Department of Economics

Working Paper

Forward Guidance Effectiveness in a New Keynesian Model with Housing Frictions

By

Stephen J. Cole and Sungjun Huh

Working Paper 2021-07

College of Business Administration

MARQUETTE UNIVERSITY

Forward Guidance Effectiveness in a New Keynesian Model with Housing Frictions

Stephen J. Cole*
Marquette University

Sungjun Huh†
Marquette University

September 8, 2021

Abstract

Housing markets are closely related to monetary policy. This paper studies the link between housing frictions and the effectiveness of forward guidance. A housing collateral constraint and forward guidance shocks are incorporated into a standard medium-scale New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model. Our main results produce a number of important implications. First, financial frictions emanating from the housing market dampen the effectiveness of forward guidance on the economy. Second, forward guidance has asymmetric effects on the welfare of lenders and borrowers when housing frictions increase. Housing frictions also attenuate the effect of forward guidance at the zero lower bound. Finally, this article provides a solution to "forward guidance puzzle" of Del Negro et al. (2012). Thus, policymakers should consider housing frictions when examining the effects of forward guidance on the economy.

Keywords: Forward guidance, Financial frictions, Housing collateral, Zero lower bound

JEL classification: E32, E44, E52, R21

**Address:* Stephen J. Cole, Assistant Professor, Department of Economics, Marquette University, P.O. Box 1881, Milwaukee, WI 53201. *Phone:* +1 (414) 288-3367. *E-mail:* stephen.cole@marquette.edu. *Homepage:* http://www.stephencoleeconomics.com.

†*Address*: Sungjun Huh, Assistant Professor, Department of Economics, Marquette University, P.O. Box 1881, Milwaukee, WI 53201. *Phone:* +1 (414) 288-6915. *E-mail:* sungjun.huh@marquette.edu. *Homepage:* https://sites.google.com/site/huhsungjun/.

1 Introduction

Monetary policy and housing markets have historically been closely related. Arguably, the Federal Reserve's excessively easy monetary policy contributed to a bubble in housing prices, which was one of the main causes of the global financial crisis in 2007-2009. To combat the recession, the federal funds rate, the conventional monetary policy instrument, was essentially set to zero from 2008 to 2015. Accordingly, the Fed conducted unconventional monetary policies such as large-scale asset purchases and forward guidance. In addition, in response to the COVID-19 induced recession, the Federal Reserve once again lowered the federal funds rate to its lowest possible level starting in May 2020, and consequently, issued guidance on the future course of policy. Abundant liquidity has flowed into the housing market again. In May 2021, Robert Shiller said, "In real terms, the home prices have never been so high." As such, monetary policy and the housing market are inextricably linked. Surprisingly, to the best of our knowledge, no theoretical research has yet examined the zero lower bound (ZLB), unconventional monetary policy, and housing markets simultaneously.

This paper studies both financial frictions emanating from the housing market and unconventional monetary policy of forward guidance announcements. A medium-scale New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model is utilized with two distinctive features. First, the borrowing capacity of producers depends on the value of collateral such as housing prices (e.g., Iacoviello, 2005; Liu et al., 2013; and Liu et al., 2016). Second, forward guidance shocks are incorporated to a Taylor type monetary policy rule (e.g., Laséen and Svensson, 2011; Del Negro et al., 2012; and Cole, 2021). The shocks are similar to "news shocks" of Schmitt-Grohé and Uribe (2012) and represent time-contingent forward guidance in which the central bank communicates a specific end date of their guidance on the interest rate.

We add to a standard New Keynesian DSGE model housing frictions similar to Liu et al. (2016). The unique components include both households and capitalists holding housing stock. The former group receives utility from housing. However, capitalists utilize housing for a different purpose. This group borrows to fund their operations, but is subject to a borrowing constraint in the spirit of Kiyotaki and Moore (1997). The extent of their credit depends on the value of their collateral, which includes both holdings of housing and capital. Thus, if a capitalist holds more housing, it can borrow more funds, increase capital, and generate economic economic activity.

An important parameter in our analysis is ζ, which can be interpreted as loan-to-value (LTV) ratio of capitalists and will be detailed later in the paper. As ζ approaches to 1, capitalists can borrow as much as the value of their collateral. This case stands for decreasing financial frictions. As ζ approaches to zero, capitalists cannot borrow at all as their collateral is not valuable. This scenario captures increased financial frictions. Moreover, in our benchmark case, we use an empirically motivated value of ζ found in prior studies.

Our results provide a number of important takeaways. First, financial frictions emanating from the housing market dampen the effectiveness of forward guidance on the economy. For instance, when the monetary authority announces interest rates will increase in the future, the impulse responses of the macroeconomic variables react more when housing frictions are reduced. Under a high value of ζ, a promise to increase the interest rate in the future decreases the value of collateral leading to drops in both output and housing. Consequently, the value of collateral decreases again giving rise to the so called "financial accelerator effect." However, under low values of ζ, collateral is minimally altered by interest rate changes leading to less of a reaction from macroeconomic variables. In addition, forward guidance has asymmetric effects on the welfare of lenders and borrowers when housing frictions increase. Lenders are better off because they consume more rather than loan. However, borrowers cannot access as much credit as before, and thus, cannot consume as much relative to the benchmark case. Lastly, financial frictions emanating from the housing sector attenuate the effect of the ZLB. This result is observed when a recessionary shock causes the economy to move to the ZLB. The central bank then issues communication such that the future path of interests rates will be zero for L periods into the future. Forward guidance produces a stimulative effect on output and inflation, but this effect is diminished when housing frictions increase.

The present paper also suggests a solution for the "forward guidance puzzle" of Del Negro et al. (2012). This phenomenon can be defined as standard DSGE macroeconomic models predicting unbelievably large responses of macroeconomic variables to relatively small forward guidance shocks. It should be noted that the paper of Del Negro et al. (2012) did not include financial frictions emanating from the housing sector. However, we present evidence that macroeconomic variables do not exhibit an unusually large response to forward guidance when housing frictions are allowed to exist.

Robustness checks also provide additional takeaways and confirm our base results. First, if the public perceives the central bank as less than fully credible in fulfilling their future interest rate commitments, the effects of forward guidance are dampened. If the collateral of capitalists becomes more valuable because the value of capital becomes as important as real estate, the effectiveness of forward guidance also increases. Firms are able to finance more since capital becomes more valuable as collateral. Finally, our results are robust when incorporating a time-varying LTV. The differences between this robustness case and our benchmark impulse responses are found to be trivial.

Overall, the results show that housing frictions can significantly influence the effectiveness of forward guidance. The effects of forward guidance on macroeconomic variables are attenuated when frictions emanating from the housing market increase. Thus, policymakers should consider housing frictions when examining the effects of forward guidance on the economy.

1.1 Previous Literature

This paper is closely related to two strands of the literature. One strand studies impacts of financial frictions on business cycles. Kiyotaki and Moore (1997) and Bernanke et al. (1999) document that financial frictions introduce a wedge between lenders and borrowers that amplifies economic fluctuations in macroeconomic models. Iacoviello (2005) and Liu et al. (2013) are two papers most closely related to our paper. Iacoviello (2005) incorporates housing collateral constraints into a DSGE model to study amplified and propagated effects of shocks. Liu et al. (2013) study a positive co-movement between investment and housing prices using a DSGE model with collateral constraints. Relative to these studies, the present paper attempts to measure the effectiveness of forward guidance with housing collateral.

The other strand of the literature regards forward guidance. Eggertsson and Woodford (2003), Kiley (2016), and Swanson (2018) study forward guidance at the ZLB. Campbell et al. (2019) find that the efficacy of forward guidance varies depending on the time horizon. Chen, Cúrdia, and Ferrero (2012) show that forward guidance can increase the positive benefits of large-scale asset purchases. Jansen and Jia (2020) study forward guidance in response to the COVID-19 induced recession.

Prior forward guidance literature has also focused on extreme responses of standard models to forward guidance. The seminal papers by Del Negro et al. (2012) and Carlstrom et al. (2015) show that standard macroeconomic models predict unusually large responses of macroeconomic variables to forward guidance, the so called "forward guidance puzzle." Bundick and Smith (2019) show this large effect with a VAR model. De Graeve et al. (2014) study threshold-based forward guidance in which guidance on future policy is tied to economic conditions. They find that this type of forward guidance can attenuate the unrealistically large responses of macroeconomic variables to forward guidance. Cole (2021) shows that a more realistic expectations formation assumed in a macroeconomic model can help solve the forward guidance puzzle. Heterogeneous expectations can also influence the effectiveness of forward guidance on the economy as shown in Andrade et al. (2019). Haberis et al. (2019) and Cole and Martínez-García (2021) focus on how central bank credibility can alleviate the extreme responses predicted by macroeconomic models.

Another closely related paper is Cole (2020) who analyzes the effectiveness of forward guidance with financial frictions depending on net worth of firms in light of Bernanke et al. (1999). Our paper differs from Cole (2020) for several reasons. First, financial frictions in our paper are induced from collateral constraints that are tied to housing prices, so the mechanisms of frictions are quite different. Second, our paper analyzes the linkage between housing markets and monetary policy. Recent papers document that housing markets are important source of economic fluctuations, so it is imperative to measure the effectiveness of forward guidance with a housing sector (e.g., Iacoviello and Neri, 2010; and Liu et al., 2013). Lastly, our paper explores the effect of forward guidance on welfare of lenders and borrowers, while welfare analysis is abstracted from Cole (2020).

The organization of the paper is as follows. Section 2 describes the model. Section 3 lays out choice of parameter values. Section 4 presents results. Sections 5 includes robustness checks while Section 6 concludes. An appendix to the paper provides log-linearized model.

2 Model

The economy consists of four types of agents: households, capitalists, final-good firms, and intermediate-goods firms. Households consume both final-good and housing services, save in the risk-free bond market, and supply labor. Capitalists receive utility by consumption and do not supply labor. They own all firms and require both internal funds and external borrowing for providing investment. Capitalists' external financing capacity depends on the value of collateral which are their holding of house and capital stock. Final-good firms buy intermediate goods in a competitive market and produce final good. Intermediate-goods firms need labor, capital, and housing as input to produce goods and introduce nominal price rigidities through their price setting strategy as in

Calvo (1983). There is a monetary authority in the model that conducts not only conventional monetary policy but also forward guidance. Moreover, without nominal rigidities and monetary policy, our model is similar to the model in Liu et al. (2016).[1] However, the forward guidance is a salient feature in our model and it has substantial effects on the economy. In all other respects, our model follows DSGE framework developed by Christiano et al. (2005) and Smets and Wouters (2007).

2.1 Households

There exists a continuum of identical households. Each household chooses consumption, housing service, labor, and savings every period to maximize the present discounted flow utility:

$$\max \; E_0 \sum_{t=0}^{\infty} \beta^t \left(\log(c_{h,t} - bc_{h,t-1}) + \vartheta \log h_{h,t} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} \right), \tag{1}$$

where E_0 is the expectation operator, $c_{h,t}$ is household consumption, $h_{h,t}$ is household holdings of housing stocks, and l_t is hours of work at time t. Parameters $\beta \in (0,1)$ is the subjective discount factor of the household, $b > 0$ determines the habit persistence, $\vartheta > 0$ is the relative weight on housing, $\chi_0 > 0$ denotes the relative weight on labor, and $\chi > 0$ is the inverse Frisch elasticity of labor supply. For simplicity, we assume that consumption and housing service are separable as in Iacoviello (2005).

The flow budget constraint each period is given by

$$P_t c_{h,t} + B_t + Q_{h,t}(h_{h,t} - h_{h,t-1}) = R_{t-1} B_{t-1} + W_t l_t, \tag{2}$$

where P_t is the aggregate price level, B_t is nominal household savings, $Q_{h,t}$ denotes nominal housing price, R_t is the gross nominal interest rate, and W_t is nominal wage at time t.

Solving the household problem with real budget constraint yields the following first order conditions:

$$c_{h,t}: \;\; \lambda_t = \frac{1}{c_{h,t} - bc_{h,t-1}} - E_t \frac{\beta b}{c_{h,t+1} - bc_{h,t}}, \tag{3}$$

$$b_t: \;\; \lambda_t \frac{1}{P_t} = \beta E_t \lambda_{t+1} R_t \frac{1}{P_{t+1}}, \tag{4}$$

$$h_{h,t}: \;\; q_{h,t} \lambda_t = E_t \beta q_{h,t+1} \lambda_{t+1} + \frac{\vartheta}{h_{h,t}}, \tag{5}$$

[1] Our model is abstract from labor search and matching for simplicity.

$$l_t: \ \lambda_t w_t = \chi_0 l_t^{\chi}, \tag{6}$$

where λ_t denotes the Lagrangian multiplier on the budget constraint, $b_t \equiv \frac{B_t}{P_t}$ is real household savings, $q_{h,t} \equiv \frac{Q_{h,t}}{P_t}$ denotes real housing price, and $w_t \equiv \frac{W_t}{P_t}$ is real wage. Equation (4) is a standard intertemporal Euler equation and equation (6) is an intratemporal Euler equation. Rewriting (5) yields the housing price:

$$q_{h,t} = E_t m_{t+1} q_{h,t+1} + \vartheta \frac{1}{\lambda_t h_{h,t}}, \tag{7}$$

where $m_{t+1} \equiv \beta \frac{\lambda_{t+1}}{\lambda_t}$ denotes stochastic discount factor of the household. Notice that equation (7) shows the current housing price is the expected infinite sum of discounted future value of "rent". We interpret the second term, $\vartheta \frac{1}{\lambda_t h_{h,t}}$, marginal rate of substitution between housing and consumption, as "rent" because it gives additional utility to home owners each period.

2.2 Capitalists

The economy is populated by a continuum of identical capitalists. At each time t, the representative capitalist chooses consumption to maximize the expected present discounted value of utility flows,

$$\max \ E_0 \sum_{t=0}^{\infty} \beta_c^t \log(c_{c,t} - b_c c_{c,t-1}), \tag{8}$$

where $c_{c,t}$ is capitalist's consumption at time t. Parameters $\beta_c \in (0,1)$ denotes the subjective discount factor and b_c is the habit persistence of the capitalist. Initially, the capitalist is endowed with housing stock $h_{c,-1}$ and capital stock k_{-1} and has no debt. As all the firms are collectively owned by capitalists in the economy, they invest and have to pay interest on borrowings. Firms distribute returns from capital, housing and profits back to capitalists each period. The capitalist's flow budget constraint is thus:

$$P_t c_{c,t} + Q_{h,t}(h_{c,t} - h_{c,t-1}) + P_t I_t + R_{t-1} B_{t-1} = B_t + P_t R_{k,t} k_{t-1} + P_t R_{h,t} h_{c,t-1} + \Pi_t, \tag{9}$$

where $h_{c,t}$ denotes capitalist's holding of housing stock, I_t is investment, B_t denotes nominal borrowings, $R_{k,t}$ is the capital rental rate, $R_{h,t}$ is the housing rental rate, and Π_t is nominal profits from firms at time t.

There is an investment adjustment cost. The law of motion for capital is thus:

$$k_t = \left(1 - \frac{\Omega}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right) I_t + (1-\delta) k_{t-1}, \tag{10}$$

where $\delta > 0$ is the depreciation rate of capital and Ω determines the cost of adjusting investment. The capitalist also faces a borrowing constraint in the spirit of Kiyotaki and Moore (1997). The borrowing capacity of capitalist depends on a fraction of value of not only capital but also housing. About 70% of commercial loans requires collateral and they mainly depend on tangible assets such as real estate and equipment.[2] The credit constraint is given by:

$$B_t \leq \zeta E_t \left(\omega_1 Q_{h,t+1} h_{c,t} \frac{1}{R_t} + \omega_2 Q_{k,t+1} k_t \frac{1}{R_t} \right), \tag{11}$$

where $Q_{k,t}$ is nominal price of capital, $\zeta \in (0,1)$ is a parameter which can be interpreted LTV, and ω_1 and ω_2 are relative weight of housing and capital in the collateral value, respectively. When ζ approaches to one, the capitalist can borrow as much as the value of the collateral meaning decrease in financial frictions. On the other hand, as ζ approaches to zero, the capitalist cannot borrow at all in spite of the collateral as it is not valuable. This implies that financial frictions are increased. As it is common in the literature, we also assume that $\beta > \beta_c$ to make equation (11) to bind.

Solving the capitalist problem subject to (9), (10), and (11) in real terms yields the following first order conditions:

$$c_{c,t}: \ \lambda_t^c = \frac{1}{c_{c,t} - b_c c_{c,t-1}} - E_t \frac{b_c \beta_c}{c_{c,t+1} - b_c c_{c,t}}, \tag{12}$$

$$b_t: \ \lambda_t^c = E_t \left(\beta_c \lambda_{t+1}^c R_t \frac{1}{\pi_{t+1}} \right) + \nu_t, \tag{13}$$

$$h_{c,t}: \ \lambda_t^c q_{h,t} = E_t \left[\beta_c \lambda_{t+1}^c \left(R_{h,t+1} + q_{h,t+1} \right) + \nu_t \zeta \omega_1 q_{h,t+1} \frac{\pi_{t+1}}{R_t} \right], \tag{14}$$

$$k_t: \ \mu_t = E_t \left[\beta_c \left(\lambda_{t+1}^c R_{k,t+1} + \mu_{t+1}(1-\delta) \right) + \nu_t \zeta \omega_2 q_{k,t+1} \frac{\pi_{t+1}}{R_t} \right], \tag{15}$$

$$I_t: \ \lambda_t^c = \mu_t \left\{ 1 - \frac{\Omega}{2} \left(\frac{I_t}{I_{t-1}} - 1 \right)^2 - \Omega \left(\frac{I_t}{I_{t-1}} - 1 \right) \frac{I_t}{I_{t-1}} \right\} + \beta_c E_t \left(\mu_{t+1} \Omega \left(\frac{I_{t+1}}{I_t} - 1 \right) \left(\frac{I_{t+1}}{I_t} \right)^2 \right), \tag{16}$$

where λ_t^c is the Lagrangian multiplier for (9), π_t is gross inflation rate, ν_t is the Lagrangian multiplier for (11), μ_t is the Lagrangian multiplier for (10), and $q_{k,t} \equiv \frac{\mu_t}{\lambda_t^c}$ is real shadow price of capital (Tobin's q) at time t. Equation (14) shows that the present housing price is determined by expected discounted future return of housing and its price plus housing price as a collateral. Other equations are fairly standard. Equation (12) is a marginal utility of capitalist's consumption; equation (15) is the capital Euler equation; and equation (16) is the investment Euler equation that makes the marginal benefit of new capital equals to the marginal cost of investment.

[2]Berger and Udell (1990) and the Flow-of-Funds by the FRB.

2.3 Final-goods Firms

A perfectly competitive final goods sector aggregates intermediate goods using a CES production function

$$Y_t = \left(\int_0^1 y_t(f)^{\frac{1}{1+\theta}} df \right)^{1+\theta} \tag{17}$$

where Y_t is the quantity of the final goods, $y_t(f)$ is an intermediate good of firm f, and $\theta > 0$ is the steady state markup. Each final good producing firm maximizes its profit given the production function (17) and the prices of intermediate and final goods. An intermediate goods producing firm f accordingly faces a downward-sloping demand curve

$$y_t(f) = \left(\frac{P_t(f)}{P_t} \right)^{-\frac{1+\theta}{\theta}} Y_t \tag{18}$$

where $P_t \equiv \left(\int_0^1 P_t(f)^{-\frac{1}{\theta}} df \right)^{-\theta}$ is the CES aggregate price of final goods and $P_t(f)$ is the intermediate goods price.

2.4 Intermediate-goods Firms

The economy also contains a continuum of intermediate goods firms indexed by $f \in [0, 1]$. They sell slightly differentiated goods under monopolistic competition. Firms produce output using housing, capital, and labor as inputs. The production function is given by

$$y_t(f) = A_t \left(h_{c,t-1}(f)^{\phi} k_{t-1}(f)^{1-\phi} \right)^{1-\eta} l_t(f)^{\eta} \tag{19}$$

where A_t is total factor productivity and $k_t(f)$, $h_{c,t}(f)$ and $l_t(f)$ are firm f's capital, housing, and labor inputs, respectively. The parameters $\eta \in (0, 1)$ and $\phi \in (0, 1)$ denote the elasticity of output with respect to labor and housing, respectively.

Total factor productivity follows an AR(1) process in logs,

$$\log A_t = \rho_A \log A_t + \varepsilon_t^A \tag{20}$$

where $\rho_A \in (0, 1]$, and ε_t^A is drawn from an *i.i.d.* white noise process with zero mean and variance σ_A^2.[3]

[3] For simplicity, we assume that the steady state growth rate of technology is zero since the aggregate housing supply is fixed in the model economy.

Cost minimization problem yields the following first-order conditions for capital, housing, and labor:

$$k_{t-1}(f):\ R_t^k = \frac{\varphi_t(f)}{P_t}\left[(1-\eta)A_t\left(h_{c,t-1}(f)^{\phi}k_{t-1}(f)^{1-\phi}\right)^{-\eta} l_t(f)^{\eta}(1-\phi)h_{c,t-1}(f)^{\phi}k_{t-1}(f)^{-\phi}\right] \tag{21}$$

$$h_{c,t-1}(f):\ R_t^h = \frac{\varphi_t(f)}{P_t}\left[(1-\eta)A_t\left(h_{c,t-1}(f)^{\phi}k_{t-1}(f)^{1-\phi}\right)^{-\eta} l_t(f)^{\eta}\phi h_{c,t-1}(f)^{\phi-1}k_{t-1}(f)^{1-\phi}\right] \tag{22}$$

$$l_t(f):\ w_t = \frac{\varphi_t(f)}{P_t}\left[\eta A_t\left(h_{c,t-1}(f)^{\phi}k_{t-1}(f)^{1-\phi}\right)^{1-\eta} l_t(f)^{\eta-1}\right] \tag{23}$$

where $\varphi_t(f)$ is the Lagrange multiplier of the cost minimization problem. Equation (21) shows that rental rate of capital equals to marginal production of capital multiplied by real marginal cost; equation (22) displays that rental rate of housing equals to marginal benefit of purchasing one additional unit of housing multiplied by real marginal cost; and equation (23) means that real wage equals to marginal benefit of an additional work hour multiplied by real marginal cost.

Dividing (21) by (22), (22) by (23), and (23) by (21) yield the housing-capital, the labor-housing, and the capital-labor ratios, respectively:

$$\frac{R_t^k}{R_t^h} = \frac{1-\phi}{\phi}\frac{H_{c,t-1}}{K_{t-1}}, \tag{24}$$

$$\frac{R_t^h}{w_t} = \frac{\phi(1-\eta)}{\eta}\frac{L_t}{H_{c,t-1}}, \tag{25}$$

$$\frac{w_t}{R_t^k} = \frac{\eta}{(1-\phi)(1-\eta)}\frac{K_{t-1}}{L_t}, \tag{26}$$

where $H_{c,t}$ is the aggregate housing of the capitalist, K_t is the aggregate capital stock, and L_t is the aggregate quantity of labor. The ratios are common across firms because they face the same rental rates. As a result, the demand functions for capital, housing, and labor are:

$$R_t^k = MC_t(1-\phi)(1-\eta)A_t\left(\frac{H_{c,t-1}}{K_{t-1}}\right)^{\phi(1-\eta)}\left(\frac{L_t}{K_{t-1}}\right)^{\eta}, \tag{27}$$

$$R_t^h = MC_t\phi(1-\eta)A_t\left(\frac{K_{t-1}}{H_{c,t-1}}\right)^{1-\phi(1-\eta)}\left(\frac{L_t}{K_{t-1}}\right)^{\eta}, \tag{28}$$

$$w_t = MC_t\eta A_t\left(\frac{H_{c,t-1}}{K_{t-1}}\right)^{\phi(1-\eta)}\left(\frac{K_{t-1}}{L_t}\right)^{1-\eta}, \tag{29}$$

where $MC_t \equiv \frac{\varphi_t}{P_t} = \left(\frac{1}{1-\phi}\right)^{(1-\phi)(1-\eta)}\left(\frac{1}{\phi}\right)^{\phi(1-\eta)}\left(\frac{1}{1-\eta}\right)^{1-\eta}\left(\frac{1}{\eta}\right)^{\eta}\frac{\left(R_t^k\right)^{(1-\phi)(1-\eta)}\left(R_t^h\right)^{\phi(1-\eta)}w_t^{\eta}}{A_t}$ denotes the real marginal cost at time t.

Each intermediate goods firm also sets the new contract price $P_t(f)$ to maximize the firm's lifetime profit in a staggered fashion: only a fraction, $1-\xi$, of firms are able to adjust its price optimally each period, while the remaining firms fix their prices over the life of the contract. Hence, the real value of the firm is given by:

$$\max_{P_t(f)} E_t \sum_{j=0}^{\infty} m^c_{t,t+j}(P_t/P_{t+j})\xi^j \left[P_t(f)y_{t+j}(f) - MC^n_{t+j}(f)y_{t+j}(f)\right], \tag{30}$$

where $m^c_{t,t+j} \equiv \Pi^j_{i=1} m^c_{t+i}$ is the stochastic discount factor of the capitalist from period t to $t+j$, and $MC^n_t(f)$ is firm-specific nominal marginal cost.

The first order necessary condition of (30) with respect to $P_t(f)$ yields the optimal price which is given by:

$$p^*_t(f) = \frac{(1+\theta)E_t \sum_{j=0}^{\infty} m^c_{t,t+j}\xi^j MC_{t+j}\pi_{t+j}^{\frac{1+\theta}{\theta}} Y_{t+j}}{E_t \sum_{j=0}^{\infty} m^c_{t,t+j}\xi^j \pi_{t+j}^{\frac{1}{\theta}} Y_{t+j}} \tag{31}$$

where $p^*_t(f) \equiv P^*_t(f)/P_t$. Note that the optimal price $p^*_t(f)$ is a markup over a weighted average of current and expected future marginal costs.

2.5 Monetary Policy

Monetary policy is conducted by à la Taylor rule with interest-rate smoothing as well as forward guidance shocks. In log-linearized form, the nominal interest rate is given by[4]

$$\hat{R}_t = \rho_i \hat{R}_{t-1} + (1-\rho_i)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t\right) + \varepsilon^{MP}_t + \sum_{l=1}^{L} \varepsilon^{FG}_{l,t-l} \tag{32}$$

where $\rho_i \in (0,1)$ is the smoothing parameter, $\phi_\pi > 0$ is the feedback coefficient on inflation, $\phi_y > 0$ is the feedback coefficient on output gap, and ε^m_t denotes unanticipated monetary policy shock. $\varepsilon^{FG}_{l,t-l}$ is forward guidance shock and is defined as a central bank promise in period $t-l$ to change the interest rates l periods later, i.e., period t. (e.g., Laséen and Svensson, 2011; Del Negro et al., 2012; and Cole, 2021). Each forward guidance shock follows an *i.i.d.* process and L denotes the length of the forward guidance horizon. Moreover, the reason for considering forward guidance shock in this way is to circumvent the indeterminacy problem.[5]

[4]The "$\hat{}$" denotes log deviations from steady-state. Note that interest rates are already percentages so we leave it as in absolute deviations.

[5]See Honkapohja and Mitra (2005), Woodford (2005), and Cole (2020) for more details.

The system of equations is also augmented with the following:

$$v_{1,t} = v_{2,t-1} + \varepsilon_{1,t}^{FG}, \tag{33}$$

$$v_{2,t} = v_{3,t-1} + \varepsilon_{2,t}^{FG}, \tag{34}$$

$$\vdots$$

$$v_{L,t} = \varepsilon_{L,t}^{FG}. \tag{35}$$

The vector $v_t = [v_{1,t}, v_{2,t}, \ldots, v_{L,t}]'$ contains all forward guidance information communicated in the past (v_{t-1}) and present ($\varepsilon_t^{FG} = [\varepsilon_{1,t}^{FG}, \varepsilon_{2,t}^{FG}, \ldots, \varepsilon_{L,t}^{FG}]'$). Moreover, equations (33) - (35) can together be rewritten to show that $v_{1,t-1} = \sum_{l=1}^{L} \hat{\varepsilon}_{l,t-l}^{R}$, which is the last term in equation (32).

2.6 Resource Constraint and Market Clearing Conditions

Combining the downward sloping demand curve and the production function yields the aggregate output equation:

$$Y_t = \triangle_t^{-1} A_t \left(H_{c,t-1}^{\phi} K_{t-1}^{1-\phi} \right)^{1-\eta} L_t^{\eta}, \tag{36}$$

where $\triangle_t \equiv \int_0^1 \left(\frac{P_t(f)}{P_t} \right)^{-\frac{1+\theta}{\theta}} df$ denotes the cross-sectional price dispersion.

The market clearing condition for the final good in a competitive equilibrium implies:

$$Y_t = C_{h,t} + C_{c,t} + I_t + \varepsilon_t^d, \tag{37}$$

where $C_{h,t}$ is the aggregate consumption of the household, $C_{c,t}$ is the aggregate consumption of the capitalist, and ε_t^d denotes demand shock which follows an AR(1) process with ρ_d and σ_d. The market clearing condition for housing sector is given by:

$$H_{h,t} + H_{c,t} = \bar{H} \tag{38}$$

where $H_{h,t}$ is the aggregate housing demand of the household and $H_{c,t}$ is the aggregate housing demand of the capitalist. Following Iacoviello (2005) and Liu et al., (2013), we assume that the aggregate housing supply is fixed at unitary.

2.7 Welfare Analysis

We can expect that the forward guidance shock together with housing collateral affects differently the wealth distribution of lenders and borrowers. To better understand this issue, we will conduct a welfare analysis. We construct a household's welfare metric as the present discounted value of household utility:

$$V_t^h \equiv E_t \sum_{j=0}^{\infty} \beta^j \left[\log(c_{h,t+j} - bc_{h,t+j-1}) + \vartheta \log h_{h,t+j} - \chi_0 \frac{l_{t+j}^{1+\chi}}{1+\chi} \right], \tag{39}$$

where V_t^h denotes welfare of the household. This can be written in a first-order recursive form:

$$V_t^h = \log(c_{h,t} - bc_{h,t-1}) + \vartheta \log h_{h,t} - \chi_0 \frac{l_t^{1+\chi}}{1+\chi} + \beta E_t V_{t+1}^h. \tag{40}$$

Similarly, capitalist's welfare can be defined as the present discounted value of capitalist utility:

$$\begin{aligned} V_t^c &\equiv E_t \sum_{j=0}^{\infty} \beta_c^j \log(c_{c,t+j} - b_c c_{c,t+j-1}) \\ &= \log(c_{c,t} - b_c c_{c,t-1}) + \beta_c E_t V_{t+1}^c, \end{aligned} \tag{41}$$

where V_t^c is welfare of the capitalist. We use these two welfare measures to analyze the impact of forward guidance to each economic agent associated with housing collateral.

3 Parameterization

The choice of parameter values for the model is presented in Table 1. The model includes thirty-seven parameters. Seven parameters (ϑ, β_c, b_c, ζ, ω_1, ω_2, and ϕ), are linked to housing sector, thirteen parameters ($\sigma_1^{FG}, \sigma_2^{FG}, \ldots\ \sigma_{12}^{FG}$ and L) are related to forward guidance, and the remaining seventeen parameters are conventional parameters in the literature.

We start with the traditional parameters. The household's discount factor, β, is set to 0.9925 implying 3 percent of real interest rate in the nonstochastic steady state. The persistence of habit of the household, b, is calibrated to 0.5, consistent with the estimate in Liu et al. (2013). The household's relative utility weight of labor, χ_0, is set to normalize labor to one in steady state. The labor margin, χ, is set to 3, which corresponds to a Frisch elasticity of labor supply of 1/3 that is about in line with the estimate in Del Negro et al. (2015).

Our choice of the production sector is also standard. The elasticity of output with respect to labor, η, is set to 0.6. We calibrate depreciation rate to 0.025, implying a depreciation of 10 percent per year, consistent with the estimate in King and Rebelo (1999). We set the steady state markup, $(1+\theta)$, to 1.1 as in Smets and Wouters (2007). The Calvo contract parameter, ξ, is chosen to be 0.65 to match autocorrelation of inflation in the data. The persistence of technology and demand shocks, ρ_A and ρ_d, respectively, are chosen to be 0.95, implying stationary processes. The standard deviations of technology and demand shocks, σ_A and σ_d, respectively, are set to 1 percent.

Monetary policy includes both standard and nonstandard parameters. The smoothing parameter, ρ_i, is set to 0.7, the feedback coefficient on inflation, ϕ_π is set to 1.5, and the feedback coefficient on output gap, ϕ_y, is calibrated to 0.1. The standard deviation of unexpected monetary shock, σ_m, is set to 1 percent which is the same size as the technology shock. The length of forward guidance, L, is chosen to 12, consistent with empirical time-contingent forward guidance by the Federal Reserve in September 2012.[6] The size of each forward guidance shock is set to 1 percent as the unexpected monetary policy shock.

Turning to the parameters related with housing sector, we use estimates from Liu et al. (2013). We set the capitalist's discount factor, β_c, to 0.945. The credit constraint (11) thus binds as the capitalist's discount factor is smaller than the household's discount factor. The capitalist's habit persistence, b_c, is set to 0.65 meaning that the capitalist depends on the past consumption level more than that of the household. The LTV, ζ, is calibrated to 0.8 which implies that the capitalist can borrow up to 80% of the collateral value. The relative weight on housing value in the collateral constraint, ω_1, is normalized to 1 and the weight on capital value, ω_2, is set to 0.1. The elasticity of investment adjustment cost, Ω, is calibrated to 0.1753. This value is relatively smaller than its typical estimates of 2 to 3 (e.g., Justiniano et al., 2010). It is, however, necessary to have a small adjustment cost parameter to match volatility of investment. This small value is also consistent with the minor role of capital in the collateral value. Lastly, the elasticity of output with respect to house, ϕ, is set to 0.07.

We provide additional justification for our chosen model. Table 2 displays the autocorrelations up to three lags of the following macroeconomic variables: output (Y), consumption (C), investment (I), housing price (q_h), wages (w), labor (L), inflation (π), nominal interest rate (R), and real interest rate (r). We compute these statistics from U.S. data across the time period *1948:Q1 - 2008:Q4* (top panel of Table 2). The data are retrieved from Bureau of Economic Analysis as well as Robert Shiller's website.[7] All series are HP filtered except for inflation and interest rates. The counterparts from our model described in Section 2 are also reported in the bottom panel of Table

[6] "the Committee also decided today to keep the target range for the federal funds rate at 0-0.25% and currently anticipates that exceptionally low levels for the federal funds rate are likely to be warranted at least through mid-2015" (Federal Reserve, 2012).

[7] http://www.econ.yale.edu/~shiller/data.htm

2. The results show that the model does a reasonable job of matching the data. The sign of the autocorrelations from our model match entirely the counterparts from the data. The persistence of variables generated by the model also are similar to those derived from U.S. data overall.

4 Results

4.1 Impulse Responses

We proceed with our main analysis examining the influence of housing frictions on the effectiveness of forward guidance. Figures 1 - 6 display the responses of select macroeconomic variables to a one standard deviation increase in the unanticipated monetary policy, four, eight, and twelve periods ahead forward guidance shocks. The solid line denotes our benchmark case of housing frictions, i.e., $\zeta = 0.8$. The dashed line represents increased financial frictions, (i.e., $\zeta = 0.0001$) in which the borrowing of capitalists is constrained as collateral is not valuable.[8]

The benchmark case of $\zeta = 0.8$ shows the effects of forward guidance on macroeconomic variables. When the central bank announces that the interest rate will increase four, eight, or twelve periods into the future, Figures 1 and 3 display that output, inflation, and investment all initially decline. Households substitute away from consumption into more housing (displayed in Figures 2 and 4), while capitalists decrease investment with the higher interest rate. Consequently, output and inflation overall decrease in response to an adverse forward guidance shock. Finally, output, inflation, and investment all decrease again around the time when the forward guidance shock is realized on the economy.

Housing variables also have similar intuitive responses to forward guidance statements. To understand these effects, it is helpful to first examine the responses to an unanticipated monetary policy shock (i.e., first column in Figures 2 and 4). When the interest rate increases, the value of collateral decreases as seen by equation (11). Accordingly, borrowing of capitalists decreases causing investment and output to drop. Given the decrease in output, housing demand from intermediate goods firms reduces along with housing prices. This result further decreases the value of collateral. Consequently, capitalists are further constrained in their borrowing generating a deeper economic downturn and giving rise to the so called "financial accelerator" effect. In addition, households

[8]The reason why we represent the increased financial frictions case with $\zeta = 0.0001$ instead of $\zeta = 0$ is as follows. Since the model is log-linearized, setting ζ to zero would imply no borrowing *but* lending could still exist. Thus, for illustrative purposes, we denote the increased financial frictions case by setting the LTV ratio to 0.0001. Section 5.3 also provides an exercise when the LTV ratio varies by time.

increase their holdings of housing services due to the decreases in housing prices from the accelerator mechanism. Thus, H_h increases while both H_c and q_h decline.

The financial accelerator reasoning can then be provided for the effects of forward guidance shocks on housing variables. Knowledge that future interest rates will increase causes the value of collateral to drop. Similar to the previous paragraph, an increase in borrowing constrained firms leads to a decline in output and house prices before the change in the interest rate is realized. The decrease in home prices further causes capitalists' value of collateral to decrease resulting in the financial accelerator effect.

What occurs if financial frictions regarding housing increase? The dashed lines in Figures 1 - 4 show the effects of forward guidance are dampened. The responses of the macroeconomic variables to forward guidance shocks four, eight, and twelve periods ahead display the same initial sign as the $\zeta = 0.8$ case, but do not react as strongly. For instance, output, inflation, and investment decline upon announcement that the interest rate will increase in the future. However, the reaction is muted when financial frictions in the housing market increase. The reason for this can be due to a constrained financial accelerator mechanism. When ζ is set to 0.0001, borrowing already reached its minimum value (or very close to zero). The capitalist can only use their internal funds for activities, and thus, the accelerator mechanism is restricted. Increases in interest rate would minimally alter, if at all, the collateral constraint. Therefore, the responses of macroeconomic and housing variables are much attenuated.

Similar results occur when examining housing market variables. As in the benchmark scenario, the response of capitalist's housing initially is negative. However, since they are constrained in their borrowing, the capitalist cannot borrow as much (household lending is less under the sufficiently tight credit constraint than the benchmark case). Consequently, the capitalist's holdings of housing does not initially change a lot implying the same for household's holdings of housing.[9]

We also analyze the effects of forward guidance on welfare of households and capitalists. Under the baseline scenario of $\zeta = 0.8$ (solid line), Figures 5 and 6 show increases in both the welfare of households and capitalists. Figures 5 and 6 display that the welfare of households rises at the beginning, but it turns to a negative response after four, eight, and twelve periods, respectively. The reason regards the response of the interest rate. In Figures 1 and 3, the interest rate (slightly)

[9]We also analyzed the $\zeta = 1$ case, that is, a looser housing credit constraint. Our benchmark takeaway is still confirmed: the effectiveness of forward guidance shock is attenuated when housing frictions are increased.

declines before the monetary authority actually increases the interest rate. As a result, households reduce lending and consume more housing increasing households' welfare (see equation (40)). After the interest rate actually rises, households have an incentive to lend more, reducing both their nonhousing consumption and housing consumption. Accordingly, after the four, eight, and twelve periods, the welfare of households turns negative. On the other hand, capitalists hold less housing because of the increase in interest rates from the forward guidance shock and the credit constraint as can be seen in Figures 2 and 4. Investment also decreases in the model due to news that interest rate will increase in the future.[10] With capitalists holding less housing and investment, consumption rises leading to increases in capitalists' welfare as shown in equation (41).[11]

This result can be also due to changes in expectations of future welfare displayed in Figures 7 and 8. With news of interest rates increasing in the future, households switch away from consumption towards more holdings of housing today and expect more future housing and consumption due to savings from expected higher interest rate. Consequently, the welfare of households enlarges via current housing and expectations of future welfare as seen by equation (40). In addition, with capitalists expecting higher holdings of housing by households in the future, the income of capitalists increases given their budget constraint and increases future consumption which increases future welfare. This increase in future welfare increases current welfare as seen in equation (41).

The effects of forward guidance on welfare, however, are asymmetric between households and capitalists when financial frictions are exacerbated. For households, their welfare is higher with increased financial frictions ($\zeta = 0.0001$). The dashed line is above the solid line in the top rows in Figures 5 and 6. When the credit channel is constrained, households do not provide as much funds to capitalists resulting in more consumption of both nonhousing goods and housing. This leads to the increase in the welfare of households as seen in equation (40). However, the welfare of capitalists is less when financial frictions are exacerbated. Under this scenario, capitalists cannot depend on external financing. Hence, consumption of capitalists decreases more which decreases the welfare of capitalists further. Their expected welfare is also not as large relative to the $\zeta = 0.8$ case as capitalists expect households not to hold as much housing in the future. The dashed line is below solid line in Figures 7 and 8). Since capitalists expect households not to hold as much housing,

[10] Our model generates positive co-movements between housing price and investment since the collateral constraint is on firms rather than households (e.g., Liu et al., 2013). However, when households are credit constrained, models can generate positive co-movements between housing price and consumption but it is hard to produce positive co-movements between housing price and investment (e.g., Iacoviello and Neri, 2010).

[11] The increase is relatively small due to high amounts of persistence in habits in consumption.

the former's expected income decreases resulting in less consumption. This outcome results in less future and current welfare as seen by equation (41).

The main results also provide a solution to the "forward guidance puzzle" as laid out by Del Negro et al. (2012). The previous phenomenon can be described as extreme responses (relative to the data) of the macroeconomic variables to forward guidance. Under the model utilized in Del Negro et al. (2012), frictions emanating from the housing market were not considered. However, in the present paper, Figures 1 - 8 show that the responses of the macroeconomic variables to forward guidance are muted if housing frictions are assumed. Thus, housing market frictions can be another solution to the forward guidance puzzle.

4.2 Constant Interest Rate

The Federal Reserve kept the federal funds rate at its lowest level in response to the 2007 - 2009 Great Recession and COVID-19 induced recession. In this section, we explore how the ZLB affects an economy with the housing frictions. To this end, we proceed in the following manner. A recession is generated by a negative technology shock in time period $T+1$. In the first scenario, the central bank communicates to the public that the interest rate will be at zero in period $T+1$ and L periods into the future. Computationally, we choose the unanticipated monetary policy and forward guidance shocks (i.e., ε_t^m, $\varepsilon_{1,t}^{FG}$, $\varepsilon_{2,t}^{FG}$, $\dots$, $\varepsilon_{L,t}^{FG}$) such that the interest rate equals zero in the current period and L periods into the future. The second scenario, generates the same recession, but the central bank does not implement any policy to keep the interest rate at zero. Finally, we calculate the difference between the two scenarios to obtain the pure effect of the central bank's forward guidance policy on the economy.

The solid lines in Figures 9, 10, and 11 show the stimulative effects of forward guidance under benchmark housing frictions. When the negative supply side shock hits the economy, output decreases but inflation increases (similar to Figure 15). According to the Taylor rule, this implies the nominal interest rate must increase. However, since the central bank promises a *relatively* lower interest rate at the ZLB, inflation increases. The Fisher equation also indicates that the higher inflation rate while nominal interest rate is fixed at zero leads to decrease in the real interest rate. Consequently, investment and output increase. With the higher output, both consumption of households and capitalists increase. As output increases, capitalists' housing increases because production sector demands more housing to produce more which pushes housing price upward.

Accordingly, households cannot hold housing as much as before.

Frictions emanating from the housing sector, however, attenuate the beneficial effects of forward guidance. This result is specifically observed in the beginnings of the forward guidance period. Since capitalists are constrained in their borrowing, the dashed line is below the solid line in the bottom panel of Figure 10. Therefore, investment is diminished with higher housing frictions. This prior result causes output and inflation to be lower under $\zeta = 0.0001$ than 0.8. Consequently, forward guidance is not as powerful at the ZLB when housing sector frictions are exacerbated.

Overall, financial frictions emanating from the housing market dampen the effectiveness of forward guidance on the economy. Under an empirically motivated LTV ratio, that is, existence of some financial frictions in the economy, the impulse responses of the macroeconomic variables (e.g., output, inflation, investment, and household lending) to forward guidance react strongly. However, when financial frictions are exacerbated, the responses are muted. Similar results are observed when the economy is at the ZLB for an extended period of time. Thus, policymakers should take into account the dynamics of financial frictions emanating from the housing market when examining forward guidance effects.

5 Robustness

5.1 Imperfect Central Bank Credibility

The credibility of the central bank plays a very important role for forward guidance to effectively work. In the benchmark model, we assumed that all economic agents completely trust the central bank's announcements about the future interest rate. However, if agents harbor doubts that the monetary authority will renege on its future interest rate statements, the predicated economic effects could be muted.[12]

We proceed to analyze the effects of central bank credibility on forward guidance with housing frictions in the following manner. Following Sims and Wu (2021), we modify the (log-linearized) monetary policy of the central bank as follows:

$$\hat{R}_t = \rho_i \hat{R}_{t-1} + (1-\rho_i)\left(\phi_\pi \hat{\pi}_t + \phi_y \hat{Y}_t\right) + \hat{\varepsilon}_t^{MP} + \gamma \sum_{l=1}^{L} \hat{\varepsilon}_{l,t-l}^{R} \tag{42}$$

where $\gamma \in [0,1]$ is an empirically motivated measure of the credibility of forward guidance announcements. $\gamma = 1$ means that all economic agents completely trust announcements of the central

[12] Cole and Martínez-García (2021) analyze credibility and forward guidance, but without a housing sector.

bank for future monetary policy. On the other hand, as this value approaches zero, no one trusts the central bank. Therefore, forward guidance has no effect on the economy. The benchmark case in Section 4 corresponds to the case where γ is set to 1 (i.e., solid line in Figure 12). For illustrative purposes, we compare the results with the case where γ is set to 0.5 (i.e., dashed line in Figure 12), meaning the economic agents trust the forward guidance announcements for 50%.

The results show a dampening effect of forward guidance on the economy with a less than fully credible monetary authority. As displayed in Figure 12, a promise to increase the interest rate in the future has a more muted effect on the macroeconomic variables. For instance, output and inflation decrease less under an imperfectly credible central bank than fully credible central bank (overall, dashed line is above solid line in first two rows of Figure 12). The initial impact on the housing of households and capitalists are also less under $\gamma = 0.5$ than $\gamma = 1$.

5.2 Alternative Parameterization for the Credit Constraint

The value of collateral depends mostly on real estate when firms want to get external financing. Indeed, some studies do not even consider capital at all in credit constraints (e.g., Iacoviello, 2005; and Iacoviello and Neri, 2010). We fixed the weight of capital to 0.1 when the weight of housing as collateral is normalized to 1 as in the estimates in Liu et al. (2016). However, there can exist cases where capital contributes more to the value of a firm's collateral (e.g., expensive machines and instruments).

This subsection examines the effectiveness of forward guidance when the value of capital as collateral becomes as important as real estate. We set ω_2 to 1 in the credit constraint and perform impulse response analyses to eight and twelve period ahead forward guidance shocks. The results are displayed in Figure 13 with the solid line representing our benchmark case and dashed line denoting $\omega_2 = 1$.

We find that the effectiveness of forward guidance is enhanced when $\omega_2 = 1$. For instance, output shows a greater response to forward guidance news relative to the benchmark case. The dashed line, overall, is below the solid line in Figure 13. Under $\omega_2 = 1$, firms are able to finance more since capital increases in value. By applying this previous logic to interest rates expecting to increase, capitalists decrease their investment more by further utilizing financial markets. In other words, the results are similar to the case when the LTV is increased.

5.3 Time-Varying Loan to Value

Prior literature finds that financial shocks are important in accounting for economic fluctuations (e.g., Nolan and Thoenissen, 2009; Jermann and Quadrini, 2012; and Liu et al., 2013). Therefore, we consider a credit shock by incorporating time-varying LTV as follows:

$$B_t \leq \zeta_t E_t \left(\omega_1 Q_{h,t+1} h_{c,t} \frac{1}{R_t} + \omega_2 Q_{k,t+1} k_t \frac{1}{R_t} \right), \tag{43}$$

where ζ_t follows an exogenous AR(1) process:

$$\ln \zeta_t = (1 - \rho_\zeta) \ln \zeta + \rho_\zeta \ln \zeta_{t-1} + \varepsilon_t^\zeta, \tag{44}$$

ζ is the nonstochastic steady state vaule of LTV ratio, $\rho_\zeta \in (-1, 1)$ is the persistence parameter, and ε_t^ζ is an *iid* shock with mean zero and variance σ_ζ^2.[13] Not surprisingly, Figure 14 shows that the difference between impulse responses of benchmark model and the model with the time-varying LTV is very trivial.[14] We find that the time-varying LTV does not change our results dramatically.

6 Conclusion

Monetary policy and housing markets have been historically linked. For instance, the Federal Reserve's easy monetary policy, arguably, created a housing market bubble causing the 2007-2009 global financial crisis. The U.S. central bank responded to the recession by lowering U.S. short-term interest rates to its ZLB and issuing forward guidance on the future course of policy. In this paper, we analyze this link between ZLB, forward guidance, and housing markets.

The results of our paper produce a number of findings. First, housing frictions dampen the effectiveness of forward guidance on the economy. For example, when the central bank announces that the interest rate will increase in the future, the impulse responses of output and inflation are diminished with an increase in housing frictions. Forward guidance has asymmetric effects on the welfare of households and capitalists depending on the degree of housing frictions. When the monetary authority promises to increase interest rates in the future, the welfare of households is higher under more frictions than not. Since households are more constrained in lending with the sufficiently tight credit constraint, they consume more leading to higher welfare. However,

[13] Following Liu et al. (2016), we set $\rho_\zeta = 0.96$. To be consistent with other standard deviations of our paper, σ_ζ is fixed to 0.01.

[14] We do not report impulse responses of the credit shock as we focus on the effect of forward guidance shock. Results are available upon request.

capitalists cannot borrow as much when financial frictions from housing sector are exacerbated. Their welfare is then lower under $\zeta = 0.0001$ than $\zeta = 0.8$. In addition, housing frictions attenuate the beneficial effects of forward guidance at the ZLB. When conventional policy of changing current interest rates is constrained by the ZLB, central bank communication that the interest rate will remain at zero for L periods into the future produces beneficial effects on the economy. However, as the value of capitalists' collateral decreases (i.e., increase in housing frictions), the positive forward guidance effects are muted. Overall, policymakers should consider housing frictions when examining the effects of forward guidance on the economy.

References

Andrade, P., G. Gaballo, E. Mengus, and B. Mojon (2019). "Forward Guidance and Heterogeneous Beliefs." *American Economic Journal: Macroeconomics 11* (3), 1–29.

Berger, Allen N., and Gregory F. Udell. (1990). "Collateral, Loan Quality and Bank Risk." *Journal of Monetary Economics*, 25(1), 21-42.

Bernanke, Ben S., Mark Gertler, and Simon Gilchrist. (1999). "Limited Borrowing Capacity of Financial Intermediaries in a Quantitative Business Cycle Framework." *Handbook of Macroeconomics*, 1, 1341-1393.

Bundick, B. and A. L. Smith (2020). "Should We Be Puzzled by Forward Guidance?" *Federal Reserve Bank of Kansas City Research Working Papers No. 20-01.*

Calvo, Guillermo A. (1983). "Staggered Prices in a Utility-Maximizing Framework." *Journal of Monetary Economics*, 12(3), 383-398.

Campbell, J. R., C. L. Evans, J. D. Fisher, and A. Justiniano (2012). "Macroeconomic Effects of Federal Reserve Forward Guidance." *Brookings Papers on Economic Activity 2012* (1), 1–80. *Project MUSE.*

Carlstrom, C. T., T. S. Fuerst, and M. Paustian (2015). "Inflation and Output in New Keynesian Models With a Transient Interest Rate Peg." *Journal of Monetary Economics 76* (November), 230–243.

Chen, H., Cúrdia, V., and Ferrero, A. (2012). "The Macroeconomic Effects of Large-Scale Asset Purchase Programmes." *The Economic Journal*, 122(564), F289-F315.

Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. (2005). "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1), 1-45.

Cole, Stephen J. (2021). "Learning and the Effectiveness of Central Bank Forward Guidance." *Journal of Money, Credit and Banking*, 53(1), 157-200.

Cole, Stephen J. (2020). "The Limits of Central Bank Forward Guidance under Learning." *International Journal of Central Banking*, 16(4), 199-250.

Cole, S.J., and Martínez-García, E. (2021). "The Effect of Central Bank Credibility on Forward Guidance in an Estimated New Keynesian Model." *Globalization and Monetary Policy Institute Working Paper*, (375).

De Graeve, F., P. Ilbas, and R. Wouters (2014). "Forward Guidance and Long Term Interest Rates: Inspecting the Mechanism." *Sveriges Riksbank Working Paper No. 292*.

Del Negro, Marco, Marc P. Giannoni, and Christina Patterson. (2012). "The Forward Guidance Puzzle." *FRB of New York Staff Report*, 574.

Del Negro, Marco, Marc P. Giannoni, and Frank Schorfheide. (2015). "Inflation in the Great Recession and New Keynesian Models." *American Economic Journal: Macroeconomics*, 7(1), 168-196.

Eggertsson, G. B. and M. Woodford (2003). "The Zero Bound on Interest Rates and Optimal Monetary Policy." *Brookings Papers on Economic Activity 2003* (1), 139–211. *Project MUSE*.

Haberis, A., R. Harrison, and M. Waldron (2019). "Uncertain Policy Promises." *European Economic Review 111*, 459–474.

Honkapohja, Seppo, and Kaushik Mitra. (2005). "Performance of Inflation Targeting Based on Constant Interest Rate Projections." *Journal of Economic Dynamics and Control*, 29(11), 1867-1892.

Iacoviello, Matteo. (2005). "House Prices, Borrowing Constraints, and Monetary Policy in the Business Cycle." *American Economic Review*, 95(3), 739-764.

Iacoviello, Matteo, and Stefano Neri. (2010). "Housing Market Spillovers: Evidence from an Estimated DSGE Model." *American Economic Journal: Macroeconomics*, 2(2), 125-164.

Janson, W., and Jia, C. (2020). "Forward Guidance during the Pandemic: Has It Changed the Public's Expectations?" *Economic Commentary*, (EC 2020-27).

Jermann, Urban, and Vincenzo Quadrini. (2012). "Macroeconomic Effects of Financial Shocks." *American Economic Review*, 102(1), 238-271.

Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti. (2010) "Investment Shocks and Business Cycles." *Journal of Monetary Economics*, 57(2), 132-145.

Kiley, M. T. (2016). "Policy Paradoxes in the New Keynesian Model." *Review of Economic Dynamics 21* (July), 1–15.

King, Robert G., and Sergio T. Rebelo. (1999). "Resuscitating Real Business Cycles." *Handbook of Macroeconomics*, 1, 927-1007.

Kiyotaki, Nobuhiro, and John Moore. (1997). "Credit Cycles." *Journal of Political Economy*, 105(2), 211-248.

Laséen, Stefan, and Lars E.O. Svensson. (2011). "Anticipated Alternative Policy Rate Paths in

Policy Simulations." *International Journal of Central Banking*, 7(3), 1-35.

Liu, Zheng, Jianjun Miao, and Tao Zha. (2016). "Land Prices and Unemployment." *Journal of Monetary Economics*, 80, 86-105.

Liu, Zheng, Pengfei Wang, and Tao Zha. (2013). "Land-price Dynamics and Macroeconomic Fluctuations." *Econometrica*, 81(3), 1147-1184.

Nolan, Charles, and Christoph Thoenissen. (2009). "Financial Shocks and the US Business Cycle." *Journal of Monetary Economics*, 56(4), 596-604.

Sims, Eric, and Jing Cynthia Wu. (2021). "Evaluating Central Banks' Tool Kit: Past, Present, and Future." *Journal of Monetary Economics*, 118, 135-160.

Smets, Frank, and Rafael Wouters. (2007). "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3), 586-606.

Swanson, E. T. (2018). "The Federal Reserve Is Not Very Constrained by the Lower Bound on Nominal Interest Rates." *Brookings Papers on Economic Activity 2018* (2), 555–572. *Project MUSE*.

Woodford, Michael. (2005). "Central Bank Communication and Policy Effectiveness." *National Bureau of Economic Research*, No. w11898.

Appendix

A Tables

Table 1: **Parameter Values**

Parameters	Descriptions	Value
Household		
β	Discount rate of household	0.9925
b	Habit persistence of household	0.5
ϑ	Relative utility weight of housing	0.0457
χ_0	Relative utility weight of labor	$\frac{\eta}{1+\theta}\frac{1-\beta b}{1-b}\left(L^{1+\chi}\right)\left(\frac{C_h}{Y}\right)^{-1}$
χ	Labor margin	3
Capitalist		
β_c	Discount rate of capitalist	0.945
b_c	Habit persistence of capitalist	0.65
ζ	Loan to value	0.8
Ω	Elasticity of investment adjustment cost	0.1753
ω_1	Weight on housing value	1
ω_2	Weight on capiatl value	0.1
Production Firms		
η	Labor share	0.6
ϕ	Housing share	0.07
δ	Depreciation rate	0.025
ξ	Calvo parameter	0.65
θ	Monopolistic markup	0.1
Monetary Authority		
ρ_i	Smoothing parameter of monetary policy	0.7
ϕ_π	Response of monetary policy to inflation	1.5
ϕ_y	Response of monetary policy to output	0.1
σ_m	Standard deviation of unexpected monetary policy shock	0.01
σ_l^{FG}	Standard deviation of forward guidance shock	0.01
L	Length of the forward guidance horizon	12
Shock Processes		
ρ_A	Persistence of technology shock	0.95
σ_A	Standard deviation of technology shock	0.01
ρ_d	Persistence of demand shock	0.95
σ_d	Standard deviation of demand shock	0.01

Table 2: **Autocorrelations of Select Macroeconomic Variables**

Lag	$\boldsymbol{Y}$	$\boldsymbol{C}$	$\boldsymbol{I}$	$\boldsymbol{q_h}$	$\boldsymbol{w}$	$\boldsymbol{L}$	$\boldsymbol{\pi}$	$\boldsymbol{R}$	$\boldsymbol{r}$
	U.S. Data: 1948:Q1-2008:Q4								
1	0.812	0.848	0.724	0.949	0.732	0.899	0.78	0.932	0.736
2	0.569	0.664	0.541	0.865	0.498	0.693	0.693	0.882	0.608
3	0.282	0.454	0.288	0.754	0.34	0.444	0.623	0.86	0.529
	Model-Implied Data								
1	0.82	0.85	0.79	0.79	0.58	0.78	0.70	0.64	0.61
2	0.51	0.6	0.43	0.56	0.31	0.59	0.51	0.37	0.37
3	0.19	0.34	0.07	0.3	0.04	0.33	0.3	0.12	0.16

B Figures

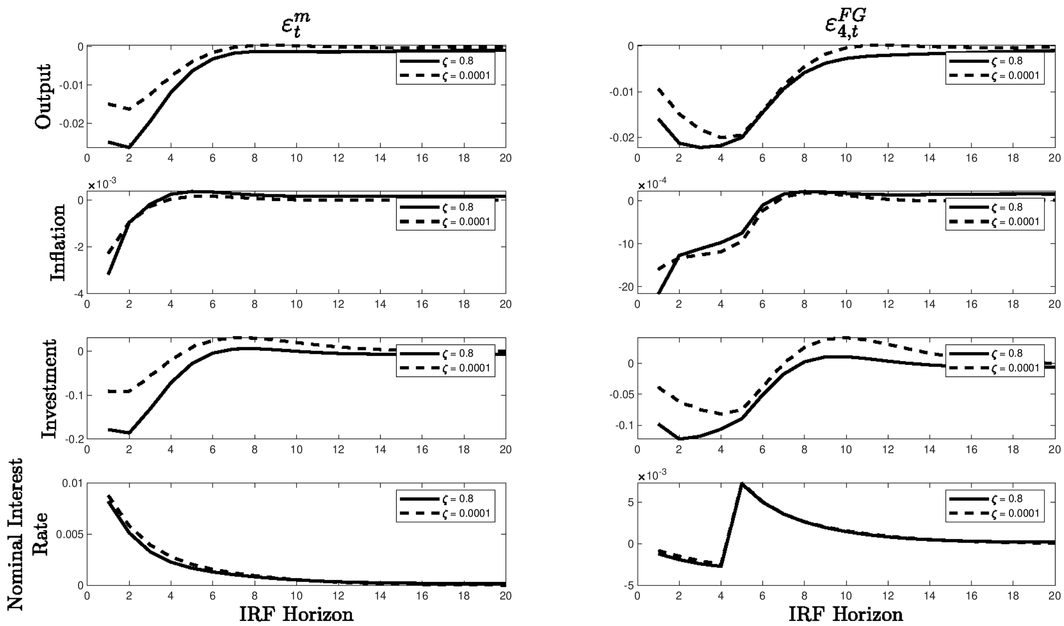

Figure 1: Impulse Responses of Macroeconomic Variables to Unanticipated Monetary Policy and Four Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

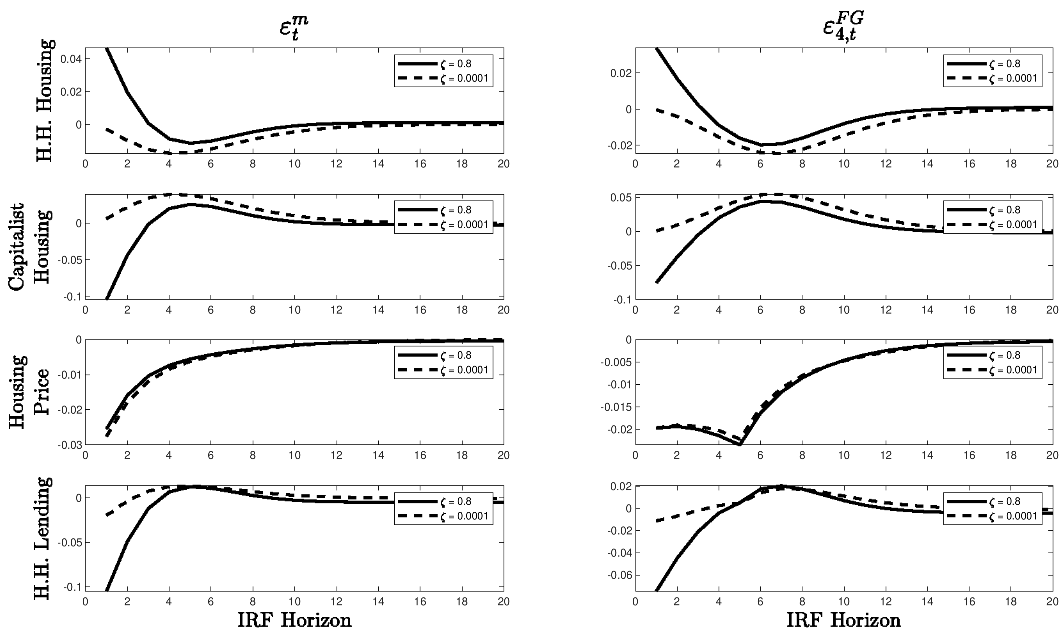

Figure 2: Impulse Responses of Housing Variables to Unanticipated Monetary Policy and Four Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

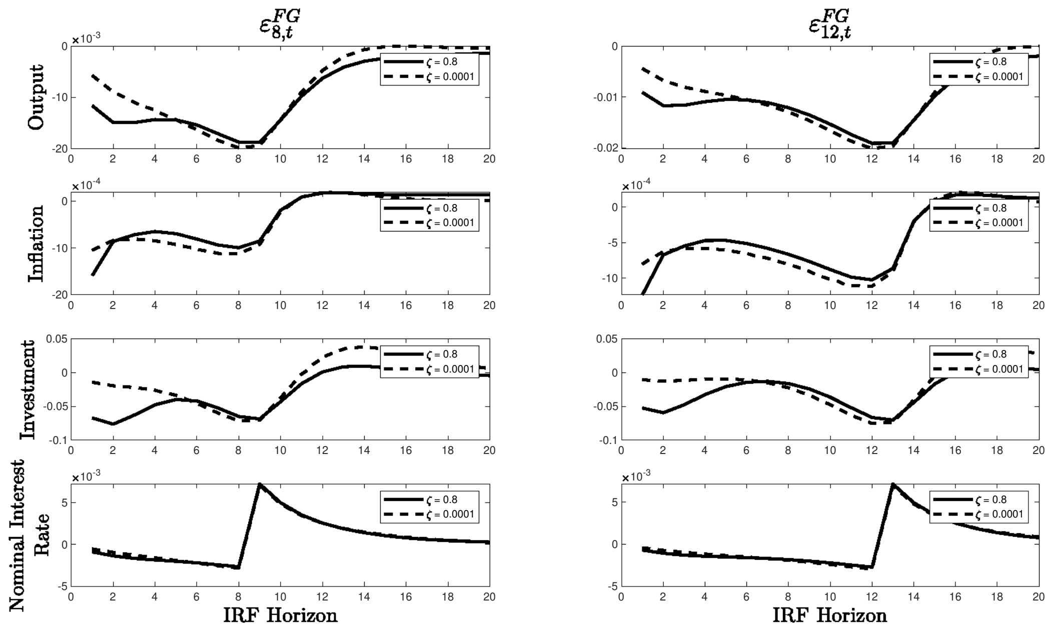

Figure 3: Impulse Responses of Macroeconomic Variables to Eight and Twelve Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

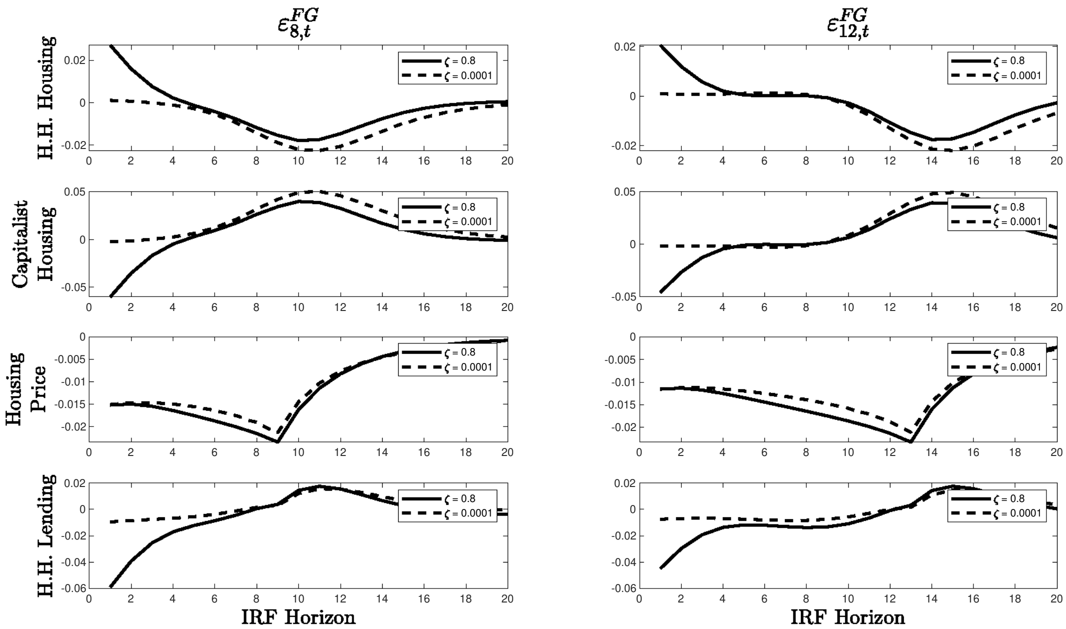

Figure 4: Impulse Responses of Housing Variables to Eight and Twelve Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

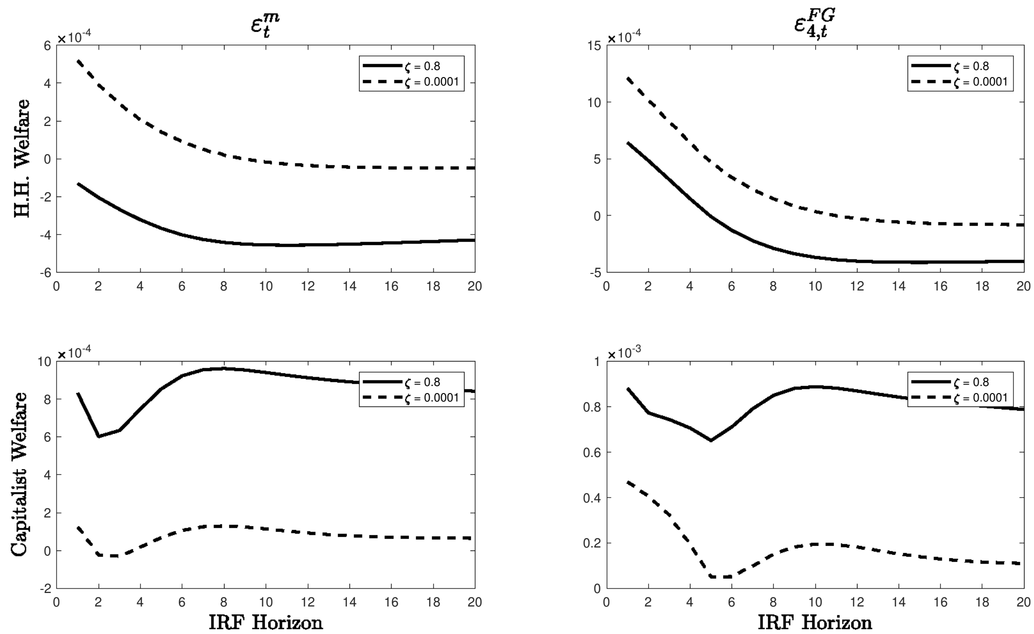

Figure 5: Impulse Responses of Household and Capitalist Welfare to Unanticipated Monetary Policy and Four Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

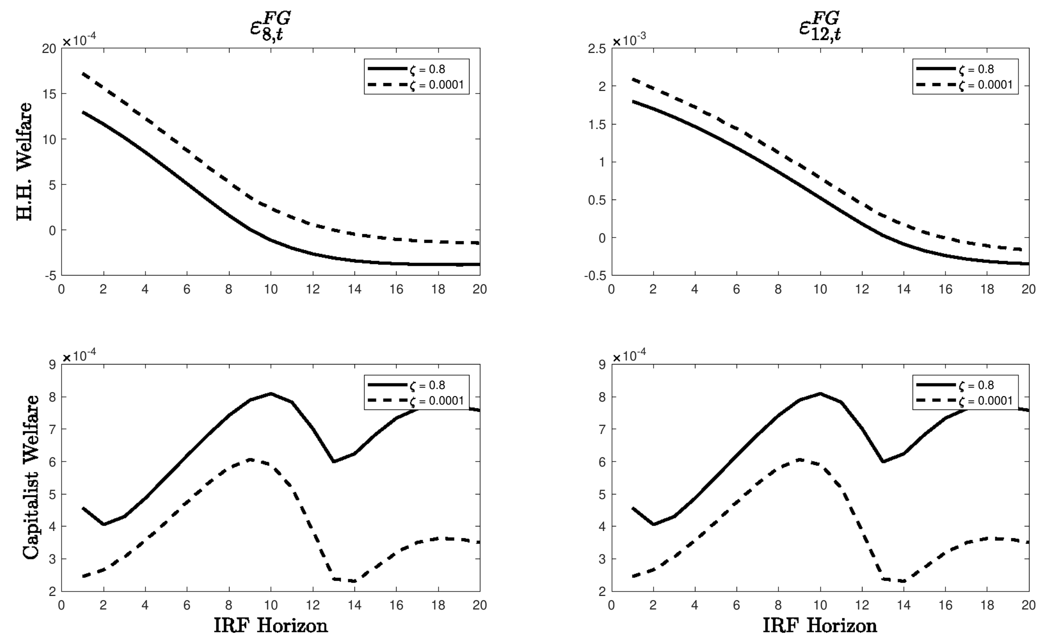

Figure 6: Impulse Responses of Household and Capitalist Welfare to Eight and Twelve Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

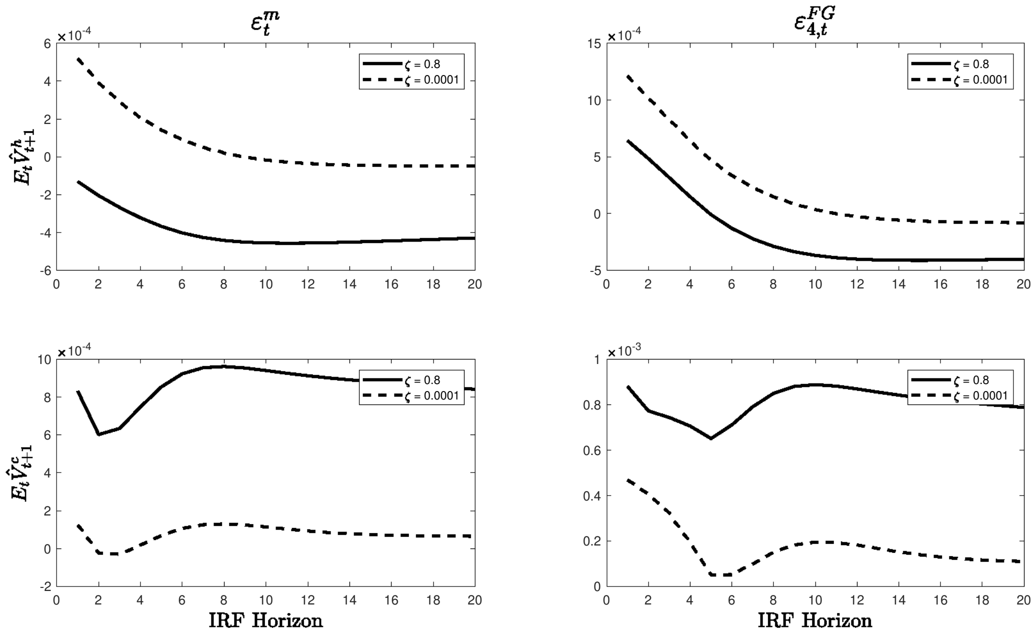

Figure 7: Impulse Responses of Household and Capitalist Expected Welfare to Unanticipated Monetary Policy and Four Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

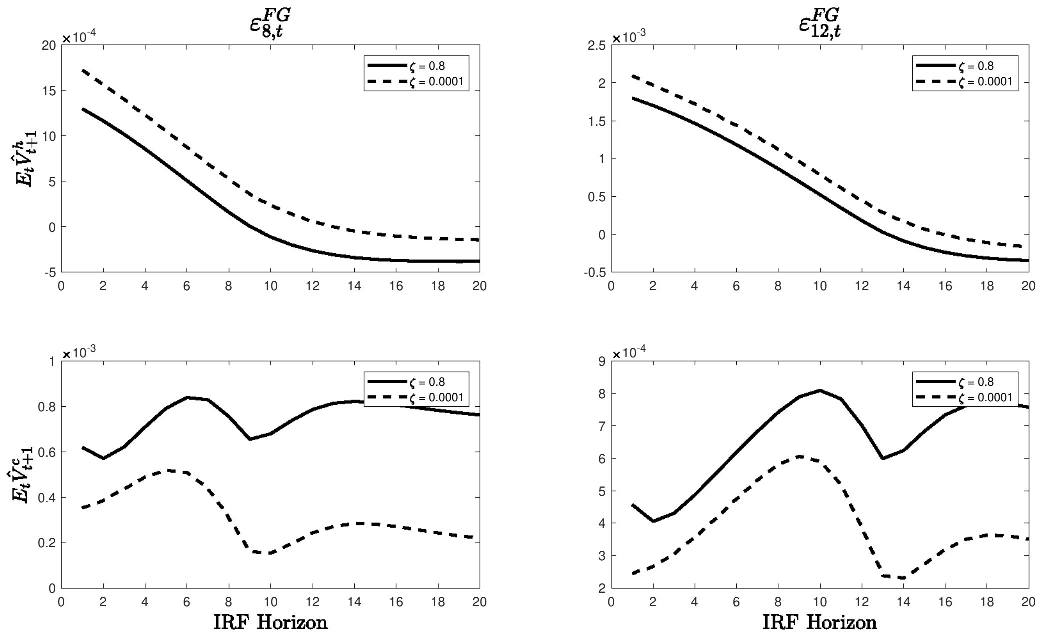

Figure 8: Impulse Responses of Household and Capitalist Expected Welfare to Eight and Twelve Periods Ahead Forward Guidance Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

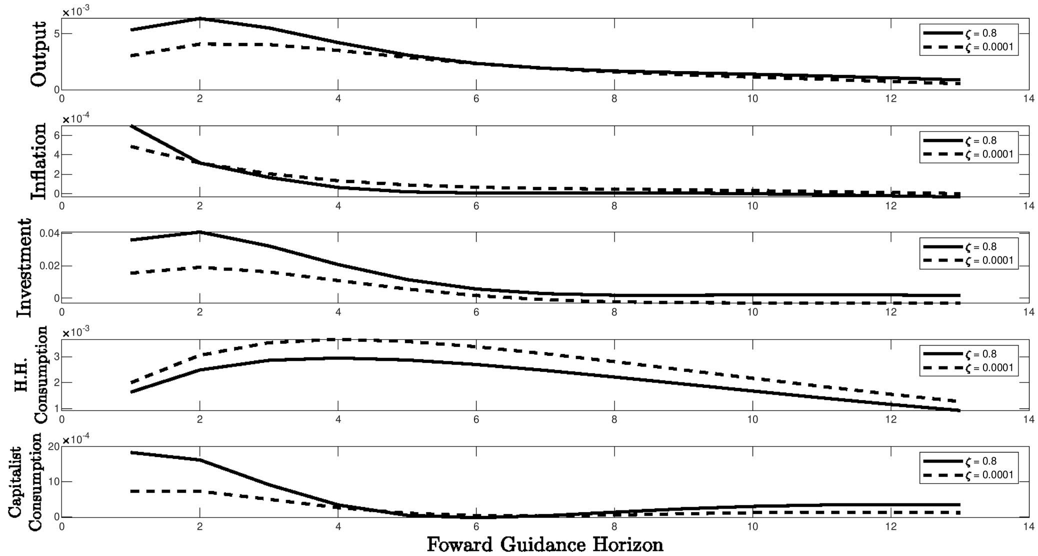

Figure 9: The Effect of Forward Guidance and ZLB on Macroeconomic Variables during a Recession

Note: This graph shows the path of the macroeconomic variables when a recession exists and the central bank communicates that the nominal interest rate will equal zero now and L periods into the future. Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

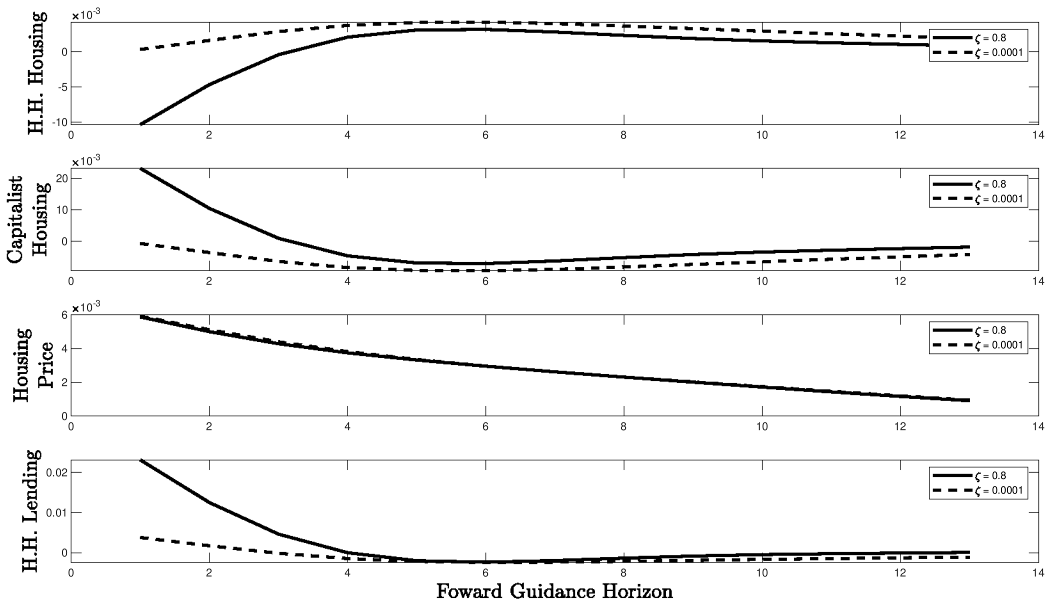

Figure 10: The Effect of Forward Guidance and ZLB on Housing Variables during a Recession

Note: This graph shows the path of the macroeconomic variables when a recession exists and the central bank communicates that the nominal interest rate will equal zero now and L periods into the future. Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

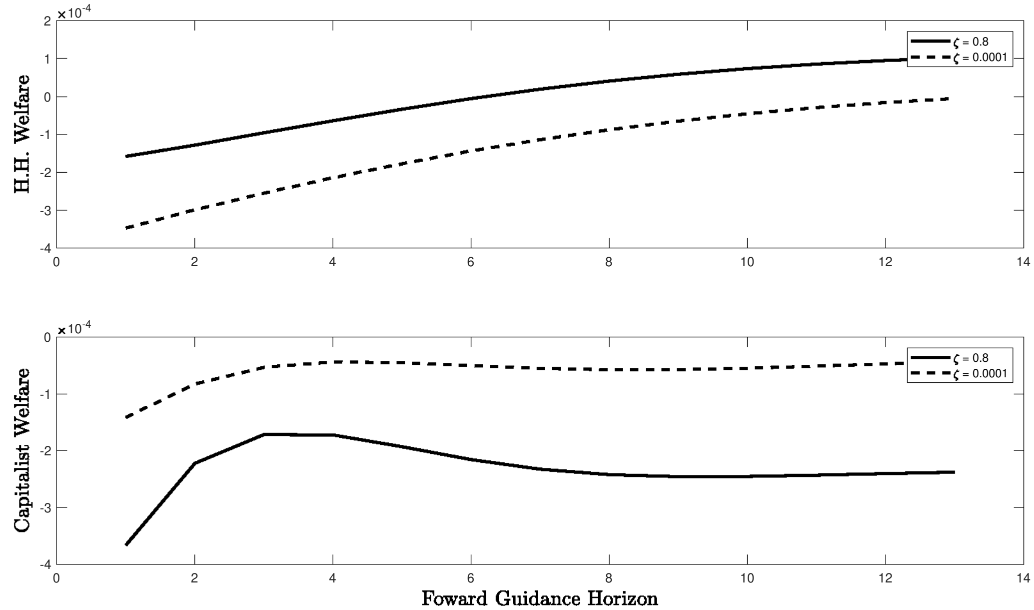

Figure 11: The Effect of Forward Guidance and ZLB on Household and Capitalist Welfare during a Recession

Note: This graph shows the path of the macroeconomic variables when a recession exists and the central bank communicates that the nominal interest rate will equal zero now and L periods into the future. Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

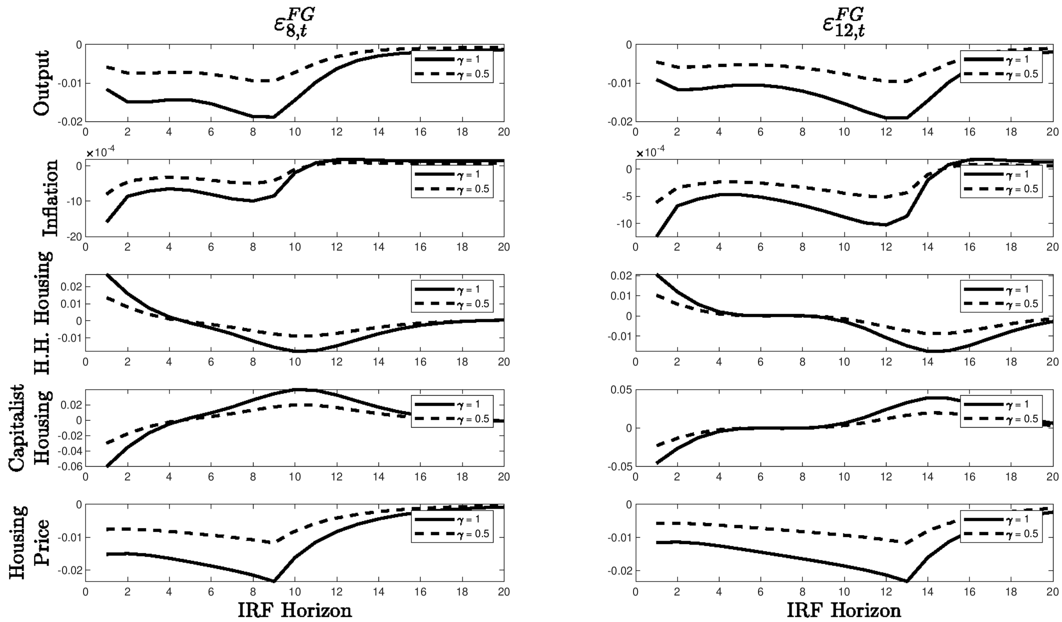

Figure 12: Impulse Responses of Variables to Eight and Twelve Periods Ahead Forward Guidance Shocks Under Perfect and Imperfect Central Bank Credibility

Note: Solid Line: $\gamma = 1$ (Perfect Central Bank Credibility); Dashed Line: $\gamma = 0.5$ (Imperfect Central Bank Credibility)

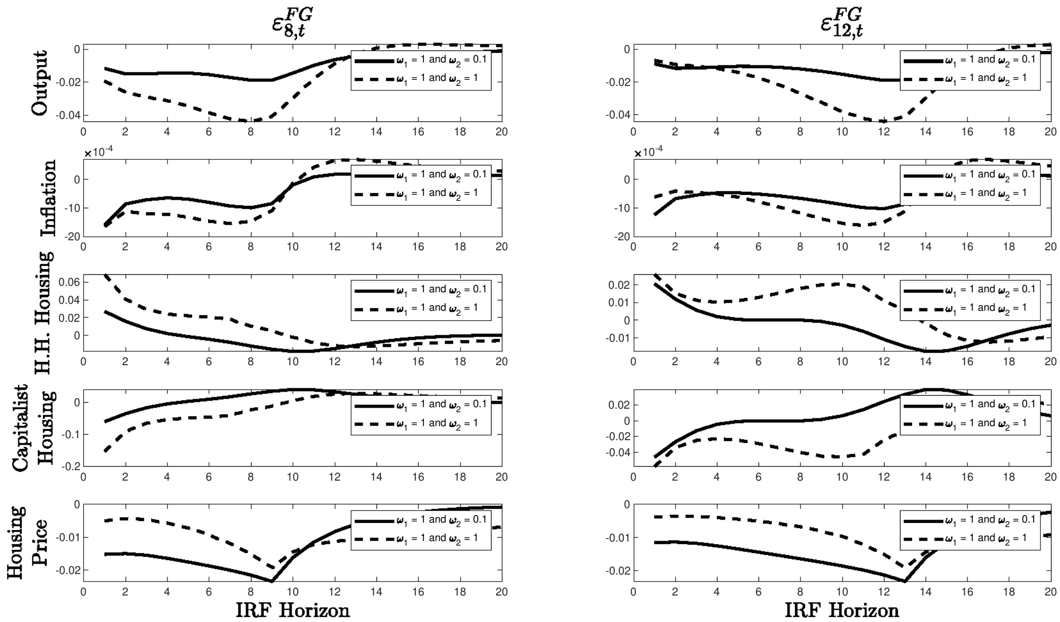

Figure 13: Impulse Responses of Variables to Eight and Twelve Periods Ahead Forward Guidance Shocks Under Benchmark and Alternative Parameterizations for the Credit Constraint

Note: Solid Line: $\omega_2 = 0.1$ (Benchmark Parameterization, Capital Contributes Less to the Value of Firm's Collateral); Dashed Line: $\omega_2 = 1$ (Alternative Parameterization, Capital Contributes More to the Value of Firm's Collateral)

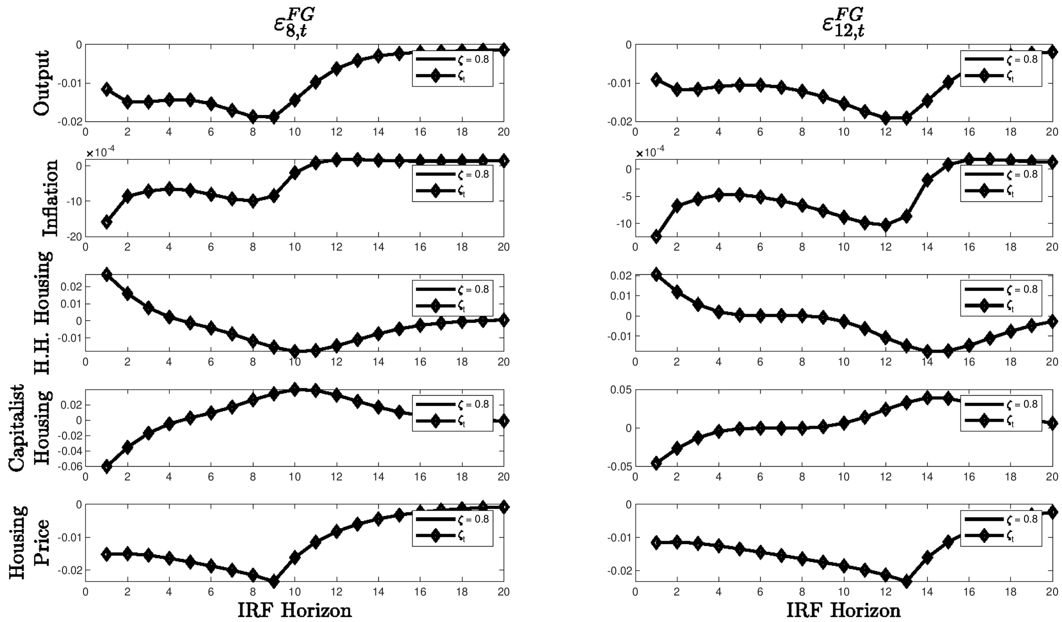

Figure 14: Impulse Responses of Variables to Eight and Twelve Periods Ahead Forward Guidance Shocks Under Benchmark and Time-Varying Loan-to-Value Ratios

Note: Solid Line: $\zeta = 0.8$ (Benchmark Case); Dashed Line: ζ_t (Time-Varying Loan-to-Value Ratio)

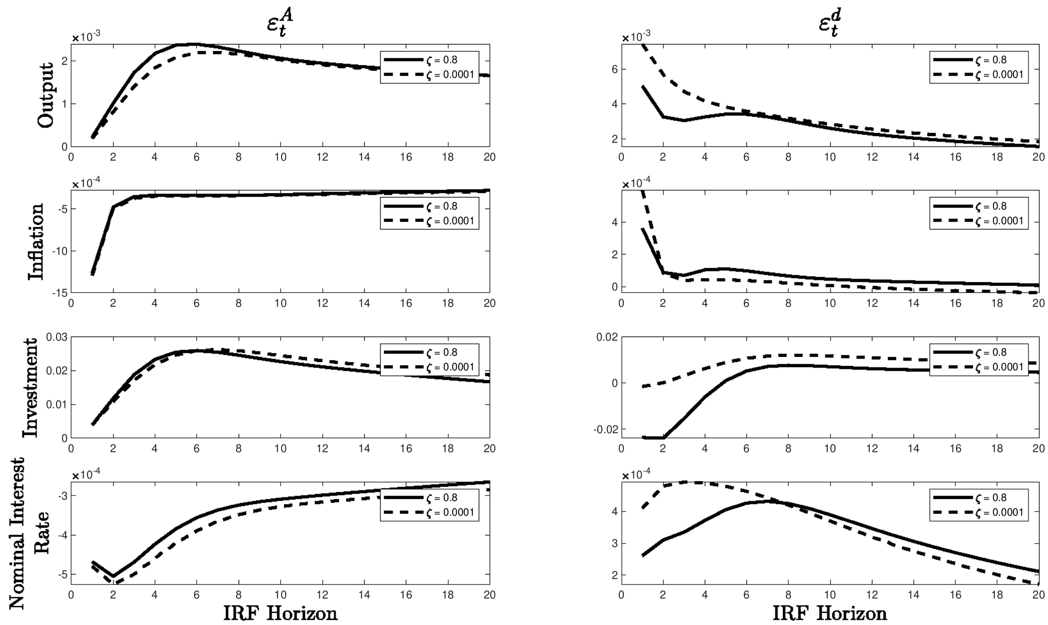

Figure 15: Impulse Responses of Macroeconomic Variables to Total Factor Productivity and Demand Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

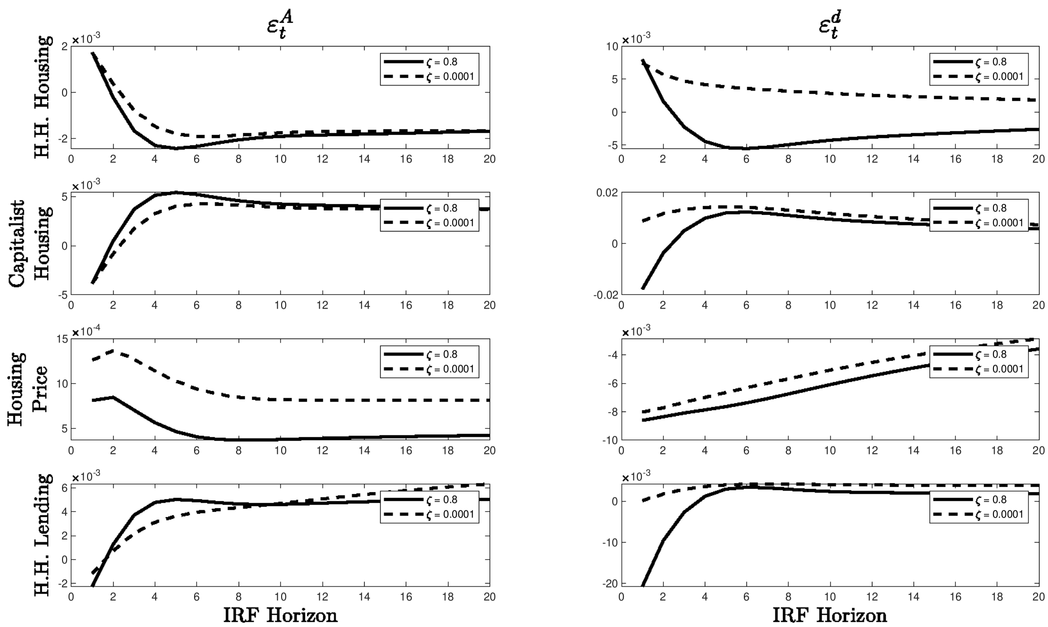

Figure 16: Impulse Responses of Housing Variables to Total Factor Productivity and Demand Shocks

Note: Solid Line: $\zeta = 0.8$ (Decreased Housing Financial Frictions); Dashed Line: $\zeta = 0.0001$ (Increased Housing Financial Frictions)

C Log-linearization

We use the hat notation to denote percentage deviations; $\hat{x}_t \equiv \frac{x_t - x}{x}$ where x denotes nonstochastic steady state value of variable x_t. Please note that interest rates are already percentages so we leave it as in absolute deviations. Lastly, we assume that $\frac{1}{r} \approx 1$.

$$\hat{\lambda}_t = \frac{-1}{(1-b)(1-b\beta)} \left[(\hat{C}_{h,t} - b\hat{C}_{h,t-1}) - b\beta(E_t\hat{C}_{h,t+1} - b\hat{C}_{h,t}) \right] \tag{45}$$

$$\hat{\lambda}_t = E_t\hat{\lambda}_{t+1} + \hat{r}_t \tag{46}$$

$$\hat{r}_t = \hat{R}_t - E_t\hat{\pi}_{t+1} \tag{47}$$

$$\hat{\lambda}_t + \hat{w}_t = \chi\hat{L}_t \tag{48}$$

$$\hat{\lambda}_t + \hat{q}_{h,t} = \frac{\beta\lambda q_h}{\beta\lambda q_h + \frac{\vartheta}{H_h}} E_t\left(\hat{\lambda}_{t+1} + \hat{q}_{h,t+1}\right) - \frac{\frac{\vartheta}{H_h}}{\beta\lambda q_h + \frac{\vartheta}{H_h}}\hat{H}_{h,t} \tag{49}$$

$$C_c\hat{C}_{c,t} + H_c q_h(\hat{H}_{c,t} - \hat{H}_{c,t-1}) + \delta K\hat{I}_t + rb\left[\hat{r}_{t-1} + \hat{b}_{t-1}\right] = b\hat{b}_t + R_k K\left(\hat{R}_{k,t} + \hat{K}_{t-1}\right) + R_h H_c\left(\hat{R}_{h,t} + \hat{H}_{c,t-1}\right) \tag{50}$$

$$\hat{b}_t = \frac{\omega_1 q_h H_c}{\omega_1 q_h H_c + \omega_2 K}\left(E_t\hat{q}_{h,t+1} + \hat{H}_{c,t} - \hat{r}_t\right) + \frac{\omega_2 K}{\omega_1 q_h H_c + \omega_2 K}\left(E_t\hat{q}_{k,t+1} + \hat{K}_t - \hat{r}_t\right) \tag{51}$$

$$\hat{\lambda}_t^c = \frac{-1}{(1-b_c)(1-b_c\beta_c)}\left[(\hat{C}_{c,t} - b_c\hat{C}_{c,t-1}) - b_c\beta_c(E_t\hat{C}_{c,t+1} - b_c\hat{C}_{c,t})\right] \tag{52}$$

$$\hat{\lambda}_t^c = \frac{\beta_c\lambda^c}{\beta_c\lambda^c + \nu}\left(E_t\hat{\lambda}_{t+1}^c + \hat{r}_t\right) + \frac{\nu}{\beta_c\lambda^c + \nu}\hat{\nu}_t \tag{53}$$

$$\hat{\lambda}_t^c + \hat{q}_{h,t} = \frac{1}{\iota_1}E_t\left[\beta_c\lambda^c R_h\left(\hat{\lambda}_{t+1}^c + \hat{R}_{h,t+1}\right) + \beta_c\lambda^c q_h\left(\hat{\lambda}_{t+1}^c + \hat{q}_{h,t+1}\right) + \nu\zeta\omega_1 q_h\left(\hat{\nu}_t + \hat{q}_{h,t+1} - \hat{r}_t\right)\right] \tag{54}$$

where $\iota_1 = \beta_c\lambda^c\left(R_h + q_h\right) + \nu\zeta\omega_1 q_h$.

$$\hat{\mu}_t = \frac{1}{\iota_2}E_t\left[\beta_c\lambda^c R_k\left(\hat{\lambda}_{t+1}^c + \hat{R}_{k,t+1}\right) + \beta_c(1-\delta)\hat{\mu}_{t+1} + \nu\zeta\omega_2\left(\hat{\nu}_t + \hat{q}_{k,t+1} - \hat{r}_t\right)\right] \tag{55}$$

where $\iota_2 = \beta_c\left(\lambda^c R_k + \mu\left(1-\delta\right)\right) + \nu\zeta\omega_2$.

$$\hat{\lambda}_t^c = \hat{\mu}_t - \Omega\left(\left(\hat{I}_t - \hat{I}_{t-1}\right) - \beta_c\left(E_t\hat{I}_{t+1} - \hat{I}_t\right)\right) \tag{56}$$

$$\hat{K}_t = (1-\delta)\hat{K}_{t-1} + \delta\hat{I}_t \tag{57}$$

$$\hat{q}_{k,t} = \hat{\mu}_t - \hat{\lambda}_t^c \tag{58}$$

$$\hat{p}_t^* = \hat{z}_t^n - \hat{z}_t^d \tag{59}$$

$$\hat{z}_t^n + \hat{\lambda}_t^c = \frac{1}{\vartheta_1}\left[\lambda^c Y\left(\hat{\lambda}_t^c + \hat{MC}_t + \hat{Y}_t\right) + \xi\beta_c\lambda^c\pi^{\frac{1+\theta}{\theta}}z^n E_t\left(\hat{\lambda}_{t+1}^c + \frac{1+\theta}{\theta}\hat{\pi}_{t+1} + \hat{z}_{t+1}^n\right)\right] \tag{60}$$

where $\vartheta_1 = \lambda^c Y + \xi\beta_c\lambda^c\pi^{\frac{1+\theta}{\theta}} z^n$.

$$\hat{z}_t^d + \hat{\lambda}_t^c = \frac{1}{\vartheta_2}\left[\lambda^c Y\left(\hat{\lambda}_t^c + \hat{Y}_t\right) + \xi\beta_c\lambda^c\pi^{\frac{1}{\theta}} z^d E_t\left(\hat{\lambda}_{t+1}^c + \frac{1}{\theta}\hat{\pi}_{t+1} + \hat{z}_{t+1}^d\right)\right] \tag{61}$$

$\vartheta_2 = \lambda^c Y + \xi\beta_c\lambda^c\pi^{\frac{1}{\theta}} z^d$.

$$\hat{\pi}_t = \frac{(1-\xi)\pi^{-\frac{1}{\theta}}}{(1-\xi)\pi^{-\frac{1}{\theta}} + \xi}\left(\hat{p}_t^* + \hat{\pi}_t\right) \tag{62}$$

$$\hat{R}_{k,t} = \hat{MC}_t + \hat{A}_t + \phi(1-\eta)\hat{H}_{c,t-1} - (\phi(1-\eta)+\eta)\,\hat{K}_{t-1} + \eta\hat{L}_t \tag{63}$$

$$\hat{R}_{h,t} = \hat{MC}_t + \hat{A}_t + (1-\phi)(1-\eta)\hat{K}_{t-1} - (1-\phi(1-\eta))\,\hat{H}_{c,t-1} + \eta\hat{L}_t \tag{64}$$

$$\hat{w}_t = \hat{MC}_t + \hat{A}_t + \phi(1-\eta)\hat{H}_{c,t-1} + (1-\phi)(1-\eta)\hat{K}_{t-1} - (1-\eta)\hat{L}_t \tag{65}$$

$$\hat{Y}_t = -\hat{\triangle}_t + \hat{A}_t + \phi(1-\eta)\hat{H}_{c,t-1} + (1-\phi)(1-\eta)\hat{K}_{t-1} + \eta\hat{L}_t \tag{66}$$

$$\hat{A}_t = \rho_A\hat{A}_t + \varepsilon_t^A \tag{67}$$

$$\hat{\triangle}_t = -\left(\frac{1+\theta}{\theta}\right)(1-\xi)\,(\hat{p_t^*}) + \xi\hat{\triangle}_{t-1} \tag{68}$$

$$\hat{R}_t = \rho_i\hat{R}_{t-1} + (1-\rho_i)\left(\phi_\pi\hat{\pi}_t + \phi_y\hat{Y}_t\right) + \hat{\varepsilon}_t^{MP} + \sum_{l=1}^{L}\hat{\varepsilon}_{l,t-l}^{FG} \tag{69}$$

$$\hat{Y}_t = \frac{C_h}{Y}\hat{C}_{h,t} + \frac{C_c}{Y}\hat{C}_{c,t} + \frac{I}{Y}\hat{I}_t + \varepsilon_t^d \tag{70}$$

$$0 = \frac{H_h}{\bar{H}}\hat{H}_{h,t} + \frac{H_c}{\bar{H}}\hat{H}_{c,t} \tag{71}$$

$$\hat{V}_t^h = \frac{1}{\omega_h}\left[\frac{1}{1-b}\left(\hat{C}_{h,t} - b\hat{C}_{h,t-1}\right) + \vartheta\hat{H}_{h,t} - \chi_0 L^{1+\chi}\hat{L}_t + \beta V^h E_t\hat{V}_{t+1}^h\right] \tag{72}$$

where $\omega_h = \log(1-b) + \log(C_h) + \vartheta\log H_h - \chi_0\frac{L^{1+\chi}}{1+\chi} + \beta V^h$

$$\hat{V}_t^c = \frac{1}{\omega_c}\left[\frac{1}{1-b_c}\left(\hat{C}_{c,t} - b_c\hat{C}_{c,t-1}\right) + \beta_c V^c E_t\hat{V}_{t+1}^c\right] \tag{73}$$

where $\omega_c = \log(1-b_c) + \log(C_c) + \beta_c V^c$. The system is also augmented with equations (33) - (35).