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Resonant Microcantilevers Vibrating Laterally in Viscous Liquid Media

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Abstract:

The characteristics of microcantilevers vibrating in the in-plane flexural mode (also known as lateral vibration) in viscous liquid media are investigated. A numerical model was utilized to determine a correction to Stokes' solution for an infinite plate to obtain an analytical expression for the hydrodynamic forces acting on a laterally vibrating microcantilever as a function of both Reynolds number and aspect ratio (thickness over width). The results allowed for the resonant frequency and quality factor to be investigated as a function of both beam geometry and medium properties. Trends in these characteristics can be used to optimize device geometry and maximize the frequency stability. As the thickness of a microcantilever is increased, both its stiffness and the medium's viscous damping increase. This will lead to an optimum quality factor in terms of the thickness. The characteristics of laterally and transversely vibrating microcantilevers with similar geometries are also compared. It is found that the resonant frequency and quality factor are higher for laterally vibrating microcantilevers (at least by a factor of 2 to 3 or higher for the Q-factor depending on the geometry) compared to those of similar beams under transverse (or out-of-plane) vibration. The improvement in sensitivity (due to the increase in frequency) and in the quality factor (thus a reduced frequency noise) are expected to yield much lower limits of detection in liquid-phase chemical sensing applications.

SECTION I. Introduction

The microcantilever is a microelectromechanical (MEMS) device that has been utilized as a highly sensitive chemical sensor platform [1], [2]. This device can be excited into resonance and shifts in the resonant frequency or quality factor can be used to detect characteristics of the operating environment. While dynamically excited microcantilevers are well suited for operation in low-viscosity media such as air, their usefulness as a sensing platform is limited when operating in viscous liquid media. This is due to the large decrease in the device's quality factor caused by the additional viscous losses in the fluid [3]. Low quality factors increase the system's susceptibility to frequency noise, raising the sensor's detection limit.

Microcantilevers are commonly vibrated in the out-of-plane mode (also commonly known as transverse flexural mode). In a fluid, this mode of vibration causes a pressure force on the top and the bottom of the beam, subjecting it to a hydrodynamic force that is partially out-of-phase with the beam's displacement. The total hydrodynamic force can be conceptually broken up into a force in-phase and a force out-of-phase with the displacement of the beam. The forces acting out-of-phase with the displacement are considered damping forces, while the ones in-phase are the inertial forces and are considered equivalent to a displaced fluid mass.

It was proposed in [3] that exciting the microcantilever in the in-plane vibration mode (or lateral vibration) instead of the out-of-plane flexural mode would reduce the amount of fluid resistance (combined effects of fluid-related inertial and viscous forces), which could potentially improve the sensitivity and limit of detection of microcantilever-based chemical sensors. However, the fluid resistance model used in that study for lateral vibrations, while including the effects of pressure resistance on the leading and trailing faces of the beam, did not include the effects of fluid shear resistance on the upper and lower faces. A more accurate calculation of the total hydrodynamic force on a microcantilever in a liquid should include both the pressure and shear stress exerted by the fluid on the beam. For a rectangular beam, due to the symmetry of the problem, the hydrodynamic forces acting in the vertical direction cancel out when the beam is laterally vibrating. This leaves only the shear force acting on the top and bottom of the beam and the pressure force acting on the leading and trailing edges. If the thickness is small enough, the shear force can be said to be the dominant source of fluid drag and cannot be neglected. The pressure force also remains significant under certain conditions even when the beam width is much greater than the thickness.

In the present study, in order to more accurately determine the advantages of using in-plane mode over out-of-plane mode for a dynamically driven microcantilever chemical sensor, relevant electrical characteristics such as the beam's resonant frequency and quality factor will be obtained for similar beams excited both laterally and transversely. However, in order to evaluate these characteristics, the hydrodynamic forces must first be calculated. Expressions for these relevant characteristics will be found in terms of the hydrodynamic forces. The hydrodynamic forces will be found using a combination of analytical and numerical techniques. An approximate analytical technique that assumes a zero-thickness beam is first used. The effects of thickness will then be investigated numerically using a finite element model and the results will be compared to the results from the approximate analytical technique. The effect of the hydrodynamic forces on the resonant frequency and quality factor can then be investigated with respect to the properties of the fluid, beam geometry, and excitation direction. The advantages of using in-plane mode cantilevers will be summarized and cantilever geometries for better sensing characteristics will be identified.

Figure 1 Geometry of a microcantilever of length L , width b , and thickness h with a deflection of $v(x,t)$

SECTION II. Theoretical Analysis

A. Equation of Motion

The geometry of a laterally excited microcantilever, with dimensions on the order of microns, is shown in Fig. 1. It is generally assumed that $L \gg b$, allowing for the application of Euler-Bernoulli beam theory. This assumption generally holds true for standard commercially available microcantilevers. The equation of motion for the laterally excited beam in a vacuum is given by

$$EI \frac{\partial^4 v(x,t)}{\partial x^4} + \rho B b h \frac{\partial^2 v(x,t)}{\partial t^2} = F_y(x) e^{j\omega t} \quad (1)$$

View Source where

$$I_{lat} = \frac{b^3 h}{12} \quad (la)$$

View Source In (1), E is the Young's modulus, ρB is the mass density of the beam, $v(x,t)$ is the in-plane (or lateral) deflection at points along the length of the beam, and $F_y(x)$ is the position-dependent forcing function per unit length operating at an angular frequency of ω_{lat} . Note that the equation for the moment of inertia, I_{lat} has the width cubed instead of the thickness (as is the case in transverse excitation), indicating that the flexural rigidity is larger for beams undergoing lateral vibration. In a vacuum, that represents the only difference between lateral and transverse excitation when solving the equation of motion.

When the microcantilever is operating in a viscous liquid medium, an additional force from the medium affects the microcantilever and the equation of motion is modified as

$$EI \frac{\partial^4 v(x,t)}{\partial x^4} + \rho B b h \frac{\partial^2 v(x,t)}{\partial t^2} = F_y(x) e^{j\omega t} + F_{\text{medium}, \text{lat}}(x,t) \quad (2)$$

[View Source](#) The force per unit length, $F_{\text{medium}, \text{lat}}$, is partially out-of-phase with the displacement, and can be represented as

$$F_{\text{medium}, \text{lat}}(x,t) = -g_{1, \text{lat}} \frac{\partial v(x,t)}{\partial t} - g_{2, \text{lat}} \frac{\partial^2 v(x,t)}{\partial t^2} \quad (2a)$$

[View Source](#) where $g_{1, \text{lat}}$ and $g_{2, \text{lat}}$ are coefficients associated with the fluidic damping force per unit length and the fluidic inertial force (displaced fluidic mass) per unit length, respectively [4], [5].

B. Resonant Frequency

The derivation of the formulae for the resonant frequency and the quality factor for an uncoated device in a viscous liquid medium are the same as those in [6], with the exception of different values for the moment of inertia (I_{lat}) and the hydrodynamic force (i.e. $g_{1, \text{lat}}$ and $g_{2, \text{lat}}$). An analytical expression for the resonant frequency is then found from the equation of motion as

$$f_{\text{res}, \text{lat}} = \frac{\alpha^2 i 2 \pi k_{\text{lat}}}{M_{\text{lat}}} \quad (3)$$

[View Source](#) where

$$k_{\text{lat}} = EI_{\text{lat}} / L^3, \quad M_{\text{lat}} = (\rho B b h L + L g_{2, \text{lat}}) + L \left((g_{1, \text{lat}} / \omega_{\text{res}, \text{lat}}) + (\omega_{\text{lat}}^2) \frac{d}{d\omega} (g_{1, \text{lat}} / \omega) \right) (m/L + g_{2, \text{lat}} + (\omega_{\text{lat}}^2) \frac{d}{d\omega} (g_{2, \text{lat}})) (g_{1, \text{lat}} / \omega_{\text{lat}}) \quad (3a) \quad (3b)$$

[View Source](#) and

$$g_{1, \text{lat}} = \pi 4 \rho L b^2 \Gamma_{\text{lat}, I}(\text{Re}, h/b) \omega_{\text{lat}}, \quad g_{2, \text{lat}} = \pi 4 \rho L b^2 \Gamma_{\text{lat}, R}(\text{Re}, h/b) \quad (3c) \quad (3d)$$

[View Source](#) where α_i is a constant dependent on the mode number ($\alpha_1 \cong 1.875$ for the fundamental flexural mode). $\Gamma_{\text{lat}, R}$ and $\Gamma_{\text{lat}, I}$ are the real and imaginary parts of the hydrodynamic function, a normalized version of the hydrodynamic force (to be defined in section D) that depends only on the aspect ratio h/b and the Reynolds number (Re) of the medium. The Reynolds number is a measure of the ratio of inertial forces to the viscous forces acting on the beam, and is defined as

$$\text{Re} = \rho L \omega_{\text{lat}} b^2 / 4 \eta \quad (4)$$

[View Source](#) where ρL and η are the mass density and dynamic viscosity of the fluid, respectively. If $\text{Re} \gg 1$, the $g_{1, \text{lat}}$ and $g_{2, \text{lat}}$ terms can be considered frequency-independent and the resonant frequency can be obtained from

$$f_{\text{res}, \text{lat}} = \frac{\alpha^2 i 2 \pi L^2 EI_{\text{lat}} (\rho B b h + g_{2, \text{lat}})}{(\rho B b h + g_{2, \text{lat}})^2 + (g_{1, \text{lat}} / \omega_{\text{lat}})^2} \quad (5)$$

[View Source](#)

As the beam gets stiffer, the resonant frequency will increase; it will decrease when the total mass or damping increases. Note that in a vacuum, the resonant frequency is independent of the thickness. If there are only small dissipative effects from the viscous liquid medium, the equation for the resonant frequency simplifies to

$$f_{\text{res}, \text{lat}} = \frac{f_{011} + L g_{2, \text{lat}} / (\rho B b h)}{\quad} \quad (6)$$

[View Source](#) [?] where f_0 is the beam's resonant frequency in a vacuum. This demonstrates that even when there is no damping, the resonant frequency is still affected by the displaced mass of the fluid. Equation (6) is identical to the one given in [5], but with $g_{2,lat}$ assuming lateral motion instead of transverse motion.

C. Quality Factor

The quality factor Q_{lat} is directly related to the ratio of the maximum energy stored in the system per cycle to the amount of energy dissipated in one cycle. When the loss is low, the quality factor can be approximated as

$$Q_{lat} = f_{res,lat} / \Delta f_{3dB} \quad (7)$$

[View Source](#) [?] where Δf_{3dB} is the 3-dB bandwidth of the system around resonance. If it is assumed that $g_{1,lat}$ and $g_{2,lat}$ are frequency-independent in the 3-dB bandwidth, an expression for the quality factor can be found from the equation of motion as

$$Q_{lat} = \frac{2(1 - g_{1,lat}/\omega_{lat} \rho_{b,h} + g_{2,lat} \dots - v)}{2} \quad (8)$$

[View Source](#) [?]

D. Hydrodynamic Force

In order to calculate the resonant frequency and the quality factor, the total hydrodynamic force acting on the fluid-beam interface must be found. The hydrodynamic force can conceptually be decomposed into a force parallel with the surface of the beam (the shear or frictional force) and a force perpendicular to the surface of the beam (the pressure force). For a beam of rectangular cross-section, due to the inherent symmetry of the problem and the direction of excitation, all the forces acting in the transverse direction will cancel each other out. Remaining are the shear force acting along the width and the pressure force acting on the thickness. It is assumed that the beam is long enough to neglect the forces acting on the surface at the free end. The various components of the hydrodynamic force are illustrated in Fig. 2.

When the beam's thickness is small compared to its width, the beam can be approximated as a laterally vibrating ribbon. If the beam is also quite wide relative to the boundary layer thickness, the problem is reduced to the steady state solution to Stokes' second problem (the hydrodynamic force acting on an oscillating infinite plane) [7]. It is known from [7] that if the incompressible version of the Navier-Stokes equation is used as the equation of motion for the fluid, the shear force per unit area can be found as

$$T = -V_0 e^{j\omega_{lat} t} \rho L \omega_{lat}^2 \dots - v(1+j) \quad (9)$$

[View Source](#) [?] where V_0 is the amplitude of the velocity at a particular point along the beam. Due to the symmetry of the problem, the total shear force will be double the force on the top of the beam. Assuming that the shear force is independent of y , the total shear force (per unit length) over a beam of width b is given by

$$F_{medium,lat} = 2 \int_{-b/2}^{b/2} T dy = -2V_0 e^{j\omega_{lat} t} \rho L \omega_{lat}^2 \dots - v(1+j)b \quad (10)$$

[View Source](#) [?]

The above equation states that the displaced mass and damping forces are equal in magnitude and are linearly dependent on both the width and the velocity V_0 , while dependent on the square-root of the viscosity, liquid mass density, and excitation frequency. The hydrodynamic function is then found as

$$F_{lat,Stokes(Re)} = F_{medium,lat} j \pi \eta Re V_0 e^{j\omega_{lat} t} = 2 \dots - v(1+j) \quad (11)$$

[View Source](#) [?]

E. Numerical Procedure

The previous analytical solution, (11), is based on the assumption that the force is uniform over the width of the beam and that the effects of beam thickness are negligible. Reference [8] made use of a numerical technique in order to find the hydrodynamic forces acting on an infinitely thin transversely oscillating beam without the assumption that the force is uniform over the width of the beam. At the time the present paper was being submitted, the authors were made aware of a very recent work [9] in which the method of [8] was extended to obtain a semi-analytical solution for the general case in which the aspect ratio of the rectangular cross section is arbitrary. In the present work, an arbitrary aspect ratio is also considered to account for the thickness effect; however, the approach taken here involved finite element modeling using ANSYS, followed by the determination of an analytical correction factor to be applied to Stokes' solution.

A 2-D mesh of fluid elements representing the cross-section of the fluid surrounding an oscillating beam was modeled with an unmeshed gap representing the beam itself. (This assumes that the beam's cross-section does not undergo deformation). The fluid was modeled as a rectangle with the cross-section of the beam having a constant width and a variable thickness. The mesh was defined so that the node density was higher near the beam and lower near the outer boundary. On the outer boundary, the fluid's displacement was set to zero. A sinusoidal velocity was imposed on the fluid-beam boundary. The amplitude of the displacement was held constant, while the frequency was varied in order to investigate the effects of different Reynolds numbers. The viscosity and mass density were set to that of water ($\rho_L=1000\text{kg/m}^3$ and $\eta=1\text{cP}$). A transient analysis was performed over 2 cycles, which was determined to be long enough (for the frequency range considered) to let the solution reach steady state. The pressure and shear forces acting on the fluid-beam interface were then extracted as a function of time, and the magnitude and phase offset from the imposed displacement determined.

Figure 2: The hydrodynamic force acting on a laterally oscillating beam can conceptually be decomposed into the forces acting parallel to the surface (the fluidic shear force) and forces acting perpendicular to the surface (the pressure). Due to the symmetry of the beam, the forces acting in the z direction cancel each other out

A convergence study was performed to identify an appropriate mesh density/distribution. The mesh density is higher around the beam cross section since higher gradients occur near the beam. The thickness of the beam and the resonant frequency were then varied to find $\Gamma_{\text{lat},R}$ and $\Gamma_{\text{lat},I}$ as a function of Re and h/b . Different mesh densities are used to insure convergence for $\Gamma_{\text{lat},R}$ and $\Gamma_{\text{lat},I}$.

SECTION III. Results and Discussion

A. Results of Numerical Simulation

Fig. 3 and 4 show the numerical results of the hydrodynamic function as a function of the Reynolds number and aspect ratio (on a log scale) for the real and imaginary part of the hydrodynamic function, respectively. The hydrodynamic function based on Stokes' theory is also plotted on the same figure. As the thickness decreases,

the two methods converge except when the Reynolds number is low. For low Reynolds numbers the assumption of uniform shear fails to account for edge effects, which are still present even as the thickness of the beam approaches zero.

B. Analytical Approximation

An analytical form of the hydrodynamic function may be obtained using both Stokes' solution and the numerical results. The ratio of the numerical results and the hydrodynamic function found from Stokes' theory is used to obtain a thickness correction factor for the real and imaginary parts of the hydrodynamic function. Fitting the trends in the ratio and assuming that the correction factor goes to one for high Reynolds numbers and thin beams, the hydrodynamic function with thickness effects taken into account is then given by

$$\text{Flat}(\text{Re}, h/b) = \frac{22 - \sqrt{\pi \text{Re}}}{2} \frac{1}{\sqrt{1 + C_R + jC_I}} \quad (12)$$

View Source [?] where the thickness correction factor for the real and imaginary hydrodynamic function are given, respectively, by

$$C_R = (1.658(h/b) + 1.83\text{Re}) \frac{1}{\sqrt{1 + 3.08(h/b) + 1}} \quad C_I = ((2.56 - 1.321(h/b)) \frac{1}{\sqrt{1 + 3.108(h/b) + 1}}) \quad (12a)(12b)$$

View Source [?]

Figures 3 and 4: The real (above) and imaginary (below) portion of the hydrodynamic function as a function of the aspect ratio and the Reynolds number found using a finite element model and compared to the thickness independent analytical results using Stokes' theory. The ratio of these two produces a correction factor that may be applied to Stokes' solution to account for the effects of thickness

For beams with an aspect ratio $h/b \geq 1/50$ and Reynolds numbers between 10 and 10,000, the fit of $\Gamma_{lat,I}$ is within 5.7%. For Reynolds numbers between 10 and 1000, the fit of $\Gamma_{lat,R}$ is also within 5.7%. At higher Reynolds numbers, the fit of $\Gamma_{lat,R}$ doesn't work well for beams with aspect ratio $h/b < 1/10$. A more complicated fitting model could be used; however, at a high Reynolds number the beam mass usually dominates the displaced fluid mass (i.e. the $\Gamma_{lat,R}$ term).

Figure 5: The simulated resonance curve of a 200×45×12 μm microcantilever vibrating both in-plane and out-of-plane in water

C. Frequency Spectrum

The solution to the equation of motion (Eq. 2) can be used to calculate the tip deflection as a function of frequency in water. It is shown in Fig. 5 that, for a particular geometry, the resonant frequency and 3-dB bandwidth are higher for the first in-plane mode compared to the first out-of-plane mode. Since the resonant frequency increases more than the 3-dB bandwidth, the quality factor (ratio of the resonant frequency and the 3-dB bandwidth) for the in-plane mode is thus higher (by a factor of 2 in this case). The resonant frequency of the in-plane mode will always be higher due to the increased stiffness.

It can also be seen from Fig. 5 that care should be taken when choosing the beam's geometry so that the first lateral mode's resonant frequency does not interfere with the resonant frequency at a higher-order transverse mode.

D. Resonant Frequency

The resonant frequency of a laterally vibrating beam is found using Eq. 3. In a vacuum, the theoretical resonant frequency is linear in b/L^2 , while independent of beam thickness. When operating laterally in a viscous liquid media, the b/L^2 dependence remains. (See Fig. 6.) However, the addition of pressure effects causes a slight dependence on the thickness. As h increases and exceeds b , the resonant frequency reduces to that of a microcantilever undergoing out-of-plane vibration with width h and thickness b . The resonant frequency of a beam vibrating out-of-plane decreases dramatically when placed into water due primarily to the large increase in the displaced fluidic mass. The result of the frequency decrease is a lower sensitivity in liquid chemical sensor applications.

Figure 6: The simulated resonant frequency of laterally vibrating microcantilevers in water as a function of b/L^2 for widths of 45, 60, 75, and 90 μm , lengths of 200, 400, 600, 800 and 1000 μm , and a thickness of 12 μm

As shown in Table I, this decrease can range from 36 to 50%, or larger for longer beams. The resonant frequency of the same beams vibrating laterally only drops by 5 to 10%, even when the thickness effects are taken into account. This trend continues even for higher viscosities as shown in Fig. 7. The resonant frequency of a $200 \times 45 \times 12$ μm beam in 70% aqueous glycerol (23 cP) vibrating in the in-plane mode will decrease by 15% while the same beam vibrating in the out-of-plane mode will see a 45% decrease in its resonant frequency. This effect is even more pronounced for a thin beam vibrating in the out-of-plane mode. The chemical sensitivity of a microcantilever chemical sensor is proportional to the resonant frequency. This also indicates that the high chemical sensitivity of a laterally vibrating microcantilever in air will be mostly retained when operating in viscous liquid media.

E. Quality Factor

The quality factor can be found using Eq. 8. The quality factor was first calculated as a function of $b^{1/2}/L$ using the numerical results for the hydrodynamic function. The results were compared to those obtained using the analytical expression with the correction factor. The calculated Q-factors were found to be within 5% of those obtained from the finite element model. From Fig. 8, Fig. 9, and Table I, it is seen that the quality factor of a laterally vibrating microcantilever is better for beams that are thicker, wider, and shorter due to their increase in stiffness. When the thickness effects are included, the quality factor decreases due to the additional fluid damping. The quality factor will still increase with increasing thickness; however, this is no longer true as h/b approaches 1. Even with the addition of the thickness effects, the quality factor is still higher than that of the same beam excited transversely. The calculated improvement ranges from a factor of 2 to 3 or higher depending on the beam geometry. It is noted that the improvement in quality factor is not as high as stated in [3] which did not consider the frictional losses.

Figure 7: Simulated resonant frequency of a 200×45×12 μm vibrating laterally and transversely in concentrations of up to 70% aqueous glycerol (23 cP). Note the drastic drop in the resonant frequency for the transverse mode compared to the lateral mode

Figure 8: Quality factor of laterally vibrating microcantilevers in water for a length, L , of 200 μm , and thickness, h , of 12 μm , and widths, b of 45, 60, 75, and 90 μm as a function of $b^{1/2}/L$

Figure 9: Quality factors of laterally vibrating microcantilevers in water with and without thickness effects. The width is varied between 45 and 90 μm, the length from 200 to 1000 μm, and the thickness of 12 μm and 14.5 μm. Experimental data are from [10]

Table I: Resonant frequencies and quality factors predicted for four sample beam geometries

Geometry $[\mu\text{m}]$	$f_{res,trans}$ (air) [MHz]	$f_{res,trans}$ (water) [MHz]	Percent Change	$f_{res,la}$ (air) [MHz]	$f_{res,lat}$ (water) [MHz]	Percent Change	Q_{trans} (water)	Q_{lat} (water)
200×45×12	0.412	0.264	−35.9%	1.547	1.411	−8.79%	17.3	34.3
200×90×12	0.412	0.214	−48%	3.095	2.934	−5.2%	22.7	60.0
200×45×6	0.206	0.102	−50.4%	1.547	1.443	−6.72%	9.1	21.2
400×45×12	0.103	0.064	−37.8%	0.386	0.347	−10.1%	9.3	17.1

SECTION IV. Conclusions

The resonant frequency and quality factor of laterally vibrating microcantilevers in a viscous liquid medium were analyzed in terms of the beam's geometry and the properties of the liquid and compared to those of transversely vibrating microcantilevers. The hydrodynamic force acting on the beam was evaluated, accounting for both the shear force and the pressure force. It was found that, even with relatively thin beams, there was a need to account for the effects of thickness to obtain accurate estimates of the hydrodynamic forces. A set of correction factors were determined for modifying Stokes' solution to include the effects of thickness. It was found that the resonant frequency for in-plane modes was higher than that of out-of-plane modes. Due to the increase in the resonant frequency, the chemical sensitivity is expected to be much larger for laterally vibrating cantilevers. Furthermore, the improvement in the quality factor and thus a reduced frequency noise are expected to yield much lower limits of detection (LOD) in liquid-phase chemical sensing applications.

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REFERENCES

- 1.M. Sepaniak, P. Datskos, N. Lavrik and C. Tipple, "Microcantilever Transducers: A new Approach in Sensor Technology", *Analytical Chemistry*, vol. 74, no. 21, pp. 568-575, 2002.
- 2.K. Goeders, J. Colton and L. Bottomley, "Microcantilevers: Sensing Chemical Interactions via Mechanical Motion", *Chemical Reviews*, vol. 108, no. 2, pp. 522-542, 2008.
- 3.I. Dufour, S. Heinrich and F. Josse, "Strong-axis Bending Mode Vibrations for Resonant Microcantilever (Bio)chemical Sensors in Gas or Liquid Phase", *IEEE International Ultrasonics Ferroelectrics and Frequency Control Joint 50th Anniversary Conference*, pp. 193-199, 2004.
- 4.S. Basak and A. Ramana, "Hydrodynamic Loading of Microcantilevers Vibrating in Viscous Fluids", *Journal of Applied Physics*, vol. 99, no. 114906, pp. 1-10, 2006.
- 5.J. Sader, "Frequency Response of Cantilever Beams Immersed in Viscous Fluids with Applications to the Atomic Force Microscope", *Journal of Applied Physics*, vol. 84, no. 1, pp. 64-76, 1998.
- 6.R. Cox, F. Josse, M. Wenzel, S. Heinrich and I. Dufour, "Generalized Model of Resonant Polymer-Coated Microcantilevers in Viscous Liquid Media", *Analytical Chemistry*, vol. 80, no. 15, pp. 5760-5767, 2008.
- 7.G. Stokes, "On the Effects of the Internal Friction of Fluids on the Motion of Pendulums", *Transactions of the Cambridge Philosophical Society*, vol. 9, pp. 8-106, 1851.
- 8.E. Tuck, "Calculation of Unsteady Flows due to Small Motions of Cylinders in a Viscous Fluid", *Journal of Engineering Mathematics*, vol. 3, no. 1, pp. 29-44, 1969.
- 9.D. Brumley, M. Willcox and J. Sader, "Oscillation of Cylinders of Rectangular Cross section Immersed in Fluid", *Physics of Fluids*, vol. 22, no. 052001, 2010.
- 10.L. Beardsless, A. M. Addous, S. M. Heinrich, F. Josse, I. Dufour and O. Brand, "Thermal Excitation and Piezoresistive Detection of Cantilever In-Plane Resonance Modes for Sensing Applications", *accepted for publication in Journal of Microelectromechanical Systems*, 2010.