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“BIG IDEAS” IN SECONDARY MATHEMATICS EDUCATION PROGRAMS

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Although research and policy documents provide recommendations to inform secondary mathematics teacher preparation, no single study has addressed the “big ideas” of courses in multiple programs and how those big ideas may be interpreted through the lens of recent research and policy documents. To answer this need, we focused on big ideas and course objectives from three courses (i.e., Linear Algebra, Secondary Mathematics Methods, and Teaching in a Diversity Society) taught in four secondary mathematics programs. Major themes emerged related to mathematical content, pedagogy, and issues of equity. We describe findings related to big ideas, course objectives, and their connections to recommendations from current policy documents. Such integration contributes to promoting dialogue related to the preparation of mathematics teachers and informing teacher educators.

Keywords: Preservice Teacher Education, Linear Algebra, Equity

Given the challenges that future mathematics teachers will face in supporting their students for successful learning, preservice teachers (PSTs) must be guided by quality instruction to meet these challenges (Association of Mathematics Teacher Educators [AMTE], 2017). Extant research has addressed aspects of knowledge that teachers need to develop related to both content and pedagogy (e.g., Ball, Thames, & Phelps, 2008; Fuson, Kalchman, & Bransford, 2005). Specifically, teachers need preparation that “covers knowledge of mathematics, of how students learn mathematics, and of mathematical pedagogy that is aligned with recommendations of professional societies” (National Research Council [NRC], 2010, p. 123). Recent policy documents have been written by mathematicians, mathematics educators, and teacher educators to focus on: (a) how secondary mathematics teachers should be prepared (e.g., National Council of Teachers of Mathematics [NCTM], 2014; AMTE, 2017), (b) how mathematics should be taught---to future teachers or any undergraduates (e.g., Mathematical Association of America [MAA], 2018; NCTM, 2014), and (c) how university instructors should address issues of equity and access to mathematics, and communicate these issues to future teachers (e.g., AMTE, 2017; MAA, 2018; NCTM, 2000; 2014). Little is known about how such recommendations are incorporated into programs of study in mathematics for teachers (NRC, 2010). Although these policy recommendations inform secondary mathematics teacher preparation, no single study has addressed the “big ideas” of courses in multiple secondary teacher preparation programs and how these big ideas are interpreted through the lenses of recent research reports and policy documents (e.g., AMTE, 2017; Ball et al., 2008; Fuson et al., 2005; MAA, 2018). This paper utilizes data from a larger study that administered a national survey and conducted case studies to describe opportunities that secondary mathematics preparation programs provided for PSTs to learn about mathematical content, teaching practices, and issues of equity. In this study, we examined three courses required at four universities (i.e., 12 courses) to address the following question: *How do course goals and big ideas of courses in secondary mathematics teacher education programs emphasize areas related to content and teaching practices necessary for future mathematics teachers as recommended by policy documents?*

Relevant Literature

The teaching and learning of mathematics for K-16 students has been studied extensively, leading to detailed conceptualizations and descriptions of essential mathematics content and student experiences. To develop our framework for this study, we examined several national policy documents (e.g., AMTE, 2017; MAA, 2018; NCTM, 2000; 2014).

The authors of MAA's (2018) *Instructional Practices Guide* argued that "professional associations in the mathematical sciences along with state and national funding agencies are supporting efforts to radically transform the undergraduate education experience" (p. vii). The guide provided overviews and vignettes of effective mathematics teaching practices for undergraduate mathematics faculty. The introductory manifesto argued that mathematics instruction should incorporate experiences that allow access to rich and deep mathematics for all students, which requires ongoing change and attention to classroom, assessment, and task design practices.

The standards described in AMTE (2017) inform the preparation of preservice mathematics teachers, including "clearly articulated expectations for what well-prepared beginning mathematics teachers need to know and be able to upon completion of a certification or licensing program and the recommended characteristics for programs to support teachers' development" (p. xii). AMTE built on existing research (e.g., Ball & Forzani, 2011; Hill, Rowan, & Ball, 2005; Shulman, 1986) and policy documents (e.g., NCTM, 2000; NCTM, 2014). For example, AMTE (2017) stated that learning to teach mathematics requires deep understanding of the content they will teach, knowledge of how students reason about mathematics, knowledge of instructional approaches that support students' mathematical learning, and awareness of the societal context in which the content is used in students' everyday life. Similarly, several researchers proposed the importance of knowledge related to mathematical content, student thinking, and instructional approaches (Ball et al., 2008; Hill et al., 2005; Shulman, 1986), as well as the societal context in which mathematics is taught (Turner, Celedón-Pattichis, Marshall, & Tennison, 2009).

Teaching for access and equity is emphasized by researchers and professional organizations, including AMTE (2017), MAA (2018), and NCTM (2000). AMTE, for example, highlighted the importance of equity as the first of five foundational assumptions about mathematics teacher preparation, stating "Although equity, diversity, and social justice issues need to be specifically addressed as standards, they must also be embedded within *all* the standards...we believe that equity must be both addressed in its own right and embedded within every standard" (p. 1). Similarly, NCTM (2000) emphasized equity as the first "principle," highlighting the critical importance of ensuring *all* students have access to mathematics programs that provides quality instruction. MAA (2018) also emphasized inequities existing in our society, encouraging instructors to provide mathematics instruction that increases access to all students. While these standards provide recommendations for mathematics educators to shape their courses, little is known about how these recommendations are enacted in specific mathematics, mathematics education, and general education courses in secondary mathematics teacher education programs.

Method

As part of a larger study, we conducted a series of interviews in secondary mathematics teacher education programs at four universities: Great Lakes University (GLU), Midwestern Research University (MRU), Midwestern Urban University (MUU), and Southeastern Research University (SRU); the institutions were chosen based on the diverse nature of their student populations, the types of communities in which they were situated, and the departmental homes

of their secondary mathematics education programs. At each university, the research team selected approximately ten required courses in the secondary mathematics teacher education program based on the likelihood that each course would offer students opportunities to learn algebra content and/or to learn to teach algebra; courses included mathematics, mathematics education, mathematics for teachers, and general education courses. For the purpose of this paper, we examined one Linear Algebra course, one Secondary Mathematics Methods course, and one General Education course related to teaching in a diversity society required at each university.

For each course, we collected a syllabus and interviewed an instructor. We asked each instructor, “What are the goals or big ideas of this course?” For mathematics content courses, we asked a follow-up question, “Do you do anything specific in this course to help prepare future mathematics teachers?” We analyzed course goals and big ideas as reported in the interview and written in corresponding course syllabi under “Course Objectives” or “Course Goals.” To clarify statements from the big ideas question and course objectives, at times we examined other elements of a course syllabus or responses to follow-up questions.

To answer our research question, multiple policy documents (e.g., AMTE, 2017; MAA, 2018; NCTM, 2000; 2014) were reviewed by the three authors. We also examined instructor responses from interview transcripts and text from corresponding syllabi, noting emergent themes that were common and different across the courses (Creswell, 2007). We focused on emergent themes that related to recommendations in policy documents. We compared themes to summarize similarities and differences between course objectives in syllabi and instructor responses to interview questions. After writing a summary of responses, we iteratively reviewed their original responses, considering what they reported through the lens of selected policy documents.

Findings

We present findings from each course type in this section: linear algebra, secondary mathematics methods, and teaching in a diverse society. We compare similarities and differences between reported big ideas and course objectives, through the lens of policy documents.

Linear Algebra

The four Linear Algebra instructors understandably described their big ideas, their goals for Linear Algebra, in similar ways. For example, instructors reported focusing on: moving from or between concrete mathematical situations and abstractions (MRU, SRU); studying systems of linear equations and their solutions (GLU, MUU, SRU); eigenvectors, eigenvalues, and eigenspaces (GLU, MUU, SRU); computational applications (MRU, SRU); and learning ideas that are needed in other areas of mathematics and other disciplines (GLU, MUU, SRU). In this section, we describe how the four Linear Algebra instructors reported their intended classroom practices, assessment practices, and course (and task) design practices.

Three of the instructors (GLU, MUU, and SRU) described their modeling of teaching strategies that they believed teachers might notice and use in their own teaching. The GLU instructor explained that in his department, the culture is that mathematicians and mathematics education specialists work well together so he incorporated teaching strategies to be consistent with experiences in pedagogy courses. The MUU and SRU instructors felt their teaching strategies would implicitly support future teachers. Instructors’ responses revealed intended instructional practices.

MAA (2018) described several strategies to support access to mathematics for all students

through classroom practices. We focused on the use of groupwork, supporting productive struggle, and supporting critical thinking and reasoning. MAA strongly recommended use of small groups as a strategy for collaborative learning. In his syllabus, the GLU instructor urged students to work together on homework. He reported using groupwork extensively in the course, saying that students worked in small groups on activities to consolidate ideas, foreshadow ideas, or discover concepts. MAA described practical tips for supporting productive struggle, practice that is essential in mathematics. In alignment with these goals, the MUU instructor described his structure of class sessions as "lively," with spontaneous discussions of student questions and struggles: "going through the challenges that they also went through and showing how they have overcome that." MAA described strategies for "responding to student contributions in the classroom," especially "creating a safe space for incorrect answers" and "focusing on reasoning" (pp. 5-6). The SRU instructor described that he pushed all of his students to explain their reasoning with a focus on explanations, proofs, counter-examples, and holding students accountable for addressing the why rather than just stating facts.

MAA (2018) recommended using multiple forms of assessment, including formative assessment cycles and summative assessment when appropriate. In their syllabi, instructors described their assessments as including: three exams and a final exam (all), weekly homework (GLU, MRU, SRU), weekly quizzes (GLU, MRU, MUU), and computer lab activities (GLU). Although neither assigned points for student journaling, the MUU and GLU instructors reported that they encouraged students to write mathematical journals. The MUU instructor's syllabus did not indicate his expectation for writing journals; however, he reported that, in the beginning weeks of the course, he frequently told students when certain ideas from their homework or class notes should be written in their journals. After several weeks, he said he would stop pointing out these ideas, expecting students to take ownership of their needs for the journals. The GLU instructor gave a clear description of expectations for mathematical communication and his expectation for students to keep "a well-organized record of all your study notes and completed problems for future reference."

Providing clear learning goals to students is recommended as a course design practice in MAA (2018). In his syllabus, the MUU instructor listed clear learning goals for students that included attention to content and process; for example, "Be able to apply some technology...to facilitate problem solving." Regarding task design, MAA drew on Stein et al. (1996) to recommend that students have opportunities to engage in high-level tasks that allow multiple solution strategies: "there is not a predictable, well-researched approach or pathway explicitly suggested by task instructions" (p. 31). MAA also drew on Boaler (2015) to recommend that instructors open tasks to provide open learning spaces using several strategies, including "open the task up to multiple methods, pathways and representations" (p. 40). The GLU instructor reported that he modeled valuing mathematical processes and encouraged students to recognize the potential for alternative, and equally valid, solutions or approaches.

Secondary Mathematics Methods

The four secondary mathematics courses share commonalities and differences in their big ideas. Common features of these courses included the practices of planning and implementing mathematical lessons, analyzing students' mathematical thinking, and exploring instructional materials. While three courses (GLU, MUU, SRU) focused more on pedagogical content (e.g., learning to assess student learning or identify appropriate questions), the MRU course centered around the reconstruction of school mathematics (e.g., ratios and proportional reasoning, integers) to envision how PSTs would communicate mathematically with their students. In this

section, we describe how these secondary mathematics methods instructors provided opportunities for PSTs to develop knowledge and practices.

Most secondary mathematics courses involved field experience components that required the design and implementation of lesson plans. AMTE (2017) recommended that effective secondary preparation programs provide PSTs with multiple opportunities to learn to teach through clinical experiences with coherent, developmentally appropriate contents. The GLU instructor, for example, mentioned a big idea addressing this recommendation: “We spend a great deal of the semester focusing on this cycle [teaching-learning cycle]... you start developing plans...then you implement the plans and then the cycle goes around.” Similarly, the SRU instructor emphasized planning and implementing mathematical lessons as a big idea. PSTs in her course had the opportunity to learn about facilitating classroom discourse and using appropriate instructional strategies in the field, while they concurrently took a methods course in which they discussed and reflected on relevant readings. Learning about teaching through the design of instruction addresses the knowledge of content and teaching (Ball et al., 2008), one of the domains of mathematical knowledge for teaching. In selecting and implementing tasks for teaching, orchestrating effective classroom discussions is essential. PSTs pose purposeful questions to probe students’ mathematical ideas and make mathematical structures visible (NCTM, 2014).

While PSTs designed, implemented, and reflected on their lessons, instructors intended that PSTs develop their understanding of student thinking. Anticipating what students are likely to think about mathematics is relevant to knowledge of content and students (Ball & Forzani, 2011; Ball et al., 2008; Shulman, 1986). AMTE (2017) recommended that well-prepared PSTs are committed to deepening their knowledge of students’ mathematical skills and dispositions. The GLU instructor addressed this need: “As you are talking to the student [during implementing your lesson plan], what information did you gather, what did it tell you about, and then what support did you need to provide as a result of that?” Similarly, the MUU instructor provided PSTs with the opportunity to interview a middle or high school student to ascertain the student’s beliefs about mathematics and knowledge of a particular mathematical topic. PSTs were to develop a series of questions and performance-based tasks that they would pose to the student during the interview. In a writing report, PSTs would describe the student’s mathematical knowledge and beliefs based on their analysis of data gathered during the interview.

All instructors incorporated opportunities for PSTs to explore a variety of instructional materials. AMTE (2017)’s standards address this opportunity that well-prepared preservice teachers analyze and discuss curriculum and standards documents. Knowing about appropriate instructional materials and their characteristics is an essential teacher knowledge to be developed (Ball et al., 2008; Hill et al., 2005). The GLU secondary mathematics methods instructor included a goal that addressed this knowledge: “to acquaint the teacher assistant with available instructional resource materials such as curricula, professional journals, and relevant research.” The MUU instructor also described a specific activity PSTs engaged in during the semester, in which they explored and critiqued textbooks. She said, “I have them look at materials [the traditional course sequence versus integrated math courses] and evaluate them, and that’s the subject of again, usually a class discussion about what they think the opportunities are that are afforded by these textbooks, versus traditional textbooks, the challenges of teaching math in this way.” PSTs in her course then observed the way integrated math curricula were being taught and evaluated the implementations of the curricular.

While all four courses emphasized PSTs' development of pedagogical content knowledge, a course offered by MRU emphasized PSTs' development of specific mathematical contents for teaching. AMTE (2017) stated that learning to teach mathematics requires "a central focus on mathematics" (p. 2) and flexible knowledge of school mathematics. The MRU secondary methods course was the only course among the four in which developing specific school mathematical concepts was a course objective. The syllabus highlighted: "The mathematical topics that we will examine are ratios and proportional reasoning,...and quadratic relationships and factoring. These are BIG ideas in middle school and early high school mathematics, and they are important for reasoning algebraically." PSTs in this course were asked to keep a three-ring binder of problems exemplifying these topics. The instructor stated in her syllabus, "One of your greatest assets in understanding students' mathematical thinking is understanding and deepening your own mathematical thinking." PSTs generated mathematical conversations with each other, reflected on their own mathematical knowledge around these mathematical concepts, and used their mathematical knowledge to design problem sequences for students. They submitted the binder of problems to receive feedback and points for thoroughness, organization, explanations and analysis of targeted problems, quality of problem sequence and discussion, and mathematical correctness.

Teaching in a Diverse Society

Each of the four SMTE programs under review required a course related to diversity; the title of this course varied across program, however to protect anonymity, we gave all courses the generic title: Teaching in a Diverse Society (TDS). What was common across these courses was that they were general education courses, taught by female instructors who were not associated with mathematics or mathematics education; therefore, the curriculum was not subject-specific and students from multiple education disciplines enrolled simultaneously. Several themes emerged from TDS course big ideas and objectives provided by the instructors.

First, the instructors emphasized their attention in the course to highlighting the vast number of ways in which diversity is present in the United States; all instructors highlighted multiple aspects of diversity. For example, when the SRU TDS instructor was asked about the big ideas of the course, she said "we talk about race and ethnicity; we talk about class, gender and sexual identity, exceptionality, like special needs students. We talk about language, geography, religion. And really, my goal at the end is that students would...be ready to teach in a diverse environment." No one aspect of diversity was mentioned in all four TDS courses; however, race, culture, ethnicity, religion, sexual orientation, ability, and language were each mentioned in three of the four courses. The second theme emphasized across the TDS courses was the idea that schools are situated in historical, socio-political, and geographic contexts. For example, the MUU instructor emphasized the local context, including "Understand the impact of family and community in the learning experiences of English language learners in the classroom" as a course outcome. Taking a more national approach to context, the GLU instructor stressed the importance of PSTs' "understanding how their work in the classroom and in the schools is a part of democratic practice in the United States."

The third and most prevalent theme highlighted across the TDS courses was the impact of diversity (theme 1) and historical, socio-political, and geographic contexts (theme 2) on educational opportunities in particular schools and for particular learners. TDS instructors highlighted strategies they used to attempt to mitigate these effects. For example, the MUU instructor discussed opportunities that she offered for PSTs to engage in investigating students' school experiences, including reading and discussing articles such as *Nothing to Do: The Impact*

of Poverty on Pupil's Learning Identities (Muschamp, Bullock, Ridge & Wikeley, 2009) and *Barbie Against Superman: Gender Stereotypes and Gender Equity in the Classroom* (Aksu, 2005). In addition, the PSTs analyzed U.S. federal laws developed to ensure all students access to education (e.g., Individuals with Disabilities Education Act [IDEA], McKinney-Vento Act [protects the rights of homeless children]).

Several TDS instructors also mentioned critical reflection as an important activity for PSTs in developing dispositions and skills for teaching diverse learners; the MRU instructor made this focus on reflection explicit in her syllabus: "We will explore various realms of diversity...As part of that exploration we will engage in significant reflection, written and oral, personal and collective, challenging our assumptions, and questioning our beliefs." Similarly the GLU course was described as: "grounded in the idea that an essential aspect of good teaching is having the time and space to reflect upon the kinds of issues that impact your pedagogy and instruction."

The attention to the impact of diversity and contextual factors on educational opportunities reported by TDS instructors is well supported by professional mathematics and mathematics education organizations, including AMTE, MAA, and NCTM. In fact, attention to historical inequities in mathematical learning opportunities is highlighted front and center in NCTM (2000) as the first "principle," in AMTE (2017) as "Assumption #1," and in the Manifesto of MAA (2018). NCTM (2000) expressed concerns about pervasive low expectations and tracking practices, and less challenging mathematics curriculum for "students who live in poverty, students who are not native speakers of English, students with disabilities, females, and many nonwhite students" (p. 13). They highlight engaging curriculum, use of technology, enhanced assessment practices, and increased attention to mathematics processes (beyond memorization and symbolic manipulation) as possible mitigators to increase equity in mathematics classrooms.

AMTE (2017) took a similarly strong stance toward the need for teacher education programs' commitment to preparing teachers who have the skills and dispositions to teach all learners: "Assumption #1: Ensuring the success of each and every learner requires a deep, integrated focus on equity in every program that prepare teachers of mathematics" (p. 1). The authors repeatedly emphasized the disparate opportunities resulting from historic discrimination and sociopolitical factors, and stressed the importance of preparing teachers who are advocates for their students with these disparities in mind: "Well-prepared beginning teachers embrace and build on students' current mathematical ideas and on students' ways of knowing and learning...They also attend to developing students' identities and agency so that students can see mathematics as components of their cultures and see themselves in the mathematics" (p. 13). The authors recommend opportunities for PSTs to critically analyze current mathematics education systems, challenge deficit views about student learning, recognize the key roles that identity and power play in mathematics education, and spend time in community settings to learn from and about students, families, and communities.

MAA (2018) continued this call for first, recognition that these systemic inequities exist, and second, action to change the status quo: "Inequity exists in many facets of our society, including within the teaching and learning of mathematics...We owe it to our discipline, to ourselves, and to society to disseminate mathematical knowledge in ways that increase individuals' access to the opportunities that come with mathematical understanding" (p. vii). The authors describe the statistical disparities of underrepresented populations among both mathematicians and university students in mathematics departments, and encourage instructors to beware of implicit and explicit messages being sent to students about who "belongs" in mathematics.

Discussion & Implications

Although recent policy documents provided recommendations about how secondary mathematics should be prepared to learn about mathematics, mathematics teaching, and equity issues in mathematics teaching and learning (AMTE, 2017; MAA 2018; NCTM 2000; 2014), little is known about how such recommendations are integrated into mathematics teacher education programs (NRC, 2010). To promote dialogue related to the preparation of secondary mathematics teachers, this study highlighted ways in which course goals and big ideas in secondary mathematics teacher education programs emphasized areas related to mathematics learning, teaching, and issues of equity and access as recommended by policy documents.

Across all course types, we noticed that many policy and research recommendations were addressed, both explicitly and implicitly. In fact, many of the “big ideas” reported with closely related to these recommendations. For example: The GLU Linear Algebra instructor pointed to the strong mathematics-mathematics education community in his department as leading him to experiment with multiple teaching strategies that he hoped would align with his students’ future teaching needs. The MRU secondary mathematics methods course centered around the reconstruction of school mathematics, such as proportional reasoning and integers, to address PSTs’ development of big mathematical ideas for teaching. The MUU TDS instructor provided opportunities for PSTs to engage with their students’ families and communities.

In this study, we confirmed that, although the four universities required similar versions of these three courses (i.e., Linear Algebra, Secondary Mathematics Methods, and Teaching in a Diverse Society), the courses were also unique. Therefore, the experiences of PSTs across these programs will be different, likely as a result of many factors, including geography, program emphases, and priorities of the course instructor.

This study is intended both to build on existing research (e.g., Ball et al., 2008; Fuson et al., 2005; Hill et al., 2005; Turner et al., 2009) and policy documents about mathematics teacher preparation (e.g., AMTE, 2017; MAA, 2018; NCTM, 2000) and to encourage researchers to explore areas that are less well investigated. We acknowledge that creating programs that are coherent across multiple departments and disciplines is often a challenge. We wonder what creating a culture of communication among the dozens of faculty who support future teachers at each university would look like. We wonder how such a culture could naturally build coherence by discussions asking: What are the big ideas of our program? What are the fundamental ideas we want threaded throughout the program? How do we ensure that students have multiple opportunities to encounter these ideas, building on each other, through the program?

From the findings of this study, next steps would include an investigation of how big ideas play out in written curriculum (e.g., textbooks, course materials, other resources) and enacted curriculum (e.g., classroom instruction) as well as an investigation of how PSTs perceive the opportunities provided throughout their secondary mathematics teacher education programs. What would be the benefits of an in-depth exploration of each of our programs? How would such a conversation get started? How would it be sustained? For example, from a stance of equity, how might TDS instructors interact with mathematics and methods instructors to stimulate conversation about opportunities for discipline-specific, equity-related experiences?

Integration between course goals and policy documents contributes to promoting dialogue related to the preparation of secondary mathematics teachers. Our study presented here, connecting big ideas of multiple courses with recommendations from research and recent policy documents, promises to inform teacher educators, especially those who are new mathematics teacher educators in the field. Future work analyzing our larger data will provide insights into

how other areas of recommendations from policy documents play out in practice; this current work is an initial step to illustrate big ideas of required courses commonly offered by the four case study universities.

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