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A Simple Dynamics Experiment Based on Acoustic Emission

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ABSTRACT

This paper describes a simple experiment well suited for an undergraduate course in mechanical measurements and/or dynamics, in which physical information is extracted from an acoustic emission signature. In the experiment, a ping–pong ball is dropped onto a hard table surface and the audio signal resulting from the ball–table impacts is recorded. The times between successive bounces, or "flight times", are used to determine the height of the initial drop and the coefficient of restitution of the impact. The experiment prompts questions about modeling the dynamics of a simple impact problem, including the use of the coefficient of restitution and the importance of accounting for aerodynamic effects.

1. Introduction

Acoustic emission is an example of an indirect measurement technique in which clues to the behavior of a physical system are obtained via sound signatures. Sound waves are often incidental manifestations of dynamic behavior, and provide the opportunity to learn much about the physical world. This paper describes an experiment in which physical information is extracted from an acoustic emission signature. It is simple to conduct, affords students the opportunity to compare their results with established theoretical concepts, and is well suited for undergraduate engineering courses in “mechanical measurements” and “dynamics”.

In the experiment, shown in the photographs of Fig. 1, a ping–pong ball is dropped from rest onto a hard table surface. The acoustic signatures of the ball–table impacts are recorded using a microphone (with an integrated pre-amplifier) attached to the sound card of a PC. A shareware software package, GoldWave,[1] is used to record, play, and analyze the audio signal picked up from the microphone. (Sound measurement is addressed in mechanical measurement textbooks see, for example.[1]) The software displays the temporal history of the bounce sounds of successive impacts, from which the times between bounces, or “flight times”, are determined. The flight times are used to calculate the height of the initial drop and the coefficient of restitution of the ball–table impacts.

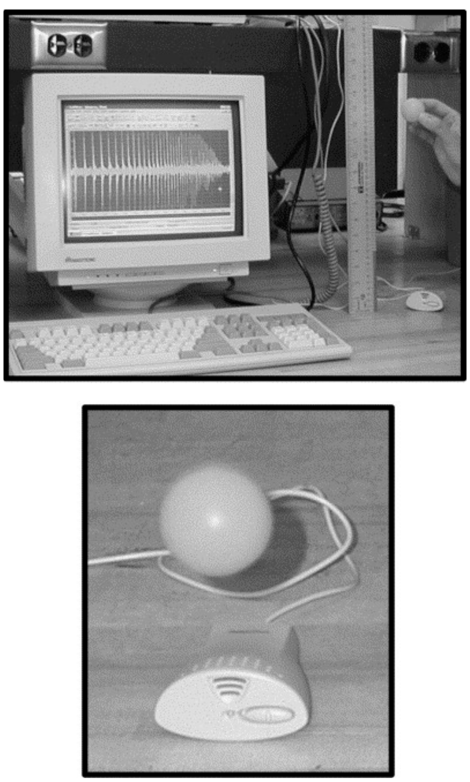

Fig. 1. Photographs of experimental setup (top) and ball falling near microphone (bottom).

1.1. Background: table tennis

The official rules of table tennis[2] specify the characteristics and properties of a ping–pong ball: “The ball shall be spherical, with a diameter of 38 mm. The ball shall weigh 2.5 g. The standard bounce required shall be not less than 23.5 cm nor more than 25.5 cm when dropped from a height of 30.5 cm on a specially designed steel block. The standard bounce required shall not be less than 22 cm nor more than 25 cm when dropped from a height of 30.5 cm on an approved table”. The rebound height depends on elastic properties of the ball and the surface of impact, and is specified in dynamics by the coefficient of restitution.

2. Theoretical Foundation

This experiment focuses on one type of collision which can be modeled classically as a direct, central impact of particles. The theory of direct, central impact of particles – which follows from Newton's laws of motion – can be found in numerous textbooks (e.g.,[2,3]) that present the fundamentals of dynamics. More advanced treatments of planar and three-dimensional impact for both particles and rigid bodies are developed in specialized textbooks (e.g.,[4,5,6]). A method for the measurement of the coefficient of restitution for collisions between a bouncing ball and a horizontal surface is provided in.[7] The derivation of this paper draws heavily from this method.

A ball is dropped from known height h_0 onto a tabletop, modeled as a massive (i.e., immobile), smooth, horizontal surface. The trajectory of the ball is depicted schematically in Fig. 2 which shows the ball height as a function of time for the first few collisions. The ball is modeled as a particle, and vertical motion only is considered in the analysis. Due to the inelastic nature of the ball–table collision, the maximum height of the ball decreases successively with each impact. The trajectory of the ball can be described in terms of the ball height from the surface as a function of time. Neglecting aerodynamic resistance, for the simplest model, the flight time from the top of the trajectory to the surface is then half of the total flight time, T_n, between the nth and (n+1)th bounce.

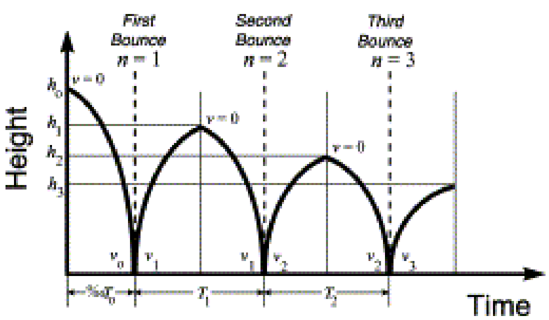

Fig. 2. Ball height vs. time for first few bounces.

For the case of no air resistance, the vertical speed v_n (i.e., the vertical component of the velocity) of the ball associated with its nth bounce is

(1) $v_n = gT_n 2 (n=1,2,3,\ldots),$

where g is the acceleration due to gravity. Assuming a constant coefficient of restitution, e,

(2) $v_n = ev_{n-1} = e^n v_0 (n=1,2,3,\ldots).$

Equating (1) and (2) yields

(3) $T_n=e^n 2v_0 g(n=1,2,3,\ldots)$,

from which

(4) $\log T_n = n\log e + \log 2v_0 g(n=1,2,3,\ldots)$.

Plotting Eq. (4) in a graph of $\log(T_n)$ vs. n yields a straight line with slope $m=\log e$ and ordinate intercept $b=\log(2v_0/g)$. The intercept can be used to determine v_0 from which the initial height h_0 can be found. In general, the height h_n reached after the *n*th bounce can be written as

(5) $h_n={v_n}^2 2g(n=0,1,2,\ldots)$,

from conservation of energy. The coefficient of restitution and the drop height can then be expressed from the slope and intercept, respectively, as

(6) $e=10^m, h_0=18g(10^{2b})$.

Although the number of bounces is infinite in theory, the total time required for the ball to come to rest and the total distance traveled are both finite (see Example 4–5 in[2]). The total time required for the ball to come to rest (assuming no air resistance) is

(7) $T_{total}=12T_0+\sum_{n=1}{}^{\infty}T_n=v_0g+2v_0g\sum_{n=1}{}^{\infty}e^n=v_0g1+2\sum_{n=1}{}^{\infty}e^n=2h_0g1+2\sum_{n=1}{}^{\infty}e^n=2h_0g1+e1-e$.

The total distance traveled along the path is

(8) $S_{total}=h_0+2\sum_{n=1}{}^{\infty}h_n=h_0 1+2\sum_{n=1}{}^{\infty}e^{2n}=h_0 1+e^2 1-e^2$.

3. Protocol

In the experiment, the procedure is to drop the ball from rest from a measured height above the table such that it lands and bounces near the microphone (see Fig. 1). The software can be initiated for recording upon dropping the ball or configured for automatic recording once a bounce sound is detected. Bounce sounds are indicated by spike amplitudes in the audio signal display, as shown in Fig. 3, and the times associated with the bounces are found. The times between bounces give the flight times. From a linear regression curve fit of the $\log(T_n)$ vs. *n* data, plotted in Fig. 4 for the first ten bounces, the coefficient of restitution *e* and height of initial ball drop h_0 can be determined from Eq. (6).

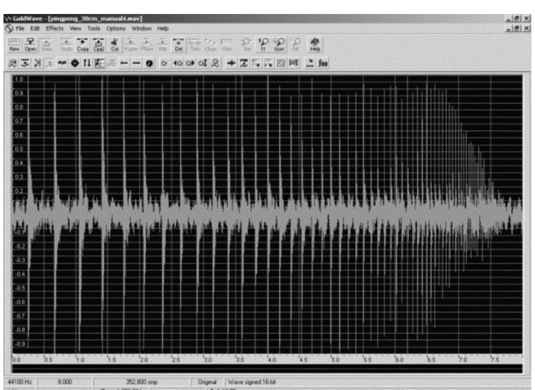

Fig. 3. Bounce history audio signal with ball–table impacts indicated by spikes (for ball dropped from 30.5 cm, bounce sounds end in 7.5 s).

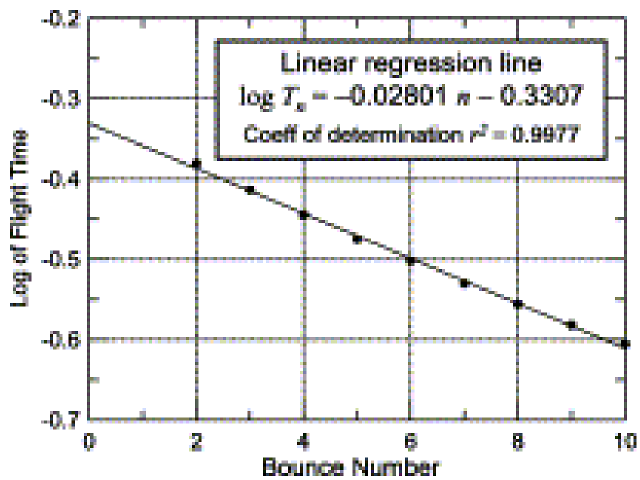

Fig. 4. Log of flight time vs. bounce number.

Students are asked to address the following sample questions:

1. Determine the range of coefficients of restitution of a ball with a “steel block” and an “approved table” based on the rules of table tennis. Compare this with the experimentally determined coefficient.
2. Determine the initial drop height h_0 of the ball as well as the height h_n after the nth bounce. Compare the calculated initial height with the measured height.
3. Determine the total time required for the ball to come to rest and the total distance traveled along the path. If, mathematically, the number of bounces is infinite, why does the ball stop bouncing? Comment.
4. Investigate the assumption that the coefficient of restitution is a constant.
   - Determine the coefficient of restitution accounting for the first ten or fifteen bounces only. Is this coefficient of restitution more accurate than one that includes the full bounce history, or one based on the last ten or fifteen bounces only?
   - Consider the possibility that the coefficient of restitution is not constant but rather a function of approach speed. Calculate the coefficient of restitution for each collision directly from the definition, $e_n = v_n/v_{n-1}$. What conclusions can you draw from plotting the results as a function of bounce number?

5. Investigate the assumption that air resistance is negligible. Develop a dynamic model that includes aerodynamic resistance and simulate its effect. Are aerodynamic effects important in the model? If so, are the effects equally significant for the beginning bounces and later bounces? Consider the possibility that the drag coefficient is not constant with velocity.
6. Determine the accumulated energy loss at the bounces. Assuming the entire loss is converted to thermal energy at the first bounce, estimate the temperature increase (taking the ball material as plastic). Assuming the entire loss is converted to acoustic energy at the first bounce, estimate the noise. Comment.
7. Does the bounce frequency change near the end of the bounces? Comment.
8. Estimate the time of contact between the ball and table at a bounce. Is it reasonable to neglect this time of contact in the model? Is the time of contact constant for successive bounces?

9. Someone suggests that the ball can be modeled as an equivalent mass-spring-damper system. Assuming a linear model, how would the mass, stiffness, and damping values be determined? Are these parameters related to the coefficient of restitution and the contact time at each bounce?
10. Someone suggests that gravity acts like a spring between the ball and the table. Is this idea reasonable? If so, does gravity act like a linear spring?
11. Is it appropriate to model the ball as a particle undergoing vertical motion only? Why does the ball rotate and migrate as it bounces? Is the motion of the ball deterministic?

Many other questions can be posed to trigger discussion and prompt student thinking about the physics of impact and assumptions of appropriate models.

4. Results And Discussion

A ping–pong ball was dropped from an initial height of 30.5 cm onto a butcher-block-top lab bench. The coefficient of restitution *e* and the height of the initial drop h_0, calculated from the linear regression curve fit of the data presented in Fig. 4, give $e = 0.9375$ and $h_0 = 26.7$ cm, or a 12.3% error in predicted drop height. The predicted coefficient of restitution is higher than the theoretical range of $0.878 \leqslant e \leqslant 0.914$ for a "steel block" and $0.849 \leqslant e \leqslant 0.905$ for an "approved table" based on the rules of table tennis. The differences in the predicted vs. actual initial heights and in the coefficients of restitution may be attributable, as indicated below, to neglecting aerodynamic drag in the analysis.

In theory, the total time required for the ball to come to rest is a function of the coefficient of restitution, as indicated in Eq. (7) and plotted in Fig. 5 for an initial height of 30.5 cm. In the experiment, the total time before the ball comes to rest is 7.50 s corresponding to a coefficient of restitution of 0.938. Similarly, the total distance traveled by the ball before it comes to rest is a function of the coefficient of restitution for a given initial height, as indicated in Eq. (8). For a drop height of 30.5 cm and a coefficient of restitution of 0.94 the total distance traversed is 4.94 m.

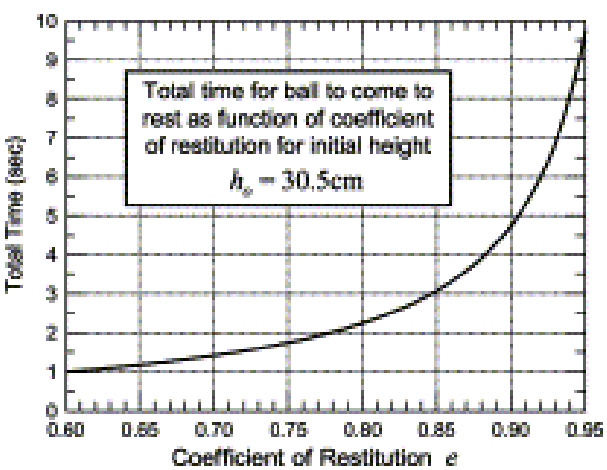

Fig. 5. Total time of bounce history vs. coefficient of restitution.

The coefficient of restitution is a composite index that accounts for impacting body geometries, material properties, and approach velocities. To investigate whether the coefficient of restitution remains constant for each ball–table impact requires a slight modification of Eq. (2),
namely, $v_n = e_n v_{n-1}$(n=1,2,3,…). where e_n is the coefficient for the *n*th bounce. Combining this equation with Eq. (1) yields an expression for the coefficient of restitution in terms of flight

times, $e_n=T_n/T_{n-1}$(n=1,2,3,…). The values plotted in Fig. 6 for the first thirty bounces, as well as the linear regression fit, indicate a trend-wise increase in the coefficient with successive bounces.

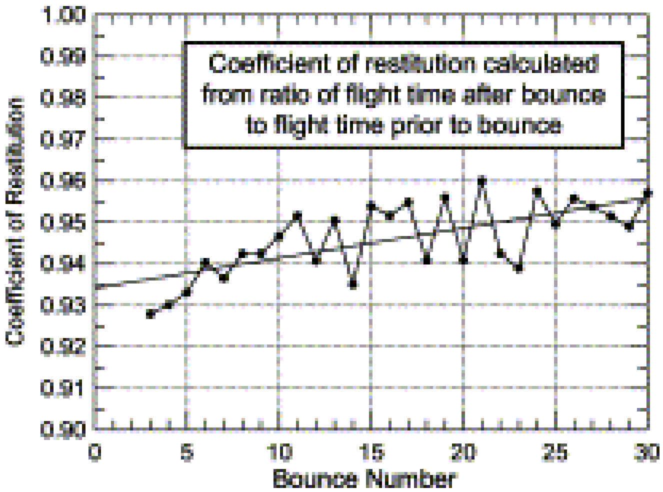

Fig. 6. Coefficient of restitution vs. bounce number.

The effect of aerodynamic drag has thus far been neglected in the analysis. Without aerodynamics in the model the acceleration of the ball during flight is constant (due to gravity only). More complete models that consider the retarding effect of drag during the flight phases can be developed. For example, Fig. 7 shows a free-body diagram of a falling ball, accounting for the body force $F_{\mathrm{body}} = mg$ (the weight) and the aerodynamic drag force $F_{\mathrm{aero}}=12\rho AC_D v^2$, where m is the mass of the ball, ρ is the density of air, A is the ball cross-sectional area, and C_D is the drag coefficient. The equation of motion for the falling ball can be written in a form that shows the aerodynamic force normalized with respect to the body force, i.e., $g(1-(F_{\mathrm{aero}}/F_{\mathrm{body}})) = dv/dt$. The magnitude of the normalized force term determines the importance of aerodynamic effects in the model.

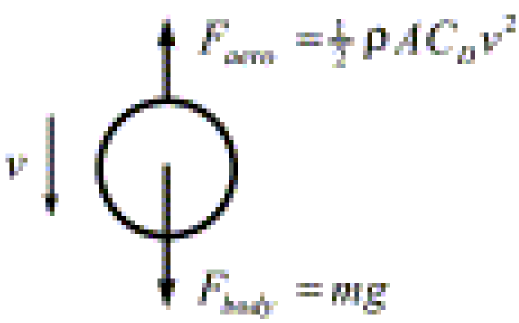

Fig. 7. Free-body diagram of falling ball.

Different aerodynamic models can be considered. One approach is to assume a constant (i.e., velocity-independent) drag coefficient. An alternative approach is to recognize that the drag coefficient of flow over a smooth sphere is a function of the Reynolds number, and hence is velocity dependent (see, for example, Fig. 9.11 of[8]). This method was adopted to develop Fig. 8, which shows the aerodynamic force normalized by the body force, i.e., the weight of the ping–pong ball (0.0245 N), as a function of ball velocity. At 2.4 m/s, the maximum approach speed for a 30.5 cm drop, the aerodynamic force is 6.4% of the body force. Fig. 9 gives the height history predicted by a dynamic simulation model that accounts for the velocity-dependent aerodynamic effects of Fig. 8 for the case of a (constant) coefficient of restitution of 0.94. Although aerodynamic effects do not dominate the trajectory dynamics, they influence the bounce history and are needed for a more accurate model.

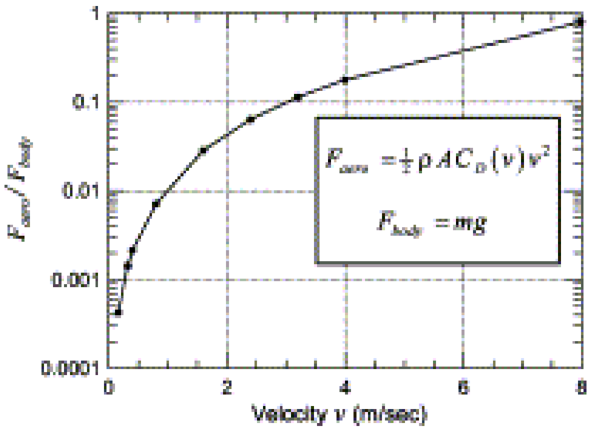

Fig. 8. Normalized aerodynamic force vs. velocity.

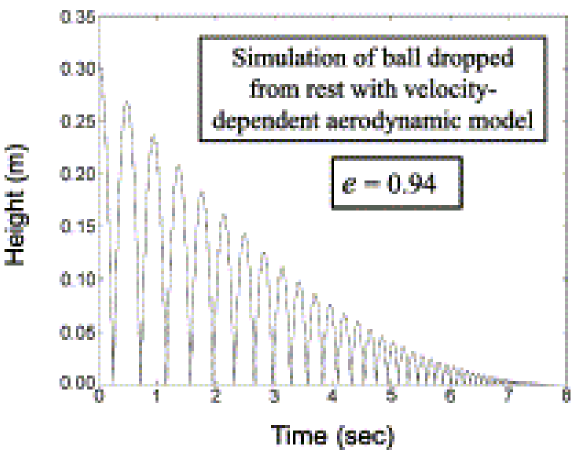

Fig. 9. Ball height vs. time for entire bounce history (simulation accounts for aerodynamic effects with velocity-dependent drag coefficient).

4.1. Value of Experiment

The experiment engages students in creative thinking about the dynamics of a simple impact problem, and brings to life equations of physics. With basic instructions provided in a laboratory manual, the experiment has proven to be "open-ended". Students are encouraged to explore different impact situations, and test their suspicions regarding the effect of height and noise of collision with the change in coefficient of restitution.

5. Conclusion

This experiment has been incorporated into a junior-level mechanical engineering course, "MEEN 120: Mechanical Measurements and Instrumentation" at Marquette University. The experiment has minimal requirements for hardware (PC with sound card, microphone, ball), software (GoldWave shareware), and time (taking approximately 5–10 min). Since it does not require any special laboratory facilities, the experiment can be conducted in a classroom using a notebook computer with a sound card and microphone. The experiment has been well received, has fostered significant discussion with students, and is suggested for use in courses in "measurements" and "dynamics".

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