Influence of Mortar Rheology on Aggregate Settlement

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The influence of the rheology of fresh concrete on the settlement of aggregate is examined. Fresh concrete exhibits a yield stress that, under certain conditions, prevents the settlement of coarse aggregate, although its density is larger than that of the suspending mortar. Calculations, based on estimates of the yield stress obtained from slump tests, predict that aggregate normally used in concrete should not sink. To test this prediction, the settlement of a stone in fresh mortar is monitored. The stone does not sink in the undisturbed mortar (which has a high yield stress), but sinks when the mortar is vibrated, presumably due to a large reduction in its yield stress. This implies that during placement of concrete, the aggregate settles only while the concrete is being vibrated. A unique experimental method for measuring aggregate settlement is also introduced and demonstrated.

**Keywords:** aggregate; concrete; rheology; settlement; vibration.

**INTRODUCTION**

Fresh concrete is a multicomponent mixture consisting of cement, water, sand, coarse aggregate, and additives such as superplasticizers, silica fume, and fly ash. When water is added to the mixture of solids, it becomes a plastic concrete mixture, which, over time, sets and becomes a hard, rock-like material due to the hydration reactions that take place in the concrete.\(^1\)

It is generally believed that in a freshly placed concrete, the heavier coarse aggregate settles down while the lighter elements such as entrapped air, water, and mortar are pushed upward, resulting in certain undesirable effects such as bleeding and the top bar effect. The top bar effect is the phenomenon in concrete beams characterized by the fact that the bond stress developed on a horizontal bar placed at the top part of the beam is smaller than that on a bar at the bottom part of the beam. It is believed that due to the aggregate settlement, the bottom horizontal bars are mostly encased in a high-strength aggregate-rich medium, while the top horizontal bars are encased in aggregate on their upper sides, but with low-strength mortar or voids on their lower sides.

It is well-known that fresh concrete exhibits a yield stress\(^2\) below which it behaves as a solid, and above which it flows as a liquid. During placement and compaction, fresh concrete is usually vibrated or tapped to make it flow more easily. Under vibration, there is a significant reduction in the yield stress\(^3-5\) and a decrease in its viscosity.\(^3\) The presence of a yield stress in a medium retards and may even prevent the natural settlement of heavier solids in it.\(^6\) A solid may or may not sink in a medium, depending on the size of the solid, the magnitude of the yield stress of the medium, and the density difference between the solid and the medium. It is not clear whether coarse aggregate settles in concrete only during placement or also after placement. Also, during placement, it is not clear whether the coarse aggregate settles all the time or only during vibration. Therefore, there is a need to understand the influence of the rheology of concrete and the placement procedures (in particular, the effect of vibration) on aggregate settlement. This understanding could help shed some light on phenomena such as the top bar effect.

**RESEARCH SIGNIFICANCE**

In this study, the rate of settling of coarse aggregate in fresh concrete is predicted and measured. Contrary to widely held beliefs, it is shown that coarse aggregate settles in the fresh concrete only while vibration is applied to the concrete; before and after vibration, there is no settling. A criterion based on the yield stress was used to predict the aggregate settlement, and simple experiments were performed to verify these predictions.

**RHEOLOGY OF FRESH CONCRETE**

The rheology of fresh concrete is complex due to its composition and the accompanying chemical changes. Previous researchers have described fresh concrete as a complex non-Newtonian material that possesses a yield stress and a shear-rate dependent viscosity. Both the yield stress and the viscosity change with time.\(^4,5,7-9\) As the concrete sets, the yield stress and the viscosity increase greatly. In practice, the flow behavior of fresh concrete is simply represented by the Bingham model\(^2,10,11\)

\[
\tau = \tau_0 + \eta_p \gamma
\]

where \(\tau\) is the shear stress, \(\tau_0\) is the yield stress, \(\gamma\) is strain rate (or shear rate), and \(\eta_p\) is the plastic viscosity. \(\tau_0\) and \(\eta_p\) are extracted from plots of shear stress versus shear rate. As shown in Fig. 1, \(\eta_p\) is the slope of shear stress-shear rate curve, and \(\tau_0\) is the intercept on the ordinate axis.

When fresh concrete is subjected to vibration, previous researchers\(^5,7,8\) observed significant changes in its rheological properties. Using an apparatus described as a two-point workability apparatus, Tattersall et al.\(^3,4\) concluded that concrete under vibration loses its yield stress. Using a similar apparatus, Kakuta and Kojima\(^7\) found that concrete under vibration changes from a thixotropic material with a yield stress to an apparently non-thixotropic, shear-thinning material with little or no yield stress. de Larrard et al.\(^5,8\) using an apparatus fitted with a grooved parallel-plates geometry, found that, under vibration, the yield stress of the concrete mixtures used in their studies decreased to half its magnitude, and in some cases, became negligible. With regards to the viscosity, some researchers\(^3,6,7\) found that, under vibration, the plastic viscosity of concrete and the concrete becomes shear-thinning. de Larrard et al.\(^5\), however, found the plastic viscosity to be unaffected by vibration. Even if the...
plastic viscosity does not decrease under vibration, the concrete would still become more workable because of the decrease in the yield stress. This is because the viscosity is the ratio of shear stress to shear rate

\[ \eta = \frac{\tau}{\gamma} \]  

Thus, the effective viscosity for a Bingham plastic material at a particular shear rate \( \gamma_0 \) is given by

\[ \eta = \frac{\tau}{\gamma_0} = \frac{\tau_0}{\gamma_0} + \eta_p \]  

This value is equal to the slope of the line from the origin to the shear stress at that particular shear rate (shown as the lightly dotted lines in Fig. 1). Under vibration, flow curves obtained for concrete are below the flow curve obtained in the unvibrated state, \( 5,12 \) at least at low shear rates. \( 3,7 \) As is evident from Fig. 1, the slope of the dotted lines at a particular shear rate \( \gamma_0 \) are smaller when the concrete is vibrated than for the corresponding case when the concrete is not vibrated. Therefore, the effective viscosity is expected to be smaller; hence, the workability of the concrete is expected to be better when it is subjected to vibration than when it is not.

The presence of a yield stress in concrete mixtures has not been explained in terms of their microstructure, and only recently has there been some work in that direction. \( 1,13 \) The concentration of solids in concrete is very high. The size of these solid particles varies anywhere from a fraction of a micron to a few centimeters, and the morphology of these particles also varies. The yield stress of concrete mixtures may have three primary contributing sources. One source is the mechanical interlocking of the larger, irregularly shaped, rough aggregate that forms the basic skeleton of the concrete network. \( 9 \) The second source of yield stress may be the attractive colloidal forces between the cement and other sub-micron particles that causes them to flocculate. \( 13 \) The third source (and the one that contributes to the ultimate hardening of the concrete mixtures and cements) is a colloidal gel of hydrated calcium silicate that forms around the cement particles as a result of the reaction between the cement and water. \( 7 \)

The causes for the decrease in yield stress due to vibration are not understood either. It is speculated that vibration deflocculates the cement aggregate and also breaks the initially weak chemical bonds resulting from gelation in fresh cement paste. The vibration may also cause the larger aggregate to jiggle, thus unlocking the initially interlocked skeleton. Therefore, the reduction in the yield stress of concrete mixtures under vibration may be related to the weakening of the mechanical and chemical bonds among its ingredients.

**CRITERIA FOR AGGREGATE SETTLEMENT**

Consider a solid sphere submerged in a Newtonian fluid (zero yield stress) with a density lower than that of the solid. When the Reynolds number is very small, the solid sinks in the fluid with a terminal velocity \( U_t \) that can be derived from the Stokes drag to be

\[ U_t = \frac{2 R^2 (\rho_s - \rho_f) g}{9 \mu} \]  

where \( R \) is the radius of the sphere, \( \rho_s \) is the density of the sphere, \( \rho_f \) is the density of the fluid, \( g \) is the acceleration due to gravity, and \( \mu \) is the viscosity of the fluid.

If the fluid possesses a yield stress, however, the sphere may not sink at all, even if its density is greater than the density of the fluid. The criterion for a sphere to settle in a fluid with yield stress is available. Beris et al. \( 6 \) predicted that a spherical particle will settle in a fluid with Bingham plastic behavior only when the dimensionless group referred to as the yield-stress parameter \( Y_g \), defined below, is less than 0.143

\[ Y_g = \frac{3 \tau_0}{2 R (\rho_s - \rho_f) g} \]  

where \( \tau_0 \) is the yield stress of the fluid. Since the unvibrated concrete possesses a significant yield stress, coarse aggregate may or may not settle in it. The previously mentioned criterion will be used as follows to determine whether coarse aggregate will settle in the concrete samples used in this study.

**PRELIMINARY ASSESSMENT**

The yield stress of the concrete mixtures used in this study, for use in Eq. (5), was estimated from slump data \( 14 \) with the
help of Fig. 2. In Fig. 2, the dimensionless slump is plotted against the dimensionless yield value. The dimensionless slump is the ratio of the slump \( s \) to the initial height \( H \) which is the height of the cone used for the slump tests. The dimensionless yield stress is the ratio of the yield stress to the product \( \rho gH \), where \( \rho \) is the density of the concrete mixture. The solid line in Fig. 2, which represents the theoretical relationship between the dimensionless slump and the dimensionless yield stress established by Schowalter and Christensen, was used to estimate the dimensionless yield stress from the dimensionless slump data.

The ratios, by mass, of cement, aggregate, sand, and water in the concrete mixture used in this study were 1.0:1.7:1.25:0.5 (C:A:S:W). When there was no vibration, the average slump from two tests was 95 mm (3.75 in.). The ASTM C 143 standard cone used in the slump tests is 305 mm (12 in.) tall, giving a dimensionless slump value of 0.31 (95/305 mm) for the concrete mixture. From Fig. 2, the dimensionless yield value that corresponds to a dimensionless slump of 0.31 was found to be 0.11. Taking the density of concrete to be 2240 kg/m\(^3\), the yield stress of the unvibrated concrete was estimated to be 740 Pa.

Using a density of 2650 kg/m\(^3\) for the aggregate and 2240 kg/m\(^3\) for the concrete mixture, Eq. (5) predicts that a common-size aggregate, with a diameter less than 50 mm (2 in.), will not sink in concrete with a yield stress of 740 Pa. In fact, the aggregate would have to have a diameter larger than 3860 mm (3.86 m, or approximately 152 in.) for it to sink. Even in concrete with a density as low as 1000 kg/m\(^3\), when the other factors remain constant, the diameter of the smallest aggregate that would sink is estimated to be 960 mm (38 in.). Therefore, based on the previous calculations, common-size aggregate should not settle in the undisturbed, unvibrated concrete.

When vibration was applied (using a wand vibrator) to the concrete following the removal of the slump cone, however, the fresh concrete completely spread over the tray in which it was placed. In this layer, the coarse aggregate went to the bottom, while the mortar remained on top. Since the fresh concrete readily flowed under vibration, thus maximizing its slump, its yield stress under vibration was considered to be very small, or even zero. It was expected that the yield stress of fresh concrete under vibration would be sufficiently small for the yield-stress parameter to be less than 0.143. Thus, the aggregate is expected to settle in fresh concrete under vibration.

To verify the previous predictions, experiments were conducted in which the settlement of an irregularly shaped stone in mortar was monitored. The purpose of these tests was to confirm the previous prediction that the stone sinks in fresh concrete only under vibration. The materials and the experimental procedure are described as follows.

**MATERIALS**

The ratios, by mass, of cement, sand, and water in the mortar used in this study were 1.0:1.25:0.42 (C:S:W). The cement used was Type III, high-early-strength portland cement. The specific gravity of this mortar was measured to be 2.17. The mortar exhibited a slump of 155 mm, and the yield stress value that was estimated using Fig. 2 was 430 Pa.

A stone with dimensions approximately 147 x 92 x 57 mm (5.8 x 3.6 x 2.2 in.) and a mass of 0.95 kg (2.1 lb) was used. This stone was irregular in shape, and was taken to be representative of the shape of the coarse aggregate used in concrete.

**EXPERIMENTAL SETUP**

The schematic of the experimental setup used is shown in Fig. 3. Well-mixed mortar was placed in a stainless-steel tank, and the surface exposed to air was flattened. Then the stone described previously, with an aluminum plate attached to its upper side, was placed gently on the surface of the mortar. The depth to which the stone sank in the mortar was measured with the help of an electrical circuit connecting a depth meter to the aluminum plate. When the bar of the depth meter touched the aluminum plate, the bulb in the circuit lit up, indicating contact between the bar and the plate. The depth meter could measure distances with a precision of 0.025 mm (0.001 in.).

The experiment was conducted by first adjusting the depth meter such that its bar just touched the aluminum plate (causing the bulb to light up). If the stone settled in the mortar, the bar of the depth meter and the aluminum plate would sepa-
rate, turning off the bulb. The bar of the depth meter could then be lowered until it again made contact with the aluminum plate, relighting the bulb. The distance by which the depth meter was lowered would be a measure of the depth to which the stone had sunk.

RESULTS AND DISCUSSION

Initially, when the mortar was not vibrated, the bulb was continuously on for about 300 s, indicating that the stone did not sink during this period of observation. When vibration was later applied to the mortar, the stone sank approximately 76 mm in 20 s. The stone kept sinking as long as vibration was applied, but immediately stopped sinking when vibration stopped.

For a yield stress of 430 Pa, Eq. (5) predicts that the aggregate will not sink unless its diameter is larger than 1920 mm (76 in.). To underscore the previous conclusion, a large rock was placed on the surface of fresh mortar (Fig. 4 (a)). The dimensions of the rock were approximately 460 x 330 x 180 mm (18 x 13 x 7 in.), which is much larger than those of most common-size aggregates used in concrete. When the mortar was not vibrated or disturbed, the rock did not sink. This can be confirmed by comparing the line markers in Fig. 4(b) and (c) taken 180 s apart. The mortar was then subjected to vibration by tapping the side of the tank with a paddle. The stone sank a little with each tap. Figure 4(d) shows the position of the stone after tapping the tank 20 times. The stone kept sinking as long as vibration was applied, but came to a stop immediately after the vibration stopped.

Fig. 4—Experimental setup of large stone placed on fresh mortar.
The settlement of the stone in the mortar under vibration is attributed to the decrease in the yield stress of the mortar. In the unvibrated state, the mortar has a yield stress high enough to prevent the stone from sinking. Under vibration, the yield stress decreases below the critical value, and the stone sinks. The mortar apparently recovers its yield stress quickly once the stress decreases below the critical value, and the stone sinks. To prevent the stone from sinking, the unvibrated mortar has a yield stress high enough to prevent the stone from further settlement. Under vibration, it may be concluded that aggregate of the size normally used in concrete will not settle under its own weight.

Although the settlement of the stone in mortar under vibration may be attributed primarily to the reduction in yield stress of the medium, in a recent paper, Wünsch developed a criterion for settlement of a sphere under vibration without assuming a decrease in the yield stress of the suspending medium. According to Wünsch, when harmonic vibrations were applied to the fluid in which the sphere was immersed, an extra stress was generated around the sphere. The extra stress exceeds the yield stress of the fluid when the frequency and amplitude of the vibration exceed a threshold value, causing the sphere to settle in the fluid. This means that even if the vibration does not completely reduce the yield stress of mortar to zero, the settlement of a stone could be enhanced by a generation of extra stress around the stone by applying vibration above a certain threshold frequency and amplitude of vibration.

The experimental results are in agreement with the predictions presented in the previous section. Since the large stone used in this experiment did not settle in the mortar without vibration, it may be concluded that aggregate of the size normally used in concrete will not settle under its own weight. This should be especially true for aggregate in concrete, rather than pure mortar, since concrete has a smaller water-solids ratio than mortar, and should, therefore, have a larger yield stress than mortar. The results of the slump tests are consistent with this expectation. The aggregate will only settle if the concrete is subjected to vibration or other such disturbance.

Prediction of settling velocity of aggregate in mortar

Normally, concrete is vibrated for about 15 to 30 s during placement. This vibration will cause the aggregate to settle in the concrete. To estimate the distance that an aggregate settles in typical concrete due to vibration, the settling velocity of the aggregate was calculated. The settling velocity of coarse aggregate 50 mm (2 in.) in diameter with a density of 2650 kg/m³ in a mortar with a density of 2170 kg/m³ is presented in Fig. 5(a). In this figure, the settling velocity for this aggregate is plotted as a function of the viscosity and the yield stress of the mortar. The settling velocity of the aggregate was computed from the values of Stokes drag coefficient \( C_S \), which, in turn, was estimated from the following asymptotic relationships established by Beris et al.\(^6\)

\[
\ln(C_S - 1) = 0.91 + 0.55 \ln N_B \quad \text{for} \quad 0.005 \leq N_B \leq 0.012, \quad (6)
\]

\[
C_S = 1 + 1.874N_B^{1/2} + 1.152N_B \quad \text{for} \quad 0.012 \leq N_B \leq 600, \quad \text{and}
\]

\[
C_S = \frac{1.031Y_g}{(Y_{gl} - Y_g)^2} \quad \text{for} \quad N_B \geq 600
\]

where \( N_B \) is the Bingham number, and \( Y_{gl} (= 0.143) \) is the limit value for the yield stress parameter \( Y_g \). The Stokes drag coefficient \( C_S \) is defined for a sphere under free fall in a fluid with a zero-shear viscosity \( \eta_0 \) as\(^6\)

\[
C_S = \frac{2R}{9\eta_0 U_t}
\]

Thus, \( C_S \) is equal to unity for spheres falling in Newtonian fluids at low Reynolds numbers.

The Bingham number \( N_B \) is a dimensionless number that is a measure of the ratio of the yield stress to the viscous stress, and is defined as\(^6,17\)

\[
N_B = \frac{2\tau_0 R}{\eta_0 U_t}
\]

Note that the yield stress parameter \( Y_g \) defined in Eq. (5) can be related to \( C_S \) and \( N_B \) by the simple relationship

\[
Y_g = \frac{N_B}{6C_S}
\]

The dashed vertical line shown in Fig. 5(a) indicates the yield stress (11.4 Pa), calculated using Eq. (5), above which an aggregate 50 mm in diameter would not sink in the mortar. The corresponding settling distances for a time period of 30 s, obtained by multiplying the settling velocity by the time, are plotted in Fig. 5(b). After placement and while the mortar is setting, the yield stress is sufficiently high to prevent the coarse aggregate from further settlement.

Using Fig. 5(b), it may be estimated that 50 mm diameter coarse aggregate could settle during vibration approximately 40 mm in 30 s. For this calculation, an estimate of the viscosity of the mortar obtained by the previously mentioned settlement experiment with the 0.95 kg (2.1 lb) stone was used. Under vibration, the stone sank 76 mm in 30 s, corresponding to a settling velocity of 3.8 mm/s. Neglecting wall effects, assuming the mortar under vibration to be a Newtonian fluid with zero yield stress, and the density of the stone to be 2650 kg/m³, the
The viscosity of the mortar was estimated using Eq. (4) to be 530 Pa.s. Since the stone was not spherical, the hydraulic radius of the stone, instead of the radius of a sphere, was used in Eq. (4). The hydraulic radius of the stone was calculated as follows

\[
a_{\text{hydraulic}} = \left( \frac{m/\rho_s}{\frac{4}{3} \pi} \right)^{1/3} = 44 \text{ mm} \quad (10)
\]

where \( m \) and \( \rho_s \) are the mass and density of the stone, respectively.

**Top bar effect**

As mentioned in the introduction, it has been previously suggested that aggregate in concrete settles, while the lighter elements, such as entrapped air, water, and mortar, rise in the fresh concrete. Such settlement is believed to be the cause of the top bar effect, as the bottom horizontal bars would be mostly encased in a high-strength aggregate-rich medium, while the top horizontal bars would be encased in a low-strength aggregate-poor medium.

During placement, concrete is typically subjected to vibration to help the concrete fill the form and become compacted. As mentioned previously, the vibration reduces the yield stress of the concrete, causing the heavier aggregate to settle and the lighter phases to rise. After the concrete is placed and while it is setting, however, in the absence of any further vibration, the concrete would regain its yield stress and the aggregate will not settle further. Therefore, if the top bar effect is due to the settlement of the aggregate, then this effect will originate only during the vibration of the concrete. Further, the application of the vibrator would have the most critical influence on the top bar effect.

**Effect of vibration parameters**

In this study, a commercial vibrator was used that does not allow the frequency or amplitude of the vibration to be varied. Consequently, it was not possible to study the effect of the frequency and amplitude of the vibration on the settling velocity of aggregate in mortar. Since the intent of this study was only to show that vibration is what causes the aggregate to settle, the influence of vibration parameters on settling will be part of a future study. Tattersall and Baker, however, reported that at low shear rates, the inverse of fluidity (which is a measure of the viscosity) decreases with increasing frequency and amplitude of the vibration. Therefore, aggregate settlement is expected to become more pronounced when the frequency and amplitude of vibration are increased.

**Effect of superplasticizers**

Superplasticizers are usually surfactants and resins that are added to reduce the surface tension of water. The addition of superplasticizers to cement paste and concrete mixtures has two consequences: a significant reduction in the yield stress, and a marginal decrease in the plastic viscosity. These decreases may be attributed to the deflocculation of cement particles caused by the superplasticizers. Since the superplasticizer is expected to reduce the yield stress, aggregate is expected to settle faster under vibration in concrete mixtures containing superplasticizers than in those without.

**FURTHER STUDIES OF CONCRETE RHEOLOGY**

To further study the effects of mortar and concrete rheology on the settlement of aggregate, a more complex experimental procedure is required. To verify the theoretical predictions, it is necessary to look inside the fresh concrete before, during, and after vibration. The authors have proposed and demonstrated a unique method of accomplishing exactly that.

The authors have successfully completed a number of pilot tests that measure the settlement of spheres in fresh mortar
and concrete. These experiments utilize a scintillation camera (Fig. 6(a)) to look inside fresh concrete at metallic spheres that have been tagged with a radioactive isotope. The camera is programmed to record an image every 1/2 s while the fresh concrete is vibrated (Fig. 6(b)). The camera easily records the location of the tagged spheres (Fig. 6(c)). For the pilot experiments, both aluminum and steel spheres were used. The aluminum (Al) has a density similar to aggregate while the steel (Fe) has a density considerably higher, permitting multiple-point verification of the theoretical equations.

A sample of the pilot study output is presented in Fig. 6(c). Each panel represents a single time step in the recorded history. The mortar specimen shown is 305 mm (12 in.) square and has a 152 mm (6 in.) thickness. Three spheres were placed on the mortar surface, as shown in the left-most panel. The spheres were allowed to sit in the fresh mortar for 15 s; the mortar was then vibrated for 30 s, and then allowed to sit for a further 15 s. As can be seen in Fig. 6(c), the spheres only settled during vibration, and were stationary both before and after. Additionally, the following observations may be made from Fig. 6(c):

1. The denser steel (Fe) sphere settles considerably faster than the aluminum (Al);
2. Greater settlement is observed closer to the vibrator (compare the two aluminum spheres);
3. There appears to be a trend for the spheres to settle both downward and toward the vibrator; and
4. The spheres do not settle without the presence of direct vibration.

These observations appear to support the theoretical and experimental results presented previously. Results of this pilot study are being prepared for publication in a companion paper.

CONCLUSIONS

Aggregate does not sink in typical concrete mixtures unless subjected to vibration. From simple theoretical predictions and experiments, it was found that a stone, larger in size than typical aggregate, placed on the exposed surface of mortar does not sink under its own weight unless the mortar is vibrated. The same conclusion applies to the settlement of aggregate in concrete mixtures. The settlements under vibration were attributed to the reduction of the yield value to a lower, or even negligible, value in concrete mixtures subjected to vibration. Finally, the origin of the top bar effect, commonly observed in beams containing horizontal bars, has been attributed to the vibration of concrete mixtures during their placement.

The authors also introduced and demonstrated a unique experimental technique for measuring aggregate settlement in concrete or mortar mixtures.

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NOTATIONS

\[ a_{\text{hydraulic}} = \text{hydraulic radius of irregular stone, mm} \]
\[ g = \text{acceleration due to gravity constant (= 9.806 m/s}^2) \]
\[ m = \text{mass of stone, kg} \]
\[ R = \text{radius of sphere, mm} \]
\[ U = \text{terminal velocity, mm/s} \]
\[ Y = \text{yield stress parameter} \]
\[ Y_f = \text{limit value of } Y \]
\[ \eta = \text{effective or apparent viscosity, Pa·s} \]
\[ \eta_0 = \text{zero-shear viscosity, Pa·s} \]
\[ \eta_p = \text{plastic viscosity, Pa·s} \]
\[ \mu = \text{Newtonian viscosity, Pa·s} \]
\[ \rho = \text{density of solid sphere or stone, kg/m}^3 \]
\[ \rho_f = \text{density of fluid or continuous medium, kg/m}^3 \]
\[ \tau = \text{shear stress, Pa} \]
\[ \tau_0, \tau_y = \text{yield stress, Pa} \]
\[ \dot{\gamma}, \dot{\gamma}_0 = \text{strain rate or shear rate, s}^{-1} \]

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