9-1-2010

Processing Induced Voxel Correlation in SENSE FMRI Via the AMMUST Framework

Daniel Rowe
Marquette University, daniel.rowe@marquette.edu

Iain P. Bruce
Marquette University - Graduate Student

Processing Induced Voxel Correlation in SENSE FMRI Via the AMMUST Framework

Daniel B. Rowe¹,² and Iain P. Bruce¹
¹Department of Mathematics, Statistics, and Computer Science, Marquette University Milwaukee, WI, USA
²Department of Biophysics, Medical College of Wisconsin, Milwaukee, WI, USA

Synopsis

Many preprocessing steps are applied prior to fMRI statistical analysis and their effects ignored. Nencka et al. (2009) presented the AMMUST-I model to examine preprocessing and reconstruction induced correlation. We extend the AMMUST-I model to include the SENSE multi coil image reconstruction method. We find induced correlations between a voxel and its unfolded counterparts. This body of work has novel hypothesis baseline fMRI implications.

Introduction

In fMRI, images of objects are Fourier encoded [1] similar to the Fourier transform in Figure 1.

\[
\begin{align*}
\mathbf{\Omega}_a + \mathbf{\Omega}_a & \ast (\mathbf{V}_a + \mathbf{V}_a) \ast (\mathbf{\Omega}_a + \mathbf{\Omega}_a)^T = (\mathbf{F}_a + \mathbf{F}_a) \\
(\mathbf{\Omega}_a + \mathbf{\Omega}_a) & \ast (\mathbf{V}_a + \mathbf{V}_a) \ast (\mathbf{\Omega}_a + \mathbf{\Omega}_a) \\
(\mathbf{\Omega}_a + \mathbf{\Omega}_a)^T & \ast (\mathbf{V}_a + \mathbf{V}_a)
\end{align*}
\]

as in Figure 3 when \( \mathbf{O}_a \) and \( \mathbf{O}_a \) are identity matrices and \( \mathbf{\Omega}_a = \mathbf{\Omega} \) however preprocessing in k-space can be performed with \( \mathbf{O}_a \) and in image space with \( \mathbf{O}_a \) and adjusting for \( T_2^* \) and \( \Delta B_0 \) through \( \mathbf{\Omega}_a \). It was shown that the reconstructed voxels covariance matrix \( \mathbf{\Sigma} \) can be represented as

\[
\text{cov}(v) = (O_a \mathbf{\Omega}_a O_a^T) \Gamma (O_a^T \mathbf{\Omega}_a^T O_a^T) = \mathbf{\Sigma}
\]

where \( \Gamma \) is the covariance matrix for the spatial frequencies [3]. With \( \Gamma = \mathbf{I} \), it was shown that preprocessing and reconstruction operations can induce spatial correlation between voxels [3].

Methods

Equation 3 is generalized to include SENSE multi coil image reconstruction

\[
v_a = (S^T \mathbf{\Psi}_a S)^{-1} S^T \mathbf{\Psi}_a a
\]

where in a given voxel, \( \mathbf{S}_a \) is the complex-valued coil sensitivity, \( \mathbf{\Psi}_a \) is the complex-valued noise covariance matrix [5], and \( a_0 \) is the vector of complex-valued aliased voxel measurements [4]. This generalization is

\[
v = P_1 \ast O_0 \mathbf{\Omega}_a P_1^T \ast O_0 \mathbf{\Omega}_a \ast f
\]

where \( f \) is the aliased spatial frequency vector for coil \( j \), \( O_{aj} \) the operations on aliased spatial frequency vector for coil \( j \), \( \mathbf{\Omega}_{aj} \) the adjusted reconstruction operator for aliased spatial frequency vector for coil \( j \), \( \mathbf{I} \) the image space operations on the reconstructed image vector from the aliased spatial frequency vector for coil \( j \), \( P_1 \) is a permutation that reorders values from by coil image to by voxel, \( u_j \) denotes an isomorphism matrix representation for a SENSE reconstruction of voxel \( q \), \( p \) is the total number of voxels, \( P_2 \) is a permutation that reorders values from by unaliased voxel to unaliased image, and \( O_0 \) are the image space operations on the combined reconstructed image vector. If Equation 6 is written as \( v = O f \), then the covariance between voxels is \( \mathbf{\Sigma} = O O^T \), with an identity spatial frequency correlation. The correlation matrix \( R \) can be found.

Results

A true noiseless vector of \( n=4 \) aliased image spatial frequencies was generated for a 96×96 Shepp-Logan phantom image scaled by 50 with an AP reduction of \( R=3 \). The coil covariance matrix is real(\( \mathbf{\Sigma}_c \)) = imag(\( \mathbf{\Sigma}_c \)) = \( \{1, \rho, \rho^2, \rho^3, 1, \rho, \rho^2, \rho^3, \rho, 1, \rho, \rho^2, \rho, 1\} \) where \( \rho = 0.33 \). In reconstruction, \( O_{ij} \) included apodization of each image with a FWHM=2 voxels, \( \mathbf{\Omega}_a = \mathbf{\Omega} \), \( u_j \) contains the true sensitivity and coil covariance matrix, while \( O_0 = \mathbf{I} \). The induced correlation image for magnitude squared data for the center voxel is in Figure 5 (left) overlaid upon the reconstructed mean image. It is apparent that there is induced local correlation from apodization and induced correlation of the center voxel of interest with two others regions from the SENSE unfolding procedure. In Figure 5 (right) the voxel standard deviation is displayed with true value being one.

Discussion

Previous work that theoretically describes induced correlation between image voxels from spatial preprocessing and reconstruction operations has been summarized [3]. This previous work has been extended to include the SENSE multi coil image reconstruction method [4]. This has null hypothesis fMRI connectivity implications as the no connectivity scenario is not for spatial correlation but is rather for the spatial correlation induced by preprocessing.

References


Supported in part by NIH EB000215 & EB007827.