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Measuring the Effects of Unconventional Monetary Policy Tools under Adaptive Learning

By

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Abstract

We compare the economic effects of forward guidance and quantitative easing utilizing the four-equation New Keynesian model of Sims, Wu, and Zhang (2023) with agents forming expectations via an adaptive learning rule. The results indicate forward guidance can have a greater influence on macroeconomic variables compared to quantitative easing, suggesting that forward guidance may have contributed to the high inflation rate after the COVID-19 related recession. Adaptive learning agents estimate a higher effect of forward guidance on the economy leading to a greater impact on expectations, and thus, contemporaneous inflation. However, the performance gap between forward guidance and quantitative easing can change. If quantitative easing includes anticipated shocks, more households finance consumption through long-term borrowing, and the central bank provides a greater percentage of liquidity in the long-term borrowing market, the performance of quantitative easing can increase, and at times, outperform forward guidance.

Keywords: Unconventional Monetary Policy, QE, LSAP, Forward Guidance, Adaptive Learning

JEL classification: E32, E52, E58, D83

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1 Introduction

In the late 1990s, two giants in macroeconomics presented different solutions while observing Japan’s zero lower-bound problems. Ben Bernanke, the former Chairman of the Federal Reserve, argued that central banks should ease credit markets by purchasing risky assets such as long-term bonds and mortgage-backed securities. At the same time, Michael Woodford proposed forward guidance, communicating the future path of monetary policy to influence people’s expectations. These two unconventional monetary policy tools were subsequently employed in response to the problems faced by the US economy during the global financial crisis in 2007-2009. Furthermore, approximately 15 years later, forward guidance and quantitative easing were concurrently implemented to combat the economic downturn caused by COVID-19.

The policy prescriptions of forward guidance and quantitative easing depend on features of the macroeconomic model chosen by the researcher. For example, standard models contain, if at all, either forward guidance or quantitative easing. However, it is important to include both policies to understand their interactions with the macroeconomic variables. Second, how agents are assumed to form expectations in the model can greatly affect policy prescriptions. Forward guidance deals with how agents’ expectations of output and inflation react to future policy promises, while guidance on quantitative easing can regard the duration of central bank long-term bond purchases. To ensure more accurate policy prescriptions, it is important to make a prudent decision on how agents construct their expectations. Thus, it is natural to explore how forward guidance and quantitative easing concurrently affect an economy when utilizing realistic expectations.

This study compares unconventional monetary policy tools within a unified structural model with agents forming expectations based on an adaptive learning rule. We build upon the four equation New Keynesian model in Sims, Wu, and Zhang (2023) by adding two additional features. First, we incorporate anticipated shocks to model both forward guidance and quantitative easing, allowing for a direct comparison of the effects of the two policies on the economy. Second, instead of assuming the standard rational expectations, we employ adaptive learning to obtain expectations in a more realistic manner. Other than these two, the model follows Sims, Wu, and Zhang (2023) which includes households divided into “parent” and “child.” The former is a standard household that can save between periods via one-period bonds. The child is characterized as impatient, and notably, funds consumption by issuing long-term bonds. Financial intermediaries are constrained
by balance sheet conditions, while the production sector operates with standard nominal price rigidities. The central bank operates under a balance sheet condition, buys bonds from the child household, and implements Taylor-type rules, considering both anticipated forward guidance shocks and unanticipated pure monetary shocks.

Agents are assumed to follow an adaptive learning rule when constructing forecasts à la Evans and Honkapohja (2001). Unlike the standard rational expectations assumption, adaptive learning agents do not have complete knowledge of the economy but rather act as real-life econometricians following the cognitive consistency principle of Evans and Honkapohja (2013). In particular, they form expectations by constructing a linear regression model, gathering data, estimating their model using OLS, and updating their forecasts each period as new data is available.

Our key findings are the following. Firstly, forward guidance has an overall more pronounced effect on inflation compared to quantitative easing. Agents are estimating a greater effect of forward guidance on output and inflation than quantitative easing leading to a higher effect on expectations and the macroeconomy. Moreover, we conjecture that the longer-lasting effect on inflation from forward guidance is (perhaps) a potential contributor to the high inflation rate after the COVID-19 recession.

We then proceed to investigate the factors that affect the performance gap between forward guidance and quantitative easing. First, if communication on the future path of quantitative easing is included, quantitative easing can perform better than before. Second, the larger the share of agents who finance their consumption via long-term bonds, the better the performance of quantitative easing. In addition, if the central bank makes up a greater share of the demand for long-term bonds relative to private financial intermediaries, quantitative easing can have a much bigger effect on macroeconomic variables than forward guidance. Finally, the degree that agents discount recent observations can affect the performance gap. If agents place less weight on recent observations, forward guidance and quantitative easing differences dissipate. If agents weigh recent observations more heavily, the results are more mixed with a shorter forward guidance horizon (slightly) widening the performance gap.

The organization of the paper is as follows. Section 1.1 provides literature review. Section 2 briefly describes the log-linearized optimal conditions of the model. Section 3 lays out our choice of parameter values. Section 4 presents the results. Section 5 concludes.
1.1 Prior Literature

While there have been numerous empirical studies analyzing the effects of these two unconventional monetary policy tools on financial markets, there are relatively few papers that specifically examine their concurrent effects on the entire macroeconomy.\textsuperscript{1} Most of the existing studies focus on either forward guidance or quantitative easing separately, rather than considering both simultaneously. For instance, Cole (2021) considers forward guidance only, while Gertler and Karadi (2011) and Sims, Wu, and Zhang (2023) focus on quantitative easing. Gertler and Karadi (2011) present a model considering the government’s direct intervention in the credit market with financial frictions, but forward guidance is abstracted. Cole (2021) examines the impact of forward guidance on the economy using adaptive learning, but quantitative easing is not included in the model. Sims and Wu (2021) develop a unified model incorporating forward guidance and quantitative easing simultaneously, but the ultimate purpose of their study is to investigate whether quantitative easing could function as an automatic stabilizer. In addition, our paper is most related to Sims, Wu, and Zhang (2023), who propose a four-equation New Keynesian model, providing a simplified framework to analyze the impact of quantitative easing on the economy. However, we incorporate anticipated forward guidance shocks in monetary policy, allowing for a direct comparison of the effects of two unconventional monetary policies on the economy. In addition, instead of assuming rational expectations, we employ adaptive learning to obtain expectations in a more realistic manner. Overall, none of these prior studies compare the effects of forward guidance and quantitative easing on output and inflation using a more realistic expectations formation assumption as we do.

We also build on the adaptive learning and policy literature. Milani (2007) shows that the adaptive learning assumption can better match the data versus the rational expectations hypothesis. Eusepi and Preston (2010) explore the value of policy communication when agents follow an adaptive learning rule. Honkaphohja and Mitra (2015) examine price-level targeting and nominal GDP targeting tools in an adaptive learning environment. Gibbs and McClung (2023) explore adaptive learning and conditions under which macroeconomic models can produce the “forward guidance puzzle”—unrealistically large responses of a model’s variables to forward guidance news.

Our paper is also related to the literature on the inflation surge that followed the COVID-19 pandemic. Gagliardone and Gertler (2023) estimate a New Keynesian model that considers

demand shocks, labor market rigidities, oil price shocks, and monetary policy. They found that the
main drivers of the sudden and persistent rise in inflation after mid-2021 are a combination of oil
price shocks and expansionary monetary policy. Ferrante, Graves, and Iacoviello (2023) find that
consumption demand shifted from the service sector to the goods sector due to COVID-19, but
labor reallocation costs prevent the easy reallocation of labor, leading to supply problems in the
goods sector and generate high inflation. Lorenzoni and Werning (2023) document that firms and
workers both try to set wages and goods prices higher than the other group expects when they have
different expectations about how much the price of labor should be relative to the price of goods.
This then triggers an increase in the overall price level in the economy. Alternatively, we investigate
the possibility that unconventional monetary policy tools could explain the high inflation rate in
the early 2020s.

2 Model

The model follows the four equation New Keynesian model in Sims, Wu, and Zhang (2023). There
are two major differences between their model and the one used in this study. First, our model
includes the anticipated forward guidance shock in monetary policy. This allows us to compare
and analyze the effects of two unconventional monetary policies on the economy. Second, instead
of assuming that economic agents form expectations following the standard rational expectations
hypothesis, we assume agents construct forecasts following an adaptive learning rule à la Evans
and Honkapohja (2001).

As the derivation of the model is well discussed in Sims, Wu, and Zhang (2023), we provide
a summary of the economic environment and log-linearized optimal conditions. The household is
divided into parent (patient) and child (impatient), where the parent engages in usual consumption,
labor, and savings decisions while the child chooses consumption and savings via long-term bond
to maximize its lifetime utility. As the child does not supply labor it uses transfer received from
the parent. Financial intermediaries that are born in one period and immediately exit in the next
period are constrained by the balance sheet condition, where they receive seed money and deposits
from the parent and hold reserves or long-term bonds from the child. Assuming a special case of
Gertler and Karadi (2011) where the exit probability is 1 simplifies the model without changing the
results dramatically. The production section is divided into three parts. The final goods market is
perfectly competitive, while imperfectly competitive retailers generate nominal price rigidities á la Calvo. Wholesale firms produce output using only labor. The central bank monetary policy tools are twofold. First, they adjust short-term interest rates following a Taylor-type rule that includes both anticipated forward guidance shocks and unanticipated pure monetary shocks. Second, they can purchase long-term bonds issued by the child resulting in a role for quantitative easing.

### 2.1 Log-linearized Equations

**IS curve:** All variables below are expressed in the percentage deviation from their nonstochastic steady states.

\[
x_t = \frac{1}{\sigma} \left( r_t^* - \hat{E}_t \pi_{t+1} - r_t^* \right) - z \left[ \hat{b}^{FI} \left( \hat{E}_t \theta_{t+1} - \theta_t \right) + \hat{b}^{cb} \left( \hat{E}_t qe_{t+1} - qe_t \right) \right] \tag{1}
\]

\[
x_t = y_t - y_{ft} \tag{2}
\]

\[
y_{ft} = \rho a y_{ft-1} + \frac{(1 + \chi)(1 - z)}{\chi(1 - z) + \sigma} \sigma a e^t_a \tag{3}
\]

Equation (1) presents an IS curve and \( x_t \) is output gap that is defined as the difference between output \( (y_t) \) and output without price rigidities and credit shocks \( (y_{ft}) \), \( r_t^* \) is the nominal interest rate, \( r_t^* \) is the natural rate of interest, \( \epsilon_t^a \) is a productivity shock, \( \pi_t \) is inflation rate, and \( \hat{E}_t \) is a non-rational expectation operator, which is discussed in detail in subsection 2.2. This equation has two major differences from an IS curve in the traditional three-equation New Keynesian model. The first difference is the inclusion of the credit condition in the financial market, \( \theta_t \), and quantitative easing, \( qe_t \), indicated within square brackets. When the current credit condition or quantitative easing exceeds their expected future values, the output gap immediately increases. The second difference is the parameter \( z \), which represents the share of the less patient child among the entire population, ranging from 0 to 1. Due to the presence of the child issuing long-term bonds, the IS curve directly influences the credit condition and quantitative easing. For instance, as \( z \) approaches 0, equation (1) converges to the conventional IS curve. \( \hat{b}^{FI} \) and \( \hat{b}^{cb} \) are parameters that represent the relative size of long-term bonds held by financial intermediaries and the central bank at the steady state, respectively. Their sum is equal to unity. Lastly, \( \sigma > 0 \) is the coefficient of risk aversion of the parent.

**New Keynesian Phillips curve:**

\[
\pi_t = \gamma \zeta x_t - \frac{z \gamma \sigma}{1 - z} \left[ \hat{b}^{FI} \theta_t + \hat{b}^{cb} qe_t \right] + \beta \hat{E}_t \pi_{t+1} \tag{4}
\]
A New Keynesian Phillips curve is presented in equation (4). \( \gamma \equiv \frac{(1-\phi)(1-\phi\beta)}{\phi} \) is the real marginal cost elasticity of inflation where \( \phi \) is the Calvo parameter, while \( \zeta \equiv \frac{\chi(1-z)+\sigma}{1-z} \) is the real marginal cost elasticity of the output gap where \( \chi \) is the inverse Frisch elasticity of the parent. Similar to the IS curve, credit condition and quantitative easing features directly affect inflation rate. Since \( z \) takes values between 0 and 1, the coefficient in front of the square brackets is negative, meaning that an improvement in credit condition or the implementation of quantitative easing leads to a decrease in the inflation rate. Moreover, increasing the output gap eventually leads to an increase in the inflation rate. However, a positive credit shock or an increase in quantitative easing can partially offset the inflationary pressures. Additionally, if the weight of consumption by the child approaches 0, equation (4) becomes identical to a Phillips curve in the three-equation New Keynesian model.

The laws of motion for productivity, credit condition, and the natural interest rate are also needed. The productivity \( (a_t) \) and credit condition \( (\theta_t) \) shocks are both assumed to follow an AR(1) process. In addition, there is a mapping from the natural rate of interest shock to potential output \( (y_f^t) \). Specifically, following Sims, Wu, and Zhang (2023), the natural rate of interest can be rewritten as:

\[
\hat{r}_t = \frac{\sigma(\rho a - 1)}{(1-z)y_f^t}
\]  

(5)

**Monetary Policy:** The monetary authority adjusts short-term rates and issues both forward guidance and quantitative easing. Specifically, the central bank adjusts interest rates via the following monetary policy rule:

\[
real_t - r_t^* = \rho_r(real_{t-1} - r_{t-1}^*) + (1 - \rho_r) [\phi_\pi \pi_t + \phi_x x_t] + \epsilon_{mp}^t + \sum_{l=1}^{L} \epsilon_{fg}^{l,t-1}
\]  

(6)

where the real interest rate \((real_t)\) is defined following the Fisher equation

\[
real_t = r_t^* - \hat{E}_t \pi_{t+1}
\]  

(7)

\( \rho_r \in (0, 1) \) denotes the smoothing parameter, \( \phi_\pi > 0 \) is the feedback coefficient on inflation, \( \phi_x \geq 0 \) is the feedback coefficient on output gap, and \( \epsilon_{mp}^t \) is unanticipated pure monetary policy shock. \( \epsilon_{fg}^{l,t-1} \) is forward guidance shock and is known prior to period \( l \) and follows an i.i.d. process (e.g., Laséen and Svensson, 2011; Del Negro, Giannoni, and Patterson, 2012; and Cole and Huh, 2023). The reason for considering forward guidance in this way is to solve the indeterminacy problem that would occur if forward guidance is modeled as pegging the path of interest rates to a specific value.\(^2\)

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\(^2\)See Honkapohja and Mitra (2005), Woodford (2005), and Cole (2021) for more details.
The form of the monetary policy rule in equation (6) follows from Martínez-García (2021) and is written in terms of deviations of the real interest rate from its natural level for the following reasons. First, returning real interest rates to their natural level is the ultimate goal for monetary policy. For example, Federal Reserve Chair Jerome Powell emphasizes the importance of the neutral federal funds rate, which occurs when “the economy operating at full strength and with stable inflation” (Powell, 2020). Importantly, the neutral federal funds rate coincides with the theoretical natural real interest rate considered in the present paper. Second, framing monetary policy in terms of real interest rates rather than nominal alleviates issues regarding the ZLB and nonlinearities in the model.

The following equations are also added to the model:

\[ v_{1,t} = v_{2,t-1} + \epsilon_{fg1,t}, \]  
\[ v_{2,t} = v_{3,t-1} + \epsilon_{fg2,t}, \]  
\[ \vdots \]  
\[ v_{L,t} = \epsilon_{fgL,t}. \]

The vector \( v_t = [v_{1,t}, v_{2,t}, \ldots, v_{L,t}]' \) contains all the information that the central bank has communicated from the past \( (v_{t-1}) \) to present \( (\epsilon_{fg} = [\epsilon_{fg1,t}, \epsilon_{fg2,t}, \ldots, \epsilon_{fgL,t}]') \). Additionally, by sequentially substituting equations (8) - (10) backward in one period, \( v_{1,t-1} = \sum_{l=1}^{L} \epsilon_{fgl,t-l} \), which is the same as forward guidance shocks in equation (6).

The evolution of the quantitative easing shock involves both contemporaneous and anticipated components. The first is a standard AR(1) process following Sims, Wu, and Zhang (2023).\(^3\) We also include anticipated shocks, a novel feature not found in the 4 equation model of Sims, Wu, and Zhang (2023). In practice, the FOMC provides information on how much and for how long the U.S. central bank will purchase certain assets. For example, on December 12, 2012, “the FOMC announces it will purchase $45 billion of longer-term Treasuries per month for the indefinite future.” Moreover, Bauer and Rudebusch (2014) describe contemporaneous and anticipated components of quantitative easing affecting the term premium. The first regards a portfolio balance channel, where the central bank increases the demand for bonds, raising bond prices and lowering bond yields. The second is the signaling channel, where quantitative easing influences expectations of

\(^3\)To bring automatic stabilization to the economy Sims and Wu (2021) set up quantitative easing to respond to the output gap and inflation gap, similar to how the short term interest rate is determined by the Taylor rule.
future short-term interest rates, thereby reducing long-term bond yields. To incorporate these channels (especially signaling), we modify a standard AR(1) process of quantitative easing so that it includes anticipated shocks that are known in advance as follows:

\[ qe_t = \rho qe_{t-1} + \epsilon_t^{qe} + \sum_{l=1}^{L} \epsilon_{l,t-l}^{qefg} \]  

(11)

Similar to forward guidance shocks in monetary policy, we add the following equations:

\[ v_{qe}^{1,t} = v_{qe}^{2,t-1} + \epsilon_{1,t}^{qefg} \]  

(12)

\[ v_{qe}^{2,t} = v_{qe}^{3,t-1} + \epsilon_{2,t}^{qefg} \]  

(13)

\[ \vdots \]

\[ v_{qe}^{L,t} = \epsilon_{L,t}^{qefg}. \]  

(14)

The vector \( v_t^{qe} = [v_{1,t}^{qe}, v_{2,t}^{qe}, \ldots, v_{L,t}^{qe}]' \) contains all the information that the central bank has communicated about quantitative easing from the past \( (v_{l,t-1}^{qe}) \) to present \( (\epsilon_t^{qefg} = [\epsilon_1^{qefg}, \epsilon_2^{qefg}, \ldots, \epsilon_L^{qefg}]') \).

Also, by sequentially substituting equations (12) - (14) backward in one period, \( v_{1,t-1}^{qe} = \sum_{l=1}^{L} \epsilon_{l,t-l}^{qefg} \), which is the same as forward guidance shocks in the quantitative easing equation (11).

### 2.2 Expectations Formation

Agents in our model are assumed to follow an adaptive learning rule when forming expectations of the output gap and inflation found in (1) and (4). Specifically, as in Evans and Honkapohja (2001) they are assumed to follow the cognitive consistency principle in which agents act like econometricians when forecasting the future. They create an econometric model, gather data, estimate parameters, and use their model to construct expectations. As new data arrive every period, adaptive learning agents update their estimates and forecasts.

Adaptive learning agents’ forecasting model - perceived law of motion (PLM) – is assumed to be of the following form:

\[ Y_t = \psi_1 + \psi_2 Y_{t-1} + \psi_3 \theta_t + \psi_4 qe_t + \psi_5 v_t^{qe} + \psi_6 v_t + \psi_7 v_{1,t-1} + \bar{\epsilon}_t \]  

(15)

\[ ^{4}\text{The adaptive learning agents are assumed to know the values of } \rho_\theta \text{ and } \rho_q, \text{ respectively. Knowledge of the values of the autoregressive parameters of structural shocks is common in the adaptive learning literature.} \]
where $Y_t = [x_t, \pi_t, i_t]'$, $\psi_1$, $\psi_2$, $\psi_3$, $\psi_4$, $\psi_5$, $\psi_6$, and $\psi_7$ are coefficients of appropriate dimensions to be estimated each period, and $\bar{\varepsilon}_t$ is the usual regression error term.\(^5\)

Adaptive learning agents utilize OLS to update their estimates of $\psi_1$, $\psi_2$, ..., $\psi_7$ at the end of every period. Specifically, the coefficients are estimated using OLS written in recursive form:

$$\Psi_t = \Psi_{t-1} + \tau_t R_{t-1}^{-1} z_t (Y_t - \Psi_{t-1}' z_t)' \quad (16)$$

$$R_t = R_{t-1} + \tau_t (z_t z_t' - R_{t-1}) \quad (17)$$

where $\Psi = [\psi_1, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7]'$ contains the coefficients in the PLM and $R_t$ is the matrix of second moments of the PLM regressors $z_t \equiv [1, Y_{t-1}', \theta_t, qe_t, v_t, v_t qe, v_t, v_1, t-1]'$.

This paper will utilize constant gain learning; therefore, the “gain” parameter $\tau_t$ in equations (16) and (17) is assumed constant, that is, $\tau_t = \bar{\tau}$. This assumption, which is common in the literature, allows agents to continually track changes in the economy (e.g., forward guidance and quantitative easing policies) over time. Moreover, as the value of $\bar{\tau}$ increases, agents place more weight on recent observations and thus are quicker to revise their beliefs. As the value of $\bar{\tau}$ decreases, agents place less weight on recent observations and the estimates of $\Psi$ do not update as quickly.

It is also important to review the timing of events:

1. Central bank announces to the public its quantitative easing and forward guidance policies which allow agents to know $qe_t$, $v_t$, and $v_t qe$.
2. The value of $\theta_t$ is realized.
3. Using previous periods estimates, $\Psi_{t-1}$ and the PLM in equation (15), agents construct forecasts:
   $$E_t Y_{t+1} = \Psi_{t-1}' z_t \quad (18)$$
4. Agents use OLS equations (16) and (17) to update their estimates of $\Psi_t$.

Overall, the model utilized in this paper consists of an IS curve, NKPC, monetary policy rule with forward guidance on the interest rate, quantitative easing including anticipated shocks,\(^5\)

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\(^5\)It should be noted that the PLM contains more variables than the MSV solution. Specifically, the lagged output gap and inflation are included in equation (15). We reason that we want adaptive learning agents to have at least some degree of similarity with forecasters in the real world. It is common for present-day forecasters to use a data-driven VAR, which includes a backward-looking component, exclusively or as part of a broader model to form expectations. Thus, by including the lagged output gap and inflation in our PLM, we can better match the data.
supplemental forward guidance equations, AR(1) equations for natural rate and credit condition shocks, and aggregate expectations based on adaptive learning. Specifically, equations (1) - (10), (11) - (14), AR(1) processes for the natural rate and credit condition shocks, and (18).

3 Calibration

Table 1 shows the choice of parameter values. Of the total twenty-one parameters, nine are traditional, three ($z, \bar{b}^F, \bar{b}^b$) are specific to the four equation New Keynesian model, two are the autoregressive coefficient of credit condition and quantitative easing, five are the standard deviations of shocks including unconventional monetary policy shocks, and the remaining is forward guidance horizon and the constant-gain parameter. $z$ is set to 0.33, which implies that about 33% of the households are the impatient child and about 67% of the households are the patient parent. The discount factor of the parent, $\beta$, is set to 0.995, meaning 2% interest rate in the nonstochastic steady state. The elasticity of intertemporal substitution, $\sigma$, is set to 1, which indicates that the parent’s utility function is logarithmic. The inverse Frisch parameter, $\chi$, is fixed to 1, implying that labor supply is unit elastic to the real wage. The Calvo parameter, $\phi$, is 0.75, which means that monopolistically competitive firms reset prices on average once every four quarters. Bond holdings of financial intermediaries and central bank are set to 0.70 and 0.30, respectively. This indicates that financial intermediaries hold about 70% of long-term bonds while the central bank holds the rest of long-term bonds in a steady state. The persistence of the shocks to interest rates, credit conditions, and quantitative easing is set to 0.8 for all three shocks. The feedback coefficients on inflation and output gap are fixed at 1.5 and 0, respectively. These parameter values are the same as those from Sims, Wu, and Zhang (2023). Moreover, the forward guidance horizon, $L$, is set to 12 implying that the central bank announces its monetary policy for the next 3 years to the public. This value is based on the FOMC statement on September 2012.

A key learning parameter in our paper is $\bar{\tau}$. This parameter controls how much agents discount previous observations. A higher value implies agents place greater weight on new information about the economy. A lower value of $\bar{\tau}$ means agents are not as sensitive to new data when updating their forecasts. Following empirical work by Milani (2007), we set $\bar{\tau} = 0.02$. In Section 4.2.4 we examine the results under higher and lower values of $\bar{\tau}$.

A note about the initial conditions and calculation of the impulse responses in the remaining
The adaptive learning model is simulated for 2,000 periods with initial values of $\Psi$ set to their rational expectations counterparts and $R$ to the identity matrix. Two series are then generated using the same shocks. However, the “impulsed” series contains an additional shock in period 2,001 while the “non-impulsed” series does not. Each series is simulated for a 20-period horizon. The impulse response of a variable to a particular shock is the difference between the impulsed series and the non-impulsed series. The average across 5,000 simulations is then calculated to arrive at the final impulse responses.

4 Results

4.1 Quantitative Easing vs. Forward Guidance

The model presented above follows the four equation New Keynesian model in Sims, Wu, and Zhang (2023), which incorporates a contemporaneous quantitative easing shock. We then included forward guidance shocks on both quantitative easing and interest rate processes. We now focus
on comparing the economic effects of a standard contemporaneous quantitative easing shock to forward guidance shocks.

Figure 1 shows responses of output, inflation, and the nominal interest rate to a favorable one standard deviation quantitative easing shock and its corresponding forward guidance shock. Since quantitative easing and forward guidance influence the economy via different channels, we perform the following normalization to fairly compare the economic effects of each shock. In each column, the size of an $l$-period ahead forward guidance shock is chosen such that the response of output $l$-periods ahead—when forward guidance is realized on the economy—is the same as the initial response to the contemporaneous quantitative easing shock. In addition, the solid lines plot responses of output, inflation, and nominal interest rate to quantitative easing while the dashed lines plot their forward guidance counterparts.

The results in the middle row of Figure 1 show a greater positive performance of a forward guidance shock on inflation than a (contemporaneous) quantitative easing shock. Forward guidance overall generates higher responses to inflation, and their effects are more persistent compared to that of quantitative easing. When the central bank announces that it will lower interest rates in the future, households increase consumption and firms raise prices in response. The positive effects on inflation continue during the forward guidance horizon. A positive quantitative easing shock also has similar initial beneficial effects on output and inflation. However, the effect on inflation is lower under a quantitative easing shock relative to a forward guidance shock. The dashed line is overall above the solid line across the forward guidance horizon.

The third row shows the response of the nominal interest rate to the two shocks. Because the Taylor rule determines the nominal interest rate, the nominal interest rate also rises sharply in response to the very high inflation caused by the forward guidance shock. In contrast, the quantitative easing shock, which generates a relatively small response to inflation, does not raise the interest rate significantly.

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6 This is calculated under the assumption of rational expectations.
7 There also exists notable differences of output’s response between quantitative easing and forward guidance shocks as well as periods where forward guidance generates higher levels of output. However, since the forward guidance shocks are normalized such that the response of output $l$-periods ahead is the same as the initial response to the contemporaneous quantitative easing shock, we choose to focus on the inflation responses in this section.
8 It is also noteworthy that the response of inflation to quantitative easing shock falls in the second period. Quantitative easing has both an inflation-boosting effect through the output gap, but also an inflation-dampening effect, as shown in equation (4). Similarly, inflation in the second period falls when there is a positive credit condition shock. The impulse responses to credit and other shocks are attached in Appendix A.
What can explain the performance gap between forward guidance and quantitative easing? To help answer this question, we plot the expectations of output gap and inflation in Figure 2. The top row displays the responses of the expected output gap and the bottom row shows the responses of expected inflation. The paths of both variables follow similar paths as contemporaneous output and inflation. Importantly, similar to its contemporaneous counterparts in Figure 1, forward guidance shocks have greater effects on inflation expectations than quantitative easing shocks. Since forward guidance includes an anticipation period leading up to when the shock is realized, forecasts are strongly affected. However, contemporaneous quantitative easing contains no anticipation period leading to a smaller influence on inflation.
Figure 2: Responses of Output Gap and Inflation to Forward Guidance Shocks on Interest Rate and Quantitative Easing

Note: \( \hat{E}_t x_{t+1} \) denotes expected output gap, \( \hat{E}_t \pi_{t+1} \) denotes expected inflation, \( \epsilon_{qe}^{\pi} \) denotes quantitative easing shock, and \( \epsilon_{fg}^{\pi} \) denotes forward guidance shock on interest rate. The expected output gap is expressed in percentage points. Expected inflation and interest rates are expressed in annualized percentage points.

The adaptive learning agents are also estimating a stronger reaction of output and inflation to forward guidance shocks relative to quantitative easing shocks. Figures 3 and 4 display the adaptive learning estimates of \( \psi_4 \) and \( \psi_6 \). \( \psi_{4(1,1)} \) and \( \psi_{4(2,1)} \) define the sensitivity of output gap and inflation to quantitative easing, respectively, while \( \psi_{6(1,*)} \) and \( \psi_{6(2,*)} \) represent their forward guidance shock counterparts. In absolute value terms, the coefficients in \( \psi_6 \) are larger than their counterparts in \( \psi_4 \). This disparity results in higher effects of forward guidance on inflation expectations in Figure 2 and contemporaneous inflation in Figure 1 relative to quantitative easing.

We also believe that the results presented in Figure 1 can shed light on the high inflation rate observed after the COVID-19 pandemic. While some studies attribute the high inflation rate to
Figure 3: Estimated Evolution of Coefficients for Quantitative Easing and Forward Guidance Shocks on Output Gap

*Note:* $\psi_{4(1,1)}$ denotes the estimated coefficient of quantitative easing on the output gap, and $\psi_{6(1,\ast)}$ denotes the estimated coefficient of forward guidance on the output gap.

Factors such as a combination of oil price shock and expansionary monetary policy (e.g., Gagliardone and Gertler, 2023), sectoral reallocation (e.g., Ferrante, Graves, and Iacoviello, 2023), and wage-price spirals (e.g., Lorenzoni and Werning, 2023), none of these studies specifically analyze the effect of unconventional monetary policy tools on inflation. However, Figure 1 shows positive effects on inflation from both forward guidance and quantitative easing possibly indicating central bank forward guidance is one of the causes of the inflation surge in the early 2020s.

A related note about the effects of both forward guidance and quantitative easing on inflation in the wake of the global financial crisis of 2007-2009 is also warranted. Even though both of the unconventional monetary policy tools were implemented, inflation was not as high as it was after the COVID-19 pandemic. However, financial frictions can have a mixed impact on the economy. Financial factors make recessions deeper, but they also help to dampen inflation due to financial
Figure 4: Estimated Evolution of Coefficients for Quantitative Easing and Forward Guidance Shocks on Inflation

Note: $\psi_{4(2,1)}$ denotes estimated coefficient of quantitative easing on inflation, and $\psi_{6(2,*)}$ denotes estimated coefficient of forward guidance on inflation.

and consumption wedges (e.g., Christiano, Eichenbaum, and Trabandt, 2015), or a higher degree of price rigidities (e.g., Del Negro, Giannoni, and Schorfheide, 2015). In addition, in response to the global financial crisis, the central bank implemented forward guidance relatively slowly. For example, major forward guidance was implemented five times between 2009 and 2015, but it was implemented just three to four times from March to December 2020 (e.g., Swanson, 2021; Clarida, Duygan-Bump, and Scotti, 2021). Thus, the higher frequency during the COVID-19 period can help shed light on the inflation surge.

4.2 Explaining the Disparity

The results in the previous section show the overall superior performance of forward guidance compared to contemporaneous quantitative easing shock. In the following subsections, we examine
factors that can affect performance gap between forward guidance and quantitative easing.

4.2.1 Guidance on Future Quantitative Easing

We have so far considered a contemporaneous quantitative easing shock, which follows from the 4 equations model of Sims, Wu, and Zhang (2023). However, the FOMC has provided information on how long the central bank will purchase certain assets. For example, on December 12, 2012, “the FOMC announces it will purchase $45 billion of longer-term Treasuries per month for the indefinite future.” As previously mentioned, Bauer and Rudebusch (2014) describe this type of quantitative easing as operating via two channels: portfolio balance channel and signaling channel. Of particular relevance to the present subsection is the latter channel in which quantitative easing influences expectations of future short-term interest rates, thereby reducing long-term bond yields. Thus, we consider anticipated forward guidance shocks to quantitative easing.

Figure 5 displays the difference between impulse responses of output, inflation, and nominal interest rate to forward guidance and quantitative easing shocks. The solid line plots the difference in the responses of each macroeconomic variable between a forward guidance shock and a contemporaneous quantitative easing shock. The dashed line is the same as the solid line except we use the anticipated quantitative easing shocks instead of the contemporaneous quantitative easing shock. Each column of Figure 5 corresponds to a forward horizon of 1, 4, 8, and 12 periods, respectively.

The results display that adding guidance on the future path of quantitative easing shrinks the performance gap between forward guidance and quantitative easing. Overall, the dashed lines for output and inflation are closer to zero than the solid lines across the impulse response horizon. Quantitative easing now works through the expectations/anticipation channel. Similar to forward guidance on interest rates in Section 4.1, agents are now expecting favorable quantitative easing in the future leading to further beneficial effects on contemporaneous variables.

4.2.2 Impatient Households

The model considered in the present paper includes two types of households. The “parent” household follows standard New Keynesian models. They consume, provide labor, and save via one-period ahead bonds. The “child” household is impatient. They do not work. To help finance their consumption, they issue long-term bonds to financial intermediaries and the monetary authority. Consequently, if a larger share of child households exists, more long-term bonds are supplied in
an economy. Thus, quantitative easing should theoretically have a more potent impact in this scenario. In the benchmark results of the present paper, the share of impatient households \((z)\) is set to 0.33 following Sims, Wu, and Zhang (2023). However, what would occur if \(z\) increases? To answer this question, we plot the difference in impulse responses between a forward guidance shock on the interest rate and a contemporaneous quantitative easing shock in Figure 6. The solid line represents the benchmark case of \(z = 0.33\). For illustrative purposes, we examine the results when \(z\) equals 0.90, which is represented by the dashed line.\(^9\)

The results show that quantitative easing can shrink the performance gap and even outperform forward guidance when the share of impatient households increases enough. This outcome is especially evident when examining output’s response. In the first row, the dashed line stays negative

\(^9\)It should be noted that the values of \(\delta^b\) decreases when \(z\) increases due to the \(\delta^b\)'s value depending on the \(z\).
throughout virtually the entire impulse response horizon indicating quantitative easing more effective on output than forward guidance. However, the solid line contains periods of positive values. Moreover, when examining the responses of inflation in the second row of Figure 6, the dashed line is overall above and more away from zero than the solid line. This is consistent with one of the results in Appendix F of Sims, Wu, and Zhang (2023).

The reasons for the previous results are twofold. First, impatient households help fund their consumption by issuing long-term bonds. With more impatient households in the economy, an increase in the number of long-term bonds supplied from a quantitative easing shock has a more potent effect on the economy than before.\footnote{Additionally, from Sims, Wu, and Zhang (2023), the reader should note that $y_t = (1 - z)c_t + zc_{b,t}$ where $c_t$ is patient households’ consumption and $c_{b,t}$ is impatient households’ consumption. Thus, output depends more on the impatient child household’s consumption as $z$ increases.} Mechanically speaking, as $z$ increases, the output gap in equation (1) becomes more dependent on quantitative easing than on the difference between the real interest rate and the natural interest rate. On the other hand, the initial impact on inflation from quantitative easing shock decreases since the coefficient of quantitative easing (i.e., $-z\gamma\sigma/(1 - z)$ in (4)) decreases when $z$ increases.

The second reason for the shrinking performance gap regards forward guidance primarily operating through patient households. In contrast to impatient households, patient households can save between periods via one-period-ahead bonds that pay a nominal interest rate. Since forward guidance operates via the expected interest rate channel, a decrease in patient households lowers the effectiveness of forward guidance on $c_t$ and $y_t$.

### 4.2.3 Central Bank Share of Bonds

The model of this paper includes long-term bonds that are supplied by impatient child households and demanded by financial intermediaries and the monetary authority. Following Sims, Wu, and Zhang (2023), the benchmark results of the paper assumed that financial intermediaries hold 70% of long-term bonds and the central bank the remaining 30% in a steady state. Accordingly, a natural question arises regarding how the performance gap between forward guidance and quantitative easing changes if the monetary authority holds a greater share of long-term bonds in the economy.

The dotted line in Figure 6 displays the results when the monetary authority is assumed to hold more of the long-term bonds than financial intermediaries. For illustrative purposes, we set $\bar{b}_{cb} = 0.90$ and $\bar{b}_{FI} = 0.10$. As in the previous subsection, each line represents the difference in...
impulse responses between a forward guidance shock on the interest rate and a contemporaneous quantitative easing shock in Figure 6.

The results show that quantitative easing has a greater effect on output and initial inflation relative to forward guidance when the central bank holds a greater percentage of long-term bonds. The dotted line is below the solid line and mostly takes on negative values across the impulse response horizon. Since the monetary authority holds most of the long-term bonds in this scenario, a change in quantitative easing will have a significant effect on impatient households’ consumption, and thus, output.\footnote{From Sims, Wu, and Zhang (2023), the reader should note that $c_{bh,t} = \bar{b}^{FG} \theta_t + \bar{b}^{cb} q_{et}$. Thus, the impatient household’s consumption increases more in response to an increase in quantitative easing as $\bar{b}^{cb}$ rises.} Consequently, the positive effects on output put upward pressure on prices causing the inflation performance gap to initially shrink compared to the benchmark case. In Figure 6, the dotted lines are initially below the solid lines. Overall, the share of long-term bonds demanded by the central bank influences the output performance gap relative to inflation.

Figure 6: Impulse Responses Under Different Parameter Values
4.2.4 Constant Gains

The constant gain ($\bar{\tau}$) is a key parameter in our model and the adaptive learning literature. It can be thought of as how much agents discount previous observations. A larger $\bar{\tau}$ implies agents place a greater weight on recent forecast error, and thus, new information (e.g., forward guidance and quantitative easing) plays a larger role in updating their coefficients. If $\bar{\tau}$ is small, new information about policies does not have as much of a role. In our baseline results, we use a value of $\bar{\tau} = 0.02$, which matches prior empirical studies. For robustness, this present section will examine the results under higher and lower values of $\bar{\tau}$.

Figure 7 displays the results under, higher (0.05), and lower (0.0001) values of $\bar{\tau}$. Similar to the previous subsection, each line represents the difference in impulse responses between a forward guidance shock on the interest rate and a contemporaneous quantitative easing shock. The solid line signifies the benchmark case of $\bar{\tau} = 0.02$; dashed line means $\bar{\tau} = 0.0001$; dotted line represents $\bar{\tau} = 0.05$.

The results suggest multiple takeaways when the value of the constant gain varies. First, across all values of $\bar{\tau}$, forward guidance and quantitative easing display qualitatively the same results. The solid, dashed, and dotted lines follow similar paths. Second, when agents place less weight on recent observations, the difference between a forward guidance shock and a contemporaneous quantitative easing shock shrinks. Overall, the dashed lines are closer to zero than the solid line in Figure 7. Since agents place less weight on recent observations, forward guidance and quantitative easing have less effect on the macroeconomic variables. Additionally, forward guidance operates via the expectations channel which is determined by the learning process. Thus, reducing the amount of learning with a lower value of $\bar{\tau}$ decreases forward guidance effectiveness more than a contemporaneous quantitative easing shock. Lastly, as the value of $\bar{\tau}$ increases to 0.05, forward guidance becomes (slightly) more effective at stimulating output than a contemporaneous quantitative easing at a one-period horizon. The dotted line is above the solid line at the peak as displayed in the first column first row of Figure 7. However, as the horizon of the forward guidance shock considered increases (move right along the columns of Figure 7), the overall performance of forward guidance relative to quantitative easing seems to be similar across different values of $\bar{\tau}$. This result is not surprising given that the coefficients governing the sensitivity of output gap and inflation to forward guidance shocks are much smaller in absolute value terms for forward guidance horizons 4, 8, and 12 relative to 1 as
shown in Figures 3 and 4.

Figure 7: Impulse Responses Under Alternative Constant Gains

5 Conclusion

This study compares the effects of unconventional monetary policy tools—forward guidance and quantitative easing—on the economy with agents forming expectations via an adaptive learning rule. We use the four-equation New Keynesian model of Sims, Wu, and Zhang (2023) and incorporate anticipated forward guidance shocks in the monetary policy rule. We find that forward guidance has a more pronounced impact on inflation compared to quantitative easing. Adaptive learning agents estimate a higher effect of forward guidance on the economy leading to a greater impact on expectations and contemporaneous inflation. We believe that our result indicates the possible contribution of unconventional monetary policies to the high inflation rate after the COVID-19 recession. In addition, the performance gap between forward guidance and quantitative easing can shrink if anticipated shocks are incorporated into quantitative easing, more households finance
consumption via long-term borrowing, and the central bank provides a greater percentage of liq-
uidity in the long-term borrowing market. Overall, it is important to concurrently analyze forward
guidance and quantitative easing in a unified framework with a more realistic form of expectations
when providing policy recommendations.
References


A. Appendix

Figure A.1: Impulse Responses of Macro variables to Other Shocks

Note: $y_t$ denotes output, $\pi_t$ denotes inflation, $r_t^s$ denotes interest rate, $\epsilon_t^r$ denotes conventional monetary policy shock, $\epsilon_t^a$ denotes technology shock, and $\epsilon_t^\theta$ denotes credit condition shock.