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Learning Dynamics in Monetary Policy: The Robustness of an Aggressive Price Stabilizing Policy

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Abstract
This paper investigates the effect of an aggressive inflation stabilizing monetary policy on the ability of agents to reach a rational expectations equilibrium for inflation and output. Using an adaptive learning framework, we develop a model that combines a real wage contracting rigidity with an interest rate rule. We show that an AR(1) equilibrium requires more aggressive monetary policy to achieve both determinacy and learnability. This model and policy findings contrast with Bullard and Mitra’s [Determinacy, learnability and monetary policy inertia (2001); Journal of Monetary Economics 49 (2002) 1105] model (no inflation persistence) and policy findings (less aggressive policy). These results suggest that aggressive policy is robust in different model specifications.

Previous article
1. Introduction

In this paper, we examine the determinacy and expectational stability condition of the rational expectations equilibrium (REE) in a simple macroeconomic model. These conditions are established using a particular monetary policy tack under an interest rate rule—the Taylor rule. In previous work, Bullard and Mitra, 2001, Bullard and Mitra, 2002 evaluate the determinacy and learnability of an REE with alternative policy rules. They use a purely forward-looking model of the economy suggested by Woodford (1999, p. 16) and combine that model with four information structures of the Taylor rule: (1) contemporaneous data, (2) lagged data, (3) forward-looking expectations, and (4) contemporaneous expectations. They find that policy rules which respond relatively aggressively to inflation with little or no reaction to the output gap generally induce both determinate and learnable REE (Bullard and Mitra, 2001, Bullard and Mitra, 2002).

We extend this line of thought by introducing a relative-real wage contracting model in combination with a Taylor rule (Fuhrer, 1995, Fuhrer and Moore, 1995, Taylor, 1993, Taylor, 1994, Taylor, 1999). This model contains the characteristic of inflation persistence since agents not only consider a forward-looking component of the inflation rate but they also are concerned with the past values of inflation. A number of studies have argued that the model without inflation inertia is not realistic (Woodford, 1999). Owyang, 2001, Siklos, 1999, Granato and Wong, 2002 also show that the inflation rate in the United States is persistent. Ironically, it is Woodford who states that “the complete absence of inertial terms in the structural equations is not entirely realistic” (Woodford, 1999, p. 13).  

McCallum (2002) also argues that Woodford (1999) model is not robust to the inflation persistence situation. McCallum (2002) solves the model of Woodford (1999) with full price flexibility and shows that an alternative AR(1) REE of inflation is explosive if Taylor rule is aggressive to the deviations of inflation (McCallum, 2002, p. 5). Comparing with the results suggested in Bullard and Mitra, 2001, Bullard and Mitra, 2002 we show that the condition for determinacy and learnability of the AR(1) REE occurs when the policymaker responds more aggressively to inflation. This result provides a stronger argument about the overall robustness of following an aggressive inflation stabilizing monetary policy.

The paper is organized as follows. We describe the model in the next section. Section 3 examines the determinacy condition of the model. Section 4 introduces an adaptive learning approach to show the E-stability condition. Section 5 concludes the paper.

2. The model

The model has three equations: a real wage contracting equation, an IS curve, and a policy rule. Following tradition, we assume that agents care about their real wages. However, instead of considering the overlapping nominal wage contracts model (Taylor, 1980) which implies a very flexible rate of inflation we generalize another contracting specification that is based on the recent contributions of Fuhrer, 1995, Fuhrer and Moore, 1995:

\[
(1) \pi_t = (1-\mu)\pi_{t-1} + \mu \delta E_t \pi_{t+1} + \gamma y_t + u_{1t}, \gamma > 0,
\]

where $\pi_t$ and $y_t$ are inflation rate and output gap respectively, $E_t \pi_{t+1}$ denotes the expected inflation rate over the next period, $\mu$ represents the weight of the expected inflation rate on the current inflation, $\delta$ is the household’s discount factor and $u_{1t} \sim \text{iid}(0, \sigma_{u1}^2)$. All variables are expressed in deviations from a steady state.
Eq. (1) nests the specifications in Fuhrer, 1995, Fuhrer and Moore, 1995, Bullard and Mitra, 2001, Bullard and Mitra, 2002 as special cases. Fuhrer, 1995, Fuhrer and Moore, 1995 assume a two-period contract model where the market price is expressed as the average of the current and the lagged contract wages. Since agents are concerned with their real wages over the lifetime of their contract, they derive the price setting rule of Eq. (1) where $\mu = 1/2$. On the other hand, Bullard and Mitra, 2001, Bullard and Mitra, 2002 follow the set up in Woodford (1999) and solely rely on the forward-looking component of inflation, $E_{t} \pi_{t+1}$. Eq. (1) therefore can be reduced to a special case where $\mu = 1$.

Eq. (2) is the “IS” curve derived from the Euler equation for consumer utility maximization (McCallum and Nelson, 1999, Woodford, 1999).

\[
(2) \quad y_t = -\beta (r_t - E_t \pi_{t+1} - r^*) + \lambda E_t y_{t+1} + u_{2t}, \beta, \lambda > 0,
\]

where $r_t$ is nominal interest rate, $r^*$ is the target real interest rate and $u_{2t} \sim \text{iid}(0,\sigma_{u22})$.

The policy rule that policymakers follow is the interest rate rule popularized by Taylor, 1993, Taylor, 1994, Taylor, 1999:

\[
(3) \quad r_t = \pi_t + \alpha_{\pi} (\pi_t - \pi^*) + \alpha_{y} y_t + r^*.
\]

Taylor (1999) argues that his interest rate rule is related to the quantity theory of money. He further asserts that his policy rule accurately describes different historical time periods when there were many different policy regimes. The Taylor rule has an important attribute in that it provides a basis for determining how aggressively policymakers respond to deviations from price and output targets.

In this vein, Clarida et al. (2000) argue that a non-aggressive monetary policy rule is a policy rule which accommodates inflationary pressure by reducing the real interest rate. This reduction of the real interest rate creates the self-fulfilling effect on agents’ expectations. It also stimulates an increase in aggregate demand and inflation from Eq. (2). On the other hand, Clarida et al. (2000) define an aggressive monetary policy rule as one where policymakers raise (lower) the real interest rate when inflationary (deflationary) pressures present themselves in the economy. We see that Eq. (3) can be categorized as the aggressive rule if both $\alpha_{\pi}$ and $\alpha_{y}$ are positive (Taylor, 1993, Taylor, 1994). Positive values of $\alpha_{\pi}$ and $\alpha_{y}$ indicate a willingness to raise (lower) real interest in response to the positive (negative) derivations from either target inflation rate ($\pi_t - \pi^*$) and output gap ($y_t$).

To solve the system, we substitute (3) into (2) and solve for $\pi_t$ and $y_t$ using the following form:

\[
(4) \quad z_t = A + BE_t z_{t+1} + Cz_{t-1} + \zeta_t
\]

where
\[ z_t \equiv \begin{bmatrix} \pi_t \\ y_t \end{bmatrix}, A \equiv W \begin{bmatrix} 0 & \delta \mu \\ \beta & \lambda \end{bmatrix}, B \equiv W \begin{bmatrix} 1 - \mu & 0 \\ 0 & 0 \end{bmatrix}, \zeta_t \equiv W \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix} \] and \[ W \equiv \begin{bmatrix} 1 - \gamma \beta \left(1 + \alpha_\pi \right) & -\gamma \\ \beta (1 + \alpha_\pi) & 1 + \beta \alpha_y \end{bmatrix}^{-1}. \]

3. Determinacy

We consider whether the REE is unique in this model. We first eliminate the constant term \( A \) by taking the difference from the mean value of the Eq. (4):

\[ \ddot{z}_t = B \dot{E} \ddot{z}_{t+1} + C \ddot{z}_{t-1} + \zeta_t, \]

where \( \ddot{z}_t = \dot{z}_t - \ddot{z} \) and \( \ddot{z} = (I - B - C)^{-1}. \)

Then we rearrange Eq. (5):

\[ \begin{bmatrix} 1 & 0 & -C_{11} \\ 0 & 1 & -C_{21} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{\pi}_t \\ \ddot{y}_t \\ \ddot{\pi}_L \end{bmatrix} = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{21} & B_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\pi}_{t+1} \\ \ddot{y}_{t+1} \\ \ddot{\pi}_{L,t+1} \end{bmatrix} + \begin{bmatrix} \ddot{\zeta}_{1t} \\ \ddot{\zeta}_{2t} \\ 0 \end{bmatrix} - \begin{bmatrix} B'_{1} \\ B'_{2} \\ 0 \end{bmatrix} \eta_{t+1} \]

where

\[ \ddot{\pi}_t \equiv \ddot{\pi}_{t-1}, C \equiv \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix}, B \equiv \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} = \begin{bmatrix} B'_{1} \\ B'_{2} \end{bmatrix}, \zeta_t \equiv \begin{bmatrix} \ddot{\zeta}_{1t} \\ \ddot{\zeta}_{2t} \end{bmatrix}, \dot{E} \ddot{z}_{t+1} = \ddot{z}_{t+1} - \eta_{t+1} \text{ and } \eta_t \sim \text{iid}(0, \sigma_\eta^2). \]

Taking inverse of the left hand side matrix and multiplying the matrix associated with the one time period forward variables gives us:

\[ J = \begin{bmatrix} 0 & \frac{\beta}{1+\beta \alpha_y} & 0 & \frac{\beta (1+\alpha_\pi)}{1+\beta \alpha_y} \\ \frac{\mu (1+\alpha_\pi) + \beta \gamma}{1+\beta \alpha_y(-1+\mu)} & \frac{\gamma \lambda}{1+\beta \alpha_y(-1+\mu)} & \frac{1+\beta \gamma (1+\alpha_\pi) + \beta \alpha_y}{(1+\beta \alpha_y(-1+\mu)} \end{bmatrix}. \]

In Eq. (6), \( \ddot{\pi}^t \) and \( \ddot{y}^t \) are free endogenous variables whereas \( \ddot{\pi}^t_L \) is a predetermined endogenous variable. It is necessary to have exactly two of the eigenvalues of \( J \) to be inside the unit circle for uniqueness. We provide the condition of determinacy in the following proposition.

**Proposition 1**

According to the model in Eq. (4) where \( \mu \in (0, 1) \), the necessary and sufficient condition for a unique REE is that...
\(\gamma \alpha_{\pi} + (1 - \delta) \mu \alpha_y > 0\)

and

\(\alpha_y > -1/\beta.\)

**Proof**

We calculate the characteristic polynomial of inverse matrix of \(J\), we have:

\[
p(L) = \frac{(L - 1)(L\beta \gamma + L\lambda - 1) - (L^2 \delta - 1)(L\lambda - 1)\mu - L\beta \gamma \alpha_{\pi} + \beta \alpha_y[1 - \mu + L(L\delta\mu - 1)])}{\delta\lambda\mu}\]

Note that \(p(1) < 0, p(0) > 0\) and \(p(-1) > 0\). By Descartes’ rule of signs (Barbeau, 1989, p. 171), there is necessarily a positive root and either two negative roots or a pair of complex conjugates. \(p(1) < 0\) and \(p(-1) > 0\) if (7) is satisfied. \(p(0) > 0\) holds by condition (8). □

According to the general relative-real wage contracting specification in Eq. (1), a lagged inflation term is included in convex combination with the expected inflation term. As \(\mu\) represents the relative weight between lagged and expected inflation, \(\mu \to 1\) nests the determinacy condition in Bullard and Mitra’s (2002, p. 1115) model.

For the case of \(\mu \in (0, 1)\), Proposition 1 tells us that in an economy with inflation persistence, monetary policy must have a larger parameter in hitting inflation targets (or both inflation and output) in order to attain a unique REE. This result provides a stronger stability condition than the one in Bullard and Mitra, 2001, Bullard and Mitra, 2002. On the other hand, if the lagged inflation component dominates in the model (i.e., \(\mu \to 0\)), the necessary and sufficient condition for determinacy requires \(\alpha_{\pi} > 0\). This result implies that monetary policy must be aggressive enough in responding to inflation target deviations (see Fig. 1).

**Fig. 1. Regions of determinacy and E-stability.**

4. An adaptive learning approach

Although the REE is determinate, it is also important to see whether agents are able to learn the REE. In this section, we analyze the expectational stability (or E-stability) condition in this model (Evans, 1985, Evans and Honkapohja, 1995, Evans and Honkapohja, 2001). We assume that agents initially might not be able to
form rational expectations. Instead, they learn in an adaptive manner and form expectations as new data become available over time. We also assume that agents make their forecasts by using recursive least squares. Suppose that agents believe that inflation and output gap follow the process (perceived law of motion or PLM):8

\[ z_t = a_{t-1} + b_{t-1} z_{t-1} + c_{t-1} \zeta_t, \]

where \( a_t, b_t \) and \( c_t \) are the coefficients updated by running recursive least squares using actual data (i.e., \( 1, z_{t-1}, \zeta_t \)) available over time. Evans and Honkapohja (2001) assume that the current value of the endogenous variable \( z_t \) is not available at the time of expectations formation. This eliminates any simultaneity problem.

The expected value of \( z_{t+1} \) at time \( t \) is:

\[ E_t z_{t+1} = (a_{t-1} + b_{t-1} a_{t-1}) + b_{t-1}^2 z_{t-1} + b_{t-1} c_{t-1} \zeta_t. \]

Inserting Eq. (10) into Eq. (4), one can solve for the actual law of motion, or ALM, implied by the PLM:

\[ z_t = (A + Ba_{t-1} + Bb_{t-1} a_{t-1}) + (Bb_{t-1}^2 + c_{t-1}) z_{t-1} + (Bb_{t-1} c_{t-1} + I) \zeta_t. \]

We define the mapping (T-Mapping) from the PLM to the ALM as:

\[ T \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} A + Ba + Bba \\ Bb^2 + c \\ Bbc + I \end{pmatrix}. \]

We obtain the E-stability conditions by deriving the differential equation from Eq. (12). Using the rules for vectorization of matrix products from Magnus and Neudecker (1999, p. 184), we have:

\[ DT_a \begin{pmatrix} a \\ b \end{pmatrix} = B \begin{pmatrix} I + b \end{pmatrix}, \]

\[ DT_b \begin{pmatrix} b \end{pmatrix} = b' \otimes B + I \otimes Bb, \]

\[ DT_c \begin{pmatrix} b, c \end{pmatrix} = Bb. \]

Eq. (13) gives us the following result:

**Proposition 2**

An MSV solution \( a, b, c \) to Eq. (4) is E-stable if all eigenvalues of the matrices \( DT_a(a, b), DT_b(b), DT_c(b, c) \), given by Eq. (13), have real parts less than 1. The solution is not E-stable if any of the eigenvalues has a real part greater than 1 (Evans and Honkapohja, 2001).

Using baseline parameters suggested by Woodford, 1999, Bullard and Mitra, 2001, Bullard and Mitra, 2002, we illustrate our results in Fig. 1. We show that the E-stability condition is consistent with the determinacy condition (7). Fig. 1 also presents how the value of \( \mu \) changes both determinacy and E-stability condition numerically.

There are three cases: (1) Bullard and Mitra’s model (\( \mu \to 1 \)), (2) Fuhrer and Moore’s model (\( \mu = 1/2 \)), and (3) a case without forward looking component (\( \mu \to 0 \)). Fig. 1 plots the regions of determinacy and expectation stability as a function of \( \alpha_p \) and \( \alpha_y \) in above cases. In the region where \( \alpha_p > 0 \), the rational expectational equilibrium is determinate and E-stable in all cases. However, the region of indeterminacy and E-unstability in Fuhrer and Moore’s specification (\( \mu = 1/2 \)) is larger than that in Bullard and Mitra’s setup without the
component of inflation persistence ($\mu \to 1$). This result suggests that if inflation persistence exists in an economy, policymakers should impose larger inflation and output gap parameters in the policy rule such that the REE can be determinate and learnable. Moreover, if inflation in an economy is solely affected by the lagged inflation ($\mu \to 0$), policymakers must be aggressive in attaining their inflation target to satisfy the determinacy and learnability condition. These results indicate there is a much broader and robust implication from an aggressive, inflation stabilizing monetary policy.

5. Conclusion
This paper extends the study of Bullard and Mitra, 2001, Bullard and Mitra, 2002 on determinacy and learnability of an REE when policymakers follow a Taylor interest rate policy rule. Instead of following a purely forward-looking model suggested by Woodford (1999), we modify a real wage contract model by Fuhrer, 1995, Fuhrer and Moore, 1995 along with a contemporaneous Taylor rule. The real wage contract model contains the characteristic of inflation persistence since agents concern themselves with both past and future inflation rates. Based on this model, we find a stronger, more robust result about aggressive, inflation stabilizing monetary policy than the one given by Bullard and Mitra, 2001, Bullard and Mitra, 2002. We show that if inflation is more persistent in an economy, the policy rule should have larger value of parameters on inflation and output derivations so that the AR(1) rational expectation equilibrium is determinate and learnable. If inflation is extremely persistent in an economy, the policy rule should respond aggressively to deviations from their inflation target for determinacy and learnability. Our finding has important implications for policy since it focuses exclusive attention on inflation stabilization. Policymakers who respond aggressively to deviations from their inflation target will be more successful in stabilizing business cycle fluctuations.

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References


Notes

1 See Evans and Honkapohja (2001).
2 We use the terms “expectational stability,” “E-stability,” and “learnability” interchangeably in this paper.
3 In this paper, we investigate the effectiveness of a policy rule with contemporaneous data only (Taylor, 1993, Taylor, 1994, Taylor, 1999). There are many studies that investigate this monetary policy rule and treat this specification as standard in the literature (see also Cecchetti and Ehrmann, 1999, Fuhrer and Moore, 1995, Bullard and Mitra, 2001, Bullard and Mitra, 2002, Taylor, 1993, Taylor, 1994, Taylor, 1999, Woodford, 1999). Taylor argues that this policy rule is derived from the quantity theory of money. He points out that it describes different historical time periods in the United States.
4 We would like to thank the referee for suggesting this reference and also in pointing out this argument.
5 McCallum (2002) shows that only the minimum state variable (MSV) solution is stationary under the set up of Woodford (1999) with full price flexibility. He then considers the other candidate solution, an AR(1) REE, and shows that the solution is explosive if the coefficient of inflation in the Taylor rule is positive.
7 Fuhrer, 1995, Fuhrer and Moore, 1995 do not set up their model as the deviation from a steady state. However, the model can be derived in that form without changing the equations being considered here.
8 In this model, we mainly focus on the minimal state variable (MSV) solutions (McCallum, 1983).