Marquette University e-Publications@Marquette

Finance Faculty Research and Publications

Finance, Department of

Winter 2020

Early Termination of Small Loans in the Multifamily Mortgage Market

Anthony Pennington-Cross *Marquette University*, anthony.pennington-cross@marquette.edu

Brent C. Smith Virginia Commonwealth University

Follow this and additional works at: https://epublications.marquette.edu/fin_fac

Part of the Finance and Financial Management Commons

Recommended Citation

Pennington-Cross, Anthony and Smith, Brent C., "Early Termination of Small Loans in the Multifamily Mortgage Market" (2020). *Finance Faculty Research and Publications*. 139. https://epublications.marquette.edu/fin_fac/139 **Marquette University**

e-Publications@Marquette

Finance Faculty Research and Publications/College of Business Administration

This paper is NOT THE PUBLISHED VERSION.

Access the published version via the link in the citation below.

Real Estate Economics, Vol. 48, No. 4 (Winter 2020): 1198-1233. <u>DOI</u>. This article is © Wiley and permission has been granted for this version to appear in <u>e-Publications@Marquette</u>. Wiley does not grant permission for this article to be further copied/distributed or hosted elsewhere without the express permission from Wiley.

Early Termination of Small Loans in the Multifamily Mortgage Market.

Anthony Pennington-Cross

Marquette University, College of Business, 303C Straz Hall, PO Box 1881, Milwaukee, WI Brent C. Smith

Kornblau Professor of Real Estate, School of Business, Virginia Commonwealth University 301 West Main, Suite 4105, Richmond, VA

Abstract

This article uses micro-level data on small (as defined by Fannie Mae) multifamily loans in the Fannie Mae loan portfolio to examine prepayment and default performance. The results document the importance of equity, as measured by the loan-to-value ratio, and contemporaneous property operating income relative to debt service obligations, as measured by the debt-to-income ratio. Our results indicate that the expiration of prepayment penalties and yield maintenance provisions lead to large spikes in prepayment and default. The results also illustrate that multifamily loans, as they are

not fully amortized, also have a substantial risk of both extension and default at term. The operating efficiency of the property, cash reserves and local economic conditions can also impact terminations.

Introduction

This article uses micro-level data on small (as defined by Fannie Mae) multifamily loans in the Fannie Mae loan portfolio to examine prepayment and default performance. The results document the importance of equity, as measured by the loan-to-value ratio, and contemporaneous property operating income relative to debt service obligations, as measured by the debt-to-income ratio. Our results indicate that the expiration of prepayment penalties and yield maintenance provisions lead to large spikes in prepayment and default. The results also illustrate that multifamily loans, as they are not fully amortized, also have a substantial risk of both extension and default at term. The operating efficiency of the property, cash reserves and local economic conditions can also impact terminations.

One of the most widely studied topics in the commercial mortgage finance literature is the early termination of loans through borrower default and prepayment. The sheer growth in commercial property debt over the last two decades is but one motivator. As of the 3rd quarter of 2015, outstanding commercial debt stood at \$2.76 trillion, and multifamily debt accounted for approximately 37% of that total, or \$1.2 trillion. Total outstanding multifamily debt has nearly doubled since early 2007 when it was approximately \$650 million. Loan purchase and securitization by Fannie Mae, Freddie Mac and private-label commercial mortgage-backed securities grew rapidly during the 1990s and accounted for more than half of the net growth in multifamily debt during that decade (Nothaft and Freund 2003). More recently, multifamily security issuance accounted for approximately 62% of all conduit and government-sponsored enterprise security issuance in 2011, compared to just 21% in 2006 (Heschmeyer 2012). Small loans (loans of less than \$5 million at origination on properties with 5–50 units) are an important segment of the multifamily Lending, small loans comprised approximately 27% of the total multifamily market by dollar volume and 81% by number of originations.

Researchers have long applied option-pricing methodologies to the analysis of commercial mortgages, including multifamily properties. Conceptually, if one could value the two embedded options, the value of the mortgage would be the difference between the option-free instrument and the joint value of the two options (LaCour-Little 2008). Ciochetti et al. (2003) find that an option framework is potentially useful for explaining early termination of commercial mortgages, relying on contemporaneous estimates of the loan to value (LTV) and debt coverage ratios (DCRs) as proxies for the value of the option to terminate the loan. But the evidence is not consistent. For example, Archer et al. (2002) find no evidence that initial LTV or the equity in the property has any impact on loan termination and only weak evidence that the property income relative to the debt service (DS) requirement (the measure of DCR) affects terminations. Ambrose and Sanders (2003) also fail to find a link between LTV and prepayment or default. Archer et al. (2002) hypothesize that initial LTV is ineffective in explaining early termination of commercial mortgages because it is endogenous to the terms of the mortgage. Rather it is a reflection of the lender's overall perception of risk in evaluating the potential loan. For example, lenders may require more equity when the property is viewed as more risky. If this type of tradeoff dominates underwriting, empirical results could even show that more equity is associated with more defaults. On the other hand, it is more difficult for a lender to require more income, as compared with

more equity, to cover the DS. It is not surprising, then, that Archer *et al.* (2002) find better empirical estimates for DCR than LTV. However, the initial DCR can also be manipulated to help meet underwriting standards for marginal applicants by adjusting the DS in order to meet the underwriting DCR requirement. This can be done by lengthening the amortization schedule of the loan, embedding no amortization features in a loan (interest-only payments) or lengthening the term of the loan.

Using loans held in portfolio and securitized by Fannie Mae, we examine the termination patterns of small loans over the years 2005 through 2011. We include seasoned loans and newly originated loans in the analyses and estimate quarterly conditional probabilities of default and prepayment for approximately 12,670 fixed rate loans with balloon payments due at term (not fully amortizing). In addition to examining the role of LTV and DCR, we estimate the effects of property characteristics, loan terms and economic factors on the incidence of repayment and default. The termination of the small loans is highly nonlinear and clumped around the expiration of prepayment penalties and yield maintenance provisions. This is consistent with the findings of Kelly and Slawson (2001) in a study of the impact that prepayment penalties have on the timing of prepayment. However, these mechanisms, which are used to make prepayment more predictable for the lender/investor, come with a cost. Near the expiration date of a loan with a prepayment penalty, the probability of default more than triples; when a yield maintenance provision expires the probability of default increases more than nine times.

There is also substantial extension and default risk when a loan is scheduled to end at term and a balloon payment (a lump sum payment to pay off the outstanding balance) is due. Results indicate that just under one half of outstanding loans at term are paid off. In addition, default probabilities spike up by ninefold at term and remain elevated thereafter. In short, if a mortgage extends because it cannot be terminated by paying off the loan at term there is likely a deficiency that leads to an increased chance of default thereafter.

Our general findings suggest that the terms of the loan, coupled with LTV and DCR, are the most important observed factors explaining early termination. This is in contrast to some earlier findings (such as Archer *et al.* 2002). Economic conditions in the space (rental) market and the labor market, as well as the performance of the property itself, also matter; but their magnitude of impact is much smaller. The extent to which our results can be generalized beyond the sample is uncertain, but our findings contribute to the limited collection of research on the determinants of multifamily mortgage termination and represent one of the few estimates for small multifamily mortgages. FNMA's book of business in the small loan market as of 2010 was \$34 billion, suggesting the impact of the portfolio is material. In the following sections, we provide a summary of the mortgage termination literature and discuss how it relates to multifamily loans; describe the empirical approach and the data; and present the results and our conclusions.

Motivation and Literature

The study of commercial and multifamily mortgages has utilized many of the concepts and lessons learned in the extensive body of single family mortgage literature. In that literature, mortgages are viewed as terminating in one of two ways, default and repayment. In the commercial mortgage market the primary motivations to pay off or repay a loan early or late include recapitalizing, equity extraction or sale. In addition to these motivations, there is consistent evidence in both the residential and commercial mortgage markets that falling interest rates can drive early terminations of mortgages as borrowers refinance their debt (for example, Abraham and Theobald 1997, Ciochetti et al. 2002). Prepayment penalties are often used in the commercial market to discourage prepayment and preserve the return to the investor, and there is strong evidence that such penalties are very effective in suppressing prepayments. Not surprisingly, when the penalty expires, prepayments spike dramatically. Prepayment penalties are typically fees charged for paying the loan off early calculated as a percentage of the outstanding balance. For example, Fu, LaCour-Little and Vandell (2003) show that conditional prepayment rates can jump from nearly 0 immediately before the penalty expires to 50% in the following month. This has been coined the "hockey stick" shape of prepayments. In addition to prepayment penalties with specified charges (e.q., 1% outstanding balance), there are alternative mechanisms for suppressing prepayments such as lock outs (no prepayment allowed), defeasance (security replacement) and yield maintenance (lost return compensation) requirements. Fannie Mae relies on both prepayment penalties and yield maintenance and in many cases uses a combination of the two constraints to protect investors in falling interest rate markets. [1] Key issues the literature does not fully address are whether and how prepayment suppression mechanisms affect default probabilities, and whether commonly accepted financial incentives to prepay and default still work when prepayment penalties and yield maintenance agreements are in effect.

Another important source of early termination is default, when mortgage payments are no longer being made. One motivation for default exists when the mortgage is worth more than the value of the property, often referred to as negative equity. Most research proxies for negative equity by examining property prices, the LTV ratio at origination (*ltv orig*), or updated current value using market statistics (the current LTV or *ltv*). The empirical evidence for commercial property loans suggests that the equity position of the property in the contemporaneous time period, not the equity position at origination, is a good proxy for the default option (Archer *et al.* 2002, Ciochetti *et al.* 2002, Ambrose and Sander 2003).

Our research also considers the ability of the property's income/cash flow to trigger default. For income-producing property, the property is the main collateral as well as the source of income to cover the debt. The DCR is typically defined as the annual net operating income (NOI) of the property divided by the annual DS (DCR = NOI/DS). If the DCR is less than 1.0, then there will be insufficient income to cover the mortgage payments. Empirical commercial mortgage papers mostly find that higher DCR reduces the probability of default (Vandell *et al.* 1993, Archer *et al.* 2002, Ciochetti *et al.* 2002, 2003, Goldberg and Capone 2002).

The effect of equity (LTV) and debt coverage (DCR) on default is likely to be highly nonlinear. Some research has used quadratic terms to highlight the lack of linearity (Ciochetti *et al.* 2002, 2003) and others have focused on the interdependence of equity and debt coverage (Goldberg and Capone 2002). For example, the option to default may not be exercised even when there is negative equity because the cash flow from the property may have substantial positive net value. Therefore, it may be necessary to have both a negative cash flow (DCR < 1.0) and negative equity (LTV > 100%) to make the default option in the money.

The loans used in this study have a balloon payment. A large segment of the literature has been concerned with the ability of the borrower to find new financing to cover the large balloon payment

due at the end of the loan (Akat *et al.* 2012, Fabozzi, Stanescu and Tunaru 2013, Levitin and Wachter 2013). Threats to investment return associated with balloon payments include refunding or extension risk (the loan lasting past the balloon date), term default (default on the balloon due date) and the costs/risks of workout on defaulted loans (minimizing losses after the default) (MacDonald and Holloway 1996, Tu and Eppli 2003, Eppli and Tu 2005, Chen and Deng 2012). This line of research emphasizes that even if the equity position and the cash flow position are acceptable, constraints on credit availability can restrict refinancing. This becomes even more challenging when lending standards are tightened and may require a substantial infusion of capital if property prices have declined. However, the literature does not provide much evidence on default and repayment probabilities at term with a balloon payment.

Our research contributes to the literature in a number of ways. First, we focus on small loans for multifamily housing, a significant segment of the commercial mortgage market that has received little attention (see Capone and Goldberg 2001 for an example). Second, two of the key papers on multifamily mortgage terminations (Archer *et al.* 2002, Golberg and Capone 2002) relied on more limited data, and Archer *et al.* were unable to find statistical significance for some key drivers of default. We directly measure NOI and DS allowing for contemporaneous estimates of DCR and updated LTV measures. Third, we can observe detailed information on the operational aspects of the property (operating efficiency, occupancy, cash reserves and capital expenditures). Fourth, little is known about how mortgage contract provisions designed to suppress prepayments affect default probabilities. This may be nontrivial because adding a constraint on one option (prepayment) may make a second option (default) more likely. Fifth, this article provides new evidence on the risk of term default and the magnitude of extension risk.

Fannie Mae and the World of Small Multifamily Loans

Fannie Mae purchases individual loans originated by approved mortgage lenders or purchases pools of previously securitized loans. [2] Servicing remains with the lender and, like underwriting, is conducted according to guidelines prepared by Fannie Mae. Lenders retain a risk position in these loans through a loss sharing agreement with Fannie Mae. Fannie Mae multifamily purchases must allow for the securitization of the loans into Fannie Mae guaranteed mortgage-backed securities (MBSs) and the sale of those MBS to investors.

In general, small multifamily mortgages are defined by the number of units a property has and the loan amount. A small loan is defined as a mortgage for an apartment building with 5–50 units and an original loan amount of \$3 million or less, except that the balance can go up to \$5 million in high-cost metropolitan areas. According to the Mortgage Bankers Association (2012, 2015), Fannie Mae's \$2.2 billion in 2009 small loan production represents 15% of the total market of multifamily loans. In 2010, Fannie Mae held a \$34 billion book of 30,000 "small" loans. It also held a \$21 billion book of 23,500 loans on 5- to 50-unit properties (12% of the multifamily book). Roughly 86% of Fannie Mae's 2009 small loan book of business met the definition of affordable housing set forth by the U.S. Department of Housing and Urban Development (FNMA 2011).[<u>3</u>]

Empirical Approach

Conceptual Relationships

In structuring our empirical analysis, we rely on the models of commercial lending originally presented in Archer *et al.* (2002) and refined in Ambrose and Sanders (2003). This model assumes that the value, or price, of a loan (P_L) is the present value of the interest and principal repayments per face dollar of the loan less the expected value of losses for default (net of insurance or guarantee proceeds) and the expected value of losses, as follows,

$$P_L = V(Pmts, Repay, y) - -V(ELoss_D, Ins, y) - -V(Eloss_{PP}, y),$$

(1)

where y is the yield to investor, *Pmts* is the scheduled payments per dollar of the loan, *Repay* is the repayment of loan principal per dollar of loan, *ELoss*_D is the expected loss from default, *Ins* is mitigation or default loss adjustment from insurance and guarantees and *ELoss*_{PP} is the incurred loss from prepayment per dollar of the loan. The risk characteristics inherent to the property (*e.g.*, LTV ratio and DCR), and other nonloan characteristics such as property operating efficiency, reserves, occupancy and capital costs all form the basis for expected losses from default

$$ELoss_{\rm D} = L(LTV_0, DCR_0, \mathbf{X}_u),$$

(2)

where LTV_0 and DCR_0 represent the origination LTV and DCR, respectively, and X_u is the property and market factors observed at origination. Expected loss from prepayments are embedded in the call option and represent the expected change in interest rates r^* and prepayment penalties, yield maintenance or lockout provisions in the loan contract:

$$ELoss_{\rm PP} = L(r *, \mathbf{Z}_u),$$

(3)

where Z_u represents penalties and lockout provisions. The loan rate, r_1 , incorporates the lender's required return plus the expected loss rates for default and prepayment as follows,

$$r_1 = R(ELoss_D, ELoss_{PP}, y) \text{ or } = R(L(LTV_0, DCR_0, \mathbf{X}_u), L(r *, \mathbf{Z}_u), y).$$

(4)

The likelihood of the mortgage being terminated via default or prepayment depends on the same initial factors that predict expected losses plus stochastic events that impact the property over the life of the loan X_p , the realized path of interest rates, and contractual constraints that influence borrower actions.[4] Further, since the prepayment and default decisions are substitutes, one depends on the other as illustrated in the following relationships:

$$\pi_{\text{Def}} = \pi_{\text{Def}}(LTV_0, DCR_0, X_u, y, \mathbf{X}_p, \pi_{\text{PPay}}),$$

(5)

$$\pi_{\text{PPay}} = \pi_{\text{PPay}}(r *, \mathbf{Z}_u, y, \mathbf{X}_p, \pi_{\text{Def}}).$$

(6)

Although underwriting procedures attempt to minimize risks, many are unobservable and require proxies in a model specification. Descriptors of the property, property performance and location controls can serve as viable proxies along with the interest rates. As Ambrose and Sanders (2003) illustrate, lenders will set the terms such as the contract rate, DCR and LTV in a combination such that the expected losses from loans of different caliber borrowers are equal.

Relationships Operationalized

Since 1981 when Dunn and McConnell presented their MBS model, and in 1986 when Green and Shoven's model examined the impact of market conditions and loan prepayment, researchers have recognized that mortgage contracts are best modeled empirically in a contingent claims framework (Dunn and McConnell 1981, Green and Shoven 1986). Each period the borrower faces a decision with three options—prepay, default or continue to make payments. Motivations for each option distinguish one from the other. For example, the borrower's option to prepay the mortgage at any time without penalty is a call option at a strike price of par while the default option is a put option at a strike price equal to the market value of the collateral property (Ambrose and LaCour-Little 2001). The multifamily mortgage is further complicated by the inclusion of provisions designed to suppress early repayment that, as will be illustrated, do influence the borrower's decision and timing of default and prepayment.

For the empirical analysis, a multinomial logit model is used to estimate default incidence based on an optimization of consumer choice similar to that established by Campbell and Dietrich (1983), then formalized by Long (1997) and since used in residential mortgage research (see An, Clapp and Deng 2010, Pennington-Cross 2010, as examples).[5]

Tables 1 and 2 list the state variables that will be tested. They are broadly grouped into: (<u>1</u>) Basic, which includes standard information about the loan and building that is known at origination and over time; (<u>2</u>) Operations, which describe the efficiency of the property operations and related expenses; (<u>3</u>) Market, which includes proxies for market conditions in the local labor and property and space markets; and (<u>4</u>) Prepayment, which includes a series of variables designed to capture the temporal impact of prepayment penalties, yield maintenance provisions and balloon requirements on default and prepayment/term repayment.

Variables	Definition
Basic	
default	1 in time period the mortgage is terminated due to default. 0 otherwise. Loans are coded
	as defaulted if they are liquidated due to foreclosure, deed in lieu of foreclosure,
	repurchase, dissolution or a discounted payoff.
paid off	1 in the time period the mortgage is terminated due to full repayment of the outstanding
	balance. 0 otherwise. Loans are coded as paid off if the loan is fully paid due to
	refinancing, repayment at term, repayment after term or a third-party sale.
loan age	Age of loan in months.

1 Table. Data dictionary—basic

upb orig	Loan amount or unpaid balance in 100,000 dollars at origination.
refi	1 if the loan is a refinance loan. 0 if the loan is for the purchase of the property.
term	Term of loan at origination in months.
recourse	1 if the lender/investor has recourse to other assets beyond the property if the loan
	defaults. 0 if the lender/investor does not have access to any assets beyond the
	property.
io	1 for an interest only mortgage. 0 for an amortizing mortgage.
dcr orig	The debt coverage ratio (annual net operating income/annual debt service) at
	origination.
dcr	The debt coverage ratio (annual debt service/net operating income) in the current year.
ltv orig	The loan to value (Itv = loan amount/property value) at origination.
ltv	The ratio of the current (quarterly) loan amount to the current property value. The
	current loan amount is reported directly in the servicing data. The current property
	value is derived using data from Costar. The value of the property at origination is
	updated to the current quarter using the change in the observed price per square foot
	of multifamily property at the state level from transactions reported by Costar. In
	some states it was necessary to aggregate up to the Census Division level due to very
	few observed transactions.
∆irate	The change in prevailing interest rates over the prior quarter. The interest rate series
	used is the 50th percentile of the observed "small loan" originated interest rate.
units	The number of units in the building or buildings that back the mortgage.
built	The year the building or average year the buildings backing the mortgage were built.
region1	1 if in the U.S. Census defined northeast region. 0 otherwise.
region2	1 if in the U.S. Census defined Midwest region. 0 otherwise.
region3	1 if in the U.S. Census defined south region. 0 otherwise.
region4	1 if in the U.S. Census defined west region. 0 otherwise.

2 Table. Data dictionary—building operations, market and prepayment

Operations	
egi/pgi	The ratio of effective gross income to potential gross income.
Oper%	The percent of income used by operating expenses.
Capex%	Capital expenditures as a fraction of effective gross income.
Reserves	Thousands of dollar amounts in the replacement reserve balance at the start of the
	year.
Market	
urate	The state of the property's location unemployment rate state expresses in
	percentages. These data are collected from the Bureau of Labor and Statistics.
Δrent	Change in prevailing average rent for multifamily property over the prior quarter. Rent
	is measured at the county level as the Fair Market Rent reported by the
	Department of Housing and Urban Development.

∆сар	The change in prevailing cap (capitalization) rates (net operating income/property					
	value) over the prior quarter. Cap rate data are generated by averaging observed					
	multifamily transaction cap rates to the state level. In some states it was necessary					
	to aggregate up to the Census Division level. These data are collected from Costa					
	For each observed transaction with a capitalization rate we cleaned the data to					
	include only those that were under \$5 million dollars and represented single					
	property transactions.					
Prepayment						
res ever	1 if the mortgage ever has a yield maintenance provision or prepayment penalty in					
	effect. 0 otherwise.					
Res on	1 in time periods when the yield maintenance provision or prepayment penalty is in					
	effect. 0 otherwise.					
Ym ever	1 if the mortgage ever has a yield maintenance provision in effect. 0 otherwise.					
Ym on	1 in time periods when the yield maintenance provision is in effect. 0 otherwise.					
Pen ever	1 if the mortgage ever has a prepayment penalty in effect. 0 otherwise.					
Pen on	1 in time periods when the prepayment penalty provision is in effect. 0 otherwise.					
Ball due	1 in the quarter that the balance of the loan is due. 0 otherwise.					

Data

Each observation in our sample includes the contemporaneous quarterly status of a single multifamily mortgage securitized or held in portfolio by Fannie Mae from 2005 through 2011. These loans were originated between January 1998 and June of 2011. Because we estimate a conditional (conditioned on being alive at the beginning of the quarter) probability of default and repayment, we can include seasoned loans in the estimation. Loans are defined as defaulted if they are liquidated due to foreclosure, deed in lieu of foreclosure, repurchase, dissolution or a discounted payoff. This grouping is necessary due to the low frequency of terminations for many of these categories. Loans are defined as paid off if the loan is fully paid due to refinancing, repayment at term or after term or a third-party sale. The impact of seasoning will be controlled for by the loan age. The extent to which some loans are still alive after the last observed date (fourth quarter 2011) will not bias the results, due to the conditional nature of the estimation, unless there is an unobserved characteristic associated with the surviving loans that is correlated with the probability of default or repayment. Since we observe many loans through term, right-hand side censoring should not be an important factor. Typically, Fannie Mae restricts purchases of small loans to standardized mortgages with a 10-year term, fixed rate, a balloon payment at term (not fully amortizing) and prepayment penalty, yield maintenance or combination of the two provisions. However, historically there has been substantial variation in these loan characteristics.

Our goal is to find the determinants of the conditional prepayment probability and default probability for the sample of mortgages. The term conditional indicates that we are studying the probability in a current quarter conditional on it surviving through the prior quarters since origination. Table 3 provides the basic summary statistics for the panel data set. Since the raw data included only a handful of adjustable rate mortgages, the data are limited to include only mortgages with fixed interest rates. If

the source for the data is anything except the Fannie Mae servicing platform, the source is also noted in Tables 1 and 2. After cleaning and coding, the primary estimation data set contains roughly 12,670 loans and 224,827 observed mortgage-quarters. For the operations data and current DRC and LTV, fewer loans are available (with reported data) ranging from approximately 5,000 to 6,000 loans.

		Cumulative Rate		Origination	
Origination Year	Mortgage Count	Default	Repayment	Orig Ltv	Orig Dcr
1998	3%	5.59%	68.76%	0.51	3.00
1999	3%	8.38%	68.86%	0.60	2.07
2000	2%	4.46%	61.61%	0.65	2.54
2001	4%	6.64%	70.48%	0.62	1.99
2002	6%	4.37%	50.19%	0.61	2.03
2003	11%	5.56%	40.87%	0.57	2.23
2004	12%	3.76%	33.38%	0.56	2.08
2005	14%	1.57%	11.72%	0.55	1.80
2006	10%	4.03%	9.74%	0.55	1.73
2007	13%	1.79%	2.86%	0.54	2.11
2008	8%	2.78%	1.98%	0.57	2.18
2009	5%	0.43%	1.15%	0.55	2.25
2010	6%	0.00%	0.00%	0.59	2.22
2011	3%	0.00%	0.00%	0.58	2.02

3 Table. Characteristics by origination year

Tables 3 and 4 provide information about the loans by origination year or cohort. While mortgages are originated from 1998 through 2011, 2003–2007 are the most prevalent origination years. The cumulative default rate varies across cohorts but older cohorts are more likely to be paid off. Payoff can be early, at term or after term. The LTV (*ltv orig*) ratio peaked for the 2001 cohort and was at its lowest for the 2007 cohort. Average DCRs (*dcr orig*) range from 1.73 in 2006 to 3.00 in 1998. Mortgage interest rates peak at over 8% for the 2000 cohort and drop down to 5.45 for the 2010 cohort. Mortgage terms in months are longer for the older cohorts and range from 104 to 118 months on average for the 2008–2011 cohorts. In the more recent origination cohorts, provisions to slow early payments on the mortgages have become increasingly prevalent. In fact, by 2009 almost all mortgages have both a yield maintenance and prepayment penalty provision scheduled at some time over the term of the loan.

Origination	Average Interest	Average Term in	Yield	Prepayment
Year	Rate	Months	Maintenance	Penalty
1998	7.21%	155	38%	30%
1999	7.64%	152	44%	43%
2000	8.10%	182	61%	32%
2001	7.22%	147	51%	37%

4 Table. Characteristics by origination year

2002	6.51%	133	48%	43%
2003	5.57%	133	43%	36%
2004	5.48%	128	64%	35%
2005	5.54%	137	81%	21%
2006	6.09%	141	78%	32%
2007	6.10%	134	88%	45%
2008	5.91%	104	87%	93%
2009	5.88%	114	97%	97%
2010	5.45%	118	100%	100%
2011	5.55%	113	99%	99%

Note: Yield maintenance and prepayment penalty indicate the percentage of loans originated in each year that have the prepayment suppression mechanism scheduled to be in effect for one or more months during the life of the loan.

Table 5 shows how frequently yield maintenance and prepayment penalty provisions are in effect across the 2005–2011 time frames for loans in our sample that are active at the beginning of each quarter. For example, in 2011, 77% of loans had yield maintenance provisions in effect and 11% of loans had a prepayment penalty in effect. Over time lenders have moved away from prepayment penalties and increased the use of yield maintenance provisions. Consistent with rising default rates, DCRs declined and LTVs increased from 2008 through 2011.

	Conditional Quarterly Rate						
Performance	Default	Paid	Dcr	Ltv	∆irate	Yield	Prepayment
Year		Off				Maintenance	Penalty On
						On	
2005	0.04%	1.51%	1.57	0.56	-0.60	60%	25%
2006	0.46%	1.24%	1.63	0.47	0.11	68%	21%
2007	0.10%	1.27%	1.76	0.52	0.12	69%	22%
2008	0.12%	1.37%	1.70	0.51	-0.05	69%	19%
2009	0.19%	0.83%	1.64	0.57	-0.06	71%	17%
2010	0.13%	1.13%	1.60	0.60	-0.43	74%	14%
2011	0.23%	1.84%	1.58	0.56	-0.54	77%	11%

5 Table. Key time-varying characteristics

Note: All statistics in this table are averages calculated over the year using quarters the loan is active, specifically at the beginning of the quarter.

There are many different variations in how prepayment penalties and yield maintenance provisions are combined in a loan. Tables 6 and 7 provide more detail about the length and timing of yield maintenance and prepayment penalty provisions. The vast majority (over 99%) of yield maintenance provisions start at the beginning of the loan. Prepayment penalties are more spread out but are clustered at the beginning of the loan and in the 9th year of the loan. For the typical 10-year term

mortgage, the penalty is scheduled to begin with one year left until term and is in effect directly after an expiring yield maintenance provision. In a different set up, the prepayment provision is in effect at the beginning of the mortgage but lasts for only a few months and is then replaced by a yield maintenance provision.

Loan Age in Years	Yield Maintenance	Prepayment Penalty
0	99.78%	44.11%
1	0.00%	0.00%
2	0.04%	0.44%
3	0.00%	0.20%
4	0.11%	7.33%
5	0.00%	1.67%
6	0.00%	6.58%
7	0.02%	0.10%
8	0.00%	0.08%
9	0.01%	35.17%
10	0.00%	0.29%
11+	0.03%	4.03%

6 Table. Distribution of provision start year

Note: Loan age in year 0 includes ages up to but not including the 12th month. Loan age year 1 includes loan ages from the 12th month up to but not including the 24th month. All other loans age in years are created using the same logic.

Year	Yield Maintenance	Prepayment Penalty
0	0.00%	50.70%
1	0.00%	0.83%
2	0.12%	1.50%
3	0.14%	0.41%
4	8.05%	12.05%
5	0.60%	9.29%
6	9.71%	6.29%
7	0.23%	3.98%
8	0.16%	0.56%
9	69.22%	2.99%
10	1.18%	2.69%
11	0.03%	0.15%
12	0.07%	0.39%
13	0.01%	0.42%
14	3.82%	5.31%
15	6.09%	0.83%
16+	0.56%	1.60%

7 Table. Distribution of provision length in years

Note: Loan age in year 0 includes ages up to but not including the 12th month. Loan age year 1 includes loan ages from the 12th month up to but not including the 24th month. All other loans age in years are created using the same logic.

Estimation Data Set

Table 8 reports the estimation data set. It is a panel data set with time being measured in quarters and the cross-section at the individual loan level. Loans are only included if they are alive at the beginning of the quarter, so all estimated termination probabilities are conditional quarterly rates. As previously noted, the performance of the loans is observed from the first quarter of 2005 through the last quarter of 2011.

Category	Variables	Mean	Std. Dev.	Min	Max
Basic	default	0.002	0.043	0	1
	paidoff	0.013	0.113	0	1
	loan age	47.566	30.191	1	341
	upb orig	12.606	8.364	0.65	50.00
	refi	0.341	0.474	0	1
	term	134.931	62.297	12	488
	recourse	0.906	0.292	0	1
	io	0.073	0.259	0	1
	dcr orig	2.111	2.660	0.8	56.35
	dcr	1.639	1.071	-4.11	56.35
	ltv orig	0.563	0.193	0.01	0.98
	ltv	0.551	0.289	0.00	2.89
	∆irate	-0.209	0.739	-3.67	1.07
	units	23.919	13	5	50
	built	1953.517	32.263	1800	2011
	region1	0.363	0.481	0	1
	region2	0.109	0.311	0	1
	region3	0.066	0.249	0	1
	region4	0.461	0.499	0	1
Operations	egi/pgi	1.033	0.617	0.28	63.09
	oper%	48.921	16.743	6	521
	capex%	2.901	8.383	0	518.15
	reserves	13.678	75.207	0	2,476.36
Market	urate	7.773	2.804	2.30	14.10
	∆rent	0.034	0.045	-0.25	0.73
	∆сар	0.302	0.851	-2.42	4.59
Prepayment	res ever	0.946	0.227	0	1
	res on	0.884	0.321	0	1
	pen ever	0.390	0.488	0	1

8 Table. Summary statistics

pen on	0.175	0.380	0	1
ym ever	0.724	0.447	0	1
ym on	0.708	0.455	0	1
ball due	0.002	0.046	0	1

Note: Some observations do not report the year built (*built*) and will be controlled for using categorical variables in the estimated specifications. The Market and Operations variables are also not available for all loans. Specifications using these variables will include substantially fewer loans. See each set of results tables for the exact number of loans used in the specification.

The first group, "Basic," includes a wide variety of basic information about the loan, property and location. On average in each quarter 0.2% of mortgages default and 1.3% are paid off. The loan age in months (*loan age*) is used to estimate a common baseline conditional quarterly probability of default or prepayment. The size of the loan is included (*upb orig*) to proxy for fixed costs or the sophistication of the owner/borrower. An indicator of whether the loan is an origination or a refinance (*refi*) is also included. Most of the loans are refinances. Since there is a longer loan history to review when underwriting a refinance, it is likely that these loans may default less frequently. The term of loans (*term*) is on average 11 and one-quarter years. However, the distribution is not uniform and is clustered at 5-, 7-, 10- and 15-year terms. The 10-year term is the most prominent and includes over half of the loans in the sample. The majority of loans, over 90%, have a recourse provision with the lender or the owner (*recourse*). Recourse rights may make the borrower more conservative and dampen termination probabilities. A small fraction of the loans are interest only (*io*). They are included because none of the loans are fully amortizing, so a nonamortizing loan has many similarities. The impact on default and prepayment likely reflects an unobserved selection associated with using an interest-only loan structure.

The two measures of the DCR are at origination (*dcr orig*) and the current or updated DCR (*dcr*). The prior literature frequently finds that higher DCRs tend to reduce defaults and increase prepayments. Unlike most prior research, which has had to estimate updated NOI, we observe the actual NOI for the property over time (annually); thus, we do not need to use any market indexes to update the debt payments and income. The last two ratios report the LTV at origination (*ltv orig*) and the current or updated LTV (*ltv*). Prior research has found that LTV at origination provides little information, but a higher current LTV should drive default probabilities up and prepayments down. On average, these small loans have significant equity with *ltv_orig* equal to 0.56. Changes in multifamily interest rates (*Lirate*) are also included to help capture the path of interest rates and proxy for the incentive to refinance. In general, we expect that declining interest rates will increase terminations via prepayments.

To help control for basic information about the property and locations, the number of units (*units*), the year the building is built (*built*) and region dummies are included. In general, building age may proxy for missing information about the operational efficiency of the property. So, it might be easier to refinance newer buildings and they may default more due to unexpected or rising maintenance costs. A series of dummy variables are included in the specification to allow for nonlinear relationships. For example, very old buildings may have special attributes that increase their desirability and financial performance. Large buildings (40–50 units in our small loan sample) may be more likely to be

professionally managed than 5-unit buildings and may be able to manage turnover more effectively. These factors could reduce default and increase repayment probabilities. This is largely an empirical question and a series of dummy variables are used to provide a flexible specification.

The prior literature has often relied on market conditions to proxy for unobserved property conditions (vacancy, income, efficiency). We include both market and observed property-specific conditions. A proxy for the vacancy of the property is created by dividing the annual effective gross income (egi) by the potential gross income (pgi), egi/pgi for the property. EGI is the total income of the property including rent and nonrent income. PGI is the total potential income the property should receive if all occupied units are rented at their current actual rents and all vacant units are rented at current market rents. We expect a property that is fully rented is more likely to meet its financial obligations and to gain access to the credit markets. A measure of operating efficiency is calculated by dividing operating income by effective gross income (oper%). A measure of capital expenditure is calculated by dividing capital expenditures by effective gross income (capex%). The amount of reserves, in thousands of dollars, at the beginning of the year (reserves) is also included. Similar to the previously mentioned scale impacts, the expected impact of these measures of property operations is an empirical question. However, it is anticipated that an efficiently operated property with smaller required capital expenditures and substantial cash on hand to cover unexpected expenses should be more likely to survive any unanticipated outcomes for the property or the market as a whole. In general, if market conditions are better we should expect the property to perform better and generate more income. Measures of the local unemployment rate (*urate*), changes in multifamily rents over the prior quarter $(\Delta rent)$ and quarterly changes in capitalization rates (Δcap) are included.

The last group, "Prepayment," includes various measures of contract provisions that can lead to rapid increases and decreases in repayment. Dummy variables indicating whether a loan has ever had a repayment provision (*res ever*) designed to suppress prepayment and whether that provision is currently active (*res on*) are included. The excluded category includes loans that did not have a scheduled prepayment provision in effect at any point during the mortgage's potential life. These restrictions are then separated into yield maintenance (*ym ever* and *ym on*) and penalty (*pen ever* and *pen on*) provisions. For the vast majority of observations, there are prepayment suppression provisions (*pen on*) and/or a yield maintenance provisions (*ym on*) in effect.

None of the loans in the sample were fully amortizing: all loans have a balloon payment due at term. As a result, a dummy variable is used indicating the quarter the loan is scheduled to end at term (*ball due*). The excluded category is all other time periods. It is anticipated that many loans will be successfully paid off. However, some borrowers will have difficulty finding other financing or have property-specific issues that lead to the extension of the loan beyond term or default. The default could occur before, at term or postterm and during an extension period.

Results and Implications

Table 9 shows the base specification results for the quarterly multinomial logit using panel data with three outcomes—continue, default or pay off the loan. There is an increased probability of default as the loan ages but, as indicated by the quadratic term (loan $age^2/100$), at a decreasing rate as time passes. This is illustrated in Figure 1 and shows the peak of the default baseline approximately two

years into the loan's life (in quarters 22 through 24). The probability of a loan being paid off also increases at a decreasing rate but the turning point is beyond most observed loan ages. For most purposes, as a loan ages the probability of it being paid off increases. There is no evidence that larger loans default or pay off at different rates, though this result may be an artifact of the fact that all the loans in the sample are small loans. Loans that are refinances are more likely to be paid off and refinanced again. Longer-term loans and interest-only loans tend to default more and pay off less. This finding likely reflects the fact that marginal loan applicants can use longer terms and interest-only loans to reduce the annual DS. Contrary to expectations, loans with recourse provisions have a higher probability of default. Again, this may reflect a selection process where lenders require recourse to help reduce their exposure to losses on more risky loans.

	Default	Paidoff		
Variable	Coeff	SE	Coeff	SE
loan age	0.050002	0.01	0.030002	0.00
(loan age ²)/100	-0.040002	0.01	-0.010002	0.00
upb orig	0.01	0.01	0.00	0.00
refi	-0.14	0.16	0.170002	0.05
term	0.000002	0.00	-0.010002	0.00
recourse	0.720002	0.30	-0.06	0.09
іо	0.730002	0.24	0.08	0.10
dcr orig	-0.03	0.05	-0.03	0.02
ltv orig	2.140002	0.39	1.250002	0.17
Δirate	-0.140002	0.06	-0.090002	0.03
res ever	-0.430002	0.16	1.060002	0.08
res on	-1.640002	0.17	-1.550002	0.06
ball due	3.960002	0.23	2.580002	0.11
5< = unit<10	0.16	0.21	-0.340002	0.09
10< = unit<20	0.18	0.19	-0.250002	0.08
20< = unit<30	0.08	0.18	-0.04	0.07
30< = unit<40	0.12	0.19	-0.02	0.07
built<1900	1.170002	0.59	0.360002	0.18
1900< = built<1925	1.180002	0.36	0.740002	0.12
1925< = built<1950	0.700002	0.37	0.590002	0.11
1950< = built<1970	0.26	0.39	0.460002	0.11
1970< = built<1980	0.820002	0.40	0.420002	0.12
1980< = built<1990	0.910002	0.40	0.480002	0.11
built missing	2.500002	0.34	1.680002	0.11
region 2	-0.590002	0.20	-0.310002	0.09
region 3	0.870002	0.20	-0.530002	0.12
region 4	-0.910002	0.14	-0.380002	0.06
constant	-10.020002	0.64	-5.880002	0.29

9 Table. Base model—termination results

Loans	12,670		
Observations	224,827		
Log of Likelihood	-14,595		

Notes: The standard errors are clustered for each loan. Categorical variables are used to describe the number of units and the year the building was built. If the conditions are met, the variable equals 1, otherwise the variables equal 0.50. Note that 105 observations do not report the year built (built missing = 1).

^{***}Indicates the coefficient is significant at the 99% level, ^{**}indicates the coefficient is significant at the 95% level and ^{*}indicates the coefficient is significant at the 90% level.

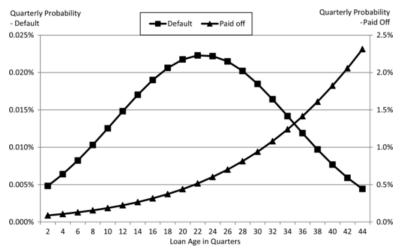


Figure 1 Baseline conditional quarterly probability. Notes: The estimated probability for a representative loan at each month using the "Base Model – Termination Results" specification. Continuous variables are evaluated at their means and categorical variables evaluated at the most prevalent category. The categorical variables are set at the following values: *recourse = 1, refi = 0, io = 0, res ever = 1, res on = 0, ball due = 0, 20< = unit<30 = 1, all other unit dummies = 0, all building age dummies = 0, region4 = 1 and all other regions = 0.*

Next, consider the three main option-related proxies—equity-LTV, cash-DCR and interest rates. The proxy for the equity position, the LTV at origination, works as expected. Loans with less equity are more likely to default. The DCR at origination is statistically insignificant for both default and the pay off. Property-specific NOI may change enough over time so that the DCR at origination is not a sufficient proxy for contemporaneous DCR. Since our sample covers a time period with a severe recession and we observe the actual NOI, this decoupling of origination versus actual DCR is more likely. Declining interest rates are associated with modest increases in the probability of paying off the loan. Provisions to suppress prepayment tend to reduce prepayments as indicated by the *res on* variable. When the loan is set to expire at term and a balloon payment is due, both defaults and pay offs increase dramatically. Later sections of this article will examine these results in more detail.

Consider the basic information about the property and the location. There is no evidence that default probabilities are related to the number of units in the building. There is only modest and inconsistent evidence across various specifications that smaller units are less likely to pay off the loan. Newer properties tend to be associated with lower probabilities of default and pay off. The region controls indicate that loans in the west and south regions tend to default the most; loans in the west region

tend to be paid off the least. These results proxy for economic conditions in the local markets and the productivity of the property.

Market Conditions and Operations

Table 10 includes more direct measures of market conditions that are likely to impact property and loan performance as well as information about the performance of the property itself. Unfortunately, for over 7,000 loans the property operating information is not available from the servicing data, reducing the sample to approximately 5,000–6,000 loans. This may impact some of the results' precision, but the results tell a similar story to those observed with the larger sample. For example, default and prepayment triggers, prepayment suppressions and balloon payment results are consistent across both samples and specifications.

	Default		Paidoff	
Variable	Coeff	SE	Coeff	SE
loan age	0.160002	0.04	0.240002	0.04
(loan age ²)/100	-0.130002	0.05	-0.160002	0.04
upb orig	-0.02	0.03	0.020002	0.01
refi	0.29	0.32	-0.590002	0.23
term	0.00	0.00	-0.010002	0.00
recourse	-1.180002	0.52	0.78	0.70
іо	-0.04	0.46	0.600002	0.23
dcr orig	-0.02	0.14	0.00	0.08
ltv orig	6.820002	1.62	-1.230002	0.68
Δirate	-0.460002	0.27	-0.720002	0.15
res ever	-0.65	1.67	0.08	0.73
res on	-1.14	0.76	-2.130002	0.27
ball due	3.110002	1.04	0.68	0.44
5< = unit<10	-0.86	0.77	-0.07	0.36
10< = unit<20	0.42	0.54	0.20	0.27
20< = unit<30	0.05	0.50	0.31	0.24
30< = unit<40	0.18	0.48	0.12	0.27
built<1900	2.080002	0.79	-0.45	0.79
1900< = built<1925	0.28	0.71	0.42	0.40
1925< = built<1950	0.13	0.73	0.38	0.40
1950< = built<1970	-0.06	0.64	0.11	0.41
1970< = built<1980	-0.08	0.67	-0.07	0.49
1980< = built<1990	0.79	0.61	0.38	0.48
region 2	-0.51	0.44	0.40	0.25
region 3	0.15	0.54	0.05	0.46
region 4	-1.560002	0.71	-0.53	0.40
egi/pgi	-3.320002	1.74	-0.40	0.50
oper%	0.030002	0.01	0.00	0.01

10 Table. Market conditions and o	norations termination results
TO TADIE. MAI KEL COMULIONS and U	perations — termination results

capex%	0.00	0.01	-0.040002	0.02
reserves	-0.030002	0.02	-0.020002	0.01
urate	0.300002	0.13	-0.10	0.08
Δrent	-7.000002	3.43	1.97	2.74
Δсар	0.12	0.17	0.01	0.10
constant	-13.490002	3.39	-9.720002	1.65
Number of Loans	4,958			
Observations	86,120			
Log of Likelihood	-1,285			

Note: Standard errors are clustered by loan. Categorical variables are used to describe the number of units and the year the building was built. No year built information is missing for this subsample.

***, ** and * indicate the coefficient is significant at the 99%, 95% and 90% level.

As anticipated, better controls for market conditions and property financials reduce the significance of building age and the number of units in the property. The results indicate that both market conditions and the performance of the property can have a nontrivial impact on loan termination probabilities. For example, property that is generating a higher proportion of its potential income is less likely to default. Or stated differently, property-specific vacancy rates matter. As expected, property that is run more efficiently and has larger cash reserves has a lower probability of default. More reserves and larger capital expenditures also reduce the probability of being paid off. In short, better managed properties with more cash on hand are less likely to default. In contrast, properties that are less prepared for large capital improvements (little cash on reserve) and properties that undertake large capital improvements are more likely to be paid off.

Market conditions only impact termination through default. Properties in locations with improving labor market conditions and rising rents are less likely to default, an effect that reinforces the importance of the individual property performance. Together the market and property results indicate that the decision to default is highest when the property itself is performing poorly and the market conditions around the property are also struggling. A struggling property is more likely to be able to right itself if the economic conditions around it are favorable.

Contemporaneous LTV and DCR

Archer *et al.* (2002) find that LTV at origination has no relationship with default while DCR at origination does. Our results are the opposite—origination LTV works as expected and DCR at origination is statistically insignificant. Given this deviation from prior published work, we have included in the Appendix an approximation of the Archer *et al.* specification on this FNMA sample. The dependent variable is coded 1 if a loan has ever been 90 days or more delinquent for the first seven years of the loans life. Loans seasoned more than two years before purchase are excluded. There is one observation per loan. Origination year dummies serve as a type of default baseline. The Archer *et al.* results find that only the DCR and the year built are statistically significant using a sample of 495 loans originated from 1989 to 1995 and observed from 1991 to 1996 from RTC securitized loans (high risk). Although we do not have direct evidence for difference in the results, there are multiple potential explanations. Our list of controls is much longer than Archer *et al.* (2002). Our performance

data include a very severe recession that had large impacts on underwriting standards, and both space and capital market conditions. This likely disentangled the origination DCR from the contemporaneous DCR as individual property NOI started to reflect market conditions. Also, the use of smaller loans makes the property in our sample more vulnerable to turnover risk and adds more volatility to NOI.[<u>6</u>] Further, our results find sensible coefficients for most variables except DCR. The DCR coefficient should be negative. This is likely due to the severe econometric problems associated with this specification. It is not a surprise that any estimation that takes loans through the great recession would find LTV or even single-family house prices to be an important part of default risk. The DCR results likely reflect the specification of the model. Finally, our data include loans selected by FNMA for securitization and as a result are of an above average quality in, among other things, LTV. The data used in the original Archer *et al.* estimation were from the Resolution Trust Corporation and represented a relatively highrisk sample of the population of all loans at the time.

Returning to the results at hand, Table 11 adjusts the DCR and LTV ratios from what was observed at origination to contemporaneous values. This should help to disentangle the endogenous relationship of these ratios identified by Archer *et al.* (2002). The results indicate that the contemporaneous or current LTV (*ltv*) and the contemporaneous or current DCR (*dcr*) both perform well and contrast with the origination specification proxies. Figure 2 illustrates the impact of these variables on the probability of default for a representative loan over a large range of values. In particular, any increase in the contemporaneous DCR reduces default probabilities substantially until the ratio is very large. While the marginal impact of *cltv* is not as large, low *cltv* is associated with very few defaults and high *cltv* (*e.g.*, negative equity) is associated with higher default probabilities. One way to read these results is that, for small loans, the ability to generate sufficient NOI to service the loan is a more important trigger of default than low or negative equity.

	Default		Paidoff	
Variable	Coeff	SE	Coeff	SE
loan age	0.160002	0.04	0.120002	0.02
(loan age ²)/100	-0.150002	0.05	-0.070002	0.02
upb orig	-0.070002	0.02	0.020002	0.01
refi	0.18	0.26	-0.860002	0.21
term	-0.01	0.00	-0.010002	0.00
recourse	-0.32	0.46	1.110002	0.54
ю	0.25	0.33	0.370002	0.18
dcr	-1.050002	0.17	0.06	0.05
ltv	2.240002	0.34	-0.660002	0.22
∆irate	-0.520002	0.21	-0.600002	0.12
res ever	-0.84	1.48	0.43	0.67
res on	-0.42	0.74	-1.970002	0.21
ball due	3.540002	0.99	0.49	0.42
5< = unit<10	-0.87	0.55	0.08	0.28
10< = unit<20	-0.38	0.43	0.21	0.21

-0.25	0.40	0.32	0.20
-0.12	0.40	0.19	0.21
0.86	0.79	-0.43	0.79
0.46	0.63	0.900002	0.38
0.38	0.64	0.810002	0.38
0.25	0.57	0.38	0.39
0.62	0.58	0.51	0.43
1.120002	0.56	0.65	0.44
-0.18	0.38	-0.01	0.19
0.26	0.40	0.15	0.31
-0.990002	0.47	-1.190002	0.22
-8.240002	1.71	-8.790002	1.13
6,137			
94,574			
-1,900			
	-0.12 0.86 0.46 0.38 0.25 0.62 1.120002 -0.18 0.26 -0.990002 -8.240002 6,137 94,574	-0.120.400.860.790.460.630.380.640.250.570.620.581.1200020.56-0.180.380.260.40-0.9900020.47-8.2400021.716,137294,5742	-0.120.400.190.860.79-0.430.460.630.900020.380.640.8100020.250.570.380.620.580.511.1200020.560.65-0.180.38-0.010.260.400.15-0.9900020.47-1.190002-8.2400021.71-8.79000294,574I.I.

Note: Standard errors are clustered by loan. No year built information is missing for this subsample. Categorical variables are used to describe the number of units and the year the building was built.

^{**}, ^{**} and ^{*} indicate the coefficient is significant at the 99%, 95% and 90% level.

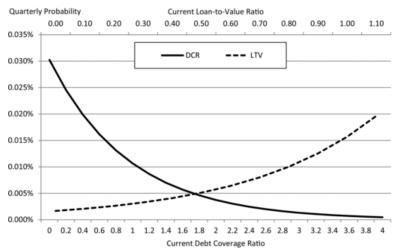


Figure 2 Current loan-to-value ratio, current debt coverage ratio and default. *Notes*: The estimated probability for a representative loan at the current debt coverage ratio (*dcr*) or the current loan-to-value ratio (*ltv*). The "Contemporaneous Ltv and Dcr – Termination Results" specification is used. Continuous variables are evaluated at their means and categorical variables evaluated at the most prevalent category. The categorical variables are set at the following values: *recourse* = 1, *refi* = 0, *io* = 0, *res ever* = 1, *res on* = 0, *ball due* = 0, 20< = unit<30 = 1, all other *unit dummies* = 0, all building *age dummies* = 0, *region4* = 1 and all *other regions* = 0.

Provisions Designed to Suppress Prepayment

Table 12 includes three different specifications to examine more closely how provisions designed to suppress prepayments impact both the probability of prepayment and default. The first specification breaks apart the impact of yield maintenance provisions from a more traditional prepayment penalty (percentage fee based on outstanding balance). For each type of provision, there is an indicator if the

loan ever has the provision and then another indicating the time periods when the provision is active or in effect. The variable "*pen*" refers to penalties and "*ym*" refers to yield maintenance. The implied reference group includes loans that do not have any prepayment suppression mechanism scheduled during the life of the loan. The primary result is that both yield maintenance and prepayment penalties suppress both prepayments and defaults when the provisions are in effect. The yield maintenance provision has a stronger impact on both types of termination. The effect on prepayments is as expected and is consistent with prior literature. It is not clear why a mechanism to reduce prepayments would also be associated with reduced defaults. One potential explanation is that the expiration of these provisions is a large event and that property owners defer making all strategic decisions until there is a viable option to change the source of equity. Borrowers with challenged properties may try to hold on until the provision expiration date only to find that other funding sources are not available. Facing this fact, default may be the only viable option remaining.

	Default		Paidoff	
Specifications	Coeff	SE	Coeff	SE
1. On and Off				
pen ever	0.380002	0.15	0.06	0.06
pen on	-1.640002	0.18	-0.430002	0.07
ym ever	0.18	0.22	1.310002	0.08
ym on	-2.580002	0.23	-2.550002	0.08
2. Prepay Penalty Timing				
ym ever	-0.03	0.24	1.460002	0.08
ym on	-2.060002	0.25	-2.590002	0.08
Years before Penalty Ends:				
4+	-0.33	0.21	-0.580002	0.09
3	-0.720002	0.30	-0.540002	0.13
2	-1.170002	0.35	-0.350002	0.11
0 and 1	-0.710002	0.29	-0.200002	0.10
Years after Penalty Ends:				
1	0.11	0.24	0.850002	0.09
2	-0.06	0.31	-0.260002	0.13
3	-0.27	0.38	0.06	0.12
4+	-0.39	0.44	0.430002	0.10
3. Yield Maintenance Timing				
pen ever	0.410002	0.15	0.160002	0.06
pen on	-1.640002	0.18	-0.530002	0.07
Years before Penalty Ends:				
4 +	-2.420002	0.16	-1.550002	0.08
3	-2.530002	0.42	-1.250002	0.11
2	-1.810002	0.37	-0.970002	0.11
0 and 1	-2.730002	0.72	-0.380002	0.10

12 Table. Provisions designed to suppress prepayments—termination results

Years after Penalty Ends:				
1	0.780002	0.24	1.900002	0.08
2 and 3	-1.190002	0.73	-0.12	0.20
4+	-0.53	0.71	0.460002	0.16

Note: Standard errors are clustered by loan. All additional variables included are dummy variables. For example, "Years before Penalty Ends: 4+" indicates time periods four or more years before the provision has expired. "Years before Penalty Ends: 3" indicates time periods in the third year before the provision expires. "Years before Penalty Ends: 0 and 1" indicates time periods in the quarter the provision expires (time = 0) and the year before the expiration. Each of the three specifications is estimated separately. The variables indicated are included in addition to the "Base Model" specification with one adjustment. The *res ever* and *res on* variables are not included in any of the three specifications in this table. The *pen ever, pen on, pen ever* and *pen on* variables are structured in the same way as the *res ever* and *res on* variables for prepayment penalties (*pen*) and yield maintenance provisions (*ym*). For all specifications the reference groups are loans that did not have a suppression mechanism ever (over the whole potential life) of the loan.

****, *** and * indicate the coefficient is significant at the 99%, 95% and 90% level.

There is likely significant strategic behavior by both lenders and borrowers as the expiration of a prepayment provision approaches. For example, the cost of the provision is typically declining over time as the outstanding balance falls and the expiration date approaches. To test for this, a series of dummy variables are created indicating time until a provision expires or how long ago a provision expired. It is expected that the effect of the provision will weaken as the provision gets closer to expiration and that there will be a large jump in prepayments just after the provision expires. Specification 2 tests this proposition for prepayment penalties and specification 3 tests for the same patterns for yield maintenance provisions. Both specifications find that the probability of prepayments starts increasing as much as three years before the provision expires. Figures 3–6 provide a visual representation of these results for a representative or "average" loan. [7] Figure 3 shows the predicted default probabilities as the loan ages and approaches the expiration of a six-year (24 quarter) prepayment penalty. These probabilities are compared to another representative loan that is identical in all ways except it has no prepayment constraint applied at any point in the term. Figure 4 repeats this process for the probability the loan is paid in full and Figures 5 and 6 repeat the process again for the yield maintenance provision. The figures show that default probabilities are low for most of the loan's life but temporarily spike up after the provision expires. The spike is largest when the yield maintenance provision expires—the estimated default probability increases more than 27 times. However, the magnitude of the default probability is still fairly modest—approximately 0.27% probability per guarter for three guarters after expiration.

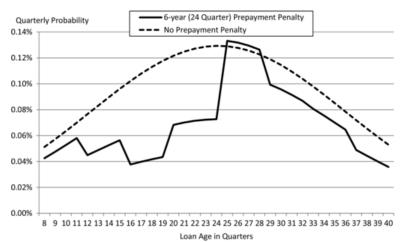


Figure 3 Six-year prepayment penalty expires and default probabilities. *Notes*: The average end of either provision (penalty or yield maintenance) is close to the end of six years of mortgage life. Therefore, the end of both provisions is set the fourth quarter of year 6 of the life of the mortgage (24th quarter). The estimated probability is calculated with all nonrelevant continuous variables at their means as loan age increases and provision expires. The categorical variables are set at the following values: *recourse* = 1, refi = 0, *io* = 0, *pen ever* = 1, *ym ever* = 0, *ym on* = 0, *ball due* = 0, 20< = unit<30 = 1, all other *unit dummies* = 0, all building *age dummies* = 0, *region4* = 1 and all *other regions* = 0. The relevant "Provisions Designed to Suppress Prepayments – Termination Results" specifications are used.

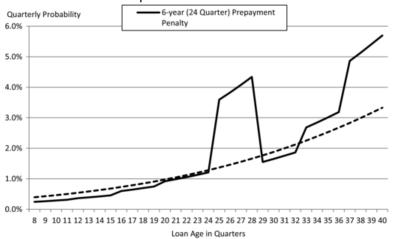


Figure 4 Six-year prepayment penalty expires and paid off probabilities. Notes: The average end of either provision (penalty or yield maintenance) is close to the end of six years of mortgage life. Therefore, the end of both provisions is set the fourth quarter of year 6 of the mortgage's life (24th quarter). The estimated probability is calculated with all nonrelevant continuous variables at their means as loan age increases and the provision expires. The categorical variables are set at the following values: recourse = 1, refi = 0, io = 0, pen ever = 1, ym ever = 0, ym on = 0, ball due = 0, 20 < = unit<30 = 1, all other unit dummies = 0, all building age dummies = 0, region4 = 1 and all other regions = 0. The relevant "Provisions Designed to Suppress Prepayments – Termination Results" specifications are used.

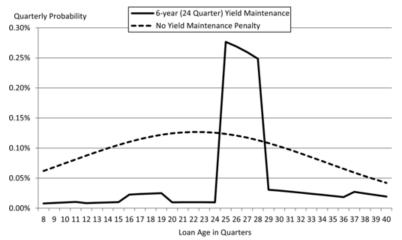


Figure 5 Six-year yield maintenance expires and default probabilities. Notes: The average end of either provision (penalty or yield maintenance) is close to the end of six years of mortgage life. Therefore, the end of both provisions is set in the fourth quarter of year 6 of the mortgage's life (24th quarter). The estimated probability is calculated with all nonrelevant continuous variables at their means as loan age increases and provision expires. The categorical variables are set at the following values: recourse = 1, refi = 0, io = 0, $ym \ ever = 1$, $pen \ ever = 0$, $pen \ on = 0$, $ball \ due = 0$, 20 < = unit < 30 = 1, all other $unit \ dummies = 0$, all building $age \ dummies = 0$, region4 = 1, and all other regions = 0. The relevant "Provisions Designed to Suppress Prepayments – Termination Results" specifications are used.

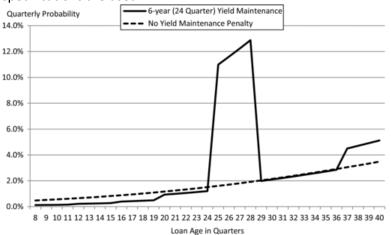


Figure 6 Six-year yield maintenance expires and paid off probabilities. *Notes*: The average end of either provision (penalty or yield maintenance) is close to the end of six years of mortgage life. Therefore, the end of both provisions is set the fourth quarter of year 6 of the mortgage's life (24th quarter). The estimated probability is calculated with all nonrelevant continuous variables at their means as loan age increases and provision expires. The categorical variables are set at the following values: recourse = 1, refi = 0, io = 0, $ym \ ever = 1$, $pen \ ever = 0$, $pen \ on = 0$, $ball \ due = 0$, 20 < = unit < 30 = 1, all other *unit* dummies = 0, all building *age* dummies = 0, region4 = 1 and all *other regions* = 0. The relevant "Provisions Designed to Suppress Prepayments – Termination Results" specifications are used.

Figures 4 and 6 document a similar pattern for prepayment. Before the provision expires the probability of prepayment is suppressed; immediately after it expires, prepayments spike up temporarily. The impact is largest for the yield maintenance provision—a more than ninefold increase. The magnitude of the prepayment effect is substantial—approximately 13% probability per quarter for three quarters. Using the exact estimates and incorporating compounding, the results imply that just

over 40% of the outstanding loans will prepay in the first year after a yield maintenance provision expires.

In summary, the provisions are very effective at suppressing prepayments and clustering the termination of loans around the provision expiration dates. This substantially reduces the risk of early payments for securities backed by these loans. Yield maintenance provisions seem to be most effective in concentrating termination into a short time frame. The primary cost is that defaults are also centered on the expiration date; however, the probability of default is much lower. For example, using a representative loan in the first quarter after a six-year yield maintenance provision, the probability of default is only 2.5% of the probability of prepayment (default probability = 0.28% and prepayment probability = 10.99%).

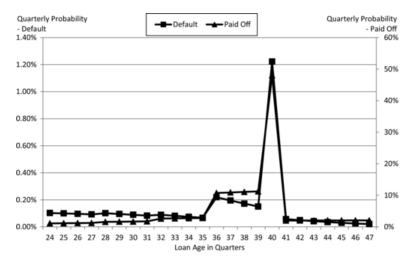
Term Default and Extension Risk

Table 13 examines default at term (the scheduled end of the loan) and risk that the loan continues beyond term (extension risk). Since none of the loans are fully amortizing, term default and extension risk are nontrivial issues necessary to value multifamily mortgages. A similar approach is taken to model any strategic behavior around the term date. The reference group is four or more years before the term (or the balloon payment date). Dummies are created to indicate if the loan is three years before the due date, two years before the due date, one year before the due date, the quarter the balance of the loan is due (term quarter) or all years or quarters after term. Because lenders and borrowers are likely preparing and communicating about the end of the loan, it is not surprising that the rate of loan pay offs and defaults steadily rises as the due date approaches. This can be seen in Table 13 and in Figure 7. Similar to the prepayment provisions, the term date is associated with a large spike in loans being paid off at term and a similar spike in loans defaulting at term. The estimated pay off probability jumps from 11.23% to 48.05% when the loan balance is due at term. At the same time, the probability of default jumps from 0.15% to 1.22%. The implication for both full payment at term (48.05) and default (1.22) is that 50.73% of loans that were open the quarter before term extend. In short, there is substantial extension risk. Of the loans that extend, the probability of default is permanently elevated relative to baseline levels but is still below preterm magnitudes.[8]

	Default		Paidoff	
Specifications	Coeff	SE	Coeff	SE
1. Balloon Timing				
Years before Balance Due:				
4+ (excluded)				
3	0.13	0.23	0.190002	0.08
2	0.26	0.25	0.570002	0.09
1	1.590002	0.17	1.960002	0.08
Quarter Balance is Due:	4.290002	0.23	3.900002	0.13
Years after Balance Due:				
1+	0.810002	0.41	0.08	0.23

13 Table. Term default and extension risk results

Note: Standard errors are clustered by loan. All additional variables included are dummy variables. For example, "Years before Balance Due: 4+" indicates time periods four or more years before the provision has expired. "Years before Penalty Ends: 3" indicates time periods in the third year before the provision expires. "Years before Penalty Ends: 1" indicates time periods in the year before the expiration. The variables indicated are included in addition to the "Base Model" specification with one adjustment. As all loans are not fully amortizing, the reference group is loans that are four or more years before term.



****, ** and ^{*} indicate the coefficient is significant at the 99%, 95% and 90% level.

Figure 7 10-year balloon (40 quarter) and default and paid off probabilities. Notes: The most prevalent term is 10 years. Therefore, the term is set to 120 months (40 quarters). The estimated probability is calculated with all nonrelevant continuous variables at their means as loan age increases and term expires. The categorical variables are set at the following values: recourse = 1, refi = 0, io = 0, $ym \ ever = 1$, $ym \ on = 0$, $pen \ ever = 0$, $pen \ on = 0$, $ball \ due = 0$, 20 < = unit<30 = 1, all other $unit \ dummies = 0$, all building $age \ dummies = 0$, region4 = 1 and all other regions = 0. The relevant "Balloon Payment – Termination Results" specification is used.

Conclusion

This article provides a detailed look at the performance of the small multifamily loan market. It represents the first research to target the small multifamily loan segment in over 10 years. In general, we find that the mortgage market functions as expected and that both borrowers and lenders react to the incentives provided by local market conditions, the property's performance and the specific contract provisions in the mortgage. The results also clarify the findings of Archer *et al.* (2002) that LTV and DCRs at origination are endogenous and not predictive of default in a meaningful way. We show that including the contemporaneous LTV and DCR addresses the endogeneity and that both have meaningful effects on default and prepayment of multifamily loans. However, small loans appear to be more sensitive to the ability to make monthly payments on the loan (DCR) than their equity position (LTV).

It is traditional for multifamily loans to have balloon payments at the end of the loan's life and prepayment penalties and/or yield maintenance contract provisions through different points in the loan's life. These provisions have profound impacts on the expected duration of the loans and the amount of credit risk. Both yield maintenance and prepayment penalties are largely successful in suppressing prepayments. When these provisions expire, prepayments increase dramatically (three to

nine times). The main benefit of these provisions is that the expected duration of a pool of loans with similar expiration dates is fairly simple to calculate. The cost of these provisions is that they also increase the probability of default as the provision expires. However, the magnitude of these probabilities is much smaller for defaults than prepayments.

Since the loans in this sample are not fully amortizing and require a large lump sum payment to pay off the remaining balance at term, it is important to examine the risk of term default and the risk of loan extension beyond term. Similar to the expiration of the prepayment suppression provisions, balloon payments concentrate a lot of terminations at one point in time. For a typical 10-year term loan with a balloon payment, 1.22% of active loans default at term and 48.05% are paid off. The remaining 50.73% extend the loan beyond its contractual length (term). For these remaining loans the probability of default is still elevated (relative to the baseline) until final termination. Hence, term default and extension risk generate a substantial risk for small multifamily loans.

In summary, the detailed contract provisions of a commercial property mortgage are extremely important. Not knowing the details of the prepayment suppression provisions could lead to gross miscalculations of expected mortgage yields and cash flows. We also find evidence that the performance of the property in terms of occupancy, operational efficiency and cash reserves has a meaningful impact on loan quality. Not surprisingly, local economic conditions also matter. In general, properties perform the worst when both the property itself is struggling and local economic conditions (labor and space markets) are poor. The contemporaneous market value of the property, DS and income and cash flow generated by the property are all key determinants in understanding when default and repayment will occur.

Acknowledgments

The authors wish to thank Fannie Mae for access to the data and continual consultation throughout the project. We also extend our appreciation to the Homer Hoyt Institute for inviting us to participate in the research project.

Appendix

Approximation of Archer et al. logit specification on the FNMA small loan data

Description	Coeff	SE	z-stat	p > z
70 < Origination LTV < = 80	0.554	0.375	1.48	0.14
80 < Origination LTV < = 85	0.874	0.431	2.03	0.04
85 < Origination LTV < = 100	1.627	0.847	1.92	0.06
1.2 < Origination DCR < = 1.4	2.225	1.086	2.05	0.04
1.4 < Origination DCR < = 1.6	2.236	1.136	1.97	0.05
1.6 > Origination DCR	1.112	1.243	0.89	0.37
At origination interest rate on mortgage—10-year treasury	0.220	0.305	0.72	0.47
constant maturity				
Loan is not fully amortizing	0.546	0.596	0.92	0.36
Loan age at securitization	-0.169	0.089	-1.89	0.06
Year Built	-0.024	0.006	-4.04	0.00

Units at origination	-0.016	0.012	-1.3	0.19
Price per unit at origination	0.000	0.000	-2.82	0.01
Percent change in FHFA metro area price index in first three years	-0.049	0.011	-4.33	0.00
of loans life				
Change in employment in first three years of loan's life	0.002	0.004	0.45	0.65
Change in 10-year treasury rates in first three years of loan's life	0.011	0.266	0.04	0.97
Origination year fixed effects (1998–2011)				
Number of obs =1,440				
LR χ ² (25)=87.25				
$Prob > \chi^2 = 0.0000$				
Log likelihood =-221.25775				
Pseudo R ² =0.1647				

Dependent variable—loan everover 90 days or more delinquent for the first seven years of the loan's life. Loans seasoned more than two years before purchase are excluded. There is one observation per loan. Origination year dummies serve as a type of default baseline. It should be noted that there is specification bias in the above results due to both left and right censoring and because the competing risk of prepayment is ignored.

Footnotes

- 1 Although it may appear abnormal for mortgage loans to have both a prepayment penalty and yield maintenance, this combination is prevalent in Fannie Mae multifamily loans, especially in the later years of the observation period. There are different calculations for the amount of prepayment premium to be paid which depends upon the date during the term of the loan when the prepayment is being made. If the borrower wants to prepay any time after the effective date of the loan and before the yield maintenance period ends, which is generally just before the last 12 months of the loan, the prepayment premium is equal to the greater of the yield maintenance calculation or 1% of the principal being repaid. If the borrower wants to pay after the yield maintenance period has ended (i.e., during the last 12 months of the loan term), then the prepayment premium is 1% of outstanding unpaid principal balance—no yield maintenance calculation. Note, Fannie Mae has used defeasance in structuring nonresidential mortgages, but there are no cases of its use in our data set.
- 2 Originally, Fannie Mae was a federal government agency. Its mandate was to act as a secondary mortgage market facility that could purchase, hold and sell FHA-insured loans, and expand liquidity in the mortgage market. Over time Fannie Mae's authority expanded and in 1984 Fannie Mae created a distinct business division to purchase multifamily loans (FNMA [16]).
- 3 In our empirical work, all loans from metropolitan areas that include Los Angeles, New York, Washington, San Francisco and San Jose are considered high-cost locations.
- 4 X_p represents the risk characteristics associated with both the put and call options in the mortgage contract, and also contains information on the volatility associated with those risk characteristics (see Kau et al. [22] for further discussion).
- 5 The annual Federal Housing Administration Actuarial Reports prepared by Integrated Financial Engineering (see [21], for example) rely on a similar bivariate model.
- 6 The impact on NOI when one tenant leaves a space can be much more severe if the property has just 5 units than if the property has 200 units.
- 7 See the note below each of the figures for the details used to define the representative loan.

8 This is because baseline probabilities are dropping due to the strong quadratic shape of the baseline.

References

- Abraham, J.M. and H. Scott Theobold. 1997. A Simple Prepayment Model of Commercial Mortgages. Journal of Housing Economics 6 : 31 – 59.
- Akat, M., B.W. Ambrose, O. Erdem and Y. Yildirim. 2012. Valuing Default and Defeasence Option for Commercial Mortgage Backed Securities. Working Paper #1, Istanbul Menkul Kiymetler Borsasi.
- Ambrose, B. and M. LaCour-Little. 2001. Prepayment Risk in Adjustable Rate Mortgages Subject to Initial Year Discounts: Some New Evidence. *Real Estate Economics* 29(2): 305 – 327.
- Ambrose, B.W. and A.B. Sanders. 2003. Commercial Mortgage-Backed Securities: Prepayment and Default. *Journal of Real Estate Finance and Economics*, 26 (2/3): 179 196.
- An, X., J.M. Clapp and Y. Deng. 2010. Omitted Mobility Characteristics and Property Market Dynamics: Application to Mortgage Termination. *The Journal of Real Estate Finance and Economics* 41 (3): 245 – 271.
- Archer, W., P. Elmer, D. Harrison and D. Ling. 2002. Determinants of Multifamily Mortgage Default. *Real Estate Economics* 30 (3): 445 473.
- Campbell, T.S. and J. Kimball Dietrich. 1983. The Determinants of Default on Insured Conventional Residential Mortgage Loans. *Journal of Finance* 38 (5): 1569 1581.
- Capone, C.A. and L. Goldberg. 2001. Renter Mobility and Multifamily Mortgage Default Risk. *Journal of Housing Economics* 10 (1): 21 40.
- Chen, J. and Y. Deng. 2012. Commercial Mortgage Workout Strategy and Conditional Default Probability: Evidence from Special Serviced CMBS Loans. NUS Institute of Real Estate Studies Working Paper IRES2010-011.
- Ciochetti, B.A., Y. Deng, B. Gao and R. Yao. 2002. The Termination of Commercial Mortgage Contracts Through Prepayment and Default: A Proportional Hazard Approach with Competing Risks. **Real Estate Economics** 30 (4): 595 – 633.
- Ciochetti, B.A., Y. Deng, G. Lee, J.D. Shilling and R. Yao. 2003. A Proportional Hazards Model of Commercial Mortgage Default with Originator Bias. Journal of Real Estate Finance and Economics 27 (1): 5 – 23.
- Dunn, K.B. and J.J. McConnell. 1981. Valuation of GNMA Mortgage-Backed Securities. Journal of Finance 36 (3): 599 616.
- Eppli, M. and C. Tu. 2005. Who Bears the Balloon Risk in Commercial MBS? The Journal of Portfolio Management 31 : 5114 5123.
- Fabozzi, F.J., S. Stanescu and R. Tunaru. 2013. Commercial Real Estate Risk Management with Derivatives. The Journal of Portfolio Management 39 (5): 111 119.
- FNMA (Federal National Mortgage Corporation). 2011. Fannie Mae's Role in the Small Multifamily Loan Market.
- FNMA (Federal National Mortgage Corporation). 2012. An Overview of Fannie Mae's Multifamily Mortgage Business. May 1.
- Fu, Q., M. LaCour-Little and K.D. Vandell. 2003. Commercial Mortgage Prepayments Under Heterogeneous Prepayment Penalty Strictures. Journal of Real Estate Research 25 (3): 245 – 275.

- Goldberg, L. and C.A. Capone. 2002. A Dynamic Double-Trigger Model of Multifamily Mortgage Default. **Real Estate Economics** 30 (1): 85 113.
- Green, J. and J.B. Shoven. 1986. The Effects of Interest Rates on Mortgage Prepayments. Journal of Money Credit and Banking 18 (1): 41 59.
- Heschmeyer, M. 2012. Could Multifamily Lead Single-Family out of its Recession? CoStar Group Press Release.
- Integrated Financial Engineering. 2015. Federal Housing Administration Actuarial Reports.
- Kau, J.B., D.C. Keenan and T. Kim. 1994. Default Probabilities for Mortgages. Journal of Urban Economics 35 (3): 278 – 296.
- Kelly, A. and V.C. Slawson, Jr. 2001. Time-Varying Mortgage Prepayment Penalties. Journal of Real Estate Finance and Economics 23 (2): 235 – 254.
- LaCour-Little, M. 2008. Mortgage Termination Risk: A Review of the Recent Literature. Journal of Real Estate Literature 16 (3): 297 326.
- Levitin, A.J. and S.M. Wachter. 2013. The Commercial Real Estate Bubble. Georgetown Law and Economics Research Paper 12-005: 83 118.
- Long, J.T.T. 1997. Regression Models for Categorical and Limited Dependent Variables. Thousand Oaks, CA : Sage Publications.
- MacDonald, G. and T. Holloway. 1996. Early Evidence on Balloon Performance, Journal of Real Estate Finance and Economics 12 (3): 279 – 293.
- Mortgage Bankers Association (MBA). 2009. Survey on Multifamily Lending. Available at www.mbaa.org.
- Mortgage Bankers Association (MBA). 2012. Commercial and Multifamily Mortgage Delinquency Rates Continue to Drop for Banks, Rise for CMBS in Second Quarter of 2012, Press Release. Available at www.mbaa.org.
- Mortgage Bankers Association (MBA). 2015. Commercial Multifamily Mortgage Debt Outstanding, Third Quarter 2015. Available at *www.mbaa.org*.
- Nothaft, F.E. and J.L. Freund. 2003. The Evolution of Securitization in Multifamily Mortgage Markets and its Effect on Lending Rates. *Journal of Real Estate Research* 25 (2): 91 – 112.
- Pennington-Cross, A. 2010. The Duration of Foreclosures in the Subprime Market: A Competing Risks Model with Mixing. *Journal of Real Estate Finance and Economics* 40 (2): 109 – 129.
- Tu, Charles C. and Mark J. Eppli. 2003. Term Default, Balloon Risk, and Credit Risk in Commercial Mortgages. *The Journal of Fixed Income* 13 (3): 42 52.
- Vandell, K.D., W. Barnes, D. Hartzell, D. Kraft and W. Wendt. 1993. Commercial Mortgage Defaults:
 Proportional Hazards Estimation Using Individual Loan Histories. *Journal of the American Real Estate and Urban Economics Association* 21 (4): 451 480.