The Nonstationarity of Money and Prices in Interdependent Economies

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Abstract
In most nations, paths of monetary aggregates and prices consistently depart from stationary trends. This paper shows that this is a fundamental implication when monetary authorities of interdependent countries seek to smooth their home output and prices in the presence of incomplete world output-market integration and structural asymmetries. Using a two-country model with interdependent output supply schedules, we show that this conclusion holds whether the exchange rate floats or is fixed. It also holds if monetary policies are coordinated. Therefore, optimal monetary policy choices by central banks yield stationary paths for money and prices only under very specific conditions.
1. Introduction

Why do central banks conduct monetary policies that produce base drift--nonstationary growth of the monetary base and other monetary aggregates? Why do they conduct policies that yield nonstationary paths for prices as well? As discussed by Goodfriend (1991) and Walsh (1990), considerable research has been directed toward answering these two questions. Walsh (1986) originally showed that permanent shifts in money demand may justify base drift as an optimal policy but did not provide an explanation for price-level non-trend-stationarities. Goodfriend (1987) motivated both base drift and nonstationary prices as natural results of monetary policy processes aimed at smoothing interest rates as well as broader objectives relating to output and price stability. Barro (1989) and Froyen and Waud (1995) have found evidence supporting Goodfriend's hypothesis for some periods in the United States. In addition, Hetzel (1995) has extended Goodfriend's framework to account for the widespread observation of "bygones" base drift, in which increases in money and prices caused by transitory disturbances lead to permanently higher paths for these variables.

As pointed out by VanHoose (1989), however, nonstationary paths for money and prices ultimately arise from a shortage of instruments relative to the full menu of policy objectives. His analysis, for instance, demonstrated that base drift and price-level non-trend-stationarity would emerge in settings in which central banks seek either to aim directly at target values for monetary aggregates or to minimize deviations of monetary aggregates from such targets. In either instance, the additional policy objectives create an instrument shortage comparable to that in Goodfriend's interest-rate-smoothing environment. Sephton (1989) made an analogous point by showing that a central bank concerned with smoothing the exchange rate in addition to prices also finds nonstationary money and price paths optimal.

More recently, Daniels and VanHoose (1995) have shown that a similar result can arise from structural and policy interdependence among economies. Their analysis demonstrates that central bank efforts to smooth both home output prices and consumer price indexes (CPIs) is sufficient to induce the optimality of base drift and price-level non-trend-stationarities even in the absence of central bank desires to limit variability of interest rates, monetary aggregates, or exchange rates. Nevertheless, the reasoning behind this conclusion again is the problem of a shortfall in the number of independent instruments relative to the overall number of central bank objectives. Daniels and VanHoose show that one way that this shortfall might be overcome is for one nation's central bank to condition its monetary policies on unexpected changes in the money stock in the other country. This essentially adds an additional independent instrument to the central bank's arsenal, thereby permitting the central bank to produce stationary money and price paths. They find, however, that with integrated financial markets, conditioning monetary policies on exchange rate innovations or on innovations in the other nation's interest rate fails to overcome the instrument shortfall if a nation's central bank already conditions its policies on the home interest rate.

This paper makes several additional contributions to our understanding of how international interdependence affects the stationarity of money and prices. The paper generalizes earlier approaches in the literature on optimal monetary policy in interdependent economies by allowing for interdependence of output supply behavior across countries. In our model, such interdependence arises in the context of an environment in which workers who consume goods of two nations desire to
index their nominal wages to unanticipated changes in consumer prices, so that aggregate supply in each nation ultimately responds to prediction errors concerning home prices, foreign prices, and the exchange rate. Additionally, we generalize the results of Daniels and VanHoose, who considered a flexible-exchange-rate setting without any cross-country aggregate supply spillovers. Here, we consider both flexible- and fixed-exchange-rate regimes and account for supply-side interdependence. We also isolate special cases in which optimal monetary policies can yield stationary money and price paths in the presence of aggregate supply interdependence. Furthermore, we analyze whether coordinating monetary policy choices leads to stationary paths for money and prices.

Our key results are as follows. We find that as long as central banks in interdependent economies are interested in smoothing unanticipated and anticipated changes in price components in order to reduce output volatility and save agents the costs of indexing nominal contracts, their optimal policy choices entail non-stationarity of money and prices under pegged or floating exchange rates. A further implication of our analysis of the fixed-exchange-rate case is that trend stationarity is a relevant policy issue for central banks even if the exchange rate is pegged contemporaneously. In addition, we find that the "bygones" base drift that has predominated in modern economies often emerges as part of the optimal monetary policy mix for interdependent economies. Finally, we demonstrate that central bank coordination of monetary policy generally fails to yield stationary paths for money and prices if financial markets are integrated. Consequently, monetary policy coordination has no benefits as far as achieving stationarity is concerned.

We are able to identify only two special cases in which trend-stationary monetary policies are optimal. A situation in which producing stationary paths for money and prices is an optimal policy is one in which world output markets are fully integrated, so that central banks are interested only in smoothing their nations' CPIs. Alternatively, central banks that pursue both output and CPI-inflation smoothing policies will find that monetary and price-level stationarity is the optimal policy if their countries are identical and experience common disturbances. In the presence of imperfectly integrated output markets and/or country-specific shocks, however, base drift and non-trend-stationary prices always emerge as the optimal monetary policies.

2. The Model and Its Solution with a Flexible Exchange Rate and Non-coordinated Monetary Policies

The model below is based on the framework developed in Daniels and VanHoose, which in turn is a melding of the two-country models of Turnovsky et al. (1988) and Turnovsky and d'Orey (1989) and the monetary policy framework of Goodfriend (1987). The structural relationships for the model are as follows:

(1a) \[ p_t^e = \alpha p_t + (1 - \alpha)(p_t^* + e_t); \quad 1/2 < \alpha < 1, \]
(1b) \[ p_t^{e*} = \alpha p_t^{e*} + (1 - \alpha)(p_t - e_t). \]
(2a) \[ y_t = cy_t^* - d_1[r_t - (E_t p_{t+1}^e - p_t^e)] + d_2(p_t^e + e_t - p_t) + \eta_t; \quad 0 \leq c \leq 1; \quad d_1, d_2 \geq 0, \]
(2b) \[ y_t^* = cy_t - d_1[r_t^* - (E_t p_{t+1}^{e*} - p_t^{e*})] - d_2(p_t^e + e_t - p_t) + \eta_t, \]
(3a) \[ m_t - p_t = y_t - br_t + \xi_t; \quad b \geq 0, \]
(3b) \(m_t^* - p_t^* = y_t^* - br_t^* + \xi_t^*\),

(4) \(r_t = r_t^* + E_t e_{t+1} - e_t\),

(5a) \(y_t = a(1 - \alpha)(p_t - E_{t-1}p_t) - a(1 - \alpha)\gamma(p_t^* - E_{t-1}p_t^*) - a(1 - \alpha)\gamma(e_t - E_{t-1}e_t) = a((p_t - E_{t-1}p_t) - \gamma(p_t^* - E_{t-1}p_t^*)); a \geq 0, 0 \leq \gamma \leq 1,\)

(5b) \(y_t^* = a(1 - \alpha)\gamma(p_t^* - E_{t-1}p_t^*) - a(1 - \alpha)\gamma(p_t - E_{t-1}p_t) + a(1 - \alpha)\gamma(e_t - E_{t-1}e_t) = a(p_t^* - E_{t-1}p_t^*) - \gamma(p_t^* - E_{t-1}p_t^*)),\)

(6a) \(m_t = m_{t-1} + \theta_1(r_t - E_{t-1}r_t) - \theta_2(m_{t-1} - E_{t-2}m_{t-1})\),

(6b) \(m_t^* = m_{t-1}^* + \theta_1^*(r_t^* - E_{t-1}r_t^*) - \theta_2^*(m_{t-1}^* - E_{t-2}m_{t-1}^*) + \theta_3^*(e_t - E_{t-1}e_t)\),

where foreign variables and policy parameters are asterisked and domestic variables and policy parameters are nonasterisked, and \(p_t\) is the log of the price level, \(e_t\) is the log of the exchange rate, computed in terms of units of domestic currency, \(y_t\) is the log of real output, \(r_t\) is the nominal interest rate, \(m_t\) is the log of the nominal money stock, \(E_{t+j}\) is the expectation operator, conditioned on information dated time \(t + j, \eta_t\) is a nominal spending disturbance, with \(E(\eta_t) = 0\) and \(E(\eta_t^2) = \sigma^2_{\eta}\), and \(\xi_t\) is a money demand disturbance, with \(E(\xi_t) = 0\) and \(E(\xi_t^2) = \sigma^2_{\xi}\). All disturbances are assumed to be independently distributed and serially uncorrelated.

Equations (1) are consumer price indexes for the two economies, where \(\alpha\) is the weight of home goods in consumption in each nation. Equation (2a) is the domestic income-expenditure equilibrium condition, in which desired expenditures on domestic goods depend positively on foreign income, negatively on the real interest rate, calculated in terms of the domestic CPI, and positively on the real exchange rate, \(p_t^* + e_t - p_t\). Equation (2b) is the analogous relationship for the foreign economy, in which foreign desired expenditures depend negatively on the real exchange rate.

Equations (3) express the demands for real money balances in the two economies, where a unitary income elasticity is a simplifying assumption that does not affect our basic conclusions. Uncovered interest parity is assumed to hold and is expressed in equation (4). Equations (5) are the aggregate supply schedules, which, following the approach outlined in Bryson et al. (1998), are derived in the Appendix under three key assumptions. First, we assume that firms in each nation produce all output at home for sale at home prices and use only home labor, which is not a mobile factor of production internationally. As a result, firms value the real wage that they pay their workers in terms of the home price level. Second, workers in each country can purchase the outputs of both countries. This means that, as in the small-open-economy model of Benavie and Froyen (1992), workers value the real wage that they earn in terms of the CPI. Third, we assume that the real wage elasticities of labor supply equal zero in both countries. This permits us to abstract from effects that changes in the expected real exchange rate otherwise would have on output supply. Although this assumption does not affect our basic results, it simplifies the exposition somewhat and, more importantly, allows us to compare our model and results with others in the literature.

In both (5a) and (5b), \(\gamma\) is the degree of nominal wage indexation to unanticipated consumer price changes, which is assumed to be identical for both countries. Note that if \(\gamma = 0\), which was assumed to hold in Daniels and VanHoose (1995), then there is no CPI indexation and the amount of output
supplied in each nation depends only on the home price prediction error. If \( \gamma > 0 \), then nominal wages in each country respond to CPI price prediction errors, which causes each nation's aggregate supply of output to depend on prediction errors of the other nation's price level and the exchange rate. For \( \gamma = 1 \), nominal wages are fully indexed to price changes that were unanticipated when nominal wage contracts were formulated.

Equations (6) are money supply rules for the central banks. The \( \theta_1 \) and \( \theta_1^* \) parameters are combination policy parameters, as in Poole (1970). Following Goodfriend (1987), the \( \theta_2 \) and \( \theta_2^* \) parameters indicate whether the nations' central banks establish stationary paths for their money stocks. If \( \theta_2 = 1 \) and \( \theta_2^* = 1 \), then money stock paths in each nation are stationary. They are nonstationary otherwise, meaning that the cumulative effects of past central bank efforts to vary their monetary instruments to stabilize goal variables would necessitate permanent deviations in the time paths of instrument values from previous trend paths. Earlier considerations of the instrument stationarity issue, which included Gramlich (1971), Holbrook (1972), and Turnovsky (1974), focused on the potential for "instrument instability," in which offsetting the cumulative effects of past instrument effects on policy goal variables would entail increasing variations in the monetary instrument. As we discuss below, instrument instability would constitute one possible form of monetary non-stationarity, which would be unstable drift of national money stocks.

Under a flexible exchange rate, the foreign central bank ignores unanticipated changes in the exchange rate, and so \( \theta_3^* = 0 \). In contrast, a fixed-exchange-rate version of the model corresponds to a situation in which \( \theta_3^* \) approaches an infinite value. We consider only these "pure" exchange-rate policies of flexible or fixed exchange rates and do not treat \( \theta_3^* \) as a choice variable that the central banks set optimally. For analyses of the optimal conditioning of monetary policy on exchange-rate innovations, see Benavie (1983) and Benavie and Froyen (1992).

Under either exchange-rate regime, the central banks seek to minimize loss functions given by

\[
(7a) \quad L = \kappa_1 \text{Var}[(p_t - E_{t-1}p_t) - \gamma (p_t^c - E_{t-1}p_t^c)] + \kappa_2 \text{Var}(E_t p_{t+1}^c - p_t^c)
\]

\[
(7b) \quad L^* = \kappa_1^* \text{Var}[(p_t^* - E_{t-1}p_t^*) - \gamma (p_t^{*c} - E_{t-1}p_t^{*c})] + \kappa_2^* \text{Var}(E_t p_{t+1}^{*c} - p_t^{*c}),
\]

where \( \kappa_1, \kappa_2, \kappa_1^* \) and \( \kappa_2^* \) are positive weights. These policy loss functions are adaptations of those proposed by Goodfriend in a closed-economy context and used by Daniels and VanHoose in an open-economy setting. In each nation, the central bank seeks to minimize a two-part objective. The first part is the variance of a linear combination of unexpected home price changes and unexpected changes in the home CPI. Based on equations (5), minimizing these variances minimizes the contributions of unexpected home price and CPI changes to the aggregate supply of output in each country. Consequently, as in Goodfriend, each central bank desires to accomplish "supply smoothing," because minimizing the variances of \( (p_t - E_{t-1}p_t) - \gamma (p_t^c E_{t-1}p_t^c) \) and \( (p_t^* - E_{t-1}p_t^*) - \gamma (p_t^{*c} - E_{t-1}p_t^{*c}) \) would, from (5), minimize the variances of the nations' output levels (which, in our stylized model, also correspond to expected squared deviations of national output levels from their full-information values). In contrast to Goodfriend, however, the fact that firms compute real wages in terms of home prices while workers calculate real wages in terms of CPIs implies that supply smoothing requires minimizing the variance of a linear combination of the home price and the CPI.
As in Goodfriend, the second part of each central bank loss function reflects each central bank’s desire to save private agents the costs that they would need to incur to index financial contracts to protect themselves against anticipated CPI inflation. Note that for \( \gamma = 0 \), these loss functions reduce to those considered by Daniels and VanHoose.

Naturally, the specific form of these loss functions shapes many of our results. For instance, as Balke and Emery (1994) have pointed out, somewhat different conclusions would follow if we were to consider intertemporal smoothing of inflation rates. As in Goodfriend’s original analysis, however, the crucial feature of the loss functions is that they imply that policymakers must confront an intertemporal tradeoff. This feature of the model thereby forces the monetary authorities to confront the stationarity issue as they formulate their optimal policies. Nevertheless, as noted by VanHoose (1989), the loss functions in (7) indicate that the central banks engage in intertemporal price smoothing on a rolling, period-by-period basis, rather than seeking to minimize the variance of the price level over an infinite horizon. If central banks actually pursued the latter objective, then the nonstationary policies that we observe would never be optimal, because they would yield infinite unconditional variances of prices.

To solve the model, we use equilibrium conditions for the domestic output and money markets and for the foreign output and money markets to obtain solutions for \( p_t, p_t^*, r_t, \) and \( e_t \). Proposed solutions for these variables are expressed as linear functions of the lagged money stocks, \( m_{t-1} \) and \( m_{t-1}^* \), the lagged money stock innovations, \( (m_{t-1} - E_{t-2} m_{t-1}) \) and \( (m_{t-1}^* - E_{t-2} m_{t-1}^*) \), and the exogenous domestic and foreign expenditure and money demand shocks. It is then possible to construct four-equation solution systems that may be solved for values of the undetermined coefficients in these proposed solutions. Using these solutions, we compute reduced-form expressions for price, exchange-rate, and CPI prediction errors and for anticipated price inflation, CPI inflation, and exchange-rate depreciation. These expressions then are substituted into the loss functions in equations (7). Because this procedure is lengthy and cumbersome, we outline the key steps in a separate appendix that is available upon request.

3. Optimal Monetary Policies in Alternative Policy Regimes
To determine the broadest possible set of circumstances under which non-stationarity of money and prices arises from the interactions of interdependent monetary policies, we consider three types of policy regimes. In the first, the foreign monetary authority fixes its country’s exchange rate, and both nations’ authorities conduct uncoordinated policies. In the second, the foreign authority permits its nation’s exchange rate to float, and the two authorities again do not coordinate their policies. Finally, we consider the implications of monetary policy coordination with either a fixed or floating exchange rate.

A Fixed Exchange Rate
In the fixed-exchange-rate version of the model, the foreign monetary authority maintains the exchange rate peg and adjusts its money stock as needed to keep the exchange rate unchanged at time \( t \), so that \( \theta_3^* \) approaches infinity. In this policy regime, the foreign monetary authority eliminates unanticipated changes in the exchange rate, and so \( e_t - E_{t-1} e_t = 0 \). The foreign instruments \( \theta_1^* \) and \( \theta_2^* \) are therefore moot.
This representation of a fixed exchange rate is common in the literature [for instance, see Gros and Lane (1992) and references therein]. It does not, however, rule out anticipated exchange-rate depreciation or appreciation. The domestic monetary authority chooses \( \theta_1 \) and \( \theta_2 \) to minimize its loss. Because the domestic authority may pursue a nonstationary monetary policy, the two currencies' values may diverge from the established parity during time \( t + 1 \). This can be seen from the fact that anticipated currency depreciation is equal to \( E_t e_{t+1} - e_t = \theta_1 (1 - \theta_2) \theta_2 (r_t - E_t - 1 r_t) \), which clearly is equal to zero only if \( \theta_2 = 1 \), or if the domestic money stock follows a stationary path.

Because the foreign central bank adjusts its money stock endogenously to eliminate exchange-rate innovations with a fixed exchange rate, \( \theta_1^{\ast} \) and \( \theta_2^{\ast} \) are irrelevant parameters for time \( t \) outcomes. Consequently, \( \theta_1^{\ast} \) and \( \theta_2^{\ast} \) have no bearing on the variance of unanticipated changes in the foreign price level and the foreign CPI [the first element of the loss function given in equation (7b)]. This is similar to the result that arises in Benavie and Froyen’s (1988) analysis of a pure nominal-interest-rate peg. In a closed-economy setting, Benavie and Froyen show that a central bank policy of pegging a nominal interest rate fails to ensure stationarity of a nation's money stock, because stationarity is irrelevant when the scope of the policy problem is limited to a single period. If the monetary authorities were interested only in stabilizing output by minimizing unanticipated price and CPI changes, then the foreign central bank’s pursuit of a fixed exchange rate would, as in Benavie and Froyen’s analysis, fail to tie the foreign money stock to a stationary path. Each period the foreign money stock would adjust as needed in light of the disturbances that might occur during time \( t \). In such a setting with a single-period smoothing objective and a fixed exchange rate, the stationarity of the foreign money stock would be a non-issue.

In our model, however, because both central banks are concerned as well about the variances of anticipated CPI inflation rates, there is an intertemporal objective. Nonstationary paths for national money stocks would induce anticipated CPI inflation volatility, and so monetary stationarity (or lack thereof) is a relevant issue. Specifically, the setting of \( \theta_1^{\ast} \) and \( \theta_2^{\ast} \) matters to the foreign authority, because these policy parameters influence the magnitude of \( Var(E_t p_{t+1}^{\ast} - p_t^{\ast}) \). The conclusion that exchange-rate pegging with intertemporal objectives makes stationarity a relevant issue is analogous to that reached by Goodfriend and by Cover and Schutte (1990) in the case of interest-rate pegging.

We consider the domestic country first and begin by solving the model for the variance of \( (p_t - E_{t-1} p_t) - \gamma (p_t^c - E_t p_t^c) \), which is equal to

\[
(8) \; \text{Var}[(p_t - E_{t-1} p_t) - \gamma (p_t^c - E_t p_t^c)] = \phi_3^{-2} \left[ (\gamma \beta_7 - \beta_1 + \phi_1 A)^2 \sigma_\eta^2 + (\gamma \beta_8 - \beta_2 + \phi_2 A)^2 \sigma_\eta^2 + \phi_3^2 A^2 \sigma_\epsilon^2 \right],
\]

where

\[
A \equiv (\beta_1 + \beta_2) d_1 \Gamma (1 - \gamma) \Delta_1^{-1},
\]

\[
\Delta_1 \equiv (\beta_1 + \beta_2) d_1 (1 + \beta_4 - \beta_5) + (\theta_1 + b)(\beta_1^2 + \beta_2^2),
\]

\[
\Gamma \equiv 1 - \theta_1 (1 - \theta_2),
\]

\[
\phi_1 \equiv \beta_1 (1 + \beta_4) - \beta_2 \beta_5,
\]
\[ \phi_2 \equiv \beta_2 (1 + \beta_4) - \beta_1 \beta_5, \]
\[ \phi_3 \equiv (\beta_1^2 - \beta_2^2), \]
\[ \beta_1 \equiv a[(1 - \alpha \gamma) + c(1 - \alpha) \gamma] + d_1 \alpha + d_2, \]
\[ \beta_2 \equiv a[(1 - \alpha) \gamma + c(1 - \alpha \gamma)] - d_1 (1 - \alpha) + d_2, \]
\[ \beta_3 \equiv a(1 - \alpha)(1 + c) \gamma - d_1 (1 - \alpha) + d_2, \]
\[ \beta_4 \equiv 1 + a(1 - \alpha \gamma), \]
\[ \beta_5 \equiv a(1 - \alpha) \gamma, \]
\[ \beta_6 \equiv a(1 - \alpha)(1 + c) \gamma + d_2 \alpha + d_2, \]
\[ \beta_7 \equiv a\beta_1 + (1 - \alpha) \beta_2, \]
\[ \beta_8 \equiv a\beta_2 + (1 - \alpha) \beta_1. \]

Minimizing (8) with respect to \( \hat{A} \) then yields \( \hat{A} = (\phi_1^2 \sigma_{\eta}^2 + \phi_2^2 \sigma_{\eta}^2 + \phi_3^2 \sigma_{\varepsilon}^2)^{-1} \times \[
\phi_1 (\beta_1 - \beta_7 \gamma) \sigma_{\eta}^2 + \phi_2 (\beta_2 - \beta_8 \gamma) \sigma_{\eta}^2 \]. \]

In addition, solving for the variance of domestic anticipated inflation yields

\[ (9) \text{Var}(E_t p_{t+1}^e - p_t^e) = \phi_3^{-2} [(\phi_1 B - \beta_7)^2 \sigma_{\eta}^2 + (\phi_2 B - \beta_8)^2 \sigma_{\eta}^2 + \phi_3^2 B^2 \sigma_{\varepsilon}^2], \]

where \( B = [\phi_3^2(1 - \theta_2) \theta_1 + (\beta_1 + \beta_2) d_1 \Gamma] \Delta_1^{-1} \). Minimizing (9) with respect to \( B \) yields \( \hat{B} = (\phi_1^2 \sigma_{\eta}^2 + \phi_2^2 \sigma_{\eta}^2 + \phi_3^2 \sigma_{\varepsilon}^2)^{-1} (\phi_1 \beta_7 \sigma_{\eta}^2 + \phi_2 \beta_8 \sigma_{\eta}^2) \). Setting \( \hat{A} = \hat{A} \) and \( B = \hat{B} \) and solving jointly for \( \theta_1 \) and \( \theta_2 \) then produces the following solution for the optimal setting for \( \theta_2 \) (the solution for the optimal value for \( \theta_1 \) is an even more lengthy expression that we do not report here but which is provided in the mathematical appendix):

\[ (10) \hat{\theta}_2 = 1 - (\beta_1 - \beta_2) d_1 (1 - \alpha) (\phi_1 \sigma_{\eta}^2 + \phi_2 \sigma_{\eta}^2) \times \left[ \left\{ (\beta_1 - \beta_2) [\phi_1 (\beta_1 - \beta_7 \gamma) \sigma_{\eta}^2 - \phi_2 (\beta_2 - \beta_8 \gamma) \sigma_{\eta}^2] \right\} - (\beta_1 - \beta_2) d_1 (1 - \alpha) (\phi_1 \sigma_{\eta}^2 + \phi_2 \sigma_{\eta}^2) \right] b - ((1 - \gamma) \phi_1 - [2 + a(1 - \gamma)](\beta_1 - \beta_7 \gamma) \phi_1 \sigma_{\eta}^2 + (1 - \gamma) \phi_2 \sigma_{\eta}^2) \left[ \right] - 1. \]

Hence, \( \theta_2 \neq 1 \), and so it is optimal for the domestic money stock to follow a non-stationary path. The reason is that the domestic authority must minimize a weighted sum of domestic price, foreign price, and exchange rate prediction errors if it is to smooth output; in addition, the domestic authority simultaneously seeks to minimize anticipated domestic CPI inflation. But the authority has only two instruments that it may aim independently at these various components of its loss function. This requires sacrificing stationarity of money and prices.

The variance of foreign anticipated inflation is

\[ (11) \text{Var}(E_t p_{t+1}^e - p_t^e) = [\phi_3 d_1 (1 - \gamma)]^{-2} \left\{ [d_1 (1 - \gamma) \beta_7 - \psi^* \phi_2 A]^2 \sigma_{\eta}^2 + [d_1 (1 - \gamma) \beta_8 - \psi^* \phi_1 A]^2 \sigma_{\eta}^2 + (\psi^*)^2 \phi_3^2 A^2 \sigma_{\varepsilon}^2 \right\}, \]
where $\psi^* \equiv (\Gamma^*) - 1[d_1(\beta_1 - \beta_2 - d_1)\theta_1^*(1 - \theta_1^*\gamma)]$ and $\Gamma^* \equiv 1 - \theta_1^*(1 - \theta_1^*\gamma)$. Minimizing (11) with respect to $\psi^*$ then yields $\psi^* = [\theta_1(\beta_1 - \beta_7\gamma)\sigma_1^2 + \theta_2(\beta_2 - \beta_8\gamma)\sigma_2^2]^{-1}d_1(1 - \gamma)(\phi_1\beta_7\sigma_1^2 + \phi_2\beta_8\sigma_2^2)$. Setting $\psi^* = \tilde{\psi}^*$ and using the solution for $A$ implies that $\theta_1^*$ and $\theta_2^*$ must satisfy

$$
(12) \hat{\theta}_1^* (1 - \hat{\theta}_2^*) = d_1[\theta_1(\beta_1 + \beta_8)\sigma_1^2 + \theta_2(\beta_2 + \beta_7)\sigma_2^2] \times \{d_1[\theta_1(\beta_1 + \beta_8)\sigma_1^2 + \theta_2(\beta_2 + \beta_7)\sigma_2^2] + (\beta_1 - \beta_2)[\phi_1(\beta_1 - \beta_7\gamma)\sigma_1^2 + \phi_2(\beta_2 - \beta_8\gamma)\sigma_2^2]^{-1} \times d_1(1 - \gamma)(\phi_1\beta_7\sigma_1^2 + \phi_2\beta_8\sigma_2^2).$$

The foreign monetary authority cannot influence the variance of the foreign price prediction error under a fixed exchange rate, and so both $\theta_1^*$ and $\theta_2^*$ are available to minimize the variance of anticipated inflation. Consequently, equation (12) gives a locus of combinations of the two instruments that is consistent with this objective. Clearly, $\hat{\theta}_2 = 1$ is not on this locus. It follows that money and price-level non-stationarity is optimal as well for the foreign monetary authority.

It can be shown that the solutions for the optimal values of $\theta_2$ and $\theta_2^*$ can assume values below unity for various ranges of parameter values. As emphasized by Hetzel (1995), optimal values of $\theta_2$ and $\theta_2^*$ that lie below one imply the sort of "bygones" base drift that typically occurs, in which monetary and price-level increases induced by random shocks permanently shift money and prices on to new, higher trend paths. As Hetzel has noted, the frameworks examined by Goodfriend and Barro and, in all but one case, by VanHoose yield "pay-later" drift of money and prices. This form of drift, rarely observed since the end of the gold standard, entails more-than-offsetting reductions in money and prices in a subsequent period following contemporaneous increases in money and prices. Pay-later base drift also can, if optimal values of $\theta_2$ or $\theta_2^*$ exceed two, yield a policy setting that entails instrument instability.

Hetzel corrected this failing of earlier published work on the stationarity issue by considering a downward-sloping IS schedule, as opposed to the horizontal IS schedule (Fisher equation) of Goodfriend. This alteration of Goodfriend's model endogenizes the real interest rate, thereby reducing the extent of subsequent money-stock adjustment following a contemporaneous price-level change. Therefore, "bygones" base drift is an optimal policy response in Hetzel's model.

Our framework, as well as the narrower model considered in Daniels and VanHoose (1995), also produces "bygones" base drift as a possible policy outcome. There are two reasons for this. One is that, like Hetzel, these models include a downward-sloping IS schedule. Consequently, the real interest rates in both countries are endogenous "shock absorbers" that give the central banks increased flexibility in determining the optimal intertemporal paths of money and prices. Another reason, however, is that the real exchange rate performs an analogous role in the two-country setting. This also reduces the extent to which the central banks are constrained in their ability to adjust the money and price paths in an effort to smooth prices across both periods. For instance, substituting the definitions of the $\beta$ coefficients in equation (12) indicates that for $\theta_1^* > 0$, $\theta_2^* < 1$ holds for greater ranges of parameter values as $d_2$, the parameter governing the sensitivity of expenditures to real-exchange-rate variations, increases in magnitude. Consequently, monetary policy responses to disturbance-induced interest-rate innovations have stronger effects, via endogenous responses in the real exchange rate, thereby enhancing the potential for such policy responses to stabilize output and anticipated CPI inflation. This gives the foreign monetary authority the flexibility to follow a policy of bygones drift as it pursues its multipart objective.
A Floating Exchange Rate

In this section, we consider the floating-exchange-rate version of the model, in which \( \theta_2 \) is equal to zero. Conducting policy analysis in this version of the model is very cumbersome, because endogenous variations in the exchange rate enlarge the scope for feedback effects between the two economies. Daniels and VanHoose (1995) computed reduced-form solutions for all four policy parameters under the restrictive assumptions \( \gamma = d_2 = a = 0 \) and \( \eta_t = \xi_t = 0 \), and they verified that both \( \theta_2 \) and \( \theta_3 \) are not equal to one for this special case of the model. Here, we consider \( \gamma, d_2, \) and \( a \) as nonzero but model asymmetric shocks as \( \eta_t = -\eta_t \) and \( \xi_t = -\xi_t \), with \( \sigma_{\eta_t}^2 = \sigma_{\xi_t}^2 \) and \( \sigma_{\eta_t}^2 = \sigma_{\xi_t}^2 \). Therefore, these disturbances represent relative shifts in relative goods and money demands, respectively. In this situation, both central banks make identical parameter choices to smooth supply and anticipated CPI inflation, and so \( \theta_1 = \theta_1^* \) and \( \theta_2 = \theta_2^* \).

Here, the variance of \( (p_t - E_{t-1}p_t) - \gamma(p_t^e - E_{t-1}p_t^e) \), which is relevant for domestic output smoothing, is equal to

\[
\text{Var}[(p_t - E_{t-1}p_t) - \gamma(p_t^e - E_{t-1}p_t^e)] = \lambda_3^{-2} [A\lambda_1 - (1 - \gamma(2\alpha - 1))]^2 \sigma_{\xi_t}^2 + A^2 \sigma_{\eta_t}^2,
\]

and the variance of domestic anticipated inflation is given by

\[
\text{Var}(E_t p_t^e - E_{t-1}p_t^e) = \lambda_3^{-2} [B\lambda_1 + 2(1 - \alpha)]^2 \sigma_{\xi_t}^2 + B^2 \sigma_{\eta_t}^2,
\]

where, for this policy problem, we define \( A \) and \( B \) as

\[
A \equiv \Delta_2^{-1} \{[1 - \gamma(2\alpha - 1)](\theta_1 + b) + 2\gamma(1 - \alpha)\}
\]

and

\[
B \equiv \Delta_2^{-1} ((2\alpha - 1)\{1 - a(1 - \gamma)\}\theta_1(1 - \theta_2) - (\theta_1 + b)) + 2(1 + \alpha)(1 + a),
\]

and

\[
\Delta_2 \equiv \lambda_1 [(\theta_1 + b) - 2a(1 - \alpha)\Gamma + \lambda_2 \lambda_3],
\]

\[
\lambda_1 \equiv a(1 + c)[1 - \gamma(2\alpha - 1)] + d_1(2\alpha - 1) + 2d_2
\]

\[
\lambda_2 \equiv d_1(2\alpha - 1) + 2[a(1 + c)(1 - \alpha)\gamma + d_2],
\]

\[
\lambda_3 \equiv (1 + a) - a\gamma(2\alpha - 1).
\]

Minimizing (13) with respect to \( A \) yields \( \hat{A} = (\lambda_3^{-4} \sigma_{\eta_t}^2 + \lambda_3^{-2} \sigma_{\xi_t}^2)^{-1} [1 - \gamma(2\alpha - 1)]^2 \lambda_1 \sigma_{\xi_t}^2 \), and minimizing (14) with respect to \( B \) yields \( \hat{B} = (\lambda_3^{-4} \sigma_{\eta_t}^2 + \lambda_3^{-2} \sigma_{\xi_t}^2) - 1[(2\alpha - 1)\lambda_1 \sigma_{\xi_t}^2] \). Setting \( A = \hat{A} \) and \( B = \hat{B} \) produces the following solution for the optimal value of \( \theta_2 \):

\[
\tilde{\theta}_2 = \tilde{\theta}_2^* = 1 - ((2\alpha - 1)(1 - \gamma)\{\lambda_3[2\gamma(1 - \alpha) + (2\alpha - 1)(1 - \gamma) b] \sigma_{\eta_t}^2 - \lambda_1(1 - \gamma)[d_1(2\alpha - 1) + 2d_2] \sigma_{\xi_t}^2\})^{-1} \times 2(1 - \alpha)\lambda_3(2\alpha - 1)(1 - \gamma)\sigma_{\eta_t}^2.
\]

Hence, with a floating exchange rate non-stationarity is the optimal policy for both central banks. Each monetary authority seeks to smooth three variables: the home price-prediction error, the home CPI-prediction error, and anticipated home CPI inflation. But each authority possesses only two instruments that it may use independently in an effort to accomplish its three-part objective. For the domestic central bank, these are its responses to domestic interest-rate innovations \( \theta_1 \) and the extent to which the path of the domestic money stock departs from its previous trend (implied by the extent to which \( \theta_2 \) departs from a value of unity). Because the domestic authority cannot use interest-rate responses alone to offset both the variance of domestic price prediction errors and the variance of domestic CPI prediction errors, it must allow the domestic money stock to depart from its past trend to
induce a change in expectations of expected future prices. This policy mix causes some variability in anticipated CPI inflation but is consistent with the domestic central bank's overall objective. The same reasoning applies for the foreign authority.

Note that in our example with relative demand shocks, equation (15) indicates that stationarity is optimal in the absence of expenditure disturbances \( (\sigma_n^2 = 0) \). The reason is that if a positive domestic money demand disturbance is accompanied by a negative foreign money demand shock of equal magnitude, then the central banks' optimal interest-rate conditioning of their monetary policies (their identical choices for \( \theta_1 \) and \( \theta_1^* \)) entails responding in equal measure, but in opposite directions, to these diametrically opposed money demand disturbances. With uncovered interest parity, these interest-rate responses jointly mitigate the effects that such money-market shocks otherwise would have on price- and CPI-prediction errors. The authorities thereby are free to minimize the variance of CPI inflation rates via stationary money paths.

Consider the polar case, however, in which there are relative expenditure shocks \( (\sigma_n^2 > 0) \) but no money demand disturbances \( (\sigma_x^2 = 0) \). With a relative shift in expenditures in favor of the domestic country \( (\eta^*_x = -\eta_x < 0) \), the real exchange rate will change even though the interest-rate responses of the authorities' monetary policies stabilize as fully as possible in the face of such shocks. This adjustment in the real terms of trade induces contemporaneous price-level and CPI innovations that can be smoothed further only if the central banks depart from stationary paths for money and prices. Consequently, \( \theta_2 \) and \( \theta_2^* \) are not equal to unity in this case. Indeed, for this special case in which only relative expenditure shocks occur, bygones drift \( (\theta_2 < 1 \text{ and } \theta_2^* < 1) \) unambiguously is the optimal policy.

Monetary Policy Coordination

Would coordination of monetary policies make stationary policies optimal? It is easy, though cumbersome analytically, to prove that the answer to this question is no. Conceptually, this result is easy to understand, however. This is particularly true for the fixed-exchange-rate version of the model. When the foreign monetary authority pegs the exchange rate, it can influence only the variance of foreign anticipated CPI inflation. At the same time, the variances of domestic price-prediction errors, CPI-prediction errors, and anticipated CPI inflation and the variances of foreign price- and CPI-prediction errors depend only on the domestic policy instruments. As a result, coordination does not permit the foreign central bank to aim its instruments toward minimizing the domestic loss. Even though policy coordination expands the number of targets toward which both authorities direct their instruments, it does not solve the instrument shortage problem that yields monetary and price-level non-trend-stationarities.

Coordination also fails to overcome the essential instrument shortage problem in the floating-exchange-rate version of the model. If the exchange rate floats, then both policymakers can influence all components of both domestic and foreign objectives. Nevertheless, each central bank essentially seeks to minimize the variances of home price prediction errors, home CPI prediction errors, and home anticipated CPI inflation. This means that there ultimately are a total of six components in the two authorities' objective functions. Even if the policymakers coordinate by aiming all four instruments \( (\theta_1, \theta_1^*, \text{and } \theta_2^*) \) toward minimizing a six-part, joint loss function, the instrument shortage remains even
with a floating exchange rate. The result is base drift and price-level non-trend-stationarity in each country.

4. Are Base Drift and Price-Level Non-Trend-Stationarity Always Optimal?

Will optimal monetary policymaking in interdependent economies always yield non-stationary paths for money and prices? Daniels and VanHoose have shown that one policy approach that could eliminate base drift and price-level non-trend-stationarity entails conditioning home monetary policies on lagged foreign monetary innovations. If each central bank were to expand its monetary policy rule in this fashion, it would condition its policy choices on the dynamic behavior of the other nation's prices, effectively expanding its set of independent policy instruments. The result would be stationarity of money and prices in both economies with either uncoordinated or coordinated policies.

In the context of the present model, there are only two additional circumstances in which the nations' money stocks and price levels will be stationary when monetary policies are optimally determined. One is if we consider the amended loss functions

\[(7a') L = \varphi_1 \text{Var}(p^c - E_{t-1}p^c_t) + \varphi_2 \text{Var}(E_{t}p_{t+1}^c - p^c_t)\]

\[(7b') L^* = \varphi_1^* \text{Var}(p^{c*}_t - E_{t-1}p^{c*}_t) + \varphi_2^* \text{Var}(E_{t}p_{t+1}^{c*} - p^{c*}_t)\]

which imply that the central banks desire to minimize both the variances of CPI prediction errors and of anticipated CPI inflation. This loss specification is not consistent with an output-smoothing goal in our model. Nevertheless, equations (7') might be motivated by appealing to a broad central bank interest in CPI smoothing for its own sake. Alternatively, the loss functions in (7') would be consistent with a model in which both workers and firms value the real wage in terms of the CPI, so that minimizing the variance of CPI prediction errors would stabilize output.

In both exchange-rate regimes, following steps analogous to those discussed above yields \(\theta_2 = \theta_2^* = 1\) as the optimal policy parameter choices. When each central bank aims to smooth only its nation's CPI, trend-stationarity of money and prices emerges as the optimal policy. The economic intuition behind this result is straightforward. Under the policy loss functions in equations (7) of the basic model, the differential computations of real wages by firms and workers cause the supplies of output in each country to depend on a weighted average of the home price and the CPI. If supply smoothing is the key justification for the central banks to care about the variance of unexpected price or CPI prediction errors, then central banks should seek to minimize the variance of an asymmetrical linear combination of the home price and CPI. This asymmetrical objective yields a shortfall in the number of policy instruments relative to the effective number of objectives, given that at each central bank there are three objectives (the variances of unexpected home price prediction errors, of CPI prediction errors, and of anticipated CPI inflation) but only two independent policy instruments (\(\theta_1\) and \(\theta_2\) in the case of the domestic central bank and \(\theta_1^*\) and \(\theta_2^*\) in the case of the foreign central bank and a floating exchange rate). In turn, this shortage of instruments relative to objectives requires each central bank to sacrifice the trend-stationarity of money and prices.

In contrast, under the revised loss functions given by equations (7'), both central banks seek to smooth their nation's CPIs. The resulting symmetry of both parts of their price-smoothing objectives removes tension between the effective number of objectives and the number of independent policy

instruments. To see this, note that the domestic and foreign losses in (7') ultimately may be rewritten as
\[ L = \kappa_1 \text{Var}(p^*_L - E_{t-1}p^*_L) + \kappa_2 \text{Var}[(p^*_L - E_{t-1}p^*_L) - (1 - \theta_2)\theta_1(r_t - E_{t-1}r_t)] \]
and \[ L^* = \kappa_1^* \text{Var}(p^*_L - E_{t-1}p^*_L) + \kappa_2^* \text{Var}[(p^*_L - E_{t-1}p^*_L) - (1 - \theta_2^*)\theta_1^*(r^*_L - E_{t-1}r^*_L)]. \] Clearly, setting \( \theta_2 = 1 \) and \( \theta_2^* = 1 \) eliminates the distinction between the objectives of minimizing the variance of CPI prediction errors and the variance of anticipated inflation. The central banks then may set \( \theta_1 \) and \( \theta_1^* \) so as to minimize the variances of CPI prediction errors.

The loss functions in equations (7) are appropriate for a setting, such as that captured by our model, in which international integration of national economies is sufficiently incomplete that home firms value real wages in terms of home prices. Although the world economy certainly has become more integrated in recent years, the degrees of factor mobility and output-market integration that would induce central banks to adopt the symmetrical, CPI-based objectives in equations (7') have not yet been achieved.

A second circumstance under which stationary paths for money stocks and price levels would be optimal in our model is if we introduce sufficient symmetries into our framework. For instance, consider the case in which \( \eta^*_t = \eta_t \) and \( \xi^*_t = \xi_t \), with \( \sigma_{\eta^*}^2 = \sigma_{\eta}^2 \) and \( \sigma_{\xi^*}^2 = \sigma_{\xi}^2 \), so that both expenditure and money demand disturbances are common world shocks. In this case, if we use the general objective functions given by equations (7), then the optimal policy settings for the central banks are \( \theta_2 = \theta_2^* = 1 \). The reason is that, because both countries are identical, optimal policy responses to common shocks are identical. Furthermore, these responses tend to stabilize both nations’ prices, so that policy actions by each authority benefit both nations simultaneously. Essentially, if such complete symmetry exists, then the two countries in our framework effectively function as a closed-economy system that actions of the monetary authorities stabilize, whether or not they are coordinated.

Therefore, our framework indicates that nonstationary paths for money and prices are likely to be the optimal coordinated or uncoordinated policies for central banks in nations that have imperfectly integrated output markets and that experience asymmetric disturbances. This conclusion follows whether or not such nations fix their exchange rates. Because incomplete market integration and country-specific disturbances characterize modern economies, our conclusion is that base drift and price-level non-trend-stationarities are unavoidable by-products in an international setting.

Throughout, we have considered only pure exchange-rate policies for both authorities. A natural question concerns how our results might be affected by considering the optimal determination of \( \theta^*_3 \) and adding an analogous \( \theta_3 \) parameter to the domestic authority’s policy rule. On the surface, it seems that this might expand the menu of available policy instruments, thereby overcoming the instrument shortage problem that the authorities face. In fact, however, we would obtain the same general results, though different final-form solutions, in the context of our model. With uncovered interest parity, once the authorities have conditioned their policies on the information content of home interest-rate innovations by choosing \( \theta_1 \) and \( \theta_1^* \) optimally, there is no policy gain that may be achieved by conditioning their policies on exchange-rate innovations. In other words, additional \( \theta_3 \) and \( \theta_3^* \) parameters would be redundant policy parameters if uncovered interest parity holds, as it does in the present framework.
This reasoning suggests that in a setting in which uncovered interest parity does not apply, conditioning monetary policy on exchange-rate innovations may permit central banks to pursue stationary policies. In such an environment, $\theta_3$ and $\theta_3^*$ parameters could be aimed independently at the policy objectives, mitigating the instrument shortage. Exploring this possibility could prove fruitful, but it is beyond the bounds of our framework.

5. Conclusion

In this paper, we have shown that in an open-economy setting, base drift and price-level non-trend-stationarity in general are likely to emerge as optimal monetary policy outcomes. We have demonstrated that when uncovered interest parity holds and when monetary authorities seek to smooth the supply of real output, this conclusion follows under either fixed or flexible exchange rates and with or without monetary policy coordination.

Trend-stationary monetary policies are unambiguously optimal only under two circumstances. One arises if central banks seek to minimize unanticipated and anticipated variability of their nations' CPIs, either because the central banks have no interest in output smoothing or because factor markets are fully integrated, so that CPI smoothing is tantamount to output smoothing. If CPI smoothing is the objective of central banks, then there is no instrument shortage problem for central banks whether or not they coordinate their policies. Consequently, their optimal policy choices yield stationary paths for money and prices. The other circumstance arises if countries experience only common disturbances. For identical countries, this leads to common policy responses that effectively yield Goodfriend's closed-economy result that price-level trend-stationarity is the optimal policy.

Throughout the literature, researchers typically hypothesize that central banks in interdependent economies seek to minimize unanticipated output volatility and variations in anticipated CPI inflation. This paper has shown that non-trend-stationarity monetary and price paths generally emerge as optimal policies when central banks in open economies possess these objectives. Furthermore, the bygones form of base drift actually experienced by many world economies can arise in such a setting, even if central banks have no ultimate desires to smooth interest rates, exchange rates, or monetary aggregates.

References


Appendix
The aggregate supply functions in equations (5) stem from the following relationships:
(A.1) $y_t = a_0 l_t, \quad y_t^* = a_0 l_t^*$, $0 < a_0 < 1$ ;
(A.2) $l_t^d = -a_1 (w_t - p_t), \quad l_t^d = -a_1 (w_t^* - p_t^*), \quad a_1 \equiv (1 - a_0) - 1$;
(A.3) $l_t^s = \omega (w_t - p_t^*), \quad l_t^s = \omega (w_t^* - p_t^*), \quad \omega > 0$;
(A.4) $\hat{w}_t = (a_1 + \omega)^{-1}[(a_1 + a\omega)p_t + \omega(1 - a)(p_t^* + e_t)],$

$\hat{w}_t^* = (a_1 + \omega)^{-1}[(a_1 + a\omega)p_t^* + \omega(1 - a)(p_t^* - e_t)];$

(A.5) $w_t = E_{t-1}\hat{w}_t + \gamma(p_t^e - E_{t-1}p_t^e), w_t^* = E_{t-1}\hat{w}_t^* + \gamma(p_t^{e^*} - E_{t-1}p_t^{e^*});$

where $l_t$ denotes the log of employment and $w_t$ denotes the log of the nominal wage rate.

Equations (A.1) are firm production functions, and equations (A.2) are the implied labor demand schedules, in which intercepts are suppressed as an analytical simplification. Equations (A.3) are labor supply schedules, in which workers compute their real wages in terms of their home CPIs. Equations (A.4) are the market-clearing, full-information wages, while equations (A.5) are the contract wages. Equations (5) then follow after substituting the expectations of (A.4) into (A.5), substituting the results into (A.2) and (A.1), defining $a \equiv a_0a_1$, and considering the limiting case in which $a$ approaches a value of zero.