2-2019

Closed-Form Solution for Curling Responses in Rigid Pavements

Jaime Hernandez
Marquette University, jaime.hernandez@marquette.edu

Imad L. Al-Qadi
University of Illinois - Urbana-Champaign

Civil, Construction and Environmental Engineering Faculty Research and Publications/College of Engineering

Journal of Engineering Mechanics, Vol. 145, No. 2 (February 2019): 04018133. DOI.

Closed-Form Solution for Curling Responses in Rigid Pavements

Jaime Hernandez
Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL
Imad L. Al-Qadi
Department of Civil and Environmental Engineering, University of Illinois at Urbana-Champaign, Urbana, IL

Abstract
Closed-form expressions for calculating stresses and displacements of partially restrained concrete pavement caused by a linear temperature gradient are presented. Translational and rotational linear elastic springs along the slab edges defined the partial restraint. In addition to plate theory behavior, the model assumes linear elastic concrete and an infinitely long slab resting on a Winkler foundation. The solutions of curling stresses and displacements were validated using the finite-element (FE) method and quantified the effect of semirigid connections, slab and foundation material properties,

and slab thickness and width on them. Rotational and translational restraints, which can be related to joint condition in concrete pavement, had significant influence on the magnitude and location of maximum curling stresses and deflections. In addition, Westergaard analysis, a particular case of the proposed solution when there is no restriction along the slab's edges, resulted into the largest deflections at the center of the slab and the lowest maximum curling stresses. Adjustment factors that convert the theoretical findings from an infinitely long slab to a square slab are proposed.

Introduction

Curling stresses develop in rigid pavement due to variation in temperature between the top and bottom of a concrete slab. Although curling stresses may not be as significant as vehicular loading stresses, they usually result in increased cracking potential and, hence, reduce pavement serviceability. Westergaard conducted one of the first studies on stresses caused by temperature differential across concrete slab thickness and proposed closed-form solutions for infinitely long and semi-infinite slabs (Westergaard 1927). In addition to plate theory assumptions [i.e., cross section before bending remains plane after bending, slab's thickness is small compared to the other dimensions, and vertical strain is negligible (Timoshenko and Woinowsky-Krieger 1959)], Westergaard assumed linear elastic homogenous concrete, linear temperature distribution throughout the slab's thickness, no loss of support between the slab and the Winkler foundation, and the slab's edges being free to rotate and translate (Westergaard 1927).

Westergaard's closed-form solutions have been improved over the years by eliminating some of their assumptions. In 1993, analytical expressions for displacements and stresses considering separation between the slab and the Winkler foundation were derived. The procedure divided the problem into two domains, one for the part in contact with the Winkler foundation (same as Westergaard's equations) and the other one for the lifted part. The equations can be applied to infinitely long and semi-infinite slabs satisfying the other Westergaard assumptions, including free rotation and displacement along the edges (Tang et al. 1993). Similarly, in 1998, Liang and Niu combined thermal and plate analysis to derive closed-form expressions for the temperature distribution and curling stresses of a three-layer concrete pavement. In addition to no restriction to displacement and rotation along the slab's edges, the main assumption to calculate curling stresses was the decomposition of total deflection into the components along each direction without any coupling (Liang and Niu 1998). Finally, Ioannides et al. (1999) combined the finite-element (FE) method and artificial neural networks to assess the implications of Westergaard's assumptions.

Even though no closed-form solution has examined the boundary conditions along the slab's edges, some other analytical approaches have addressed that assumption. A three-dimensional FE model including finely meshed dowel bars was used to calculate curling stresses and deflections (William and Shoukry 2001). The main purpose of the finely meshed dowel bars was to investigate the stress state around the dowel bars under various temperature profiles. The frictional interaction between the dowel bars and concrete allowed for partial contact caused by curling, which resulted in a reduction of slab deflection. The curling stresses at the center of the slab and along the edges also changed when dowel bars were included (William and Shoukry 2001). Similarly, Wang and Chen modified NIKE3D, a three-dimensional finite-element software developed by the Federal Aviation Administration (FAA) for the analysis of airfield rigid pavement, to improve curling calculations. The study considered the effect of the interaction between multiple slab as shear stress by using linear spring elements (Wang and Chen 2011). The influence of dowel-bar looseness on principal stresses in concrete slab and the impact of the vertical stress on the base layer has also been examined using a finite-element model. Looseness, which is directly related to the edge condition, increased the maximum principal tensile stress (Davids 2000) and decreased the load transfer efficiency (Kim and Hjelmstad 2003). Additionally, FAARFIELD, the airfield pavement design software developed by the FAA, was utilized to conclude that (i) stress-based load transfer efficiency, $LTE(S)$, is relevant for thickness design of rigid pavements; and (ii) temperature gradient in the slab influences $LTE(S)$ (Joshi et al. 2012). As for the dowel bar, the stiffness matrix, which does not depend on a fine mesh for studying the interaction with concrete and load transfer mechanism, was proposed by Guo et al. (1995). The stiffness matrix of the dowel bars, which was incorporated in a finite-element code, was derived assuming three segments for the dowel bar, two embedded in the concrete and one along the join spacing. A reasonable agreement was reported between measured and calculated dowel-bar responses.

Curling in concrete pavement has also been studied using instrument responses. For instance, curling and temperature measurements indicated that the effect of high positive temperature gradients can be decreased because of the built-in curling (Yu et al. 1998), and measurement of temperature and strain distribution along the slab's depth showed that they are nonlinear, mainly at the edges of the slab (Wei et al. 2017). Furthermore, the curling calculated based on deflections at the slab corners identified upward slab curling and built-in curling as relevant for top-down cracking (Beckemeyer et al. 2002). Similarly, using measured deflections at the center of the slab, it was found that curling caused by a positive temperature differential is slightly higher than that by a negative one (Siddique et al. 2005). Not many studies using experimental measurements have focused on the slab's edge condition. One of the exceptions is the work of Asbahan and Vandenbossche (2011), who determined the slab's curvature from strain gauge, temperature, moisture, and surface profile readings. Two types of slabs, restrained (with tie and dowel bars) and unrestrained (no dowel or tie bars), were instrumented. The restrained slab showed 60% lower curvature than the unrestrained one, which underscores the relevance of the boundary conditions along the edges of the slab.

Even though Westergaard assumptions have been relaxed and studied from the analytical, numerical, and experimental points of view, no solution has quantified the effect of a partial restriction of the slab's edges on the curling stresses and deflections, which is the main contribution of this study. Maintaining all other Westergaard assumptions unmodified, closed-form solutions for curling stresses and deflections of an infinitely long slab that is partially restrained to displacement and rotation are derived. After detailing the procedure to find the solutions, the FE method validated the proposed model. Subsequently, the effect of partial restraint along the edges, geometry, and material properties of the systems on slab displacements and stresses are studied. The paper concludes by calculating adjustment factors that relate curling responses of square to infinitely long slabs.

Structural Model, Deflection, and Curling Stresses

Consider a slab extending to infinity along the x -direction, with width b along the y -direction, and thickness h as shown in Fig. 1. The slab is made of linear elastic material with elastic modulus E, Poisson's ration ν, and coefficient of thermal expansion α. The plate is supported by an elastic foundation with modulus of subgrade reaction k that does not allow separation. Furthermore, the slab is subjected to a linear temperature gradient, where the difference in temperature between the slab's top and bottom is ΔT.

The vertical displacement and rotation along the edges parallel to the x-axis are restrained by linear elastic springs. Along the edge $y = b/2$, the translational and rotational linear springs have a magnitude per unit length of S_a and κ_a, respectively. Similarly, S_b and κ_b are the rotational and translational springs per unit length along the edge $y = -b/2$. Elastic constraints have the advantage of capturing classical boundary condition. For instance, if S_a, κ_a, S_b, and κ_b are zero, there is no restriction to movement, and the edges are free. Conversely, if the magnitude of the springs is very high, the edges are fully restrained, so they are clamped. Finally, if the edges are free to rotate and cannot displace, they are pinned.

Assuming that the slab's cross section before and after bending are plane, h is small compared to the other dimensions, and vertical strain is negligible, it can be found that for an infinitely long slab subjected to linear temperature gradient, the bending moment with respect to the y-axis is the following (Timoshenko and Woinowsky-Krieger 1959; Westergaard 1927):

(1)

$$M_y = -\frac{Eh^3}{12(1-\nu^2)}\left[\frac{d^2w}{dy^2} + \frac{(1+\nu)}{h}\alpha\Delta T\right]$$

The equilibrium of a differential element in the slab provides $d^2M_y/dy^2 = kz$, so the differential equation for the vertical deflection is

(2)

$$l^4\frac{d^4w}{dy^4} + w = 0$$

where $l = \sqrt[4]{D/k}$ is the radius of relative stiffness and $D = Eh^3/12/(1-\nu^2)$ is the slab's bending stiffness. Eqs. (1) and (2) were used by Westergaard (1927). The shear and bending moments per unit length along the partially restrained edges of the slab are

(3)

$$V(b/2) = -S_a w(b/2)$$

(4)

$$V(-b/2) = S_b w(-b/2)$$

(5)

$$M(b/2) = \kappa_a \theta(b/2)$$

(6)

$$M(-b/2) = -\kappa_b \theta(-b/2)$$

The general solution of Eq. (2) is

(7)

$$w(y) = C_1 \cosh\frac{y}{l\sqrt{2}}\cos\frac{y}{l\sqrt{2}} + C_2 \cosh\frac{y}{l\sqrt{2}}\sin\frac{y}{l\sqrt{2}} + C_3 \sinh\frac{y}{l\sqrt{2}}\cos\frac{y}{l\sqrt{2}} + C_4 \sinh\frac{y}{l\sqrt{2}}\sin\frac{y}{l\sqrt{2}}$$

where C_1, C_2, C_3, and C_4 are constants found by solving the linear system of equations resulting from replacing $w(y)$ from Eq. (7) in the boundary conditions in Eqs. (3)–(6). The solution for the slab's vertical displacement is

(8)

$$w(y) = \frac{1}{\det(A)}\frac{(1+\nu)\alpha\Delta T}{h}l^2\left[c_1 \cosh\frac{y}{l\sqrt{2}}\cos\frac{y}{l\sqrt{2}} + c_2 \cosh\frac{y}{l\sqrt{2}}\sin\frac{y}{l\sqrt{2}} + c_3 \sinh\frac{y}{l\sqrt{2}}\cos\frac{y}{l\sqrt{2}} + c_4 \sinh\frac{y}{l\sqrt{2}}\sin\frac{y}{l\sqrt{2}}\right]$$

The Appendix presents the system of equations, its solution, and the expressions for c_1, c_2, c_3, c_4, and $det(A)$ as a function of $R_a = \kappa_a l/D$, $R_b = \kappa_b l/D$, $T_a = S_a l^3/D$, and $T_b = S_b l^3/D$. Once deflection is calculated, rotation, curvature, bending moment M_y from Eq. (1), shear force, and stresses on top of the slab $\sigma_y = 6M_y/h^2$ can also be computed using the deflection $w(y)$. The stresses in the y -direction are

(9)

$$\sigma_y(y) = -\frac{E\alpha\Delta T}{2(1-\nu)}\left[1 - \frac{1}{\det(A)}\left(c_1 \sin\frac{y}{l\sqrt{2}}\sinh\frac{y}{l\sqrt{2}} - c_2 \cos\frac{y}{l\sqrt{2}}\sinh\frac{y}{l\sqrt{2}} + c_3 \cosh\frac{y}{l\sqrt{2}}\sin\frac{y}{l\sqrt{2}} - c_4 \cos\frac{y}{l\sqrt{2}}\cosh\frac{y}{l\sqrt{2}}\right)\right]$$

Westergaard Case

In the case studied by Westergaard (1927), the infinitely long slab is free to displace and rotate along its edges (i.e., $R_a = R_b = T_a = T_b = 0$). Consequently, using the formulas in the Appendix gives

(10)

$$\frac{c_1}{\det(A)} = 2\cosh\lambda\frac{\sin\lambda - \cos\lambda\tanh\lambda}{\sinh 2\lambda + \cos 2\lambda}$$

(11)

$$\frac{c_2}{\det(A)} = 0$$

(12)

$$\frac{c_3}{det(A)} = 0$$

(13)

$$\frac{c_4}{\det(A)} = -2\cosh\lambda \frac{\sin\lambda + \cos\lambda\tanh\lambda}{\sinh 2\lambda + \cos 2\lambda}$$

So the displacement becomes

(14)

$$w(y) = \alpha\Delta T(1+\nu)\frac{l^2}{h}\frac{2\cosh\lambda}{\sinh 2\lambda + \cos 2\lambda}\left[(\sin\lambda - \cos\lambda\tanh\lambda)\cosh\frac{y}{l\sqrt{2}}\cos\frac{y}{l\sqrt{2}} - (\sin\lambda + \cos\lambda\tanh\lambda)\sinh\frac{y}{l\sqrt{2}}\sin\frac{y}{l\sqrt{2}}\right]$$

which can be reduced to

(15)

$$w(y) = -\alpha\Delta T(1+\nu)\frac{l^2}{h}\frac{2\cosh\lambda\cos\lambda}{\sinh 2\lambda + \cos 2\lambda}\left[(-\tan\lambda + \tanh\lambda)\cosh\frac{y}{l\sqrt{2}}\cos\frac{y}{l\sqrt{2}} + (\tan\lambda + \tanh\lambda)\sinh\frac{y}{l\sqrt{2}}\sin\frac{y}{l\sqrt{2}}\right]$$

Eq. (15) matches the equation reported by Westergaard (1927).

Demonstration Example

A typical rigid pavement is used to demonstrate the applicability and validity of the proposed solution. The concrete slab has elastic modulus $E = 28\ \text{MPa}$, coefficient of thermal expansion $\alpha = 9 \times 10^{-6} 1/°\text{C}$, and Poisson's ratio $\nu = 0.15$. The slab rests on an elastic foundation with modulus of subgrade reaction $k = 0.0542\ \text{N/mm}^3$; the slab's thickness and width are $h = 200\ \text{mm}$ and $b = 4.0\ \text{m}$, respectively. The dowel bars have diameter and spacing of 31.8 and 305 mm, respectively. The dowel bars are made of steel with an elastic modulus of 200 GPa and a Poisson's ratio of 0.30. In addition, the width of the join is 6.35 mm, and the dowel–concrete interaction coefficient is $407.3\ \text{N/mm}^3$ (Guo et al. 1995). The temperature on top of the slab is 10°C lower than that at the bottom. The objective is to calculate the deflection and curling stresses of the slab assuming that (i) edges are free to rotate and displace (Westergaard case); and (ii) one edge is elastically restrained to rotation and translation with the dowel configuration described previously, and the other edge adjoins a bridge abutment that provides no restriction to rotation or translation.

The elastic constrain provided by the dowel system was calculated following the procedure proposed by Guo et al. (1995). The procedure calculates a stiffness matrix by dividing the dowel into three segments, two inside the concrete and one spanning the join's width. The stiffness matrix is given by the following:

$$\mathbf{S}_c = \begin{bmatrix} \mathbf{T_1} & 0 \\ 0 & \mathbf{T_2} \end{bmatrix}\left(\begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} \end{bmatrix} - \begin{bmatrix} \mathbf{K_{11}} + \mathbf{T_1} & \mathbf{K_{12}} \\ \mathbf{K_{21}} & \mathbf{K_{22}} + \mathbf{T_2} \end{bmatrix}^{-1}\begin{bmatrix} \mathbf{T_1} & 0 \\ 0 & \mathbf{T_2} \end{bmatrix}\right)$$

where

$$\mathbf{T_1} = \frac{2\beta^2 E_d I}{C_1^2 + c_1^2}\begin{bmatrix} 2\beta(S_1C_1 + s_1c_1) & -(S_1^2 + s_1^2) \\ -(S_1^2 + s_1^2) & \dfrac{S_1C_1 - s_1c_1}{\beta} \end{bmatrix}$$

and

$$\mathbf{T_2} = \frac{2\beta^2 E_d I}{C_2^2 + c_2^2}\begin{bmatrix} 2\beta(S_2C_2 + s_2c_2) & S_2^2 + s_2^2 \\ S_2^2 + s_2^2) & \dfrac{S_2C_2 - s_2c_2}{\beta} \end{bmatrix}$$

with $S = \sinh\beta L$; $C = \cosh\beta L$; $s = \sin\beta L$; and $c = \cos\beta L$. Subscripts 1 and 2 represent the left and right segment of the dowel bar, respectively, which are embedded in the concrete, and L corresponds to the distance that is embedded in the slab. In addition, $\beta = (k_b/4E_d I)^{0.25}$, k_b is the product of the dowel–concrete interaction coefficient and the dowel diameter, E_d is the elastic modulus of the dowel bar, and I is the dowel bar's moment of inertia. For the dowel characteristics in this example, $\beta = 23.861/\mathrm{m}$.

The terms K_{11}, K_{12}, K_{21}, and K_{22} are defined by the segment of the dowel bar between slabs as follows:

$$\mathbf{T_1} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix} = \frac{E_d I}{l^3(1+\phi)}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & (4+\phi)l^2 & -6l & (2-\phi)l^2 \\ -12 & -6l & 12 & -6l \\ 6l & (2-\phi)l^2 & -6l & (4+\phi)l^2 \end{bmatrix}$$

where l = length of the bar between slabs (i.e., width of join); $\phi = 12E_d I/GA_s l^2$; and A_s = dowel's cross-sectional area effective in shear. After replacing all the variables, the stiffness matrix is (force in kilonewtons and distance in meters)

$$\mathbf{S}_c = \begin{bmatrix} 11{,}023.2 & -194.8 & -11{,}023.2 & 264.8 \\ -194.8 & 8.4 & 194.8 & -9.6 \\ -11{,}023.2 & 194.8 & 11{,}023.2 & -264.8 \\ 264.8 & -9.6 & -264.8 & 11.3 \end{bmatrix}$$

The translational and rotational restraints per unit distance are obtained by dividing entries 11 and 22 by the dowel-bar spacing:

$S_a = 11023.3/0.305 = 36.1\ \mathrm{N/mm/mm}$, and $\kappa_a = 8.4/0.305 = 27533.1\ \mathrm{N \cdot mm/mm/rad}$.

An FE model with the same characteristics was created in the software Abaqus. A slab three times longer than its width (i.e., 12.0 m long) represented infinite length. The model used square shell elements with a 32-mm side, four nodes, and Gaussian quadrature for section integration. Elastic connectors with constants obtained by multiplying the magnitude per unit length and the length of the shell elements represented the elastic restraints. For instance, for the translational spring along edge a, $S_a = 36.1\ \ \mathrm{N \times mm/mm}$, so the input for Abaqus is $36.1\ \ \mathrm{N \times mm/mm \times 32.0\ \ mm} = 1{,}156.5\ \ \mathrm{N/mm}$. The foundation element modeled the Winkler foundation, and the linear temperature variation was specified using the temperature gradient through the slab thickness ($0.05°\mathrm{C/mm}$).

The agreement between the proposed closed-form solution and the FE method is excellent, as is shown in Fig. 2. The figure presents the variation of the deflection and the curling stresses obtained from Eqs. (8) and (9), respectively. The horizontal axis indicates the slab's width, with $y = 0$ being the slab's center, and the vertical axis representing the vertical deflection and curling stresses for top and bottom plots, respectively. The match was slightly better for deflection than for stresses. For instance, at the center of the slab, the differences in deflection for the free and the partially restrained slab were 3.1% and 2.7%, respectively. On the other hand, the difference in stress was 5.5% for the free slab, and 4.4% for the slab with semirigid connections.

Asymmetrical behavior for the partially restrained slab can also be inferred from Fig. 2. Unequal semirigid connections along the slab's edges caused the asymmetry. The maximum responses are not at the center anymore, as for the free slab. On the contrary, the maximum is at $y = 247.6\ \mathrm{mm}$ for deflection and $y = 158.7\ \mathrm{mm}$ for curling stresses. The critical curling stresses increased by 13% for the elastically restrained slab compared with the Westergaard case. Transitioning from free to elastically restrained slab affects the magnitude and variation of curling stresses and deflections along the slab's width. The following section elaborates on the effect of semirigid connection on curling stresses and deflections for a broad range of scenarios.

Effect of Semirigid Connections

A parametric study was performed to evaluate the importance of the semirigid connections on curling stresses and deflections. The analyzed cases included 36 combinations of rotational and translational semirigid connections at edges a and b for a fixed ratio between the slab's width and radius of relative stiffness. The ratio was $b/l = 5$, which is recommended by design procedures to reduce the likelihood of transverse cracking (FHWA 1990). The restraint parameters R_a, R_b, T_a, and T_b were 0, 1, and 100; $R_a = 0$ represents no restriction to rotation along edge a, while $R_b = 100$ indicates full restriction. Theoretically, full restriction is given by $R_a = \infty$, but preliminary analysis showed that the difference between $R_a = 100$ and $R_a = \infty$ is insignificant.

Deflection

Fig. 3 presents the deflection across the slab's width for various combination of boundary conditions. The vertical axis shows the normalized deflection $\bar{w} = w/w_o$ with $w_o = \alpha\Delta T(1 + \nu)l^2/h$; the horizontal axis is the position along the slab's width normalized with respect to the width, $\bar{y} = y/b$. The normalized parameters for the translational restriction are fixed in each subplot, and the six lines correspond to different combinations of the normalized parameters for rotational restriction.

Rotational springs affected vertical deflection around the slab's center more than translation springs. On one hand, deflection was always zero when $R_a = R_b = 100$ regardless of the degree of translational restraint. On the other hand, no restriction to rotation resulted in the highest deflection around the center of the slab. As degree of rotational restriction increased, deflection gradually changed from the maximum value to zero. This can be proved by comparing $\bar{w}$ at the center of the slab for three cases. First, the normalized deflection is 0.419 when $T_a = T_b = R_a = R_b = 0$ (Westergaard case). Second, if the normalized rotational springs is changed to $R_a = 0$ and $R_b = 100$, $\bar{w}$ becomes 0.195, which is a 53% reduction. And third, if the rotational springs are maintained at zero, and $T_a =$

0 and $T_b = 100$, the normalized deflection is 0.382, a decrement of only 8.8% compared to Westergaard case. These results are the consequence of bending being the main deformation mechanism of slabs.

Even though translational springs were less relevant than rotational ones for deflection, the difference between T_a and T_b defines if Westergaard's solution is conservative. As mentioned previously, the maximum $\bar{w}$ for free edges was 0.419. The magnitude of $\bar{w}$ for the free case is smaller than 0.434, which corresponds to $T_a = 0$ and $T_b = 100$ (highest difference between translational springs). However, if $T_a = 100$ and $T_b = 100$, $\bar{w} = 0.365$, which is smaller than that for the free case. Consequently, regarding deflection, Westergaard is more conservative if there is high translational restriction at the edges of the slab, but it is not conservative if the difference between T_a and T_b is large.

Semirigid connection also affects the location of maximum deflection. As expected, the largest deflection only occurred at the center of the slab if the boundary conditions were symmetric (i.e., $R_a = R_b$ and $T_a = T_b$). The difference between deflection at the center and maximum deflection can be significant. The highest ratio between maximum deflection and deflection at the center was 1.89 for $T_a = 100$, $T_b = 100$, $R_a = 0$, and $R_b = 100$. The difference between the normalized rotational springs was the controlling factor for the discrepancy between maximum deflection and deflection of slab's center.

Curling Stresses

Fig. 4 presents the curling stresses along the slab's width. As for deflection, the horizontal axis is the ratio between the transverse location and the slab's width. The vertical axis indicates the stresses normalized with respect to the stress for a fully restrained slab $\sigma_o = E\alpha\Delta T/2/(1-\nu)$. The arrangement of the plots regarding the semirigid connections is the same as in Fig. 3.

Fig. 4 confirms the expected behavior for extreme values of R_a and R_b. First, whenever the rotational restrictions were zero (i.e., $R_a = 0$ or $R_b = 0$), the curling stresses along the corresponding edge were zero. On the contrary, if the restriction along both edges is high (i.e., $R_a = 100$ and $R_b = 100$), the curvature of the slab is unchanged, so the stresses are constant and equal to $E\alpha\Delta T/2/(1-\nu)$, as indicated by $\bar{\sigma} = 1$. The result holds for any combination of the translational springs because the deflection is zero when $R_a = R_b = 100$ regardless of T_a and T_b.

The ratio $\bar{\sigma}$ along the edges was affected differently by the translational and rotational connections. As the translational restraint increased, the ratio decreased. For instance, for $R_a = 0$ and $R_b = 1$, if $T_a = T_b = 0$, $\bar{\sigma} = 0.631$ at $y/b = -0.50$. On the other hand, if T_a was kept at zero and T_b was increased to 1, the ratio decreased by 14% to 0.545. The curling stress ratio would decrease an extra 16% to 0.462 if T_b changed from 1 to 100. On the contrary, as the degree of rotational restriction increased, $\bar{\sigma}$ would be raised as well. If $T_a = T_b = 1$, $\bar{\sigma} = 0.532$ at edge $-b/2$ when $R_a = 0$ and $R_b = 1$, which is higher than the ratio when the edge is free to rotate ($\bar{\sigma} = 0$ when $R_a = R_b = 0$) and smaller than when the edge is fully restrained to rotate ($\bar{\sigma} = 1.05$ when $R_a = 0$ and $R_b = 100$).

The variation of the curling stresses with respect to semirigid connections along the center of the slab is different than along the edges. Considering the same cases as in the previous paragraphs, for $R_a = 0$ and $R_b = 0$, if $T_a = T_b = 0$, $\bar{\sigma} = 0.713$ at $y/b = 0$. The ratio increased to 0.816 if the dimensionless translational spring along edge a was maintained at 0 and increased to 1 along edge $-b/2$. If T_b was raised to 100 and $T_a = 0$, $\bar{\sigma} = 0.886$. In other words, when both edges were free to rotate, curling stresses at the center of the slab increased 24% between free and full translational restraint, and most of the increment occurred between $T_b = 0$ and $T_b = 1$. Taking Westergaard's case as a reference, the influence of rotational springs can also be inferred. If T_a and T_b were kept at zero, $\bar{\sigma}$ changed from 0.713 to 0.804 if R_b is changed from 0 to 1 and $R_a = 0$ was held constant (13% increment). In addition, if R_b became 100, the ratio increased by 8% to 0.867. In general, the influence of rotational and translational springs on the curling stresses at the center of the slab decreased as the edge restrictions became greater.

From the practical perspective, the preceding observations highlight the relevance of properly characterizing the load transfer efficiency, not only of the shear force but also of the bending moment between concrete slabs when calculating curling stresses. The Westergaard case predicts the lowest curling stresses, which means the free-edge assumption is not conservative and can result in premature deterioration of concrete pavements.

Curling Stresses and b/l Ratio

The influence of the semirigid connections on curling stresses also depends on geometry and material properties. These variables are encompassed in the ratio b/l, which depends not only on the slab's thickness, width, and material properties, but also on the modulus of subgrade reaction. In addition, design guidelines recommend limiting b/l to 5 to reduce transverse cracking (FHWA 1990).
Fig. 5 shows the relevance of b/l on curling stresses; it shows the variation of the normalized maximum curling stress $\bar{\sigma}_{\max} = \sigma_{\max}/\sigma_o$ for a wide range of b/l and the same values of the semirigid connections as in Figs. 3 and 4.

Large b/l ratios suggest the smallest effect of boundary conditions on curling stresses. For every combination of T_a and T_b, the largest $\bar{\sigma}_{\max}$ at b/l=10b/l=10 was found for $R_a = R_b = 100$. Specifically, when $R_a = R_b = 100$ and $T_a = T_b = 0$, the maximum curling stresses are 6.6% higher than σ_o. Large b/l represents very long slabs, where the maximum curling stresses is obtained by assuming full restriction at the edges, thus explaining the minimal effect of boundary conditions on $\bar{\sigma}_{\max}$.

In general, as the degree of translational restriction increased, $\bar{\sigma}_{\max}$ increased and the corresponding b/l decreased. For instance, if T_a is fixed at 0 and T_b is changed among 0, 1, and 100, the ratios between the maximum curling stress and σ_o are 1.079, 1.127, and 1.240, respectively. Furthermore, the value of b/l at each maximum decreased: $b/l = 4.45$ for $\bar{\sigma}_{\max} = 1.079$, $b/l = 3.90$ for $\bar{\sigma}_{\max} = 1.127$, and $b/l = 3.35$ for $\bar{\sigma}_{\max} = 1.240$. The highest ratio, $\bar{\sigma}_{\max} = 1.431$, requires three conditions: $T_a = T_b = 100$, the difference between rotational springs at both edges was the highest ($R_a = 0$ and $R_b = 100$), and $b/l = 1.65$. In summary, special attention should be given to load transfer efficiency assessment when calculating curling stresses of short slabs. The only case with no influence of translational restriction on $\bar{\sigma}_{\max}$ is when edge rotation is fully restrained (i.e., $R_a = R_b = 100$), in which case the ratio is 1.

Edge conditions where rotation and/or displacement are either free or fully restrained can result in under- or overprediction of $\bar{\sigma}_{\max}$ depending on the magnitude of b/l. The b/l changed with T_a and T_b, being highest for $T_a = T_b = 0$, and lowest for $T_a = T_b = 100$. Similarly, if rotation is fully restrained (i.e., $R_a = R_b = 100$), three ranges are identified. First, if b/l is sufficiently small, $\bar{\sigma}_{\max}$ is the highest among all values of R_a and R_b. Second, for intermediate b/l, $\bar{\sigma}_{\max}$ is higher than any case with one edge not fully restrained to rotation. Third, for high b/l magnitudes, full rotational restriction provides the smallest $\bar{\sigma}_{\max}$.

Even though design guidelines restrict b/l to 5, the effect of the rotational springs on the normalized maximum curling stress is highly sensitive to the ratio between the slab's width and the radius of relative stiffness if $b/l < 5$. For instance, if $T_a = T_b = 1$ and $b/l = 3.5$, the maximum and minimum $\bar{\sigma}_{\max}$ are 1.143 and 0.546, respectively, a difference of 0.597. If b/l is changed to 4.0, the maximum is 1.120 and the minimum changes to 0.679, which represents a difference of 0.441, 27% smaller than for $b/l = 3.5$.

Adjustment Factor for Square Slab

Since the presented solution assumes an infinitely long slab, an adjustment factor (AF) is proposed to modify the maximum stresses for a square slab. First, the maximum curling stress for the infinitely long slab was calculated using Eq. (9), while for the square slab it was obtained using the FE method. The adjustment factor AF is defined as the ratio between the maximum curling stress of the square slab over the one calculated using Eq. (9) and was calculated for 720 cases that resulted from the combination of (i) six rotational springs pairs ($R_a = 0 - R_b = 0$, $R_a = 0 - R_b = 1$, $R_a = 0 - R_b = 100$, $R_a = 1 - R_b = 1$, $R_a = 1 - R_b = 100$, and $R_a = 100 - R_b = 100$); (ii) six translational spring pairs ($T_a = 0 - T_b = 0$, $T_a = 0 - T_b = 1$, $T_a = 0 - T_b = 100$, $T_a = 1 - T_b = 100$, $T_a = 1 - T_b = 100$, and $T_a = 100 - T_b = 100$); (iii) four moduli of subgrade reaction ($k = 0.01$, 0.05, 0.1, and $0.02\ \mathrm{N/mm^3}$); and (iv) five thickness ($h = 100$, 200, 300, 400, and 500 mm). Fig. 6 presents the variation of AF with b/l.

Two zones can be distinguished when analyzing the effect of b/l on AF. First, as expected, the wider the slab (large b/l) the smaller the difference between square and infinite geometries. The AF for relatively large b/l is not exactly 1 because of numerical differences between the FE and the analytical solution. Also, $AF \approx 1$ requires a significant b/l, approximately 7 or larger. Second, AF has a wide variation when $b/l < 7$, where AF can reach values higher than 2 and as low as 0.

Considering the case $b/l = 5$, which is recommended in rigid pavement design, three main observations can be made. First, for all combinations of T_a and T_b, the effect of rotational springs can be divided into two groups: when $R_b = 100$, AF is high; and when $R_b \neq 100$, AF is low. Second, the smallest quotient between the minimum and maximum AF is 1.11 when $T_a = T_b = 0$. Third, the slab fully restrained against translation (i.e., $T_a = T_b = 100$) results in the largest change in AF; it is 0.35, a 42% increment.

The influence of rotational and translational springs on AF is interconnected. As explained, for $b/l = 5$, the lines corresponding to $R_b = 100$ tended to be close for all T_a-T_b combinations; however, the lines did not always represent high AF, mainly for low b/l. Changes in translational springs are associated with changes in the magnitude of the lines, while variations of the rotational springs change the shape of the lines. Fig. 6 also shows that the only case providing adjustment factors smaller than 1 was when $R_a = R_b = 0$. Hence, the infinite slab always provides smaller values than the square one if slab edges are free to rotate. In general, whether or not the infinite-slab assumption is conservative depends on b/l and the semirigid connections.

Summary and Conclusions

A closed-form solution for curling responses of slab-on-subgrade rigid pavement considering generalized boundary conditions was derived using plate theory. The derivation adopted the assumptions of the classical work of Westergaard except for the slab's edge condition; the edges were partially restrained to displacement and rotation by linear elastic springs. After validating the solution using the FE method, the equations quantified the effect of edge restrictions on curling stresses and displacements for a wide range of material properties and geometries. To implement the closed-form solution to real-life cases, adjustment factors were calculated to link curling stresses of an infinitely long slab and a square slab.

Comparison with the FE method showed a difference of around 5% for stresses and displacements, with slightly better agreement for displacements. It was found that the elastic restraints affect the magnitude and location of maximum deflection, with rotational springs having more influence than translational ones at the slab's center. In addition, for small ratios between the slab's width and radius of relative stiffness, semirigid conditions greatly affect the quotient between the maximum curling stresses and the curling stresses of a fully restrained slab. Finally, maximum curling stresses in square slabs are usually higher than those for an infinitely long slab; the difference heavily depended on the boundary conditions and b/l ratio. This study also presents an adjustment factor for the currently used approach to analyze maximum curling stresses in rigid pavement with square slabs.

The results show that Westergaard analysis is not conservative, and that the degree of relevance of the semirigid connections depend on material properties and geometry. More generally, it is imperative to assess and quantify joint condition and their ability to transfer shear force and bending moment when performing curling analysis.

Appendix.

Linear System of Equations and Solution

Linear systems of equations in matrix form are as follows:

(16)

$$[A] \times [C] = [b]$$

(17)

$$\begin{bmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \\ A_{41} & A_{42} & A_{43} & A_{44} \end{bmatrix} \times \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$$

Entries in the coefficient matrix are as follows:

(18)

$$A_{11} = R_b \cosh\lambda \sin\lambda + \sqrt{2}\sinh\lambda \sin\lambda - R_b \cos\lambda \sinh\lambda$$

(19)

$$A_{12} = R_b \cos\lambda \cosh\lambda + R_b \sin\lambda \sinh\lambda + \sqrt{2}\cos\lambda \sinh\lambda$$

(20)

$$A_{13} = R_b \cos\lambda \cosh\lambda - \sqrt{2}\sin\lambda \cosh\lambda - R_b \sin\lambda \sinh\lambda$$

(21)

$$A_{14} = -R_b \cosh\lambda \sin\lambda - R_b \cos\lambda \sinh\lambda - \sqrt{2}\cos\lambda \cosh\lambda$$

(22)

$$A_{21} = R_a \cosh\lambda \sin\lambda + \sqrt{2}\sinh\lambda \sin\lambda - R_a \cos\lambda \sinh\lambda$$

(23)

$$A_{22} = -R_a \cos\lambda \cosh\lambda - \sqrt{2}\cos\lambda \sinh\lambda - R_a \sin\lambda \sinh\lambda$$

(24)

$$A_{23} = -R_a \cos\lambda \cosh\lambda + \sqrt{2}\sin\lambda \cosh\lambda + R_a \sin\lambda \sinh\lambda$$

(25)

$$A_{24} = -R_a \cosh\lambda \sin\lambda - R_a \cos\lambda \sinh\lambda - \sqrt{2}\cos\lambda \cosh\lambda$$

(26)

$$A_{31} = -\cosh\lambda \sin\lambda - \cos\lambda \sinh\lambda + \sqrt{2}T_b \cos\lambda \cosh\lambda$$

(27)

$$A_{32} = -\cos\lambda \cosh\lambda - \sqrt{2}T_b \sin\lambda \cosh\lambda + \sin\lambda \sinh\lambda$$

(28)

$$A_{33} = \cos\lambda \cosh\lambda - \sqrt{2}T_b \cos\lambda \sinh\lambda + \sin\lambda \sinh\lambda$$

(29)

$$A_{34} = -\cosh\lambda \sin\lambda + T_b \sinh\lambda \sqrt{2}\sin\lambda + \cos\lambda \sinh\lambda$$

(30)

$$A_{41} = \cosh\lambda\sin\lambda + \cos\lambda\sinh\lambda - \sqrt{2}T_a\cos\lambda\cosh\lambda$$

(31)

$$A_{42} = -\cos\lambda\cosh\lambda - \sqrt{2}T_a\sin\lambda\cosh\lambda + \sin\lambda\sinh\lambda$$

(32)

$$A_{43} = \cos\lambda\cosh\lambda - \sqrt{2}T_a\cos\lambda\sinh\lambda + \sin\lambda\sinh\lambda$$

(33)

$$A_{44} = \cosh\lambda\sin\lambda - \sqrt{2}T_a\sinh\lambda\sin\lambda - \cos\lambda\sinh\lambda$$

Entries in the b matrix are as follows:

(34)

$$b_1 = \sqrt{2}(1+\nu)\alpha\Delta T\frac{l^2}{h}$$

(35)

$$b_2 = \sqrt{2}(1+\nu)\alpha\Delta T\frac{l^2}{h}$$

(36)

$$b_3 = 0$$

(37)

$$b_4 = 0$$

Terms in the solution for displacements are as follows:

(38)

$$
\begin{aligned}
c_1 = {} & [R_a(T_a + 2T_b) + R_b(2T_a + T_b) - 4T_aT_b + 2]\sin^3\lambda\sinh\lambda \\
& + [-R_a(T_a + 2T_b) - R_b(2T_a + T_b) - 4T_aT_b - 2]\sin\lambda\sinh^3\lambda \\
& + 2[R_aT_a + R_bT_b - 2]\sin\lambda\sinh\lambda \\
& + \sqrt{2}[-R_aT_aT_b + R_a - R_bT_aT_b + R_b - 2(T_a + T_b)]\cos^3\lambda\sinh\lambda \\
& + (-R_aT_a - R_bT_b - 2)\cos^3\lambda\cosh\lambda + \sqrt{2}(R_a + R_b + T_a + T_b)\cos\lambda\sinh^3\lambda \\
& + \sqrt{2}[T_a(R_aT_b + R_bT_b - 1) - T_b]\cos\lambda\sinh\lambda + (R_aT_a + R_bT_b + 2)\cos\lambda\cosh^3\lambda \\
& - \sqrt{2}[R_aT_aT_b + R_a + R_bT_aT_b + R_b + 2(T_a + T_b)]\sin\lambda\cosh^3\lambda \\
& + \sqrt{2}(R_a + R_b - T_a - T_b)\sin^3\lambda\cosh\lambda + \sqrt{2}[T_a(R_aT_b + R_bT_b - 1) - T_b]\sin\lambda\cosh\lambda \\
& - 3[R_a(T_a + 2T_b) + R_b(2T_a + T_b) - 4T_aT_b + 2]\sin\lambda\cos^2\lambda\sinh\lambda \\
& + 3\sqrt{2}(-R_a - R_b + T_a + T_b)\sin\lambda\cos^2\lambda\cosh\lambda \\
& + 3\sqrt{2}[R_a(T_aT_b - 1) + R_b(T_aT_b - 1) + 2(T_a + T_b)]\sin^2\lambda\cos\lambda\sinh\lambda \\
& + 3\sqrt{2}(R_a + R_b + T_a + T_b)\cos\lambda\sinh\lambda\cosh^2\lambda + 3\sin^2\lambda\cos\lambda\cosh\lambda(R_aT_a + R_bT_b + 2) \\
& + 3(R_aT_a + R_bT_b + 2)\cos\lambda\sinh^2\lambda\cosh\lambda \\
& - 3[R_a(T_a + 2T_b) + R_b(2T_a + T_b) + 4T_aT_b + 2]\sin\lambda\sinh\lambda\cosh^2\lambda \\
& - 3\sqrt{2}[R_aT_aT_b + R_a + R_bT_aT_b + R_b + 2(T_a + T_b)]\sin\lambda\sinh^2\lambda\cosh\lambda
\end{aligned}
$$

(39)

$$
\begin{aligned}
c_2 = {} & \sqrt{2}(R_a - R_b)(T_aT_b - 1)\sin^3\lambda\sinh\lambda + \sqrt{2}(-R_a + R_b - T_a + T_b)\sin\lambda\sinh^3\lambda \\
& + \sqrt{2}[T_a(-R_aT_b + R_bT_b - 1) + T_b]\sin\lambda\sinh\lambda \\
& + [R_a(T_a + 2T_b) - R_b(2T_a + T_b)]\cos^3\lambda\sinh\lambda + \sqrt{2}(R_a - R_b - T_a + T_b)\cos^3\lambda\cosh\lambda \\
& + [R_b(2T_a + T_b) - R_a(T_a + 2T_b)]\cos\lambda\sinh^3\lambda + (2R_aT_a - 2R_bT_b)\cos\lambda\sinh\lambda \\
& - \sqrt{2}(R_a - R_b)(T_aT_b + 1)\cos\lambda\cosh^3\lambda + \sqrt{2}(R_aT_aT_b - R_bT_aT_b + T_a - T_b)\cos\lambda\cosh\lambda \\
& + (R_bT_b - R_aT_a)\sin\lambda\cosh^3\lambda + (R_aT_a - R_bT_b)\sin^3\lambda\cosh\lambda \\
& - 3\sqrt{2}(R_a - R_b)(T_aT_b - 1)\sin\lambda\cos^2\lambda\sinh\lambda + 3(R_bT_b - R_aT_a)\sin\lambda\cos^2\lambda\cosh\lambda \\
& + 3[R_b(2T_a + T_b) - R_a(T_a + 2T_b)]\sin^2\lambda\cos\lambda\sinh\lambda \\
& + 3[R_b(2T_a + T_b) - R_a(T_a + 2T_b)]\cos\lambda\sinh\lambda\cosh^2\lambda \\
& + 3\sqrt{2}(-R_a + R_b + T_a - T_b)\sin^2\lambda\cos\lambda\cosh\lambda \\
& - 3\sqrt{2}(R_a - R_b)(T_aT_b + 1)\cos\lambda\sinh^2\lambda\cosh\lambda \\
& + 3\sqrt{2}(-R_a + R_b - T_a + T_b)\sin\lambda\sinh\lambda\cosh^2\lambda + 3(R_bT_b - R_aT_a)\sin\lambda\sinh^2\lambda\cosh\lambda
\end{aligned}
$$

(40)

$$
\begin{aligned}
c_3 = {} & \sqrt{2}(R_a - R_b - T_a + T_b)\sin^3\lambda\sinh\lambda + \sqrt{2}(R_a - R_b)(T_aT_b + 1)\sin\lambda\sinh^3\lambda \\
& + \sqrt{2}(R_aT_aT_b - R_bT_aT_b + T_a - T_b)\sin\lambda\sinh\lambda + (R_bT_b - R_aT_a)\cos^3\lambda\sinh\lambda \\
& - \sqrt{2}(R_a - R_b)(T_aT_b - 1)\cos^3\lambda\cosh\lambda + (R_bT_b - R_aT_a)\cos\lambda\sinh^3\lambda \\
& + \sqrt{2}(-R_a + R_b - T_a + T_b)\cos\lambda\cosh^3\lambda + \sqrt{2}(R_aT_aT_b - R_bT_aT_b + T_a - T_b)\cos\lambda\cosh\lambda \\
& + [R_a(T_a + 2T_b) - R_b(2T_a + T_b)]\sin\lambda\cosh^3\lambda \\
& + [R_a(T_a + 2T_b) - R_b(2T_a + T_b)]\sin^3\lambda\cosh\lambda + 2(R_aT_a - R_bT_b)\sin\lambda\cosh\lambda \\
& + 3\sqrt{2}(-R_a + R_b + T_a - T_b)\sin\lambda\cos^2\lambda\sinh\lambda \\
& + 3[R_b(2T_a + T_b) - R_a(T_a + 2T_b)]\sin\lambda\cos^2\lambda\cosh\lambda + 3(R_aT_a - R_bT_b)\sin^2\lambda\cos\lambda\sinh\lambda \\
& + 3(R_bT_b - R_aT_a)\cos\lambda\sinh\lambda\cosh^2\lambda + 3\sqrt{2}(R_a - R_b)(T_aT_b - 1)\sin^2\lambda\cos\lambda\cosh\lambda \\
& + 3\sqrt{2}(-R_a + R_b - T_a + T_b)\cos\lambda\sinh^2\lambda\cosh\lambda \\
& + 3\sqrt{2}(R_a - R_b)(T_aT_b + 1)\sin\lambda\sinh\lambda\cosh^2\lambda \\
& + 3[R_a(T_a + 2T_b) - R_b(2T_a + T_b)]\sin\lambda\sinh^2\lambda\cosh\lambda
\end{aligned}
$$

(41)

$$
\begin{aligned}
c_4 = {} & (R_aT_a + R_bT_b + 2)\sin^3\lambda\sinh\lambda + (R_aT_a + R_bT_b + 2)\sin\lambda\sinh^3\lambda \\
& + \sqrt{2}(R_a + R_b - T_a - T_b)\cos^3\lambda\sinh\lambda \\
& + [R_a(T_a + 2T_b) + R_b(2T_a + T_b) - 4T_aT_b + 2]\cos^3\lambda\cosh\lambda \\
& + \sqrt{2}[R_aT_aT_b + R_a + R_bT_aT_b + R_b + 2(T_a + T_b)]\cos\lambda\sinh^3\lambda \\
& + \sqrt{2}[T_a(R_aT_b + R_bT_b - 1) - T_b]\cos\lambda\sinh\lambda \\
& + [R_a(T_a + 2T_b) + R_b(2T_a + T_b) + 4T_aT_b + 2]\cos\lambda\cosh^3\lambda \\
& + 2(R_aT_a + R_bT_b - 2)\cos\lambda\cosh\lambda + \sqrt{2}(R_a + R_b + T_a + T_b)\sin\lambda\cosh^3\lambda \\
& + \sqrt{2}[R_a(T_aT_b - 1) + R_b(T_aT_b - 1) + 2(T_a + T_b)]\sin^3\lambda\cosh\lambda \\
& + \sqrt{2}[T_b - T_a(R_aT_b + R_bT_b - 1)]\sin\lambda\cosh\lambda - 3(R_aT_a + R_bT_b + 2)\sin\lambda\cos^2\lambda\sinh\lambda \\
& - 3\sqrt{2}[R_a(T_aT_b - 1) + R_b(T_aT_b - 1) + 2(T_a + T_b)]\sin\lambda\cos^2\lambda\cosh\lambda \\
& + 3\sqrt{2}(-R_a - R_b + T_a + T_b)\sin^2\lambda\cos\lambda\sinh\lambda \\
& + 3\sqrt{2}[R_aT_aT_b + R_a + R_bT_aT_b + R_b + 2(T_a + T_b)]\cos\lambda\sinh\lambda\cosh^2\lambda \\
& - 3[R_a(T_a + 2T_b) + R_b(2T_a + T_b) - 4T_aT_b + 2]\sin^2\lambda\cos\lambda\cosh\lambda \\
& + 3[R_a(T_a + 2T_b) + R_b(2T_a + T_b) + 4T_aT_b + 2]\cos\lambda\sinh^2\lambda\cosh\lambda \\
& + 3(R_aT_a + R_bT_b + 2)\sin\lambda\sinh\lambda\cosh^2\lambda + 3(R_a + R_b + T_a + T_b)\sqrt{2}\sin\lambda\sinh^2\lambda\cosh\lambda
\end{aligned}
$$

(42)

$$
\begin{aligned}
\det(A) = {} & -[R_a(R_b(T_aT_b - 2) + T_a + 2T_b) + R_b(2T_a + T_b) - 2T_aT_b + 1]\cos(4\lambda) \\
& - [R_a(R_b(T_aT_b + 2) + T_a + 2T_b) + R_b(2T_a + T_b) + 2T_aT_b + 1]\cosh(4\lambda) \\
& + [-R_a(R_b(T_a + T_b) - T_aT_b + 1) + R_bT_aT_b - R_b + T_a + T_b]\sqrt{2}(\sin(4\lambda) \\
& - [R_a(T_b(R_b + T_a) + R_bT_a + 1) + R_bT_aT_b + R_b + T_a + T_b)]\sinh(4\lambda) \\
& + 2(R_aT_a - 1)(R_bT_b - 1)
\end{aligned}
$$

Acknowledgments

The authors would like to acknowledge the financial support provided by the Federal Aviation Administration, especially to Dr. Navneet Garg. This project was conducted in cooperation with the

Illinois Center for Transportation. The contents of this paper reflect the view of the authors, who are responsible for the facts and the accuracy of the data presented herein. The contents do not necessarily reflect the official views or policies of the Illinois Center for Transportation or the Federal Aviation Administration. This paper does not constitute a standard, specification, or regulation.

References

Asbahan, R. E., and J. M. Vandenbossche. 2011. "Effects of temperature and moisture gradients on slab deformation for jointed plain concrete pavements." *J. Transp. Eng.* 137 (8): 563–570. https://doi.org/10.1061/(ASCE)TE.1943-5436.0000237.

Beckemeyer, C., L. Khazanovich, and H. Thomas Yu. 2002. "Determining amount of built-in curling in jointed plain concrete pavement: Case study of Pennsylvania 1-80." *Transp. Res. Rec.* 1809: 85–92. https://doi.org/10.3141/1809-10.

Davids, W. G. 2000. "Effect of dowel looseness on response of jointed concrete pavements." *J. Transp. Eng.* 126 (1): 50–57. https://doi.org/10.1061/(ASCE)0733-947X(2000)126:1(50).

FHWA (Federal Highway Administration). 1990. "Technical advisory T 5040.30 concrete pavement joints." Accessed May 7, 2017. https://www.fhwa.dot.gov/pavement/t504030.cfm.

Guo, H., J. A. Sherwood, and M. B. Snyder. 1995. "Component dowel-bar model for load-transfer systems in PCC pavements." *J. Transp. Eng.* 121 (3): 289–298. https://doi.org/10.1061/(ASCE)0733-947X(1995)121:3(289).

Ioannides, A., C. Davis, and C. Weber. 1999. "Westergaard curling solution reconsidered." *Transp. Res. Rec.* 1684: 61–70. https://doi.org/10.3141/1684-08.

Joshi, A., Y. Mehta, D. Cleary, S. Henry, and C. Cunliffe. 2012. "Load transfer efficiency of rigid airfield pavement: Relationship to design thickness and temperature curling." *Transp. Res. Rec.* 2300: 68–74. https://doi.org/10.3141/2300-08.

Kim, J., and K. Hjelmstad. 2003. "Three-dimensional finite element analysis of doweled joints for airport pavements." *Transp. Res. Rec.* 1853: 100–109. https://doi.org/10.3141/1853-12.

Liang, R. Y., and Y.-Z. Niu. 1998. "Temperature and curling stress in concrete pavements: Analytical solutions." *J. Transp. Eng.* 124 (1): 91–100. https://doi.org/10.1061/(ASCE)0733-947X(1998)124:1(91).

Siddique, Z. Q., M. Hossain, and D. Meggers. 2005. "Temperature and curling measurements on concrete pavement." In Proc., 2005 Mid-Continent Transportation Research Symp. Ames, IA: Iowa State Univ.

Tang, T., D. G. Zollinger, and S. Senadheera. 1993. "Analysis of concave curling in concrete slabs." *J. Transp. Eng.* 119 (4): 618–633. https://doi.org/10.1061/(ASCE)0733-947X(1993)119:4(618).

Timoshenko, S. P., and S. Woinowsky-Krieger. 1959. *Theory of plates and shells*. New York: McGraw-Hill.

Wang, Q., and Y. Chen. 2011. "Improvements to modeling of concrete slab curling by using NIKE3D finite element program." *Transp. Res. Rec.* 2226: 71–81. https://doi.org/10.3141/2226-08.

Wei, Y., X. Gao, F. Wang, and Y. Zhong. 2017. "Nonlinear strain distribution in a field-instrumented concrete pavement slab in response to environmental effects." *Road Mater. Pavement Des.* 1–14. https://doi.org/10.1080/14680629.2017.1395353.

Westergaard, H. 1927. “Analysis of stresses in concrete pavements due to variations of temperature.” In Vol. 6 of Proc., Sixth Annual Meeting of the Highway Research Board, 201–215. Washington, DC: Highway Research Board.

William, G. W., and S. N. Shoukry. 2001. “3D finite element analysis of temperature-induced stresses in dowel jointed concrete pavements.” *Int. J. Geomech.* 1 (3): 291–307. https://doi.org/10.1061/(ASCE)1532-3641(2001)1:3(291).

Yu, H., L. Khazanovich, M. Darter, and A. Ardani. 1998. “Analysis of concrete pavement responses to temperature and wheel loads measured from intrumented slabs.” *Transp. Res. Rec.* 1639: 94–101. https://doi.org/10.3141/1639-10.