10-2019

Mathematical Modeling Opportunities Reported by Secondary Mathematics Preservice Teachers and Instructors

Hyunyi Jung
Marquette University, hyunyi.jung@marquette.edu

Eryn Stehr
Georgia Southern University

Jia He
Augusta University

School Science & Mathematics, Vol. 119, No. 6 (October 2019): 353-365. DOI.

Mathematical Modeling Opportunities Reported by Secondary Mathematics Preservice Teachers and Instructors

Hyunyi Jung
Department of Mathematical and Statistical Science, Marquette University, Milwaukee, Wisconsin
Eryn M. Stehr
Department of Mathematical Sciences, College of Science and Mathematics, Georgia Southern University, Statesboro, Georgia
Jia He
Department of Mathematics, Augusta University, Augusta, Georgia

ABSTRACT
Engaging with mathematical modeling can support learners to collaboratively explore mathematics in integrated ways as well as generate mathematical ideas and representations that may be useful in everyday life. Although several studies provide diverse insights into teaching and learning mathematical modeling, research has yet to be conducted on the mathematical modeling learning

opportunities available to secondary mathematics preservice teachers (PTs) in mathematics and education courses in teacher education programs. This study investigates the mathematical modeling learning opportunities reported by 48 instructors and ten focus groups of 37 PTs. Multiple data sources (e.g., interview transcripts, syllabi, tasks, and exams) collected from universities were used to achieve triangulation in this case study of secondary preparation programs. When asked about mathematical modeling, both PTs and instructors reported rich examples of mathematical modeling from the opportunities afforded by their respective programs. Both also reported modeling experiences that were not mathematical modeling, such as word problems, representations, or demonstrations. Along with the study's particular themes and examples, common mathematical modeling opportunities recalled by PTs and instructors are elaborated in our findings. This study intends to begin a discussion of possible pathways for providing rich opportunities for PTs to engage in mathematical modeling.

Keywords

algebra; learning processes; math/math education; teacher education; teachers and teaching

INTRODUCTION

Engaging with mathematical modeling can support learners to collaboratively explore mathematics in integrated ways as well as generate ideas and representations that may be useful outside of classrooms and in everyday life (Lesh & Doerr, [14]). Mathematical modeling, an iterative process of developing mathematical representations to provide insights into real-world problem-solving situations, is a mathematical practice every student is expected to develop (Consortium for Mathematics and its Applications & Society for Industrial and Applied Mathematics [COMAP & SIAM], [5]; Lesh, English, Riggs, & Sevis, [15]; National Governors Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], [4]). Despite this, a major challenge for teachers "is the 'conceptual fuzziness' about what counts as a modeling activity" (Cai et al., [3], p. 146). Because the term *modeling* is used in various ways in mathematics and education courses, preservice teachers (PTs) do not naturally distinguish between *mathematical* and *nonmathematical* modeling. For example, Anhalt and Cortez ([1]) found their PTs initially viewed mathematical modeling as using representations of mathematical objects or demonstrating mathematical procedures.

To support PTs' understanding of mathematical modeling, the Association of Mathematics Teacher Educators (AMTE) recommended that teacher education programs provide PTs with consistent opportunities to engage in mathematical modeling (Association of Mathematics Teacher Educators [AMTE], [2]). Research studies on teacher education programs showed that learning opportunities varied considerably according to programs' emphases, courses, and field experiences (e.g., Gansle, Noell, & Burns, [9]). Several studies provide valuable insights into secondary mathematics teachers' learning of mathematical modeling as one of the outcomes (e.g., Anhalt & Cortez, [1]; Doerr & English, [8]; Tan & Ang, [23]). However, research into experiences or perceptions of learning mathematical modeling (e.g., learning opportunities) in secondary preservice programs has yet to be conducted.

In this article, we attempt to close this research gap by exploring mathematical modeling across five teacher preparation programs. Specifically, we focus on mathematical modeling opportunities reported by 37 PTs and 48 course instructors of mathematics and education courses. To consider mathematical modeling experiences across an entire teacher preparation program, we triangulated

PTs' reports of *experienced* modeling encounters in all the required courses with instructors' reports of *intended* mathematical experiences within individual courses, comparing the reports with corresponding instructional materials. Our research questions were: (a) What are PTs' recollections of mathematical modeling experiences across the whole of their teacher preparation programs? (b) What mathematical modeling opportunities do instructors report that they intended to provide in their courses? and (c) What are common mathematical and nonmathematical modeling opportunities reported by both instructors and PTs?

THE PROCESS AND PURPOSE OF MATHEMATICAL MODELING

The *Common Core State Standards in Mathematics* (*CCSSM*) described mathematical modeling in both content and practice standards (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA & CCSSO], [4]). The *CCSSM* recommended that students identify and select variables to formulate a model; analyze and perform operations; and interpret, validate, improve, and report conclusions. Some *CCSSM* modeling cycle elements are less emphasized than others (Meyer, [17]; Pollak & Garfunkel, [20]). Pollak and Garfunkel described teachers' conceptions of the mathematical modeling process as often lacking cycle elements, namely: (a) making choices and assumptions to create a model; (b) validating a model by considering a solution's mathematical correctness; and (c) making sense of solutions within a modeling context. Similarly, Meyer ([17]) found many tasks in Algebra 1 and Geometry textbooks lacked components of identifying variables in a situation and validating conclusions. Connecting deficits across these studies, we argue that PTs may need experiences that include decision-making about assumptions and variables that translate a real-world situation into a mathematical model. They also need experiences of validating models and conclusions by comparing mathematical correctness and the practical implications of a solution.

In addition to elements of the modeling cycle, PTs need experiences with modeling as both a *vehicle* and as *content* itself (Julie, [13]). According to Julie, the purpose of mathematical modeling as a *vehicle* is to support "learning of mathematical concepts, procedures and at times justification," rather than to gain a deeper understanding of mathematical modeling itself (p. 2). In contrast, modeling as *content* entails "scrutiny, dissection, critique, extension, and adaptation" of models, with a view to coming "to grips with the underlying mechanisms of mathematical model construction," rather than that "certain mathematical concepts or procedures should be the outcome." (p. 3).

Julie ([13]) argued that teachers exposed to mathematical modeling only as a vehicle often sought out existing formulas rather than attempting to describe and analyze the context, leading to a superficial interaction with the mathematical structures of situations. This result follows a similar structure to that of many textbooks: students learn concepts first and then use those contextualized problems to reinforce and apply procedures or formulas, rather than deeply exploring the mathematics embedded in situations. This pattern for teachers and students may lead to answer-seeking rather than creative problem solving and sense-making.

The first sentence of the Standards for Mathematical Practice 4: Model with Mathematics reads: "Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life" (NGA & CCSSO, [4]). Lesh et al. ([15]), by contrast, argue that this sentence can be misleading and does not capture the essential role of mathematical modeling as content in and of itself, but may contribute to the use of modeling as a way to reinforce and apply mathematical concepts or procedures. Indeed, *mathematizing* situations (e.g., describing, quantifying, categorizing, or systematizing) (Lesh & Sriraman, [16]) is closely related to Julie's ([13]) description of modeling as content and to CMA and SIAM's ([5]) recommendation for learners to experience mathematical modeling as a process.

TEACHERS' LEARNING OF MATHEMATICAL MODELING

Both AMTE ([2]) and the Conference Board of the Mathematical Sciences (CBMS, [21]) emphasized the need for teachers to learn mathematical modeling. Pollak and Garfunkel ([20]) reported that practicing teachers believed the curriculum presents modeling as a vehicle to learn mathematics rather than as content itself. To teach mathematical modeling as content, teachers need to develop specialized knowledge related to task selection, implementation, and assessment (Doerr, [7]). Doerr recommended that teachers understand characteristics of situations that support student engagement with model development through iterative cycles of mathematical modeling. Doerr further recommended that teachers have opportunities to engage with diverse mathematical modeling tasks.

Secondary mathematics teachers' learning via mathematical modeling tasks and teaching of mathematical modeling has been examined in several studies (e.g., Anhalt & Cortez, [1]; Doerr & English, [8]; Tan & Ang, [23]). Anhalt and Cortez ([1]) examined PTs' evolving understanding of mathematical modeling as they participated in a mathematical modeling module in their secondary teacher education program. Throughout the module, PTs developed a conception of mathematical modeling as a process involving making assumptions and revising results that connected to real-life situations (Anhalt & Cortez, [1]). Tan and Ang ([23]) described secondary mathematics teachers' development of mathematical modeling teaching practices during a school-based professional development program. Doerr and English ([8]) examined the impact of mathematical modeling tasks on the learning of secondary teachers. They described the new mathematical understanding that two secondary mathematics teachers developed as they observed and listened to students rather than evaluating student ideas. These teachers noticed diverse ways in which students' mathematical thinking developed through engaging with the authentic problem-solving scenarios of mathematical modeling tasks (Doerr & English). These three studies are examples of research that has provided critical information about secondary mathematics teachers' learning of mathematical modeling as one of the outcomes. As we focus more on the experiences and perceptions of PTs' learning of mathematical modeling, we aim to describe the mathematical modeling learning opportunities available to future secondary teachers across their programs.

METHOD

This case study is a component of an NSF research project, Preparing to Teach Algebra, which focused on PTs' opportunities to learn and teach algebra. We used Project Preparing to Teach Algebra data collected from five universities. The universities were selected with the intention of identifying the broad attributes of secondary teacher preparation programs. Table presents the characteristics of each program, including the average number of graduates per academic year, the degree a student obtains upon graduation, the program's home department within the university, the university's geographic location and locale, and the race/ethnicity of enrolled undergraduate students.

Table 1. Case study secondary mathematics teacher education program characteristics

University	**GLU**	**MRU**	**MUU**	**SRU**	**WUU**
Avg. graduates	34	22	12	39	30
Degree upon completion	4-year BA	4-year BA	4-year BA	4-year BA	None (Postbaccalaureate)
Basic Carnegie classification	Master's: Larger Programs	Doctoral: Highest Research	Master's: Larger Programs	Doctoral: Highest Research	Master's: larger programs
Academic home	Math Dept.	College of Ed	College of Ed	College of Ed	College of Ed
U.S. region	Great Lakes	Midwest	Midwest	Southeast	Western
Locale	Small City	Mid-size City	Large City	Mid-size City	Large City
Race/Ethnicity	5.4% Latin@	6.0% Latin@	37.8% Latin@	6.1% Latin@	63.9% Latin@
81.9% White	72.4% White	30.5% White	71.6% White	6.5% White	

1 Average annual number of secondary mathematics program graduates across three academic years prior to study.

2 See Carnegie classifications for sources & definitions (http://carnegieclassifications.iu.edu/).

3 See National Center for Education Statistics Common Core of Data sources & definitions: http://nces.ed.gov/ccd/.

4 We use the @ sign to include all gender identifications.

Prior to receiving research funding, we selected three universities (i.e., Great Lakes University [GLU], Midwestern Research University [MRU], and Midwestern Urban University [MUU]) to represent distinct university profiles. Our selection factors included the average number of graduates, the research profile, the locale (size of city), and the diversity of the student population. We chose two additional programs (i.e., Southeastern Research University [SRU] and Western Urban University [WUU]) based on our national survey responses, recommending we add geographic and racial/ethnic diversity to our selection. We constructed university pseudonyms to describe the geographic location and research profile. For example, Southeastern Research University is in the southeastern region of the United States and is a research university.

Selected courses and participants

Project Preparing to Teach Algebra researchers worked with a site coordinator from each university, chosen for his/her knowledge about their secondary mathematics education program, to select the courses with the greatest potential to answer the research questions of the larger study, including PTs' opportunities to learn about mathematical modeling. The overarching goal of the research team was to gather data that would provide a sense of each secondary teacher education program as a whole. Hence, we were interested in PTs' experiences at program level rather than at the level of individual courses. We held focus groups of senior PTs who were nearing completion or had recently completed the program, asking them to reflect on experiences across all required program courses. PTs could have shared detailed experiences from courses if we had interviewed them throughout each course. Rather than capturing all experiences, we focused on key ideas remaining in PTs' memories, assuming those experiences to be more likely to impact future teaching. We similarly interviewed instructors of selected courses, not to detail delivery of each course, but to gain emphases across each program.

The site coordinator helped identify and communicate with the instructor and PT participants. Table shows the number of instructors of each course type interviewed at each university (rows 2–5) and the number of PTs who participated in the focus group interviews (final row).

Table 2. Number of instructor and focus group interviews at each university

Course type	**GLU**	**MRU**	**MUU**	**SRU**	**WUU**	**Total**
Mathematics	5	4	6	5	0	20
Mathematics-for-teachers	1	2	0	3	0	6
Mathematics education	3	5	3	4	1	16
General education	1	1	1	1	2	6
Total number of instructor interviews	10	12	10	13	3	48
Total number of focus group interviews	2 (6)	2 (8)	2 (8)	2 (8)	2 (7)	10 (37)

5 Number in parentheses represents the total number of PTs who participated in the focus group interviews.

Preservice teacher focus groups

We conducted 10 focus group interviews with 37 PTs across the five universities. The focus group method was used to identify PTs' perspectives during group interaction, which captured both consensus and divergent views regarding their shared experiences (Creswell, [6]). At each university, two groups of PTs consented to participate in the focus group interviews. Participating PTs may not have been enrolled in courses taught by participating instructors at the time of the interviews and focus groups. When any PT mentioned the name of a participating instructor, we compared the PTs' reported experiences with the instructor's intended mathematical modeling opportunities and tasks. During the interviews, PTs were provided with a handout listing all required courses of the program to help them reflect on modeling experiences across the entire program.

Instructor interviews

Our selection of courses included all required mathematics-for-teachers courses, all required mathematics education courses, one required general education course (e.g., Teaching in a Diverse Society), and a selection of required mathematics courses above the Calculus level. The exception is WUU, for which we selected two required general education courses with the required secondary mathematics methods course. As shown in Table , the number of required courses varied across universities. To preserve the confidentiality of our chosen programs, we renamed courses with generic course names. Table shows the selected courses, with their generic names, and the type of course as indicated by the site coordinator. The table additionally states, at the time of the study: the rank of each participating instructor, the number of years each had taught at the current institution, the total number of years of teaching experience (as reported by the instructor), and the terminal degree and emphasis for each.

Table 3. University instructor background at time of study

Generic course name	**Course type**	**Rank**	**Yrs. at univ.**	**Yrs. teaching**	**Terminal degree (Emphasis)**
Great Lakes University					
Linear Algebra	M	Full	13	22	PhD Math (Topology)
Abstract Algebra	M	Full	10	13	PhD Math (Social Theory)
Geometry	M	Full	9	20	PhD Math (Geometry)
Probability and Statistics	M	Asst	1	10	PhD Stats (Stat'l Computing)
Capstone	M	Full	15	22	PhD Undergrad Math Teach
Math for Sec. Teachers	MfT	Assoc	15	25	PhD Math (Geometry)
Mid. School Math Methods	ME	Assoc	12	25	PhD Math Teacher Education
Secondary Math Methods	ME	Assoc	12	25	PhD Math Teacher Education
Student Teaching Seminar	ME	Assoc	15	25	PhD Math (Geometry)
Teaching in Diverse Society	GE	Assoc	2	10	PhD Teaching & Curriculum
Midwestern Research University					
Linear Algebra	M	P-D	2	2	PhD Math (Topology)
Algebra for Teachers	MfT	Full	25	37	PhD Math (Algebra)
Geometry for Teachers	MfT	Full	4	40	PhD Math (Algebra)
Probability and Statistics	M	Full	11	16	PhD Math (Analysis)
Reasoning and Proof	M	Full	34	37	PhD Math (Algebra)
Mathematical Modeling	M	Full	25	37	PhD Math (Algebra)
Algebra in the Curriculum	ME	Assoc	19	27	PhD Math Education
Modeling in the Curriculum	ME	Adj	15	36	MS Math & Stats
Calculus in the Curriculum	ME	Adj	15	36	MS Math & Stats
Secondary math Methods I	ME	Asst	5	17	PhD Math Education
Secondary Math Methods II	ME	Assoc	19	27	PhD Math Education

Teaching in Diverse Society	GE	Assoc	12	22	PhD School Administration
Midwestern Urban University					
Linear Algebra	M	Full	22	34	PhD Math (Analysis)
Abstract Algebra	M	Asst	3	10	DA Math (Math Ed)
Geometry	M	Lect	1	4	PhD Math (Algebra)
Probability and Statistics	M	Full	27	41	PhD Math (Combinatorics)
Discrete Mathematics	M	Assoc	9	14	PhD Math (Topology)
History of Mathematics	M	Lect	1	4	PhD Math (Algebra)
Secondary Math Methods	ME	Asst	8	12	PhD Education
Field Experience Seminar	ME	Asst	8	12	PhD Education
Student Teaching Seminar	ME	Lect	11	44	MA Math
Teaching in Diverse Society	GE	Asst	2	10	PhD Educational Policy
Southeastern Research University					
Linear Algebra	M	Instr	20	20	PhD Math
Abstract Algebra	M	Assoc	32	35	PhD Math (Algebra)
Geometry	M	Instr	20	20	PhD Math
Probability	M	Full	18	18	PhD Applied Math
Reasoning and Proof	M	Lect	2	2	DA Math (Analysis)
Secondary Math Methods I	ME	GA	2	10	(PhD Math Teacher Educ)
Secondary Math Methods II	ME	Full	27	34	PhD Math Education
Secondary Math Methods III	ME	Asst	6	6	PhD Education
Sec. Math Connections I	MfT	Asst	3	3	PhD Math (Math Ed)
Sec. Math Connections II	MfT	Full	11	11	PhD Science and Math Ed
Sec. Math Connections III	MfT	Asst	3	3	PhD Math Education
Student Teaching Seminar	ME	Full	27	34	PhD Math Education
Teaching in Diverse Society	GE	Full	2	3	PhD Education
Western Urban University					
Teaching Eng. Lang. Learn.	GE	Asst	5	13	PhD English Education
Teaching in Diverse Society	GE	Full	17	22	PhD Education
Secondary Math Methods	ME	Full	14	30	PhD Math Education

6 Course type: M refers to mathematics courses; MfT to mathematics-for-teachers; ME to mathematics education; GE to general education courses.
7 Rank: Assoc. = associate professor; Asst. = assistant professor; Full = full professor; GA = graduate assistant; Instr. = instructor; Lect. = lecturer; P-D = post-doctoral fellow.

Mathematics-for-teachers courses were identified as such by the program coordinator. Mathematics-for-teachers courses were required by only three of the five participating universities (i.e., GLU, MRU, SRU). In the United States, mathematics-for-teachers courses are mathematics courses designed for future teachers that include types of mathematical knowledge identified as useful to teachers. At SRU, for example, one mathematics-for-teachers course syllabus explained that it: "explores various secondary mathematics topics with a modeling and data analysis approach and an explicit focus on ways of reasoning that connect critical concepts of secondary mathematics."

WUU's program was a unique postbaccalaureate program that admitted only PTs with a Bachelor-level mathematics degree (or its equivalent). Mathematics instructors from WUU were not interviewed because the site coordinator emphasized that PTs had completed the mathematics requirement through a variety of programs. We included WUU in our study to represent the variety of secondary mathematics education programs offered in the United States.

Another notable distinction was that GLU's secondary mathematics teacher education program is housed within the Mathematics Department, while other programs are located within the Colleges of Education. GLU mathematics instructors reported that they frequently communicated with mathematics education instructors about their teaching.

Course materials

The Project Preparing to Teach Algebra research team collected a course syllabus and instructional materials from each course instructor. After each interview, the research team requested additional course materials (e.g., mathematical tasks) based on interview responses.

Instructor interviews and PT focus groups

The Project Preparing to Teach Algebra interview and focus group protocols were semi-structured. All interviewers, including the authors of this study, were trained to follow interview protocols containing consistent questions for instructors and PTs. Both interview protocols included a parallel list of questions, including questions about emphasis on mathematical modeling in a program (for PTs) or a course (for instructors), any mathematical modeling activities PTs experienced, and any opportunities to learn to teach mathematical modeling. The mathematical modeling handout given to both instructors and PTs is shown in Appendix A. When PTs or instructors provided a task or problem without detailed descriptions, follow-up questions were asked to gain an understanding of the task or the nature of the problem.

Data analysis

To achieve triangulation, we conducted iterative analyses of multiple data sources (e.g., interview transcripts, syllabi, and tasks) (Creswell, [6]). We coordinated the data sources by writing a summary document that included each task or activity, along with any corresponding transcript excerpts from both instructors and PTs, PT notations, and written course materials from instructors. Each team member developed individual summary documents. In some cases, PTs mentioned participating instructors by name and described modeling experiences that corresponded to tasks also described by the instructor. We documented any such opportunities. We then discussed and reached agreement about elements of the summary documents, arranging them into combined summaries (Creswell).

The authors used the constant comparative method (Glaser & Strauss, [10]) to classify each PT's focus on his/her experiences related to mathematical or nonmathematical modeling. For example, a GLU PT reported looking at Dan Meyer's blog on "shooting basketballs into a hoop and seeing what the equation would have to be for the person shooting to actually make the ball." We identified this PT report as mathematical modeling because the PT explored visual data (i.e., the basketball video) and then represented the data using an equation. As nonmathematical modeling, a WUU PT reported using a Venn diagram to "represent A as this part of the circle, B as this part of the circle, and therefore A union B is this part of the circle... you're seeing the variables and you're creating a model." We classified this report as a nonmathematical modeling experience, because it focused on representing numbers using a Venn diagram in a nonproblem-solving situation.

We iteratively reviewed, revised, and grouped our notes on these transcript except into conceptual themes (Glaser & Strauss, [10]). Mathematical modeling themes included: (a) engaging with real-life problems that encouraged PTs or their students to make choices and develop representations (Consortium for Mathematics and its Applications & Society for Industrial and Applied Mathematics [CMA & SIAM], [5]). Nonmathematical modeling themes included: (b) representing numbers or operations visually; (c) discussing instructor demonstrating a teaching process; (d) writing and explaining proofs; and (e) using real-life contexts to introduce math concepts. For example, the basketball and Venn diagram tasks described above were classified as (a) and (b), respectively. A count of PTs' reporting each theme is presented in Table of Findings. Additional themes regarding PTs' access to mathematical modeling experiences emerged from instructor transcripts. During this iterative process of analysis and grouping, we validated (or revised) our coding using evidence drawn from corresponding course materials. Instructors reported that mathematical modeling experiences: (f) included discussing parts of the mathematical modeling cycle in class; (g) depended on PTs' classroom placements or on lessons developed by the PTs; and (h) were emphasized very little or not at all. A count of instructors' reporting of each theme is presented in Table of Findings.

Table 4. Themes for opportunities to learn mathematical modeling recalled by PTs

	Universities (total number of PTs)	**GLU**	**MRU**	**MUU**	**SRU**	**WUU**	**Total**
(6)	**(8)**	**(8)**	**(8)**	**(7)**	**(37)**		
Mathematical modeling opportunities	Engaged with real-life problems that encouraged PTs or their students to make choices and develop representations	4	5	2			11
Nonmathematical modeling opportunities	Represented numbers or operations visually	4		2	6	3	15
Discussed instructor demonstrating a teaching process				4	3	7	
Solved or taught word problems				3	1	4	
Wrote and explained proofs			4			4	
Used contexts to introduce math concepts	1			2		3	

8 Numbers in parentheses indicate the total number of PTs. Not all PTs shared modeling experiences.

Table 5. Themes among the opportunities to learn mathematical modeling recalled by instructors

	Universities (total number of instructors)	**GLU**	**MRU**	**MUU**	**SRU**	**WUU**	**Total**
(10)	**(12)**	**(10)**	**(13)**	**(3)**	**(48)**		
Mathematical modeling opportunities	Engaged with real-life problems that required PTs or their students to make choices and develop representations	4	3				7
Nonmathematical modeling opportunities	Represented numbers or operations visually				1	1	2
Discussed instructor demonstrating a teaching process						0	
Solved or taught word problems	1		1			2	
Wrote and explained proofs			1			1	
Used contexts to introduce math concepts						0	
Instructor discussed parts of mathematical modeling cycle in class		4	2	2		8	
Opportunities depended on PTs' classroom placements or developed lessons	2	1	2	4	1	10	
Instructor reported very little or no emphasis on mathematical modeling	3	4	4	6	1	18	

9 Numbers in parentheses indicate the total number of instructors interviewed.

FINDINGS

This section first presents PTs' recollections of their experiences with mathematical modeling, followed by instructors' reports of mathematical modeling opportunities they intended to provide. We close this section by describing all mathematical and nonmathematical modeling opportunities reported by both PTs and instructors.

PTs' experiences with mathematical modeling

As described in the Method section, several themes emerged from PTs' responses to questions about mathematical modeling experiences from program courses. Table lists the themes (in the first and second columns) and a count of PTs whose responses corresponded to that theme.

Mathematical modeling reported by PTs

In this section, we describe tasks reported by PTs at three programs (i.e., GLU, MRU, MUU) that allowed the PTs or their students to make choices and develop representations (e.g., graphs and tables) to interpret real-life situations.

At GLU, four PTs discussed Dan Meyer's tasks used in their Middle School Math Methods course. One task featured a dynamic parabola with a basketball context (blog.mrmeyer.com/2010/wcydwt-will-it-hit-the-hoop/) and another focused on the time to fill a water tank (mrmeyer.com/threeacts/watertank/). One PT described a CBR (calculator-based ranger) experiment from the same course, collecting motion data and identifying results based on graphical or tabular data representations. Another GLU PT described a Secondary Mathematics Methods sales tax project for which she collected food prices and calculated sales tax.

Five PTs at MRU described a Markov Chain project from Mathematical Modeling. They had the opportunity to select their own project related to their lives and future careers. They discussed several additional tasks from Modeling in the Curriculum. In these tasks, they collected and represented data to solve problems. In the same course, PTs described a Coke Can task in which they optimized the dimensions of a 355-mL Coca-Cola can.

At MUU, two PTs reported creating mathematical modeling projects in their student teaching experiences. A PT described choosing a mathematical content area around which to develop a real-world project in her Student Teacher Seminar. She chose trigonometry and developed a task involving measuring the school building's height using a clinometer.

Nonmathematical modeling reported by PTs

When asked about modeling opportunities, PTs in four programs (all but MRU) provided examples of representing numbers or operations visually. For example, two PTs at MUU reported drawing a Venn diagram or using algebra tiles to visually represent the distributive property. PTs at SRU and WUU described an instructor demonstrating a teaching process, which is a common modeling misconception (AMTE, [2]). SRU PTs described teachers thinking aloud and managing disruptive behavior.

SRU and WUU PTs reported solving or teaching word problems as modeling opportunities. SRU PTs referenced student teaching experiences in which they taught word problems. While their examples involved real-life contexts, the problems did not require students to make assumptions that could lead to multiple correct solutions (CMA & SIAM, [5]). PTs at MUU described writing, explaining, and finalizing proofs in Abstract Algebra as modeling opportunities. Finally, SRU PTs reported using real-life contexts to introduce mathematical concepts. They described student teaching experiences in which they used a marine biology context to introduce exponents and logarithms.

Instructors' intended mathematical modeling opportunities

Instructors described mathematical modeling opportunities they intended to provide in their courses and provided corresponding course materials. Overall, our analyses of instructor interviews, syllabi, tasks, and exams confirmed that few courses across the five teacher preparation programs focused on mathematical modeling. Additionally, the few mathematical modeling opportunities varied greatly across programs. Seven instructors (from GLU or MRU) reported opportunities in which PTs engaged with real-life problems that encouraged them to make choices and develop representations. Five instructors (from GLU, MUU, SRU, or WUU) reported nonmathematical modeling opportunities (e.g., representing numbers or operations visually, solving word problems, writing proofs), as shown in Table.

Along with mathematical and nonmathematical modeling opportunities, eight instructors reported that, although modeling was not a focus of their course, they discussed parts of the mathematical modeling cycle in class (e.g., "It's kind of like a build-up process. How do you set up variables? Something to do with skills in modeling using mathematical formulas."). Ten field-based course instructors (e.g., Student Teaching Seminar, Field Experience Seminar) mentioned that any modeling opportunities depended on PTs' classroom placements or the lessons the PTs developed. Eighteen instructors (e.g., Abstract Algebra, Discrete Mathematics, General Education) reported very little or no emphasis on mathematical modeling.

Mathematical modeling reported by instructors

Linear Algebra was a required course at all universities (except WUU's postbaccalaureate program). While three Linear Algebra instructors (i.e., MRU, MUU, and SRU) reported a low emphasis on mathematical modeling, the GLU Linear Algebra instructor reported a high emphasis. He described several lab activities as mathematical modeling tasks. For example, he described his Google's Page Rank task (deidentified task: figshare.com/s/d8d83ff2cf2c474e971e), which provides background information about Google's algorithm to determine page ranks of web pages. We identified this context as having "intrinsic value or meaning for students" (CMA & SIAM, p. 8), given that students have likely performed at least one Google search. The GLU instructor elaborated on elements of the mathematical modeling process included in the task:

> They [PTs] create a simple model and then they see that there are problems with that model and they have to fix the model. They...see what Google is doing is...computing a Markov chain. The students...[know] a Markov chain will oftentimes converge, but in certain conditions it doesn't. And in Google, you encounter those conditions and so they have to think about how to fix that...by taking this [their understanding of Markov chains] and using it to improve the model they have.

In his description, the instructor described PTs creating and improving a model, which are critical elements of the mathematical modeling cycle (NGA & CCSSO, [4]).

The GLU Math for Secondary Teachers course instructor described mathematical modeling tasks. For example, his Profitability of Movies Task (deidentified task: figshare.com/s/d8d83ff2cf2c474e971e) begins with data from blockbusters (e.g., Titanic, The Phantom Menace, Harry Potter). The blockbuster data may have intrinsic value to students (CMA & SIAM, [5]), who are likely interested in movies. A portion of the task required PTs to act as a movie executive, making predictions and choices (Pollak & Garfunkel, [20]). This task featured mathematical modeling as a vehicle, rather than as content (Julie, [13]). Several task questions scaffolded the PTs' use of linear and quadratic regressions. The task also provided guidelines for the PTs to modify the activity for future use, as well as considerations about teaching mathematical modeling.

Nonmathematical modeling reported by instructors

Some instructors described nonmathematical modeling opportunities when asked about modeling. Instructors reported that PTs represented numbers or operations visually (e.g., "paper strips physically and also number lines and these strip diagrams. They're drawing models all the time."). Some instructors referred to contextualized word problems as mathematical modeling opportunities (e.g., "A manager for a large insurance company needs to form a six-person committee of 18 women and nine men. If the committee consists of two women and the remainder are men, what is the probability of such a committee?"). Several of the course instructors described mathematical modeling using an alternative perspective; accordingly, PTs may not have had the necessary opportunities to develop a consistent understanding of mathematical modeling across their required courses. In contrast, there were some instances in which the instructors and PTs provided shared examples of mathematical and nonmathematical modeling opportunities, as described below.

Common modeling opportunities reported by both instructors and PTs

A few modeling opportunities were described by both instructors and PTs. For example, MRU PTs specifically mentioned participating instructors of Mathematical Modeling and Modeling in the Curriculum courses, and their recollections aligned with instructors' reports.

A Mathematical Modeling course objective was: "to analyze or try to solve a real problem using mathematical tools by first formulating the problem and making assumptions to make a model,...[and] apply analytic or simulation methods and interpret the result." The instructor asked PTs to select a project related to their lives and future careers. The instructor and two PTs each referenced the Markov Chain and linear programming tasks from their Linear Algebra textbook. One PT explained that they modeled "student tracking through high school and college using a Markov chain. So it helped me learn about how something as specific as a Markov chain can help to describe how people cycle through a system like education." This example indicates that PTs noticed that the modeling projects helped them learn how specific mathematical concepts may support real-life analyses and problem solving.

Modeling in the Curriculum was one of three one-credit courses at MRU specifically designed to support PTs' understanding of connections between college-level and 7th–12th-grade mathematics. PTs took the three courses while they were enrolled in a corresponding mathematics content course (e.g., Linear Algebra and Mathematical Modeling). The Modeling in the Curriculum syllabus included several learning objectives that the instructor discussed with PTs in class. Two course objectives focused on modeling as *content* (i.e., "Recognize and apply the power of simulations to model real situations"; "Learn how to use various technologies effectively as tools in the modeling process"). Two objectives emphasized modeling as a *vehicle,* describing the use of linear regression, median fit lines, and nonlinear models for data. Three objectives emphasized teaching modeling by (a) creating modeling activities connecting concepts in multiple mathematical areas or with other disciplines, (b) developing rubrics to assess modeling lessons, and (c) developing lessons using modeling as a vehicle for mathematical content. The instructor discussed in class that mathematical content can be taught using the context of modeling, and shared the course emphasis of "creating [modeling] situations where middle school and high school students 'wrestle' with mathematical ideas before they are taught the fundamental skills and concepts associated with those ideas." The instructor and PTs reported this emphasis on wrestling with mathematical ideas in the context of modeling. PTs described engaging with a Coke Can problem that required them to mathematize an authentic context. The task asked them to report, to a Coca-Cola company, multiple strategies to determine can dimensions that required as little material as possible while still containing 355 ml of Coke. The instructor described discussing why treating the Coke can as a cylinder was an assumption and not a fact. One PT described finding the surface area and volume of a Coke can:

> We had a piece of paper that was the same size and you had to cut out the corners to make a box and you had to figure out how much you would need to cut out of the corners to maximize the volume. It was just like...completely figure it out yourself...everybody made different sizes and then we graphed...how much we cut off and then the surface [area] or the volume that resulted. That was interesting.

The instructor mentioned one team of PTs who graphed a parabola based on the data they collected (with diameter as the independent variable and surface area as the dependent variable).

This instructor shared other tasks with authentic contexts and complete modeling cycles. The Burning Candles task involved recording the times and changing heights of a burning candle (a similar task is found at: https://tapintoteenminds.com/3act-math/candles-burning/). The Calculator-Based Ranger (CBR) task included collecting data using a CBR, creating graphical and tabular representations, and interpreting results. The Taxi Cab task began with an authentic context: owners of a taxi company kept data on locations of high taxi use. Students were asked to use the dataset to draw a transition diagram and write a transition matrix.

At times, PTs and their course instructors described corresponding nonmathematical modeling examples. WUU PTs and a Secondary Mathematics Methods course instructor referenced by the PTs all described modeling as representing numbers or operations visually without a problem-solving context. Similarly, four MUU PTs and the referenced instructor reported proofs as an example of modeling. The PTs explained how they wrote, explained, and finalized proof-writing in their Abstract Algebra course, mentioning the instructor by name. The instructor also described proof-writing activities as modeling opportunities. Such examples suggest instructors' misconceptions impact PTs' understanding of mathematical modeling.

DISCUSSION

Given the research gap in the context of exploration of mathematical modeling across a number of teacher preparation programs, we focused on the mathematical modeling opportunities reported by

37 PTs and 48 course instructors in five case study programs. By examining PTs' reports on their experiences with mathematical modeling in their teacher preparation programs, we aimed to identify the core experiences that PTs recalled among many other experiences. Eleven PTs recalled their experiences of engaging with real-life problems that encouraged them or their students to make choices and develop representations. Given that prior research showed that teachers often omitted the decision-making aspect from the mathematical modeling cycle (Pollak & Garfunkel, [20]), these reports from PTs are noteworthy.

We reported alignments between PTs' recollections and instructors' reports. Several instructors and PTs responded in ways that revealed common misconceptions of modeling, including: (a) represented numbers or operations visually; (b) involved an instructor demonstrating a teaching process; (c) solved or taught word problems; (d) wrote and explained proofs; and (e) used contexts to introduce math concepts. The nature of the reported nonmathematical modeling opportunities may be related to the varying definitions of modeling found in the extant literature. Regarding the first misconception (i.e., represented numbers or operations visually), mathematical modeling may also involve the use of representations. Using representations is a part of the full mathematical modeling process, rather than mathematical modeling itself (Anhalt & Cortez, [1]; Smith, [22]). The second, a teacher demonstrating a lesson or effective teaching practices, is another common misconception (AMTE, [2]; National Council of Teachers of Mathematics, [19]). The third misconception (i.e., solved or taught word problems) has also been discussed in literature; while both problems involve a real-life context, a word problem often does not require students to make assumptions and frequently has one correct solution (CMA & SIAM, [5]). Ideally, instructors of mathematics and mathematics education courses required by teacher preparation programs would be supported in collaboratively sharing the correct conception of mathematical modeling and preparing PTs to learn about and to learn to teach a robust conception of mathematical modeling.

While few courses in the selected programs focused on mathematical modeling, instructors at MRU and GLU provided several rich mathematical modeling tasks and included their emphasis on modeling in their syllabi. PTs at MRU and GLU demonstrated accurate understandings of mathematical modeling. We argue that their understanding was closely tied to the rich opportunities provided by these instructors and the unique program feature. MRU is a unique program in that it required PTs to concurrently take a one-credit College of Education Modeling in the Curriculum course and a Mathematics Department Mathematical Modeling course. Several PTs recalled experiences from these two courses. The instructors emphasized modeling as content in their syllabi and interviews, and provided several opportunities for PTs to engage with mathematical modeling. GLU is also a unique program in that mathematics instructors spontaneously told interviewers that they made changes in their teaching strategies and pedagogy, based on discussions with their mathematics education colleagues. One of such examples is that GLU Linear Algebra instructor, who reported collaboration with mathematics education colleagues, provided rich mathematical modeling problems. On the other hand, Linear Algebra course instructors in the other four programs reported a low emphasis on mathematical modeling. PTs' effective teaching of mathematical modeling required coordination across teacher preparation program courses (e.g., AMTE, [2]; CBMS, [21]; NGA & CCSSO, [4]). Efforts toward interdisciplinary collaborations should include ongoing conversations between instructors of both mathematics and education courses.

Another possible teacher preparation implementation is to incorporate mathematical modeling in required courses, which provides access to quality tasks and resources (e.g., Anhalt & Cortez, [1]; Julie, [13]). Based on the written tasks and verbal reports provided by instructors, we shared sample mathematical modeling tasks with online links when accessible. Additional resources regarding mathematical modeling can be found by searching online for *Small Group Mathematical Modeling*, *Case Studies for Kids*, and *Pedagogy in Action*. Sample tasks can also be found in reports and publications (e.g., CMA & SIAM, [5]; Gould, Murray, & San Fratello, [11]; Hirsch & McDuffie, [12]; Moore, Doerr, Glancy, & Ntow, [18]). Available resources used to create rich modeling problems need to be shared, especially with new teacher educators and mathematics education researchers.

LIMITATIONS

We acknowledge the limitations of this study. We may have found more opportunities if we had interviewed instructors of mathematics courses that WUU PTs had taken. Often times, it seems difficult to connect mathematics courses and education courses in post-baccalaureate programs like WUU because PTs complete a mathematics degree in a mathematics department (at WUU or other schools) and then move on to the education department for the postbaccalaureate degree in teaching. Future studies could explore the different nature of opportunities that postbaccalaureate programs may offer to PTs and their impact on the learning of PTs. We may have also identified more opportunities had we explored all required mathematics courses, observed class activities, or interviewed PTs at multiple points in their program, in addition to interviewing instructors and senior-level PTs. The overarching goal of the research team was to gather data that would provide a sense of each secondary teacher education program as a whole. This goal allowed us the breadth to compare programs, rather than deeply considering connections between teachers' learning and their opportunities to learn, which have appeared in other valuable studies (e.g., Anhalt & Cortez, [1]; Doerr & English, [8]; Tan & Ang, [23]). Although this choice certainly obscured such connections, it revealed variations across universities with respect to rich mathematical modeling tasks, opportunities to encounter certain elements of the modeling cycle, and misconceptions of the meaning of modeling. For example, we saw a valuable example of mathematics content connected between a mathematics department course and a concurrent education course in MRU's Mathematical Modeling and Modeling in the Curriculum courses. We would have missed variation and uniqueness if we had focused on fewer programs.

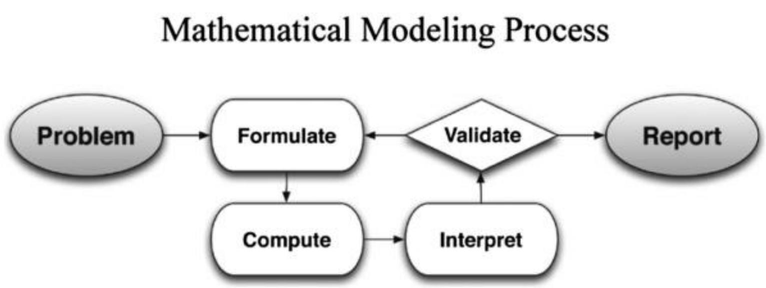
SIGNIFICANCE AND CONCLUSION

Efforts to restructure a course in which PTs could engage with full modeling cycles have been well documented (e.g., Anhalt & Cortez, [1]; Julie, [13]); however, the mathematics education field has yet to disseminate results from a larger study focusing specifically on the mathematical modeling opportunities offered in teacher education programs. The scope of our analysis—which included instructors of mathematics, mathematics-for-teachers, mathematics education, and general education courses, along with focus groups of senior PTs—allowed for comparisons across programs of five diverse universities. We found that 23 of the 48 interviewed instructors reported modeling opportunities that were nonmathematical in nature or reported that modeling was emphasized very little or not at all. Although perhaps not all courses can integrate modeling, Linear Algebra is one example of a course that might benefit from modeling experiences. Yet, only one of four Linear Algebra instructors integrated modeling experiences into his course. Given the new AMTE ([2]) recommendation that programs provide PTs with consistent mathematical modeling opportunities, our investigation of such opportunities is timely and necessary.

Pursuant to the recommendations of policy documents (e.g., AMTE, [2]; CBMS, [21]), and mathematics education researchers (e.g., Julie, [13]), our findings highlight the critical need for educators to intentionally incorporate mathematical modeling into curricula. This requires that mathematics, mathematics–for-teachers, mathematics education, and general education instructors collaborate to create coherent opportunities across programs. We acknowledge that many secondary education programs in the United States merit further investigation. Our study is an initial effort to investigate modeling experiences reported by PTs and instructors from five diverse secondary teacher preparation programs, and to begin a discussion on possible pathways to provide PTs with rich opportunities to engage with mathematical modeling.

A. Appendix Preservice teacher focus group and instructor interview modeling handout

GRAPH



- Identifying and selecting variables
- Formulating a model by creating and selecting appropriate representations
- Analyzing and performing operations to draw conclusions
- Interpreting the results of the mathematics
- Validating the conclusions, possibly improving the model
- Reporting on the conclusions and the reasoning behind them

REFERENCES

Anhalt, C., & Cortez, R. (2016). Developing understanding of mathematical modeling in secondary teacher preparation. Journal of Mathematics Teacher Education, 19 (6), 523 – 545. https://0-doi-org.libus.csd.mu.edu/10.1007/s10857-015-9309-8

Association of Mathematics Teacher Educators. (2017). Standards for preparing teachers of mathematics. Retrieved from https://www.amte.net/standard

Cai, J., Cirillo, M., Pelesko, J. A., Borromeo Ferri, R., Borba, M., Geiger, V., ... Kwon, O. N. (2014). Mathematical modeling in school education: mathematical, cognitive, curricular, instructional, and teacher education perspectives. In P. Liljedahl, C. Nicol, S. Oesterle, & D. Allan (Eds.), Proceedings of the Joint Meeting of the 38th conference of the international group for the psychology of mathematics education and the 36th conference of the North American chapter of the psychology of mathematics education (Vol. 1, pp. 145 – 172). Vancouver, Canada : PME.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). Common core state standards for mathematics. Washington, DC : Author. Retrieved from https://www.corestandards.org

Consortium for Mathematics and its Applications & Society for Industrial and Applied Mathematics. (2016). Guidelines for assessment and instruction in mathematical modeling education. Retrieved from https://www.siam.org/reports/gaimme.php

Creswell, J. W. (2009). Research design: Qualitative, quantitative, and mixed methods approaches (3rd ed.). Thousand Oaks, CA : Sage Publications.

Doerr, H. M. (2007). What knowledge do teachers need for teaching mathematics through applications and modelling? In W. Blum, P. L. Galbraith, H. W. Henn, & M. Niss (Eds.), Modelling and applications in mathematics education (pp. 69 – 78). New York, NY : Springer.

Doerr, H. M., & English, L. D. (2006). Middle grade teachers' learning through students' engagement with modeling tasks. Journal of Mathematics Teacher Education, 9 (1), 5 – 32. https://0-doi-org.libus.csd.mu.edu/10.1007/s10857-006-9004-x

Gansle, K. A., Noell, G. H., & Burns, J. M. (2012). Do student achievement outcomes differ across teacher preparation programs? An analysis of teacher education in Louisiana. Journal of Teacher Education, 63 (5), 304 – 317. https://0-doi-org.libus.csd.mu.edu/10.1177/0022487112439894

Glaser, B., & Strauss, A. (1967). The discovery of grounded theory. Chicago, IL : Aldine.

Gould, H., Murray, D., & Sanfratello, A. (2012). Mathematical modelling handbook. Bedford, MA : The Consortium for Mathematics and its Applications.

Hirsch, C. R., & McDuffie, A. R. (2016). Annual perspectives in mathematics education 2016: Mathematical modeling and modeling mathematics. Reston, VA : National Council of Teachers of Mathematics.

Julie, C. (2002). Making relevance in mathematics teacher education. In I. Vakalis, D. Hughes Hallett, D. Quinney, & C. Kourouniotis (Compilers), Proceedings of 2nd International Conference on the Teaching of Mathematics. New York, NY : Wiley [CD-ROM].

Lesh, R., & Doerr, H. M. (2003). Beyond constructivism: Models and modeling perspectives on mathematics teaching, learning, and problem solving. Mahwah, NJ : Lawrence Erlbaum Associates Inc.

Lesh, R., English, L. D., Riggs, C., & Sevis, S. (2013). Problem solving in the primary school (K-2). Mathematics Enthusiast, 10 (1–2), 35 – 60.

Lesh, R., & Sriraman, B. (2010). Theories of mathematics education: Seeking new frontiers. Berlin, Germany : Springer Science.

Meyer, D. (2015). Missing the promise of mathematical modeling. Mathematics Teachers, 108 (8), 578 – 583. https://0-doi-org.libus.csd.mu.edu/10.5951/mathteacher.108.8.0578

Moore, T. J., Doerr, H. M., Glancy, A. W., & Ntow, F. D. (2015). Preserving pelicans with models that make sense. Mathematics Teaching in the Middle School, 20 (6), 358 – 364. https://0-doi-org.libus.csd.mu.edu/10.5951/mathteacmiddscho.20.6.0358

National Council of Teachers of Mathematics. (2000). Principles and standards for school mathematics. Reston, VA : Author.

Pollak, H., & Garfunkel, S. (2014). A view of mathematical modeling in mathematics education. In Proceedings from Teachers College Mathematical Modeling Oktoberfest. New York, NY : Teachers College Columbia University.

Conference Board of the Mathematical Sciences. (2012). The mathematical education of teachers II. Providence, RI and Washington, DC : American Mathematical Society and Mathematical Association of America

Smith, J. P. (2015). On models and modeling: A commentary. Mathematics in Michigan, 48 (1), 8 – 9.

Tan, L. S., & Ang, K. C. (2016). A school-based professional development programme for teachers of mathematical modelling in Singapore. Journal of Mathematics Teacher Education, 19 (5), 399 – 432. https://0-doi-org.libus.csd.mu.edu/10.1007/s10857-015-9305-z