equity Market with a Model Breakdown Detection Process

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FACTOR BASED STATISTICAL ARBITRAGE IN THE
U.S. EQUITY MARKET WITH A MODEL
BREAKDOWN DETECTION
PROCESS

by

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ABSTRACT
FACTOR BASED STATISTICAL ARBITRAGE IN THE
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Many researchers have studied different strategies of statistical arbitrage to provide a steady stream of returns that are unrelated to the market condition. Among different strategies, factor-based mean reverting strategies have been popular and covered by many. This thesis aims to add value by evaluating the generalized pairs trading strategy and suggest enhancements to improve out-of-sample performance. The enhanced strategy generated the daily Sharpe ratio of 6.07% in the out-of-sample period from January 2013 through October 2016 with the correlation of -.03 versus S&P 500. During the same period, S&P 500 generated the Sharpe ratio of 6.03%.

This thesis is differentiated from the previous relevant studies in the following three ways. First, the factor selection process in previous statistical arbitrage studies has been often unclear or rather subjective. Second, most literature focus on in-sample results, rather than out-of-sample results of the strategies, which is what the practitioners are mainly interested in. Third, by implementing hidden Markov model, it aims to detect regime change to improve the timing the trade.

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I.   Introduction

Since Wall Street Quant Nunzio Tartaglia led his quantitative group with physicists, computer scientists, and mathematicians, at Morgan Stanley to search for arbitrage opportunities in the market in 1980s, many different statistical arbitrage strategies have been studied (Gatev et al, 2006). Commonly, statistical arbitrage refers to taking advantage of assets that are "statistically mispriced" and believed to revert to back to their equilibrium values. Many different combinations of assets have been observed to exhibit mean-reverting nature, such as foreign exchange rates (Engel 1994) or equities (Bock, 2008). Among many statistical arbitrage strategies, the pairs trading strategy is simple but one of the most well-known strategies. It generates profits off mean-reversion of spreads between two stocks by buying the relative losers and selling the relative winners. Gatev at al. (2006) presented a pairs trading strategy that yielded annualized excess returns of 11%, while showing that simple mean reversion of individual stocks is not the main driver of the performance.

Although many strategies present successful results, there seems to be three areas that can be further improved. First, many factor-based statistical arbitrage strategies seem to be unclear about the factor selection process. For example, Avellaneda and Lee (2010) explains that PCA factors that explain 55% of variance were used in their statistical arbitrage model because it performed better than other models. Avellaneda and Lee (2010) discuss how difficult it is to interpret equity return PCA factors, unlike how interest rate curves can be explained with three PCA components of level, spread, and curvature. Second, most literatures might suffer from data-snooping bias as most of them present in-sample performance results. Third, by implementing hidden Markov model, it

aims to detect regime changes to improve the timing the trade. This paper aims to add

value by addressing these three issues.

II.　　Statistical Arbitrage

Many researchers have studied different strategies of statistical arbitrage to provide a steady stream of returns that are unrelated to the market condition. Statistical arbitrage refers to the umbrella term that include many different forms of pairs trading strategies, such as distance strategy, cointegration strategy, or stochastic control approach (Krauss, 2015). Gatev et al. (2006) applied the distance strategy to U.S. stocks from 1962 to 2002. In this method, at each trading period, one year cumulative returns for each stock are collected. Then the sum of Euclidean squared distance for all possible pairs is calculated. When the distance between pairs becomes larger than the estimated threshold, a trade is opened. When the distance closes, the trade gets closed. This simple strategy generated annualized excess returns of 11%. Hong and Susmel (2003) applied cointegration approach to 64 different American Depository Receipt shares of Asian equity markets and showed annualized profits over 33%. Lo and MacKinlay (1990) applied a simple contrarian approach. In this approach, among individual U.S. equities returns, at each rebalancing interval, they will purchase securities that have performed relatively worse compared to others and short-sell securities that have performed relatively better – expecting them to fall. Due to positive cross-autocovariances among securities, the strategy performed well. Even if the returns of each securities cannot be correctly forecasted, an investor can still generate profits if relative performance can be correctly forecasted by cross-relationships. Khandani and Lo (2007) offered two possible explanations on why this strategy worked. First, the market often overreacts. Second, this strategy provides liquidity to the market.

This thesis extends a generalized version of pairs trading strategy, trading clusters of stocks versus another cluster of stocks. Several generalizations have been described. A generalized version of the pair trading strategy that we will build on was proposed by Avellaneda and Lee (2010). Avellaneda and Lee (2010) first decompose stock returns into returns that are explained by systematic exposures and returns that are idiosyncratic. These idiosyncratic portions of the returns are summed up cumulatively. By fitting the cumulative idiosyncratic portions of the returns into Orstein-Ulhembeck process, the mean reverting speeds and the standard deviations are estimated. Trading signals are generated based on how far the cumulative residuals have deviated compared to their corresponding standard deviations. With this strategy, Avellaneda and Lee (2010) demonstrated an annualized Sharpe ratio, risk adjusted performance measure that can be calculated by dividing the return by the standard deviation, of 1.44 from 1997 to 2007.

Liew and Roberts (2013) extended Avellaneda and Lee (2010) methodology by applying Black and Litterman framework. Liew and Roberts (2013) employ exchange traded funds as observable systematic factors in the market. In setting trading rules, Liew and Roberts applied the Black Litterman framework while estimating an Orstein and Ulhembeck process to determine the parameters of the mean-reversion process. Liew and Roberts (2013) suggest that a mean-reversion strategy might be profitable because of premiums received by providing liquidity in the market by selling when others are buying and buying when others are selling. Masindi (2014) applied the model by Avellaneda and Lee (2010) to South African equity market from 2001 to 2013.

III. Generalized Pairs Trading Model

This study extends the generalized pairs trading model by Avellaneda and Lee (2010) as noted above. Let $x_1, x_2, ..., x_n$ be the returns of the stocks that are in our investable universe with a length of **t** time periods and $X_{t \times n}$ be a matrix of returns. $x_1, x_2, ..., x_n$ are standardized by their sample means and standard deviation so that each column of $X_{t \times n}$ has a mean of zero and a standard deviation of one. If we let $F_{t \times r}$ be a matrix of r factors that indicate systematic movements in the equity market, $X_{t \times n}$ can be expressed in the following way.

$$X = F \times \beta + \varepsilon$$

where β is a k by n matrix that indicates stocks sensitivities to factors and ε is a t by n matrix that includes components of stock returns that are not explained by the factors.

Arbitrage Pricing Theory by Ross (1980) suggests that the expected returns of equity returns are determined by systematic factor exposures only and idiosyncratic parts of the returns are expected to be zero. Ross relies on three assumptions. First, factors that can explain systematic returns exist, such as exposures to the job market. Second, if investors build a large enough portfolio, idiosyncratic risks can be diversified away. Third, market participants are likely to take advantages of any mispriced assets, therefore making them hard to persist. In such market, any non-zero idiosyncratic returns are not sustainable. Since Ross (1980), many studies have attempted to apply different observable systematic factors. Benaković and Posedel (2010) emphasize industrial production, interest rates, and oil prices to decompose stock returns. Chen, Roll and Ross

(1986) use bonds spread, interest term structure, industrial production growth, inflation, and NYSE stock market returns to decompose returns of a portfolio. Although observable factors offer insights, many tend to suffer from multicollinearity issues and subjectivities in factor selection process as the arbitrage pricing theory does not need any particular variable to be used (Azeez, 2006).

The generalized pairs trading strategy by Avellaneda and Lee (2010) relies on the idea that no non-zero idiosyncratic returns are sustainable in the long run and was tested with both observable factors and statistical latent factors. It is implemented as follows. First, U.S. equities that exceed 1 billion dollars in market capitalization were selected. Second, exchange trade funds were selected as observable factors and Principal Component Analysis was performed to extract latent factors. Third, systematic returns were removed from each stock returns to extract idiosyncratic returns. By regressing the original dataset with either observable factors or a selected number of extracted PCA factors, residuals, which indicate the portion of the returns not explained by the systematic factors, are extracted from the original dataset. The cumulative series of these idiosyncratic returns are believed to fluctuate over time but have unconditional mean of zero. Trading signals are generated if cumulative residuals go above or below pre-determined threshold. For example, high cumulative residuals indicate that their stock returns that were not explained by systematic exposures have been consistently high and are likely to decrease going forward. The strategy with PCA factors generated an average annual Sharpe Ratio, calculated by dividing the return by the standard deviation, of 1.44 from 1997 to 2007 and the strategy with factors based on existing exchange traded funds generated an average annual Sharpe Ratio of 1.1 from 1997 to 2007.

This paper implements a similar trading algorithm to U.S. equities. Instead of testing both observable factors and PCA factors, this study focuses on PCA factors. The daily stock returns of the constituents of S&P 500 index from 2004 January through 2016 October were used in the analysis. The data was imported through Pandas, a Python data analysis toolkit. The prices were transformed into returns by taking the log difference. After excluding stocks without full samples, 420 securities were included in the analysis. The actual estimation of the model was conducted by using the data from 2004 January through 2012 December and the data from 2013 January through 2016 October were used to conduct an out-of-sample analysis. The returns were standardized prior to the estimation. Principal component analysis was conducted to extract systematic factors from the data. After determining the appropriate number of factors to be used, idiosyncratic returns were extracted by removing systematic returns from each security. Whenever the cumulative residuals are above or below determined threshold, trades are executed. Whenever a warning signal is generated from the failure detection algorithm, a security is removed from the investment universe.

IV.   Factor Analysis

Factor analysis is the most important part of this trading strategy as it plays a direct role in the creation of mean-reverting variable. To remove systematic returns based on factor exposures from the individual equity returns, we first need factors. In this study, Principal Component Analysis factors were generated from the original dataset. Principal Component Analysis has been a popular method to reduce dimensions of the asset returns (Avellaneda and Lee, 2010). PCA factors can be generated as follows. First, the empirical correlation or covariance matrix is calculated. Next, through singular value decomposition, it can be decomposed into eigenvectors and eigenvalues. Eigenvalues are then ranked in a decreasing order. Then the original data matrix X can be expressed in the following way.

$$F = X \times W$$

where F denotes the matrix of principal component score vectors and W denotes the matrix of vectors of factor loadings. Then the ith component can be found by multiplying the original data by the ith estimated loadings.

$$F_i = X \times W_i$$

The following scree plot [Figure 4.1]   illustrates the largest 30 eigenvalues in the data. After the first factor, which is likely to illustrate the general market movement, we see a gradual decrease in the variance.
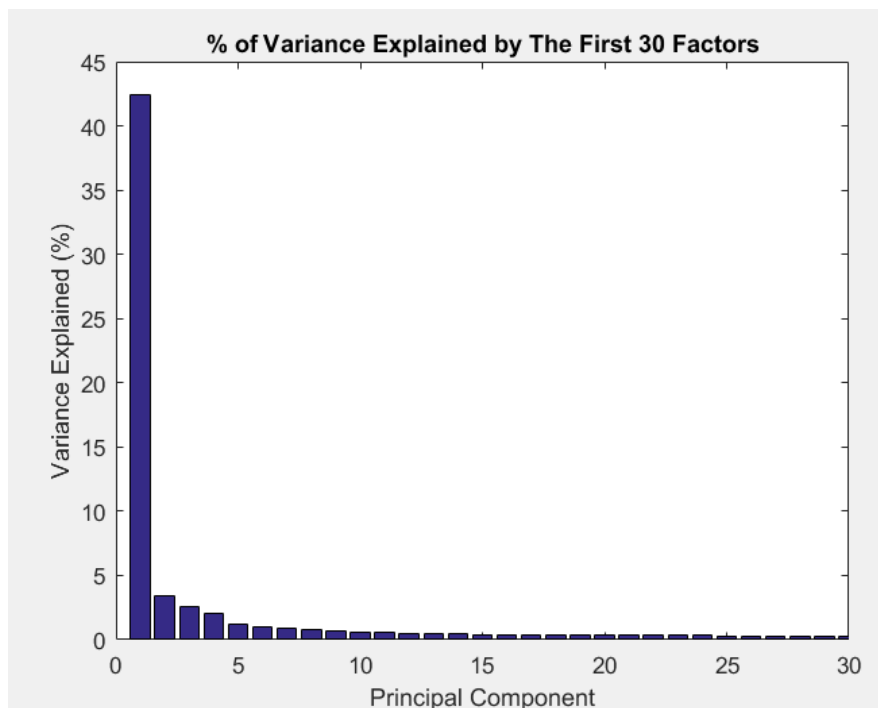
Figure 4.1: % of Variance Explained by The First 30 Factors

After eigenvalues are then ranked in decreasing order, depending on how much variance in the data needs to be explained, a certain number of factors can be chosen chronologically. In stock market universe, it is well known that the first factor, the component with the highest eigenvalue, is associated with the general market movements. There are two main advantages of using PCA factors over macroeconomic factors in finding systematic factors. First, this approach does not require to rely on a subjective exogenous factor selection process. Second, the factors are guaranteed to be independent with each other. From the correlation matrix, eigen-decomposition

algorithms draw eigenvectors one by one that are orthogonal to each other. This results in factors that are uncorrelated with each other. This prevents any issues that can arise from multicollinearity.

PCA factors do come with some disadvantages as well. One of the main disadvantages is that it can be unclear how many factors should be chosen. Often, either a fixed number of factors are selected or the number of factors that explains a pre-determined amount of variance. Avellaneda and Lee (2010) selected the number of factors that explained 55% of the total variance of the correlation matrix and suggested that it provided the superior performance compared to selecting a fixed number of factors, such as 15 factors, or different numbers of factors that explain different amount of total variance, such as 75% of the total variance. Josse and Husson (2011) note that if the number of factors are too small, not enough information would be analyzed and if the number of factors are too large, too much noise will be included in the analysis.

V.   Selecting the right number of factors

Avellaneda and Lee (2010) suggested that the performance of the strategy was superior when PCA factors explained 55% of the total variance. In this study, a standardized method of determining an appropriate number of factors by Bai and Ng (2002) was implemented. There are several advantages of applying the approach taken by Bai and Ng (2002) to the generalized pairs trading strategy. First, the approach does not require homoscedasticity across time or cross-section. As many stock returns often demonstrate heteroscedasticity, this is quite necessary. Second, it does not require sequential limits. For example, the approach by Connor and Korajczyk (1993) assumes that the number of cross-section converges to infinity with a fixed number of observation period, then the number of observation period converges to infinity.

Bai and Ng (2002) approaches the problem as a model selection problem and point out that Akaike information criterion (AIC) and Bayesian information criterion (BIC), typically used for model selection problems, do not yield robust results in selecting the appropriate number of factors when the data are large in dimensions in time and cross-section. The problem with estimating the appropriate number of factor arises from the fact that the theory established for classical models do not hold well when both time dimension and cross-section dimension approaches infinity. For example, the previous matrix form of return dataset can be written in the following way for the i th asset.

$$X = F \times \beta + \varepsilon$$

$$x_i = \mu_i + \beta_{i1} \times f_1 + \beta_{i2} \times f_2 + \dots + \beta_{ir} \times f_r + \varepsilon_i$$

μ_i is the mean return on the security i. β_i is the sensitivity of the security i to

factors. ε_i is the idiosyncratic portion of the returns. In theory, the appropriate number of

factors can be found by comparing eigenvalues of the covariance matrix of the data

because if the data are truly represented by r number of factors, only the first r number of

largest eigenvalues should diverge as the cross-sectional dimension size increases to

infinity (Bai and Ng, 2002). However, this is not a realistic solution as the estimation of

the covariance matrix is often an ill-posed problem, which does not necessarily result in

only r eigenvalues to diverge. Bai and Ng (2002) suggest a penalty function that is a

function of the size of the cross section, the number of the observations, and the number

of selected factors to penalize for overfitting. Bai and Ng (2002) proposes estimating r by

solving the following optimization function.

$$PC(k) = V(k, F^k) + kg(N, T)$$

$$V(k, F^k) = min \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (X_{it} - \lambda_i^{k'} F_i^k)^2$$

k indicates the number of factors that are being estimated. $V(k, F^k)$ indicates the

sum of squared residuals. $kg(N, T)$ indicates the penalty function. The authors suggest

two crucial conditions that the penalty function needs to meet as $lim_{N,T\to\infty}$. (i) $g(N, T) \to$

0. (ii) $C_{NT}^2 \times g(N, T) \to \infty$, where $C_{NT}^2 = min\{\sqrt{N}, \sqrt{T}\}$. The penalty functions that meet

these two conditions will ensure that any under-parameterized or over-parameterized

models will not be selected. The authors suggest six functional forms of loss functions

that meet these two conditions. The first three are named as PC criteria.

$$PC_{p1}(k) = V(k, F^k) + k\widehat{\sigma^2} \left(\frac{N+T}{NT}\right) ln\left(\frac{NT}{N+T}\right)$$

$$PC_{p2}(k) = V(k, F^k) + k\widehat{\sigma^2} \left(\frac{N+T}{NT}\right) ln(C_{NT}^2)$$

$$PC_{p3}(k) = V(k, F^k) + k\widehat{\sigma^2} ln\left(\frac{ln(C_{NT}^2)}{C_{NT}^2}\right)$$

These criteria generalize the idea from Mallow's C_p, shown below.

$$C_p = \frac{SSE_p}{S^2} - N + 2P$$

where SSE_p is the error sum of squares for the model with P number of regressors, N is the sample size, S^2 is the residual mean square with the complete set of regressors, and P is the number of regressors. Bai and Ng (2002) applies the same idea by multiplying the penalty function by $\widehat{\sigma^2}$ to scale. The three criteria will likely to be asymptotically equivalent but will have different properties in finite samples. Of the three different PC methods, PC3 method is likely to be less robust when N or T is small. The next three criteria extend the idea of Akaike information criterion (AIC) and Bayesian information criterion (BIC), which are penalized log-likelihood measure to select the appropriate number of parameters. AIC and BIC are of the following forms, where L is the likelihood, n is the number of data points, and k is the number of parameters estimated.

$$AIC = 2k - 2ln(L)$$

$$\text{BIC} = \ln(\text{n})\text{k} - 2\ln(\text{L})$$

By extending the ideas from these two information criteria, Bai and Ng (2002) suggest the next three criteria.

$$IC_{p1}(k) = \ln(V(k, F^k)) + k\left(\frac{N+T}{NT}\right)\ln\left(\frac{NT}{N+T}\right)$$

$$IC_{p2}(k) = \ln(V(k, F^k)) + k\left(\frac{N+T}{NT}\right)\ln(C_{NT}^2)$$

$$IC_{p3}(k) = \ln(V(k, F^k)) + kln\left(\frac{\ln(C_{NT}^2)}{C_{NT}^2}\right)$$

The main advantage of these three panel information criteria (IC_p) is that scaling by multiplying by variance is not necessary. In PC criteria, a maximum allowable number of k needs to be determined to properly scale the penalty term. In IC, the scaling is implicitly performed by the logarithmic transformation of V. Robustness test through simulations by Bai and Ng (2002) suggests that PC and IC methods suggested the number of factors that were close to the true number of factors, whereas the traditional AIC and BIC tend to suggest the number of factors that are too often bigger than the true number of factors. In this study, all three PC criteria and IC criteria were tested to determine the appropriate number of factors to be used.

VI.   Empirical Analysis

S&P 500 daily stock returns from the estimation period of 2004 January to 2012 December were standardized and analyzed. The dataset includes 430 stock returns with 3230 estimation periods. Three IC criteria, three PC criteria, and one BIC, noted as BIC 3, and one AIC, noted as AIC3, are computed to compare.

$$\text{AIC}_3(k) = \text{V}(k, F^k) + k\widehat{\sigma}^2 \left(\frac{2(N + T - k)}{NT} \right)$$

$$\text{BIC}_3(k) = \text{V}(k, F^k) + k\widehat{\sigma}^2 \left(\frac{(N + T - k)\ln(NT)}{NT} \right)$$

The sample periods include two different periods. January 2004 through December 2012 includes the entire sample period. January 2009 through December 2012 was also tested to test for robustness of the estimation. The maximum number of factors to be tested is set at 50.

The Suggested Number of Factors		
N = 430	Jan 2004 - Dec 2012	Jan 2009 - Dec 2012
IC1	7	7
IC2	7	7
IC3	9	10
PC1	11	11
PC2	10	11
PC3	12	20
AIC3	50	50
BIC3	5	5

Table 6.1: The Suggested Number of Factors

Although three IC and three PC criteria yield different conclusions across different sample sizes, they are generally consistent. As Bai and Ng (2002) suggested, the criteria PC3 is likely to yield less robust results compared to the other two when N or T is small, as shown above. This results suggest that this set of data might require somewhere between seven to twelve factors. Since the market and economy evolves over time, it is hard to conclude if the estimated results from the longer sample is necessarily more correct than the estimated results from the more recent but shorter sample. Based on this estimation, twelve factors were selected, which explained 57% of the variance in the sample. This is consistent with 55% of Avellaneda and Lee (2010).

VII. Extracting Idiosyncratic Returns and Creating Trading Signals

After the appropriate number of factors r was estimated to be 12, the first twelve PCA factors were regressed on each stock returns to estimate individual sensitivities to each systematic factors. The residuals ε_i from each Ordinary Least Square regression were collected to form a residual matrix E. Cumulative impacts of idiosyncratic components were gathered by taking the cumulative summation of the residual matrix, noted as C.

$$x_i = \mu_i + \beta_{i1} \times f_1 + \beta_{i2} \times f_2 + \dots + \beta_{ir} \times f_r + \varepsilon_i$$

$$E = [\varepsilon_1, \varepsilon_2, \varepsilon_3, \dots \varepsilon_n]$$

$$C_{ti} = \sum_{t=1}^{t} \varepsilon_{ti}$$
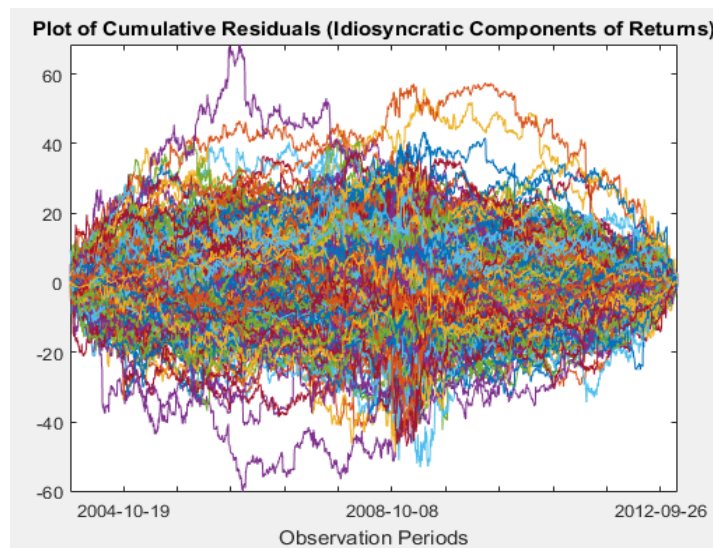
Figure 7.1: Plot of Cumulative Idiosyncratic Components of Returns

For each cumulative residual set, the empirical standard deviation is estimated to set up a trading rule. Simply, if the cumulative residual is lower than -1 standard deviation, we hold a positive position. If the cumulative residual is higher than 1 standard deviation, we hold a negative position, which can be achieved by short-selling the security.

$$Z_i = \sqrt{\frac{1}{N-1}\sum_{t=1}^{N}(c_{it} - \mu_i)^2}$$

$$S_{it} = 0 \ and \ B_{it} = 0 \ unless$$

$$S_{it} = -1 \ if \ c_{it} > Z_i$$

$$B_{it} = 1 \ if \ c_{it} < -Z_i$$

Z_i is the standard deviation for the ith cumulative residuals, c_i. N is the size of the sample. μ_i denotes the mean of the ith cumulative residual. S_{it} and B_{it} indicate the trading signals of the ith asset at time t. 1 indicates that we choose to hold the security and -1 indicates that we choose to short-sell the security to benefit from the decrease in the price. The returns for the portfolio can then be calculated as following

$$P_t = \frac{1}{2\sum_{i=1}^{n}|S_{i,t-1}|}\sum_{i=1}^{n}S_{i,t-1} \times x_{it} + \frac{1}{2\sum_{i=1}^{n}|B_{i,t-1}|}\sum_{i=1}^{n}B_{i,t-1} \times x_{it}$$

where x_{it} is the return of ith asset at time t, n is the total number of securities, P_t is the

realized return of the portfolio at time t.

The first part of the return above indicates the returns generated by short positions

and the second part indicates the returns generated by the long positions. There exists a

time lag between the position and realized returns to avoid looking-ahead bias. The

portfolio return at time t is based on the information up to the previous period. Each short

and long position are weighted so that the portfolio is staying market neutral, instead of

taking excessive positions in long, short, or both. The one of the main goal of statistical

arbitrage strategy is to generate stable stream of profits that are uncorrelated to the

market. This can be tested by measuring correlations of returns to the market portfolio

(S&P 500) and sensitivities. Sensitivities are calculated by regressing each time series

with S&P 500 returns.

Correlations Comparison	Portfolio (Short Portion)	Portfolio (Long Portion)	Portfolio	S&P 500
Portfolio (Short Portion)	1.00			
Portfolio (Long Portion)	(0.96)	1.00		
Portfolio	0.21	0.08	1.00	
S&P 500	(0.95)	0.96	(0.04)	1.00

Table 7.2: Portfolio Correlation In-Sample

Sensitivity Comparison	Beta Coefficient	R^2	Standard Deviation	Sharpe Ratio
Portfolio	(0.23)	0.00	0.22%	35.37%
Portfolio (Short Portion)	(1.67)	0.91	0.76%	3.26%
Portfolio (Long Portion)	1.72	0.93	0.75%	7.24%
S&P 500	1.00	1.00	1.33%	1.42%

Table 7.3: Portfolio Statistics In-Sample

The first table [Table 7.2] is the correlation table and the second [Table 7.3] is the results of regressing each series to S&P 500 returns. As expected, short position return is negatively correlated to S&P 500 whereas long position return is positively correlated to S&P 500. Portfolio and S&P 500 is not correlated as desired, shown by the correlation of -.04 and R^2 of 0. Short position beta of -1.67 and long position beta of 1.72 suggest that our short and long components might be more sensitive to the market than the market portfolio, although the entire portfolio is market-neutral. The below plot [Figure 7.4] shows the cumulative log returns of in-sample performance and the benchmark (S&P 500). The statistical arbitrage strategy yielded a lot higher returns than the S&P 500 index. The Sharpe ratio, calculated by dividing the mean return by standard deviation, of the strategy was 35.37% versus 1.42% of S&P 500.
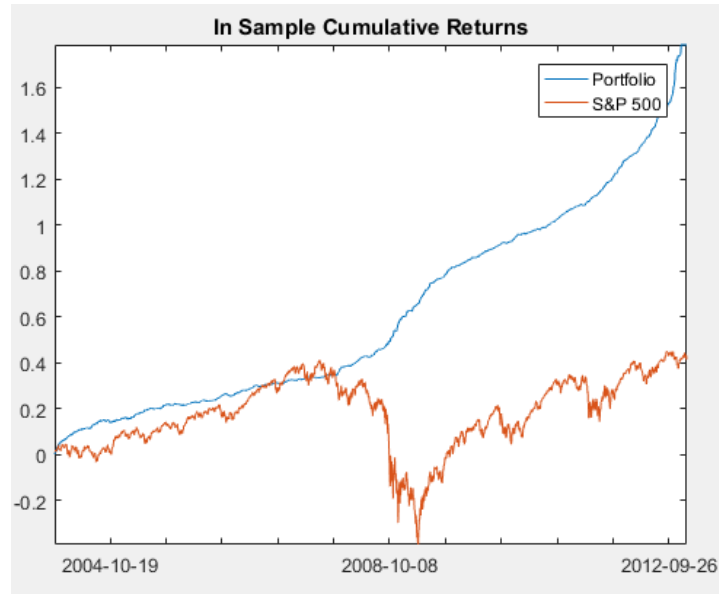
Figure 7.4: Portfolio Performance In-Sample

VIII.   Out of Sample Forecast and Performance Analysis

Although the in-sample results look impressive, often practitioners are mainly interested in strategy that can perform robust results out-of-sample. One of the biggest challenge of a strategy that is based on the cumulative residuals of regressions is a lack of signal at the beginning of the out-of-sample period.
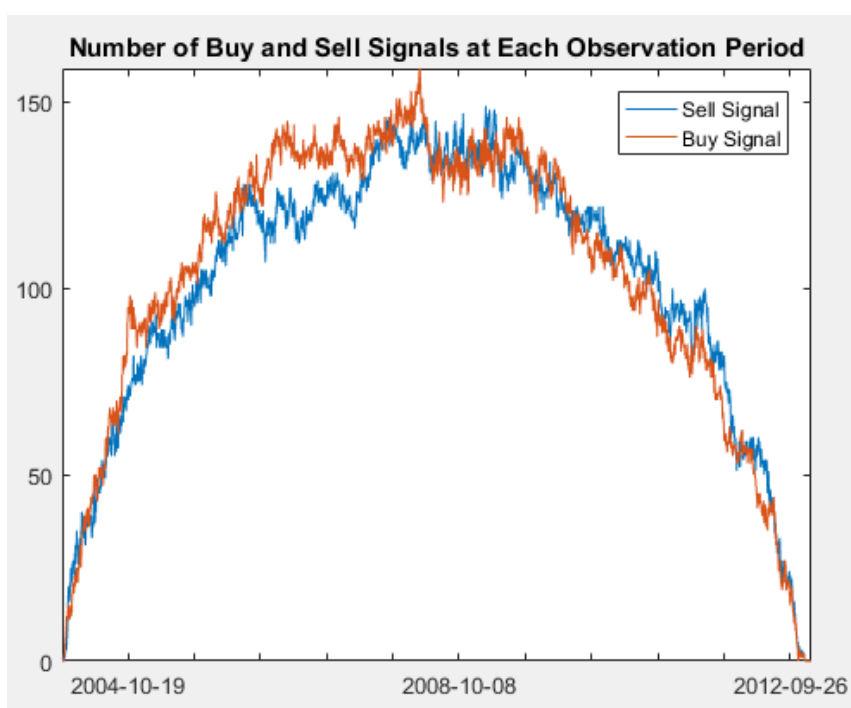
Figure 8.1: Number of Buy and Sell Signals at Each Observation Period

The above chart [Figure 8.1] shows the number of buy and sell signals in the estimation period. At the end of the estimation period, cumulative residuals for all securities will be at zero, by the nature of the linear regression, which prevents us to make any investment decision for the out-of-sample period. Therefore, we need a process

to generate factors, extract idiosyncratic returns, and cumulate idiosyncratic returns, without any looking-forward bias. This can be achieved in the following way at each out-of-sample period.

Step 1. Generate factors

$$F_{t+1,i} = X_{t+1} \times W_i$$

where $F_{i,t+1}$ is the ith factor in $t + 1$ period, which is the beginning of the forecasting period. X_{t+1} is the returns observation, transformed by subtracting by the estimation period mean and dividing by the estimation period standard deviation to duplicate the standardization procedure that took place in the estimation procedure. W_i is the factor loadings for the ith component. The length of estimation period is denoted as t, and the $t + 1$ denotes the first period of the out-of-sample period. The first 12 components are generated to form 1 by 12 matrix F_{t+1}.

Step 2. Extract idiosyncratic returns by subtracting systematic portions of the returns.

$$\varepsilon_{t+1,i} = X_{t+1,i} - F_{t+1} \times \beta_i$$

where $\varepsilon_{t+1,i}$ is the idiosyncratic return for the ith asset at the period $t + 1$, β_i is the previously estimated sensitivities to systematic factors for the ith asset.

Step 3. Cumulate idiosyncratic returns at each step and follow the same trading rules discussed previously.

The below chart [Figure.8.2] shows the cumulated idiosyncratic returns over the out-of-sample period.
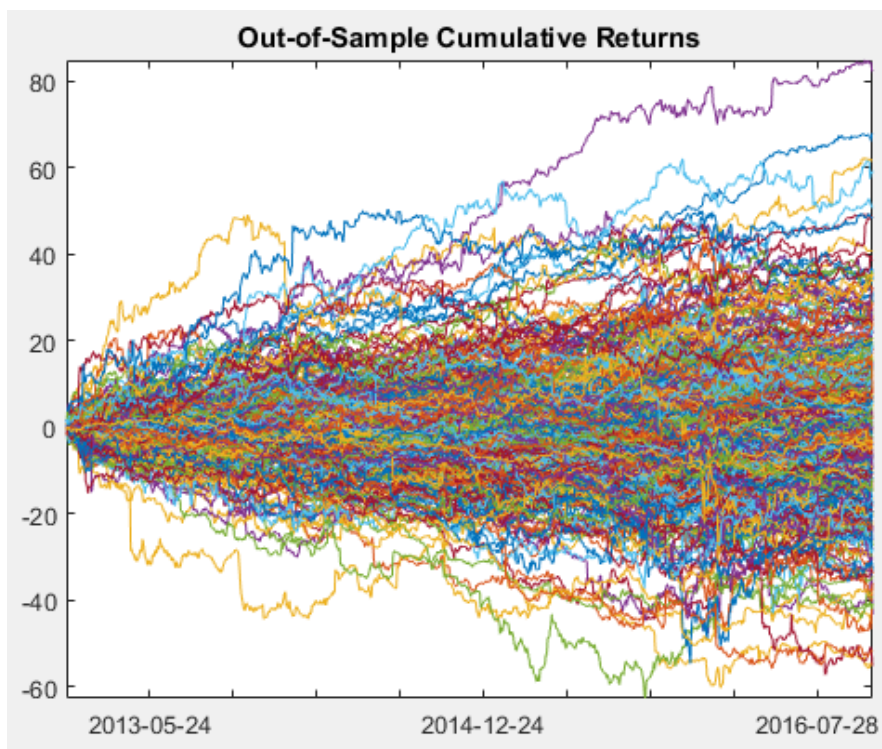
Figure 8.2: Out-of-Sample Cumulative Idiosyncratic Returns

The chart below [Figure 8.3] shows the performance of the strategy and the performance of the benchmark over the out-of-sample period. The portfolio performed significantly worse compared to the benchmark. Cumulative log return over the period for the benchmark was 47.56% while the strategy generated 3.45% only. On the risk-adjusted measure, the portfolio Sharpe ratio decreased from 35.37% in the in-sample period to 1.84% in the out-of-sample period. What did go wrong?
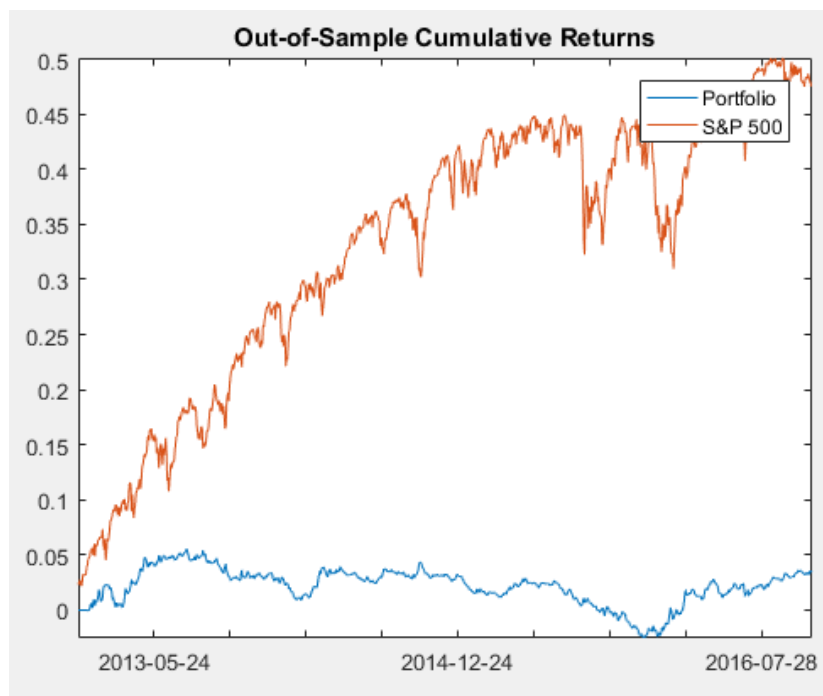
Figure 8.3: Out-of-Sample Performance

First, we need to check if the return data and the factors from the out-of-sample period are significantly different from the ones from the in-sample period. If out-of-sample factors were no longer stationary, were not centered at zero, or had different standard deviations, extraction of systematic returns might not have been calculated appropriately. The table below [Table 8.4] shows the mean and the standard deviation of the factors from each period. Stationary of the data were also checked to see if the out-of-sample factors were not stationary anymore. If out-of-sample factors were non-stationary, it could have a detrimental impact on the trading strategy as idiosyncratic residuals might become trend-stationary, instead of being centered at zero. The Augmented Dickey-Fuller

test was conducted to test for the null hypothesis of a presence of a unit root to test for stationarity.

Factor Number	Mean		Standard Deviation		Stationary		P value	
	In-Sample	Out-of-Sample	In-Sample	Out-of-Sample	In-Sample	Out-of-Sample	In-Sample	Out-of-Sample
1	0.0000	0.1535	13.5146	8.1029	TRUE	TRUE	0.001	0.001
2	0.0000	-0.0907	3.8392	2.3240	TRUE	TRUE	0.001	0.001
3	0.0000	-0.2034	3.2998	3.1833	TRUE	TRUE	0.001	0.001
4	0.0000	0.0076	2.9769	2.0501	TRUE	TRUE	0.001	0.001
5	0.0000	0.1351	2.2948	2.0975	TRUE	TRUE	0.001	0.001
6	0.0000	0.0783	2.0529	1.3555	TRUE	TRUE	0.001	0.001
7	0.0000	-0.1206	1.9483	1.5516	TRUE	TRUE	0.001	0.001
8	0.0000	-0.0330	1.7899	1.1707	TRUE	TRUE	0.001	0.001
9	0.0000	0.0133	1.7629	1.5203	TRUE	TRUE	0.001	0.001
10	0.0000	-0.0688	1.5699	1.0748	TRUE	TRUE	0.001	0.001
11	0.0000	-0.0906	1.5087	1.0498	TRUE	TRUE	0.001	0.001
12	0.0000	0.0254	1.4284	0.9899	TRUE	TRUE	0.001	0.001

Table 8.4: Out-of-Sample Factor

The null hypothesis of a presence of a unit root was rejected for all 12 factors in both periods, suggesting that stationary is maintained in the out-of-sample period. Out-of-sample factors exhibit means that are slightly different from 0 and standard deviations that are generally smaller than the standard deviations from the in-sample period. This can suggest that either the estimated factor loadings were not robust or the short-term market condition for the out-of-sample might be different from the long-term market condition of the estimation period. To test that, a further analysis on the first factor,

which explains 42% of the total variance and 75% of variance explained by the first 12

factors, was conducted by regressing it against the market, represented by the benchmark

S&P 500 index returns.

$$F_1 = \alpha + \beta \times M$$

where F_1 is the first factor and M is the standardized benchmark (S&P 500 returns).

	In-Sample	Out-of-Sample
Intercept (α)	0.0000	0.0113
Beta (β)	0.0724	0.0744
R^2	0.9570	0.9640
Mean of M	0.0000	0.0228
Standard Deviation of M	1.0000	0.614

Table 8.5: In-Sample and Out-of-Sample Comparison

To be consistent with the portfolio strategy, benchmark returns were standardized

first. Both in-sample period returns and out-of-sample period returns were standardized

by the in-sample estimated mean and standard deviations. The consistency in R^2 and the

beta coefficient in both periods suggest that the first factor appears to be robust in terms

of reflecting the general movements in the market. The mean of M in the out-of-sample

period is higher than the mean from the in-sample period. The standard deviation of M in

the out-of-sample period is lower than the standard deviation from the in-sample period.

These suggest that the general market movements in the out-of-sample period have been

characterized by higher average returns with lower volatility compared to the estimation

period. This can be analyzed further by comparing the performance of the long portion of

the portfolio, which benefits when the selected securities increase in prices, and the short

portion of the portfolio, which benefits when the selected securities decrease in prices.

Correlations Comparison	Portfolio	Portfolio (Short Portion)	Portfolio (Long Portion)	S&P 500
Portfolio	1.00			
Portfolio (Short Portion)	0.33	1.00		
Portfolio (Long Portion)	0.10	(0.91)	1.00	0.93
S&P 500	(0.17)	(0.95)	0.93	1.00

Table 8.6: Out-of-Sample Correlation

Sensitivity Comparison	Beta Coefficient	R^2	Standard Deviation	Sharpe Ratio
Portfolio	(0.72)	0.03	0.19%	1.84%
Portfolio (Short Portion)	(1.69)	0.91	0.46%	-4.32%

Portfolio (Long Portion)	1.73	0.86	0.44%	5.37%
S&P 500	1.00	1.00	0.82%	6.03%

Table 8.7: Out-of-Sample Performance Analysis

The correlation between the long portion of the portfolio and S&P 500 is .93 and the correlation of the short portion of the portfolio and S&P 500 is -.95. This shows that the relationship between our long and short positions to the market movement has not changed compared to the estimation period. However, the correlation between the portfolio strategy and S&P 500 is -.17, which is lower than the estimation period correlation of -.04 between them. This change in correlation is likely to be the result of the underperformance of the portfolio, rather than the portfolio becoming more negatively correlated with the market in the forecasting period as the sensitivities of our short and long components to the market, measured by beta, and correlation structure have not changed. Our portfolio appears to remain uncorrelated to the market as desired. The low R^2 value of .03 from the regressing the out-of-sample portfolio results with S&P 500 reinforces this conclusion. Analysis can be further decomposed into long side of the positions and short side of the positions to locate where the losses might be coming from.

During the out-of-sample period, the total cumulative log return from the long positions was 22.69% with a Sharpe ratio of 5.37%. The total cumulative log return from the short positions was -19.24% with a Sharpe ratio of -4.32%. These two components add up to the total cumulative return of 3.45% for the portfolio. Clearly, most underperformance came from the short positions. This illustrates a typical case of a mean-reversion failure due to a prolonged directional movement in the market. As the

market moved upward for an unusually long period, the sell strategy greatly suffered. In

the next section, we will first discuss how to ensure that our strategy maintain

profitability even in directional markets without sacrificing returns excessively. Then we

will also discuss if any other enhancements can be made to further improve out-of-

sample performance.

IX.   Failure Detection and Strategy Improvement Techniques

There are some previous studies on how to improve statistical arbitrage strategies or detect potential big losses. Some try seek the optimal threshold to enter and exit with respect to transaction cost to enter and exit. Leung and Li (2015) derive the optimal entry and exit prices with respect to transaction cost by maximizing the expected difference between the maximum expected profit and the distance between the current price and the transaction cost. Leung and Li (2015) extends the model by incorporating a stop-loss constraint to ensure that each position does not lose more than a certain amount. This paper approaches the problem from a slightly different perspective. Rather than finding an optimal threshold with respect to a given transaction cost, this paper aims to focus on improving and testing the fundamental forecasting ability of the strategy.

Yeo and Papanicolaou (2016) points out that how few literature covers the risk of relying on the mean reverting assumptions of the idiosyncratic returns in statistical arbitrage literatures. Yeo and Papanicolaou (2016) suggest to control the risk of the statistical arbitrage strategies by selecting securities that show high mean-reversion speeds and selecting securities that showed a high goodness-of-fit. By testing the strategy with the daily returns of 378 stocks in S&P 500 constituents from 2000 through 2014, Yeo and Papanicolaou (2016) showed that the suggested strategy provided higher Sharpe ratio. Mean-reversion speeds were estimated by fitting the mean-reverting cumulative residuals into an Ornstein-Uhlenbeck process. Goodness-of-fit was measured by comparing R^2 values of the Ornstein-Uhlenbeck process. Based on out-of-sample test results, Yeo and Papanicolaou (2016) suggest that both mean-reversion speed control and R^2 control boosted the performance.

To improve the out-of-sample performance of the statistical arbitrage strategy implemented in this study, we can start by searching for any patterns in successful positions and unsuccessful positions. Furthermore, instead of looking at the aggregate performance, each security performance is analyzed. The aggregate performance can be decomposed into the following way, where $B_{i,t}$ indicates return generated by ith security in time t by taking a long position in the ith security. S indicates returns generated by taking short positions.

$$Perf = \sum_{i=1}^{n}\sum_{t=1}^{T} B_{i,t} + \sum_{i=1}^{n}\sum_{t=1}^{T} S_{i,t}$$

The out-of-sample portfolio returns can be sorted by their performance in the following way. The below chart [Figure 9.1] illustrates aggregate cumulative performance over the out-of-sample period per security.
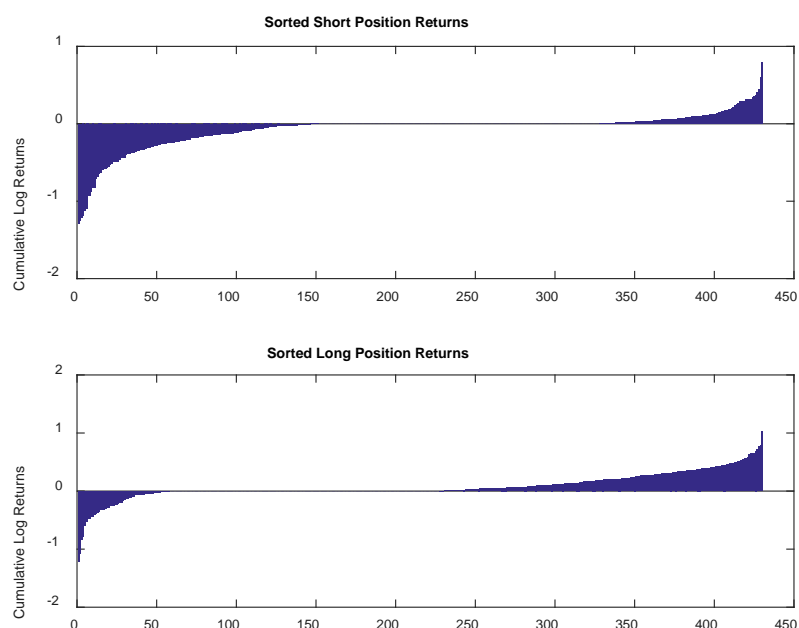
Figure 9.1: Out-of-Sample Returns Per Securities

First, we can check if assets that fitted better in the estimation period tend to perform better in the out-of-sample period. Two different goodness-of-fit can be compared. First, R^2 from the systematic exposure measuring step can be compared. Higher R^2 means that a larger part of returns was explained by systematic factors. The out of sample performances were regressed in the following way. Z indicates a 430 by 1 vector in which each element indicates the cumulative out-of-sample performance for each security. Z_l indicates cumulative returns generated by the long positions and Z_s indicates cumulative returns generated by the short positions. R indicates a 430 by 1 vector that includes individual R^2 from the estimation of the systematic exposures. The figure [Figure 9.2] below illustrates a scatterplot of returns and R^2. Each regression

yielded R^2 values of .006 and .048, suggesting that securities that were fitted well with

systematic factors did not necessarily performed better out-of-sample.

$$Z_l = \alpha_l + \beta_l \times R + \varepsilon_l$$

$$Z_s = \alpha_s + \beta_s \times R + \varepsilon_s$$

Figure 9.2: Out-of-Sample Returns Versus Goodness of Fit

Next, R^2 and mean reversion speed from fitting OU process in the cumulative

idiosyncratic process can be compared as Yeo and Papanicolaou (2016) suggested.

Ornstein-Uhlenbeck process is a stochastic process that is similar to an auto-regressive

process in the discrete time series realm. It illustrates a time series that follow Brownian

motion but shows a mean reverting tendency in the long run (Masindi, 2014). By fitting

the cumulative idiosyncratic returns into Ornstein-Uhlenbeck process, we can measure

standard deviations and mean reverting speeds. Yeo and Papanicolaou (2016) suggested

that cumulative idiosyncratic returns with faster mean-reversion speeds and higher

goodness-of-fit are likely to generate superior performances. Faster mean-reversion

speeds suggest that mean reversion will take place quickly and higher goodness-of-fit

suggests that the time series is more likely to follow the Ornstein-Uhlenbeck process

instead of the geometric Brownian motion with a unit root. The Ornstein-Uhlenbeck

process can be expressed in the following way where C_i is the cumulative idiosyncratic

returns for the ith asset, m_i is the mean reversion level for the ith asset, κ_i is the mean

reversion parameter for the ith asset, and σ_i is the standard deviation for the ith asset, and

W_i is the Brownian motion (Wiener) process.

$$d C_i(t) = \kappa_i \left(m_i - C_i(t) \right) dt + \sigma_i dW_i(t), \quad \kappa_i > 0$$

The parameters for the Ornstein-Uhlenbeck process can be estimated as a discrete

autoregressive process with lag one. Estimated mean reversion speeds and goodness-of-

fit were regressed with the out of sample performance in the same way as the equation

above. The below table [Table 8.10] shows the result. Mean reversion speed was

measured as $\frac{1}{\kappa_i}$. As low R^2 for four different regressions illustrate, mean reversion speed

and goodness-of-fit of Ornstein-Uhlenbeck process do not seem to be correlated with out-

of-sample performance, unlike as Yeo and Papanicolaou (2016) suggested.

$$Z_l = \alpha_l + \beta_l \times R + \varepsilon_l$$

$$Z_s = \alpha_s + \beta_s \times R + \varepsilon_s$$

	R^2 of the Regression Result	
	Short Positions Returns	Long Positions Returns
R = Goodness of Fit of OU Process	0.0016	0.0148
R = Mean Reversion Speed	0.0010	0.0186

Table 9.3: OU Process and Out-of-Sample Returns

There might be several reasons why screening method by Yeo and Papanicolaou (2016) did not seem consistent in this dataset. First, estimation window and the selected individual stocks are different. Second, instead of screening with parameters based on the entire sample period, Yeo and Papanicolaou (2016) estimated the parameters with different estimation windows and made stock selections at each time step. Yeo and Papanicolaou (2016) explain that the mean-reversion speed is normalized by the estimation window since the estimated mean-reversion parameter usually depends on the length of the estimation window.

The strategy tends to perform worse out-of-sample when cumulative idiosyncratic returns do not oscillate as they did in-sample and no longer show mean-reverting nature. Checking for stationarity can inform us whether the time series is likely to be stationary

or not. Augmented Dickey-Fuller test was conducted to test for this. The Augmented

Dickey-Fuller test on the cumulative idiosyncratic returns in-sample yielded the

following results. Out of 430 securities, the null hypothesis of a unit root was rejected in

195 securities, suggesting that these time series are stationary. The rest 235 securities

failed to reject the null hypothesis of a unit root. The average achieved returns per

securities that were stationary were compared with the average achieved returns per

securities that were not stationary. The cumulative returns per security were divided by

the number of time periods to compare in-sample and out-of-sample performance fairly.

As shown below, returns generated by the securities that were stationary outperformed.

This is not surprising as the strategy relies on buy-low and sell-high concept.

			Stationary	Not Stationary
In-Sample	Number of Securities		195	235
	Average Returns Per Security Per Time-Period	Short Positions	0.01429%	0.00347%
		Long Positions	0.02894%	0.01789%
Out-Of-Sample	Number of Securities		24	406
	Average Returns Per Security Per Time-Period	Short Positions	-0.00017%	-0.00704%
		Long Positions	0.00600%	0.00769%

Table 9.4: Out-of-Sample Returns and Stationarity

However, out-of-sample analysis yielded an interesting result. Only 24 out of 430 cumulative idiosyncratic returns of the securities were considered as stationary. Furthermore, returns underperformed compared to in-sample returns across both stationary and non-stationary time series. Whether taking positions only when the time series is believed to be stationary can add value was further tested by testing a modified version of the strategy. The strategy is performed as followed. At each time step, stationary test for the individual cumulative idiosyncratic returns is conducted. If it is considered as stationary, the same one standard deviation trading rule is followed. If the time series is not considered as stationary, no trading decision takes place. Out of 966 time-periods in the out-of-sample period, this strategy was implemented starting with 16th time-period to ensure that there are enough observations to conduct the ADF test.

$$S_{it} = 0 \; and \; B_{it} = 0 \; unless$$

$$S_{it} = -1 \; if \; c_{it} > Z_i \; and \; \pmb{x}_{i,1:t} \; is \; stationary$$

$$B_{it} = 1 \; if \; c_{it} < -Z_i \; and \; \pmb{x}_{i,1:t} \; is \; stationary$$

$$P_t = \frac{1}{2\sum_{i=1}^{n}|S_{i,t-1}|}\sum_{i=1}^{n} S_{i,t-1} \times x_{it} + \frac{1}{2\sum_{i=1}^{n}|B_{i,t-1}|}\sum_{i=1}^{n} B_{i,t-1} \times x_{it}$$

The performance result suggests that this is not likely to be a superior strategy. The average returns per security per time-period for the short positions was -.1700% and 0.0797% for the long positions. Although the long positions returns were better, the short positions returns were significantly worse, which caused the total cumulative return of the strategy to wind up at -94.18%. Although the stationarity is necessary for the strategy to perform well, the actual implementation of it is not easy without looking-back bias. If

the times series to be invested can be selected by the stationarity of the full-length time series, it can enhance the performance, as shown in the above table. However, at each time step, without looking forward in the future, the best information about the stationarity of the time series can be only estimated by the time series up to that point, which might not reflect if the time series will stay stationary throughout the trading period.

Markov regime switching model was tested to improve the strategy as well. Many pairs trading strategies often assume that spreads between two cointegrated stocks can oscillate around a mean of the spread. However, fundamental change in the company or the market structure might cause the spread to no longer revert to the historical mean or revert to a different equilibrium level. Bock and Mestel (2008) applied Markov regime switching model with switching mean and variance to improve pairs trading strategy. Markov chains were originally developed as a part of extension of the law of large numbers to dependent events (Merrill, 2010). Markov chain introduce the concept that, instead of a sequence of random observations generated by one state, there might be multiple states that generates random variables and the determination of current states might depend on what the previous states were.

In finance, hidden Markov Models are more often used as most of states are unobservable. We can assume that the current observations are generated by an unobservable state S_t. S_t emits observations based on its distribution. At each time step, based on transition probabilities, the state might change and the probability distribution will also change accordingly. Often, the transition from one state to another state is simplified and assumed to be dependent on only the previous state. Instead of assuming

that the transition of states are deterministic, HHM assumes that there must have been

predictable stochastic process that causes states to shift from one to another (Hamilton,

2005). Although we cannot directly observe states, we can estimate the states based on

observed emissions. Consider the following process where $K_t = 1,2$ and ε_t follows a

normal distribution with zero mean and variance given by σ_K^2.

$$X_t = \mu_{K_t} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \sigma_{K_t}^2)$$

This is a simple case of how normally distributed variable can behave across

different latent regimes (Perlin, 2015). This process can be estimated by Bayesian

inference or maximum likelihood. In this study, maximum likelihood estimation method

of Perlin (2015) was implemented. The log likelihood function can be estimated as

follows.

$$\ln L = \sum_{t=1}^{T} \ln \sum_{j=1}^{2} (f(X_t \mid K_t = j, \Theta) \Pr(K_t = j))$$

Θ indicates the set of parameters. Likelihood function in each state are weighted

averaged by the probabilities of each states. Although it is possible to estimate the model

with many regimes, estimating parameters accurately becomes difficult as the number of

regimes increase. Therefore, most HHM applications assume two or three different

regimes (Hamilton, 2010). In this study, HHM is implemented to check if it can improve

the trading strategy. States and parameters were estimated for each idiosyncratic return. Then, the adjusted trading rule was applied to check if knowing the current state of the idiosyncratic returns can improve the performance. The states and trading rules are as follows.

$$X_{i,t} = \mu_{K_{i,t}} + \varepsilon_{i,t}$$

$$\varepsilon_{i,t} \sim N(0, \sigma_{K_{i,t}}^2)$$

$$S_{it} = 0 \ and \ B_{it} = 0 \ unless$$

$$S_{it} = -1 \ if \ c_{it} > Z_i \ and \ \mu_{K_{i,t}} < 0$$

$$B_{it} = 1 \ if \ c_{it} < -Z_i \ and \ \mu_{K_{i,t}} > 0$$

$$P_t = \frac{1}{2\sum_{i=1}^{n}|S_{i,t-1}|}\sum_{i=1}^{n} S_{i,t-1} \times x_{it} + \frac{1}{2\sum_{i=1}^{n}|B_{i,t-1}|}\sum_{i=1}^{n} B_{i,t-1} \times x_{it}$$

where X indicates idiosyncratic return, $\mu_{K_{i,t}}$ indicates expected idiosyncratic return for ith asset at time t in state K.

This is based on assumption that there might be two different states that idiosyncratic returns are generated from and we can benefit from factoring that into the trading strategy. The strategy goes as follows. In the original strategy, if cumulative residual of ith asset at time t reaches the level that is higher than the Z score, the sell signal was generated. In this Markov enhanced version, sell signal is only generated if the expected value of the idiosyncratic returns is less than zero. The rationale behind this is that, even if the cumulative idiosyncratic returns might be higher than the threshold and we expect it to come down, the idiosyncratic returns might be in the state where

cumulative idiosyncratic returns are expected to continue to rise. This can be viewed consistent with the "momentum" trading strategies. The same logic applies to buy signals.

The signal generation procedure was conducted in the following way. 430 Each stock's in-sample period idiosyncratic returns and out-of-sample period idiosyncratic returns were combined and the hidden Markov model was fitted. After gathering filtered state probabilities and expected idiosyncratic return parameters, the trading signals were generated at each time step. The performance of the new strategy is shown below.
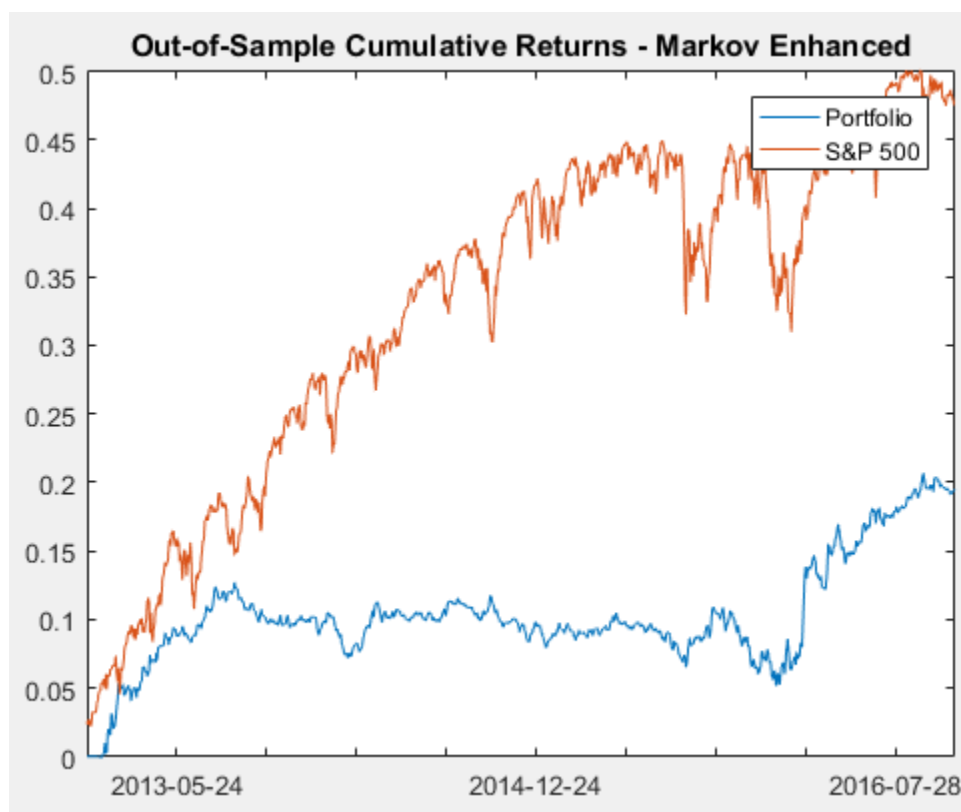

Figure 9.5: Enhanced Out-of-Sample Returns

The enhanced strategy returned 19.49% cumulative returns among the period compared to the benchmark performance of 47.56%. The Sharpe ratio was 6.07% compared to the Sharpe ratio of 6.03% for the benchmark. The correlation between two returns was -.03. Low correlation and the satisfactory Sharpe ratio suggest that this strategy can add value. Although the absolute performance is low, if the stream of returns is not correlated to the market and has a high Sharpe ratio, the leverage can be often used to enhance the magnitude of the performance. Both long position returns and short position returns appeared acceptable as shown below.

Correlations Comparison	Portfolio	Portfolio (Short Portion)	Portfolio (Long Portion)	S&P 500
Portfolio	1.00			
Portfolio (Short Portion)	0.18	1.00		
Portfolio (Long Portion)	0.46	(0.79)	1.00	
S&P 500	(0.03)	(0.93)	0.82	1.00

Table 9.6: Enhanced Out-of-Sample Returns Correlations

Sensitivity Comparison	Beta Coefficient	R^2	Standard Deviation	Sharpe Ratio

Portfolio	(0.08)	0.00	0.33%	6.07%
Portfolio (Short Portion)	(1.57)	0.87	0.49%	-2.05%
Portfolio (Long Portion)	1.25	0.68	0.54%	5.60%
S&P 500	1	1	0.82%	6.03%

Table 9.7: Enhanced Out-of-Sample Returns Statistics

A few things need to be noted. First, short position returns have improved but it still yields negative returns. However, as the goal of the strategy is to provide a positive return net of short and long positions, this is not as big of a concern. Second, both betas of short and long position returns are over 1 in absolute values, suggesting that the each components of the strategy might be riskier than the benchmark. Third, most importantly, this might suffer from a forward looking bias. The filtered probability of the states and the expected value parameter $\mu_{K_{i,t}}$ for the Hidden Markov Models for each stock were estimated with the idiosyncratic returns from both in-sample and out-of-sample periods. To truly test this strategy in out-of-sample period, the estimation of filtered probability and the expected values has to take place at each time step for each stocks. However, this was computationally too expensive for the scope of this study. Each estimation took roughly 30 seconds, which took a total of 215 minutes (30*430/60) for 430 securities. To repeat this at each 966 time periods in the out-of-sample period would have taken 3461 hours without any parallel computing.

X.   Conclusion

The purpose of this thesis was to evaluate a statistical arbitrage strategy and suggest enhancements to improve out-of-sample performance by extending the generalized pairs trading model by Avellaneda and Lee (2010). By removing systematic returns from stock returns, we extracted idiosyncratic returns. Based on previous empirical findings and theoretical support (Arbitrage Pricing Theory), we constructed a trading strategy that assumes the mean reversion of cumulative idiosyncratic returns of stocks.

Implementation of the strategy to U.S. equities from 2004 January through 2012 December yielded a daily Sharpe ratio, calculated by dividing daily returns by daily standard deviation, of 35.37% versus 1.42% of the benchmark S&P 500. As desired, implementation of long and short positions resulted in an uncorrelated strategy, as shown by the correlation of -.04 during the period.

However, the out-of-sample performance result did not appear impressive. From January 2013 through October 2016, the portfolio Sharpe ratio decreased from 35.37% in the in-sample period to 1.84% in the out-of-sample period while the Sharpe ratio of S&P was 6.03%. Stationarity of factors, stationarity of cumulative idiosyncratic returns, goodness of estimations, mean reverting speeds of Ornstein-Uhlenbeck process, and Hidden Markov regimes were analyzed to enhance the original strategy. Hidden Markov regime switching model was the only enhancement that improved the result. The enhanced strategy generated the Sharpe ratio of 6.07% while still uncorrelated to the market. However, it should be noted that it might have suffered from a forward looking bias.

There are several areas of this study that can be further improved. For example, it will be valuable to test if any specific sectors yield better results. Some sectors are known to be more cyclical and some are known to be less cyclical. Factors can be further studies as well. Unlike interest rates factor models, equity PCA factors are more difficult to tie with economic theories. The first factor is likely to represent the general market movement. It might be valuable to test if any pattern can be found between factors and stocks. For example, one can test if stocks with high leverage have positive correlation with any of the factors. Lastly, testing different estimation windows can yield interesting insights on the ideal length of data to capture both long enough and relevant enough data.

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