Processing Induced Voxel Correlation in SENSE FMRI Via the AMMUST Framework

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Introduction:

It is well known that magnetic field gradients can be used to Fourier encode the complex-valued spatial frequencies of an object. From a two dimensional array of the complex-valued spatial frequencies for an object S_C, a complex-valued image V_C can be reconstructed via inverse Fourier transform as

$$\underset{n_y\times n_x}{V_C} = \underset{n_y\times n_y}{\Omega_{Cy}} \quad \underset{n_y\times n_x}{S_C} \quad \underset{n_x\times n_x}{\Omega'_{Cx}} \tag{1}$$

where Ω_{Cx} and Ω_{Cy} are complex-valued Fourier transform matrices. This reconstruction process is nearly always modified with image adjustments and preprocessing to reconstruct complex-valued images for subsequent (generally magnitude-only) statistical analysis. It has been shown that the two dimensional inverse Fourier transformation process in Equation 1 can be equivalently represented with a real-valued isomorphism representation [2]. This isomorphism representation stacks the real part of the rows of the array of spatial frequencies S_C upon the imaginary part of the rows of S_C to form a vector s that is pre-multiplied by an inverse Fourier transform reconstruction matrix Ω to form a vector ν of voxel values as

$$\underset{2n_xn_y\times 1}{\nu} = \underset{2n_xn_y\times 2n_xn_y}{\Omega} \quad \underset{2n_xn_y\times 1}{s} . \tag{2}$$

In Equation 2, both s and ν contain real and imaginary measurements. This establishes a linear relationship between spatial frequencies and voxel values. With both preprocessing and reconstruction operations quantified as linear matrix operators, Equation 2 is generalized as

$$\underset{2n_xn_y\times 1}{\nu} = \underset{2n_xn_y\times 2n_xn_y}{O_I} \quad \underset{2n_xn_y\times 2n_xn_y}{\Omega_a} \quad \underset{2n_xn_y\times 2n_xn_y}{O_k} \quad \underset{2n_xn_y\times 1}{s} \tag{3}$$

where O_k are potential operations in k-space, Ω_a is an inverse Fourier transform matrix that can be adjusted for T_2* and ΔB_0, and O_I are potential image-space operations in the AMMUST framework [3]. It was shown that the covariance matrix Σ of the reconstructed voxels can be represented as

$$\Sigma = O_I\Omega_a O_k \Gamma O'_k \Omega'_a O'_I \tag{4}$$

where Γ is the covariance matrix for the spatial frequencies [3]. From Equation 4, a correlation matrix can be formed. With uncorrelated spatial frequencies in Equation 4, Γ=I, it was shown that preprocessing and reconstruction operations can induce spatial correlation between voxels [3].

Methods:

The model in Equation 3 can be generalized to include the voxelwise SENSE multi coil image reconstruction method $\nu_C = (S_C^H \Psi_C^{-1} S_C)^{-1} S_C^H \Psi_C^{-1} a_C$ where in a given voxel, S_C is the complex-valued coil sensitivity, H denotes the transposed complex conjugate, Ψ_C is the complex-valued noise covariance matrix [5], and a_C is the vector of complex-valued aliased voxel measurements [4]. This generalization is

$$\nu = O_I P_2 \begin{pmatrix} u_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & u_p \end{pmatrix} P_1 \begin{pmatrix} O_{I1}\Omega_{a1}O_{k1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & O_{I1}\Omega_{an_c}O_{kn_c} \end{pmatrix} \begin{pmatrix} s_1 \\ \vdots \\ s_{n_c} \end{pmatrix}, \tag{5}$$

where n_c denotes the number of coils, s_j the aliased spatial frequency vector for coil j, O_{kj} the operations on aliased spatial frequency vector for coil j, Ω_{aj} the adjusted reconstruction operator for aliased spatial frequency vector for coil j, O_{Ij} the image space operations on the reconstructed image vector from the aliased spatial frequency vector for coil j, P_1 is a permutation that reorders values from by coil image to by voxel, u_q denotes an isomorphism matrix representation for a SENSE reconstruction of voxel q, p is the total number of voxels, P_2 is a permutation that reorders values from by unaliased voxel to unaliased image, and O_I are the image space operations on the combined reconstructed image vector. If Equation 5 is written as $\nu = Os$, then the covariance between voxels is $\Sigma = OO'$, with an identity spatial frequency correlation. The correlation matrix R can be found.

Results and Discussion:

A true noiseless vector of n_c=4 aliased image spatial frequencies was generated for a 48×48 image with a reduction factor of 3. The coil covariance matrix is real(Ψ_C)=imag(Ψ_C)=[1,ρ,ρ^2,ρ; ρ,1,ρ,ρ^2; ρ^2,ρ,1,ρ; ρ,ρ^2, ρ,1] where ρ=0.33. In reconstruction, O_{kj} included apodization of each image with a FWHM=2 voxels, Ω_{aj}=Ω, u_j contains the true sensitivity and coil covariance matrix, while O_I=I. An image of the induced correlation for magnitude squared data (whuch is asymptotically equivalent to magnitude data) from preprocessing for the center voxel is presented in Figure 1 superimposed upon the reconstructed mean image. It is readily apparent that there is induced local correlation from apodization and induced correlation of the center voxel of interest with two others regions from the SENSE unfolding procedure.

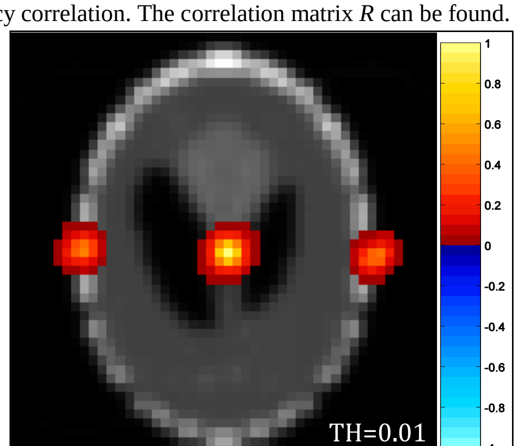

Figure 1. Induced correlation for the center voxel.

Conclusions:

Previous work that theoretically describes induced correlation between image voxels from spatial preprocessing and reconstruction operations has been summarized [3]. This previous work has been extended to include the SENSE multi coil image reconstruction method [4]. This has null hypothesis fcMRI connectivity implications as the no connectivity scenario is not for no spatial correlation but is rather for the spatial correlation induced by preprocessing.

References:

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