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Keynes's *Treatise on Probability* 100 Years Later: Small vs. large worlds and closed vs. open systems

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Keynes's *Treatise on Probability* 100 Years Later: Small vs. large worlds and closed vs. open systems

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Abstract: The meaning and significance of Keynes's *Treatise on Probability* has changed over the 100 years since its publication. Initially it stood on its own as an original contribution to probability theory. After *The General Theory* some saw the *Treatise* strengthening Keynes's later arguments. Yet by the time New Classical Economics became dominant it became largely ignored. This paper attributes that later rejection to the mainstream economics' reliance on Savage's subjective expected utility restriction of probability thinking to what he called small worlds. It argues that his small worlds-large worlds distinction produces a small worlds-closed worlds conception of economics the mainstream employs, and that Keynesian economic thinking and the *Treatise* employ a large worlds-open worlds conception of economics. It frames this open-closed opposition in terms of two contrasting conceptions of science from 1930s system theory, and argues that in economics it is the basis for two conceptions of time: a static, before-after view of temporal sequences and a dynamic, past-present-future view of temporal sequences. The paper then shifts to how Sraffa explained the relationship between production and distribution as an interaction between a relatively closed production system open to distributional forces, shows an analogous view exists in the later thinking of Wittgenstein with whom Sraffa interacted, and then argues Keynes's thinking in the *Treatise* employs a similar Cambridge understanding whereby our probability judgments are relatively closed but also open to fundamental uncertainty.

Keywords: Keynes, *Treatise on Probability*, Savage, small worlds, closed worlds, open systems, closed systems, Sraffa, Wittgenstein

JEL codes: B29, B30, C10, C11

1. The meaning and significance of Keynes's Treatise 100 years since publication

When we judge the meaning and significance of important works in the history of economics and philosophy, on the one hand we should proceed in an historical way since all important works in that history have changed their meaning and significance over time. Thus I begin in section 2 with a brief history of the changing views of the meaning and significance of J. M. Keynes's *Treatise on Probability* since its publication. On the other hand, we should also judge the meaning and significance of important works in the history of economics from our present vantage point since that allows us to see economics as a whole as the product of its history, and the judgments we are thereby able to make about it on this basis give us another view of important works in its history. Thus, I go on to argue that although Keynes's *Treatise* has a relatively unappreciated meaning and significance today that reflects the dominance of Bayesianism in current mainstream economics – a state of affairs that depends in part on neglecting the probability thinking of the *Treatise* – there are reasons to think this neglect may not persist in the future.

Section 3 discusses how the kind of world which mainstream thinking in the postwar has assumed we occupy is the small worlds conception adopted by L. J. Savage who developed the approach to probability that underlies mainstream economics. Savage distinguished small worlds and large worlds, where the latter better describes the kind of world we occupy in both the *Treatise* and *General Theory*. If we suppose, then, that we actually occupy large worlds, the probability thinking mainstream economics employs is inadequate to the task of explaining the behavior of economic agents, while Keynes's *Treatise* probability thinking provides foundations that can assist us in that task. Were economics to adopt a large worlds point of view, the *Treatise* would acquire new meaning and significance.

To explain the difference between small and large worlds, section 4 argues that representations of small worlds are closed world conceptions and representations of large worlds are open world conceptions. A distinction between open and closed kinds of science was developed in the 1930s by Ludwig Bertalanffy who characterized physics as a closed science and sciences such as biology that investigate living systems as open sciences. Since then the open-closed distinction has been employed by many others in both economics and philosophy of science.

Postwar mainstream economics, as it developed as a rational expectations alternative to Keynes's thinking, can then be seen to be constructed upon a closed, small worlds conception of economics in virtue of its reliance, like physics, on highly determinate models in which the economy is always in equilibrium. A closed, small world in this

sense is one which the world is highly predictable and stable. An open, large world by comparison is one in which the pathways economies follow are often unpredictable, subject to unexpected adjustments and redirections, and may often not be in equilibrium. That latter sort of world, is for Keynes one in which economic policy is needed to counter economic downturns with high human cost. In closed, small worlds, no such need exists, and indeed according to the Lucas critique is not even feasible.

Drawing on the philosophy of time, section 4 then identifies a key difference between these two views as follows. The former closed-small worlds conception employs a static, before-after view of temporal sequences while the latter open-large worlds conception employs a dynamic, past-present-future view of them. For the former, there is no history; we simply have a succession of equilibrium states purportedly 'linked' by exogenous shocks. Since shocks are by definition unexplainable, they are not subject to investigation. For the latter, what happens at one time endogenously determines what happens later. Thus, policy interventions make it possible to change how the economy moves from one state to another, as whether it moves to a new equilibrium state.

Section 5 goes beyond a basic opposition between open and closed science conceptions to describe a more complex relation between open and closed systems, namely, a world in which some systems are relatively closed and some largely open. This view was articulated by Piero Sraffa in the early 1930s to explain the interaction between production treated as a relatively closed system and distribution treated as a largely open system. At Cambridge, a similar view of how a relatively closed system interacts with a largely open system was employed by Ludwig Wittgenstein in his famous critique of the idea of a private language and his language-game analysis.

Section 7 returns to the *Treatise*, and argues that Keynes thinking can be interpreted as a contribution to open-closed system theorizing – a Cambridge tradition in methodological thinking manifested in Sraffa and Wittgenstein's thinking to which Keynes was an original contributor. Probability judgments are a relatively closed system of reasoning when decision-makers rely on generally accepted numerical probabilities, but in circumstances where they cannot, Keynes argued, those types of judgments interact with an open system of reasoning and fundamental uncertainty comes into play. That is, Keynes reasoned in terms of multiple domains of thinking about probability judgments, or different systems of thinking, which for decision-makers included "pretty, polite techniques which [attempt] to deal with the present by abstracting from the fact that we know very little about the future" (Keynes 1973c, p. 115) and a variety of more 'impolite techniques of decision-making' that we often need to fall back upon (see Zappia 2015, 2016).

2. The changing meaning and significance of Keynes's Treatise

How has the meaning and significance of Keynes's *Treatise on Probability* ([1921] 1973b) changed over the years since its publication? Prior to Keynes devoting himself to economics, its meaning and significance was restricted to the importance ascribed to it by a relatively small, heterogeneous community of researchers investigating probability theory. Much of our current thinking about probability came after Keynes's *Treatise*, so how the book was judged at the time by a few commentators, among whom there was limited consensus regarding its contribution, has been largely forgotten. Keynes's view – his logical theory of probability – was novel and its philosophical motivations, rooted in Cambridge philosophy of G.E. Moore and Bertrand Russell (Davis 1994), made it difficult for many to judge. Not a few commentators dismissed it after Frank Ramsey criticized it, and Keynes appeared to make concessions to him – though how serious they were is subject to debate.

After Keynes turned to economics he gave little attention to probability theory. At the time probability was also yet to play an important role in economic theory, so the *Treatise* seemed to bear on little. Attention to it consequently remained confined to a still comparatively obscure domain of statistical and quantitative thinking, and even there Keynes's view did not have great influence. This apparent unimportance was reinforced when fifteen years after publication of the *Treatise* the *General Theory* ([1936] 1973b) appeared and made little use of it. It was as if Keynes himself believed it had little to offer in the more practical world that had preoccupied him since turning to economics.

This possible assessment was sustained for the next half century before there was any substantial interest in and re-appraisal of the meaning and significance of the *Treatise*. While Keynes's concept of uncertainty in *The General Theory* for many seemed to have had antecedents in the *Treatise*, how and in what ways had not be systematically investigated. In the 1980s and 1990s, however, a whole new literature on 'Keynes and philosophy' developed which actively debated the relationship between the *Treatise* and *The General Theory*.[1] Though there was disagreement over the nature of connection between the two books – the continuity vs. discontinuity debate – the shared judgement was that the *Treatise* contributed important philosophical foundations for *The General Theory*.

[1] See Lawson, 1985, 1988; Carabelli, 1988; O'Donnell, 1989; Bateman and Davis, 1991; Davis, 1994; Mizuhara and Runde, 2003; Bateman, 1996.

Yet this literature ultimately ceased to be an active area of investigation. Some of the people involved in it turned to other subjects, and not much of what this research produced had an influence on Keynesian economists or on economics. Indeed, other conceptual foundations for Keynes's *General Theory* concept of uncertainty were investigated – nonergodicity (see Davidson 1995) and hysteresis (see Cross 1993) – which did not depend on Keynes's early thinking about probability. Thus, by the new Millennium, the meaning and significance of the *Treatise* again ceased to be of much concern, not only in Keynesian economics, but also in 'Keynes and philosophy' research.

That this came about should not be surprising. The debate over the relationship between the two books was internal to Keynes scholarship and a highly specialized topic within it, and was consequently well removed from the general concerns of practitioner economists. It can also be argued that the 'Keynes and philosophy' research focus was too narrow for those who were interested in the philosophical foundations of post-Keynesian thinking. Post-Keynesianism is not just post-Keynes but also about other currents in Cambridge, especially Kalecki, Robinson, Sraffa, and others. Thus, to the extent that philosophical thinking was regarded as important to post-Keynesian economists, it concerned a wider collection of ideas than the 'Keynes and philosophy' literature involved.

Yet there was an important countervailing force operating in postwar economics potentially working against this loss of interest in the *Treatise*: the rise of expectational analysis in economics and the attendant revision of explanations of behavior it entailed. However, while the subject of the *Treatise*, probability, became central to the development of economics, the book itself continued to be ignored by most economists. Why?

3. The small worlds exclusion of the Treatise from mainstream economics

When expectations and probability theory became central to postwar economic thinking, it did so in the form of subjective expected utility theory as principally developed by Savage (1954). Following Ramsey's (1926) and de Finetti's (1937) approaches to probability, Savage's theory, or Bayesian decision theory as it is now generally termed, combines decision-makers' individual utility functions, or personal preferences as structured by a set of consistent axioms people's preferences to are said to obey, and the probabilities they subjectively place on possible future events to produce numerical expected utility values for their possible choices (see Karni, 2008).

While it is not often emphasized – though see Binmore (2007) – Savage recognized that his theory only holds in what he called small worlds in which decision-makers could reasonably assign subjective probabilities to future events. What he distinguished as large worlds were effectively the sorts of circumstances that Keynes (and also Frank Knight) had argued generated fundamental uncertainty or somehow failed to allow for assignment of subjective probabilities. Keynes did not characterize the world to which his thinking applied as a special type of world nor did he make use of a distinction like the one Savage later advanced. There was simply one world in which people reasoned probabilistically. He believed there were circumstances in which decision-makers could reliably assign numerical probabilities to future events. However, he also believed there were circumstances in which they either could not assign numerical probabilities or indeed even assign any probabilities to possible future events, as he later emphasized in his retrospective review of his *General Theory* ([1937] 1973c).

Savage recognized there were circumstances in which his theory did not apply, but was not very clear about what non-small worlds involved nor about when we might be in them. The closest he came to saying anything about this was in his informal discussion of two proverbs he suggested justified re-interpreting non-small/large worlds as small worlds: 'look before you leap' and 'you can cross that bridge when you get to it.' Essentially the thrust of these proverbs was that, were decision-makers to encounter circumstances which were not immediately like small worlds, thus large worlds, they could reconstruct them as small ones. After all, assigning probabilities to future events was a subjective matter, and it was thus up to the decision-maker to determine how the circumstances encountered should be valued. Thus for Savage, large worlds *per se* really do not exist and his theory was in his view and for most Bayesians today comprehensive of all circumstances in which decision-makers made probability judgments.

If we focus, then, on where specifically Keynes and Savage disagreed about the scope of probability judgments, two differences are fundamental. First, for Keynes probability concerned how people rationally understand the world, not just personally appraise it, and thus probability relations are seen by him to be objective, not subjective. This implies that Savage's reliance on subjectivity to justify treating his large worlds as small ones would have been questionable for Keynes. For him, there could always be circumstances in which people simply could not form probability judgments because how people rationally understand the world is always limited. Second, Keynes denied – when we can form probability judgments – that all probability relations must be susceptible of numerical representation. Some are qualitative in nature – we think something is more probable than something else but cannot say in what degree – and yet are still rational. This is especially important, we will see below, because it was

inconsistent with how mainstream economics later used Savage to represent economies as fully quantifiable systems.

The *Treatise*, then, remained on the periphery of economics, at least for the first half century of the postwar period. Economics was dominated by a vision of the economy as occupied solely by subjective, utility-maximizing agents whose market relationships comprehensively explained the economy. The apex of this vision was refinement of the general equilibrium model of the economy in rational expectations theory, the culmination of which was the famous Lucas critique that argued policy interventions in the economy aimed at altering how the economy functioned were ineffective because economic agents' rational expectations would anticipate and offset them. On this view, the ultimate and only drivers of how the economy worked were the so-called 'deep parameters' that acted on markets and lay outside them (preferences, technology, and resource constraints). Since these factors are beyond human control, it followed that the economy operated as a self-contained system which, expressed quantitatively in market price terms, rested on an inaccessible underlying reality which was its essential foundation. An important implication of this was that markets were essentially self-regulating so that the economy must always be in or inevitably tending to a settled state of equilibrium.

However, the two key differences between Keynes's *Treatise* and Savage's or Bayesian theory give us two reasons to believe this postwar model of the economy is mistaken. When probability judgments are rational in the sense of being judgments people should logically make given the state of knowledge about the world, they remove agents from their private subjective spaces and place them in an intersubjective world in which what counts as likely has arbiters beyond personal opinion and the Lucas critique breaks down.[2] When probability judgments are non-numerical – particularly in conditions where they concern decision-making over far in the future types of events – the high level of determinacy rational expectations models assume no longer operates, and the economy may accordingly fail to be in a state of equilibrium.

Thus, the postwar rise of expectational analysis in economics, though it might have rehabilitated interest in Keynes's thinking about probability in the *Treatise*, instead required it be excluded from attention because the revision of explanations of decision-maker behavior that analysis would entail were not in keeping with the mainstream vision of the economy as a self-contained, closed system that operated independently of human intervention. Keynes of course was always an activist with respect to policy.

[2] See Gillies and Ietto-Gillies (1991) for the argument that Keynes's understanding of probability has an intersubjective basis.

He believed that the economy could be made to work better in many ways, most notably in using demand management to address unemployment and the human suffering it produced. The spirit of his thinking was dominant in postwar Keynesian economic policy until the resurgence of what he would have called, and what rational expectations theorists indeed call, 'classical' economics.

This change in attitudes regarding the scope and nature of economics was in part the product of an increasingly widespread belief in postwar advanced economy societies that markets worked well when left to themselves. Yet there was also a change within economics regarding what sort of science economics should be thought to be that played a role in this shift. Broadly speaking, as many have noted, it turned on the notion that economics was intrinsically a quantitative science, meaning by this that decision-maker behavior and market relationships could be fully represented in mathematical terms. This change, as it materialized especially in rational expectations theory, depended on Savage's small worlds view of the world. In the following section, then, in order to link that view with the idea of economics as a quantitative science, I reframe the small worlds-large worlds distinction as one between closed and open worlds conceptions of science, and use this contrast to contrast two different conceptions of the scope and nature of economics, particularly as concerns their respective treatments of time.

4. Open-system vs. closed-system conceptions of economics – and the role of time

There are many ways to conceptualize the differences between open and closed system conceptions of science and in economics.[3] The distinction was first influentially employed by Ludwig von Bertalanffy (1968 [1934]), an early general systems theory thinker of the interwar period, who described different types of sciences according to whether they were characterizable as closed or open. He associated the former paradigmatically with physics, and argued that the physical sciences are closed because their principles work 'in isolation' from their particular environments, or alternatively they work the same way in all environments. In contrast, living systems, especially those seen as undergoing growth such as studied in biology, are said to be open because they need to be explained in terms of their interaction with their environments which influence their functioning.

If we take, then, as our point of entry the role different interpretations of probability play in economics, it makes sense to interpret the open and closed system distinction in

[3] In economics, see in particular Boulding (1956) and Chick and Dow (2005).

terms Savage's small worlds-large worlds distinction. Indeed, when we focus on how Savage's theory was incorporated into mainstream economics, we can see that the small worlds idea plays an important role in advancing a science conception of economics very much like Bertalanffy's closed worlds type of science. Economics is a closed science because its small worlds basis ensures that it principles work the same way in all environments. In contrast, for Keynes economics is like Bertalanffy's open worlds type of science because how economies develop depends on the environments they occupy. Keynes's probability thinking in the *Treatise* captures this with its large worlds conception of the different kinds of decision-making people encounter.

Small worlds in Savage's subjective expected utility, where Bayesian theory applies, allow us to develop an economic theory that emphasizes the predictability and stability of the economy. Since small worlds are closed worlds, they exclude the possibility of highly unpredictable developments that create disequilibrium states of affairs. In contrast, since Keynes's logical approach to probability allows for unpredictable developments that create disequilibrium states of affairs. That is, the basic difference between these two conceptions of economic science concerns the degree of determinacy economic representations of the world create in the assumptions they make about economic behavior and agent interaction where this especially concerns the weight placed on the equilibrium concept.

To deepen this difference, I argue that at the root of this difference between the small, closed worlds conception of economics and the large, open worlds conception of it lies two opposed ways they represent time. In the philosophical theory of time since Aristotle (1984) and the ancient Greek philosophers there have been two main kinds of temporal sequences of events used to explain time: before-after sequences and past-present-future sequences (see Emery, Markosian, and Sullivan 2020). The before-after sequence ignores the passage of time or the flow of time idea, because it is basically a comparative conception or more of a logical ordering idea. In contrast, the past-present-future sequence emphasizes the passage or flow of time, because those periods are connected in virtue of their related meanings.

I argue, then, that the before-after temporal sequence is characteristic of mainstream economics in that its small worlds-closed worlds, Bayesian reasoning lends itself to a static view of the economy that places great weight on economies being in equilibrium. Economies are either always close to and tending toward or (in rational expectations theory) always in equilibrium. Thus, all we can do is compare in a before-after way how economies change, and there is no way to observe the passage of time. Further, what moves an economy from one equilibrium state to another are exogenous shocks arising outside the economy (at the level of underlying parameters). Since these shocks

are not part of the economic process, the economy can only be represented as a sequence of equilibrium states proceeding in a before-after manner. This is less a conception of time than of an out-of-time relational ordering. As Bertalanffy said of physics, since for mainstream economics economies are self-contained processes whose deep parameters stand outside how they work, they all work in basically the same way and in all economies no matter what the time or place.[4]

In contrast, the past-present-future temporal sequence is characteristic of Keynesian (and much heterodox) economics, because its large worlds-open worlds reasoning supports a dynamic view of the economy as a process driven by endogenous factors. What moves an economy from one state to another are what has happened in the past, how that changes the present for decision-makers today, and how this influences how the future will develop tomorrow. Time is a process whose passage is central to our understanding of the economy. As in Bertalanffy's thinking, the economy is more like a living system that undergoes growth and change, and in which its being out of equilibrium or in disequilibrium endogenously sets the stage for what will subsequently occur. The nature of the environment at hand is accordingly fundamental to how economies change. If mostly math matters for mainstream economics, for Keynesian (and heterodox) economics mostly history matters.

5. Sraffa on commodity values as a relatively closed but open system

Bertalanffy's systems approach broadly distinguishes kinds of sciences according to whether they are either open or closed. Yet in a more fine-grained analysis we can also distinguish them and systems more generally according to how closed or how open they are. Then, if closed systems are not entirely closed, but mostly so, and open systems are not entirely open, but still largely so, we may investigate how relatively closed systems interact with largely open ones. Thus in science, the principles of physics also operate within biological systems, which are nonetheless still governed principally by their interaction with their environments, but are also affected in their functioning by physical principles. Just, then, as these the two types of science systems interact, so it can be argued that not fully closed and not fully open types of systems interact in economic life. This type of relationship is especially pertinent to economics which treats prices in a relatively systematic way and allows they are acted upon by a variety of non-price economic and social factors.

[4] Aristotle argued in his famous sea battle tomorrow thought experiment that strict adherence to the before-after temporal sequence committed one to fatalism as well (see Gale, 1967, 9-23; Davis, 2021).

This more complex kind of systems thinking can be developed in different ways, but one way it can be argued it was developed in Cambridge in the interwar period was in how Piero Sraffa treated the system of production determining commodity values as closed but at the same time also open, thus as only a relatively closed (Davis 2012, 2018). Sraffa recognized that one needed to know what wages or profits are to fully determine commodity values. In Classical economics, commodity values were understood in cost of production terms, but wages and profits were understood to result from the struggle over distribution. Thus, a relatively open system acted upon and completed the determination of a relatively closed system.

In his unpublished 1931 'Surplus Product' paper, Sraffa saw that if the system of production is fully complete and closed in itself in the sense that in cost of production terms commodity values fully determine commodity values – what he termed an objectivist or natural science point of view – economies could not be seen to generate a surplus, which he believed was characteristic of commodity producing economies.

> If one attempts to take an entirely objectivist point of view, the very conception of a surplus melts away. For if we take this natural science point of view, we must start by assuming that for every effect there must be a sufficient cause, that the causes are identical with their effects, and that there can be nothing in the effect which was not in the causes; in our case, there can be no product for which there has not been an equivalent cost, and all costs (=expenses) must be necessary to produce it (Sraffa 1931, D3/12/7: 161; also quoted in Kurz and Salvadori 2008, 268).

Sraffa's reaction to this problem is interesting. In rejecting Marshallian marginalism, he was committed to a Classical economics cost of production explanation of commodity values. He had at first assumed on objectivist grounds that the production of commodities as a natural system constituted a closed system that it was somehow acted upon, as a whole, by factors external to it. In this respect his view was not unlike rational expectations theory's idea that deep parameters affect the price system from outside. Yet Sraffa, who took history seriously, assumed that distributional struggles outside that 'closed system,' or the 'economic field' as he referred to it, had to be incorporated into the determination of commodity values. Commodity values could not be independent of the relationship between profits and wages. He expressed this metaphorically in the following way:

> There must be a leak at one end or the other: the 'closed system' is in communication with the world.

He then went on to say,

> When we have defined our 'economic field', there are still outside causes which operate in it; and its effects go beyond the boundary (D3/12/7: 161 (3-5); quoted in Kurz and Salvadori, *Ibid.*).

What is particularly interesting here from a systems perspective is that in his characterization of how the 'closed system' is in communication with the world he chose to say that the "outside causes" associated with distribution "operate *in*" (my emphasis) the 'economic field' and not simply upon it from outside that field. The latter view is the rational expectation theory deep parameters view, but here Sraffa rejects it. His position, then, is that the economic field was both closed and yet somehow at the same time open system affected internally by what lies "outside" it.

The refinement Sraffa consequently makes on Bertalanffy's systems view starts with a critical evaluation of closed science conceptions, arguing that in the case of economics they need to also be seen as only relatively closed based on their relationship to what lies outside them. If in a fully closed system all causal relationships are internal to it, in an only relatively closed system those causal relationships interact with other factors not internal to it, which in coming from outside the system, involve a different kind of influence and have a different causal nature that somehow determine changes in the system's internal causal relationships.

In this interactionist systems view, two types of causal systems affect and reinforce one another. Distributional struggles for Sraffa were historical and open in nature, but also affected by what commodity values prevailed. Thus they constituted a largely but not fully open system. At the same time, endogenizing distributional values to the relatively but not fully closed system of commodity values explained them over time as an historical outcome. In this respect, Sraffa's systems thinking shared some of the features of Bertalanffy's thinking, but in focusing on the interaction of different kinds of systems went beyond it.

Returning to the two different representations of time, while we can apply the before-after temporal sequence to Sraffa's analysis, because he still sees commodity values as

susceptible to equilibrium analysis, yet the state of distributional struggles at any one time makes the past-present-future temporal sequence important and show how history also matters. We can make relatively determinate before-after equilibrium comparisons of prices and values in economies, but these comparisons need to be framed in passage of time past-present-future terms. Sraffa achieves this type of explanation through his particular use of the open-closed systems method.

6. Wittgenstein on language-games as a relatively closed but open system

The change in Wittgenstein's philosophical thinking from his early to later views is associated with his changed view of the nature of language. A key step in his rejecting his early views in favor of those advanced in his *Philosophical Investigations* (1953) was his critique of the idea of a private language (see Hintikka and Hintikka 1986, 243), which he came to believe had been implicit in his early thinking. A private language is one in which a person has an entirely independent, individual association of meanings with words within some fully private, inner mental state. Wittgenstein decided, however, that this idea was incoherent because he believed language meanings are inherently shared and social. To capture an alternative understanding of language meanings he needed to explain their nature, and one important step in doing so was to treat language meanings as if they were governed by how language is used in what he called language-games, which like ordinary games were rule-governed. Rules, he then observed, combine both rule-following and rule-interpretation. The latter involved being able to judge how a rule applies in different, often new circumstances, and thus when it does applies and when it does not.

In this way, rule-followers are like decision-makers who need a means of deciding what they should do. To the extent that they understand a language rule, they establish its scope but also its limitations. Thus, like Sraffa's relatively closed system determining commodity values, language rules are a relatively closed system that guides people's choices – relatively closed because they are still subject to interpretation. Thus, like how Sraffa's system of production is only relatively closed because also open to the struggle over distribution, Wittgenstein's language-games are only relatively closed because also open to a less tightly structured open type of system in which people debate the nature of meanings built into rules. In effect, that larger, less structured debate over the nature of meanings operates outside yet also within the only relatively closed way that rule-based language games are played.

Wittgenstein gave special credit to Sraffa in the Preface to his *Philosophical Investigations*.[5] However, whether Sraffa influenced Wittgenstein or whether Wittgenstein influenced Sraffa, or whether they influenced one another can be debated, and more interesting is the general similarity between how they thought in a systems sort of way. This suggests that they shared kind thinking that existed in Cambridge in the 1930s. Of course, this was when Keynes, who knew them both, developed his economic thinking resulting in his *General Theory*. In the next section, then, I argue that a similar kind of reasoning was already present in his *Treatise*, and this suggests that the three of them shared a set of underlying philosophical views that were applied and expressed in their three different domains of investigation.

7. Keynes on probability judgments as a relatively closed system open to uncertainty

Some of interpreters of Keynes's *General Theory* argue that its reference to animal spirits and arguments about convention imply Keynes believed behavior in financial markets was driven by speculation and was irrational (e.g., Akerlof and Shiller 2009). This would suggest that he gave up his logical theory of probability in the *Treatise* altogether and replaced it with his concept of uncertainty. Thus, in terms of the small-closed worlds versus large-open worlds dichotomy this would mean that, given the central role investment plays for Keynes, economics should assume we only occupy large-open worlds and can ignore small-closed ones.

There are two problems with this view. First, Keynes never said he had given up his thinking in the *Treatise.* Even his concession to Ramsey's criticisms is measured and limited (see O'Donnell 2021). Second, in his own personal investment behavior Keynes made regular use of standard methods for assessing the values of stocks (see Marcuzzo 2019). He allowed speculation could play a role in determining stock values but still began with a traditional, expected value analysis of the underlying firm. Thus, I argue it is more reasonable to say that he treated probability theory as a relatively closed framework for examining what might occur but a framework that at the same time that needed to be seen as open to speculative forces and uncertainty where probability judgments were qualitative or not possible. That is, his view was not that we must either fully embrace a subjectivist conception of probability or see fundamental uncertainty as the only determinant of how economies function, but rather that we need

[5] After crediting Frank Ramsey, Wittgenstein wrote: "Even more than this – always certain and forcible – criticism I am indebted to that which a teacher of this university, Mr. P. Sraffa, for many years unceasingly practiced on my thoughts. I am indebted to *this* stimulus for the most consequential ideas of this book" (1953, x[e]).

to somehow find a reasonable means of combining the former with the latter, and additionally especially develop "a criterion for decision-making under uncertainty alternative to the maximisation of subjective expected utility" (Zappia 2016, 836-7).

This is not to say that Keynes reasoned about probability theory as a relatively closed but open system in precisely the same way as I argued above that Sraffa and Wittgenstein did. The subject at hand for Keynes was different, and the *Treatise* was written well before the idea of systems began to be used in philosophical thinking. What can be argued he shared with them was the idea that the world possesses distinguishable domains of activity which are different from each other in regard to the principles that explain activity within them, and how they can be understood, yet which also interact with each other. One domain – a relatively closed one in virtue of its systematic character and its degree of analytical development – was much of our well-worked out probability thinking. The other domain that interacted with it was our thinking about where probability judgments were difficult to make and there existed uncertainty about many possible events – a more unstructured yet distinguishable domain. For Sraffa, this latter, less tightly structured domain was the nature of distributional struggle; for Wittgenstein it was the continual evolution of language meanings; for Keynes it was the inevitability of our being uncertain about many things and the many ways in which people seek to deal with this.

For all three, the challenge was explain the interaction between relatively closed more tightly structured domains or systems and largely unstructured domains or systems – and explain the ways in which they were connected and how the latter modified and entered into the former. For Keynes, this was a matter of how one made probabilistic decisions when uncertainty interfered with doing so in a still reasonable if not fully logical way. Consider again his emphasis on non-numerical probabilities. They fall short by the standard of Bayesian thinking but that conception presupposes we do not encounter them and only live in small worlds. Thus, when we acknowledge that we also live in large worlds, Bayesians have nothing to offer, though non-numerical, qualitative probability judgments often provide us reliable means for making decisions. Indeed, frequently it is sufficient to simply know something is more or less probable to determine a course of action. Or consider Keynes's insistence that sometimes we have no basis at all for making probability judgments. This tells us that our decision-making needs to be built around different principles of reasoning, such as generalized caution and/or insuring ourselves against potentially costly consequences of our actions. This is

not quite a method of probability thinking, but it bears a close connection to it in regard to expectation formation.[6]

We might therefore argue that Keynes reasoned in terms of a multi-faceted agenda of types of prescribed behaviors whereby people continually assess how the specific circumstances they encounter require different strategies and approaches to decision-making. We cannot count on there being just one size-fits-all kind of way people form expectations and act upon them that suits all circumstances. Taking uncertainty thinking as a highly unstructured but distinguishable domain with which we must contend, we need to employ a variety of diverse, connected strategies to deal with how it affects our decisions. Our system of probabilistic decision-making is a powerful tool reflecting centuries of thinking about the nature of probability judgments, but it will always be an only relatively closed system.

In closing, I return to the issue of the meaning and significance of Keynes's *Treatise* 100 years since its publication. Its neglect by mainstream economists and their confidence in Bayesian thinking reflects their commitment to thinking the world is small and closed, and thus stable and predictable. That view increasingly seems to be a fantasy in a world of financial crises, pandemics, and environmental threat. Bayesianism is thus a questionable project that was perhaps born of a postwar desire after the horror of the World War to see the world as a peaceful and unthreatening place. If economics, then, were to begin to see the world we live in more as it is now, in particular as a large, open world in which economies are not simple self-governing market machines that can be disrupted by unexpected developments, then the value of Keynes's arguments in the *Treatise* may enjoy new respect and its meaning and significance over the next 100 years be much greater than now thought. If so, once again history will matter in economics.

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[6] See Carabelli's (2021) interpretation of the *Treatise*. She argues that of the different types of probability judgments, some can be dealt with in a formal way and others not. The former refer us to a more tightly structured domain of reasoning while the latter refer us to probability judgments that are more indeterminate and subject to uncertainty considerations.

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