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Abstract
Description of tire model development using the finite element (FE) method is presented. Three-dimensional tire-pavement contact stresses were predicted for braking, traction, and free rolling using the FE method. Measured load-deflection curves, contact area, and contact stresses were used for model outcome validation. Slide-velocity-dependent friction and accurate input regarding geometry and material properties were considered. The developed tire model, which helped in studying contact stresses variation in each direction, was used to explain the various phenomena taking place at the

tire-pavement interface during straight-line rolling. The analysis matrix includes nine rolling conditions and various loads, tire inflation pressures, and speeds. Vertical contact stresses were not significantly affected by speed or slip ratio; however, contact stresses were greatly modified along the in-plane directions by rolling conditions. Analytical expressions were introduced to represent vertical and longitudinal contact stresses for full braking and full traction. Formulas are presented for low speed and full braking, which are relevant for roadway intersections design.

Introduction

The tire-pavement contact phenomenon has important implications not only on vehicle safety, maneuvering, and fuel consumption, but also on pavement response and damage quantification. Pavement surface distresses are relatively easier to remedy; therefore, pavement design methodologies focus on localizing damage on the upper pavement layers (NAPA 2013). In addition, appropriate characterization of contact loading has proven relevant when evaluating stresses and strains in pavement structures, primarily at points close to the surface (Al-Qadi and Yoo 2007). This study addresses the characterization of tire-pavement interaction using finite elements and experimental measurements for validation.

The finite element (FE) modeling of tires has continuously evolved as computational powers have increased (Ghoreishy 2008). Incompressible hyperelastic rubber and rebar elements for tire reinforcement have been considered to show the low effect of friction coefficient on vertical contact stresses and its relevant influence on contact shear (Ghoreishy et al. 2007). Similarly, gains of longitudinal contact stresses with the increase of friction coefficient have been reported (Wang and Wu 2009). However, influence of tire inflation pressure on transverse contact stresses was not observed.

Tire FE models have been applied in pavement analysis (Wang et al. 2012). Even though rubber materials were assumed linear elastic with a Poissons ratio of 0.49, large displacement and geometric nonlinearity were considered. The model was calibrated with load deflection curves and validated using measured contact stresses. Various rolling conditions were analyzed (e.g., free-rolling, braking, traction, cornering). Increasing longitudinal contact stresses were observed during braking. The model was improved to include velocity-dependent friction (Wang et al. 2014). It was shown that braking and traction resulted in higher longitudinal contact stresses, but smaller transverse contact stresses compared with the free-rolling condition. During cornering, in-plane contact stresses were greater than the contact stresses in the free rolling condition. The study concluded that constant friction coefficient is acceptable for free rolling and small-slip angles, but not for braking and traction.

A racing tire was modeled giving special attention to braking (Gruber et al. 2012). The model assumed rubber as hyperelastic (Mooney-Rivlin) in addition to constant friction coefficient ($\mu = 0.5$) and speed ($V = 19.4\ \mathrm{m/s}$). The results obtained agree with the trends reported in the literature. The important influence of carcass deformation on shear stress distribution was reported. Based on these results, a physical tire model was developed (Gruber and Sharp 2012), which divided the tire into three main components: rigid wheel, flexible carcass, and rubber tread. The approach proposed was used to assess the significance of contact patch geometry, contact pressure distribution, carcass flexibility, rolling radius variation, and friction coefficient on shear forces at tire-surface interface. Rolling radius and friction were found to be essential for determining the magnitude and distribution of shear forces.

In this study, a wide-base truck tire is modeled to study the three-dimensional (3D), tire-pavement contact stresses. A detailed description of the developed FE model is presented. The contact stresses are studied at braking, free rolling, and traction. Special attention is given not only to contact stresses magnitude, but also to the shape of variation along the contact length. Finally, mathematical expressions with potential use in the design of road intersection are presented. Work is currently underway at the University of Illinois at Urbana-Champaign to apply a similar methodology to the modeling of conventional dual tire assembly. This will allow the comparison of both tires and their effect on pavement performance.

Finite Element Model

Geometry

The modeled wide-base truck tire (WBT) is 445 mm wide; the ratio between the tires height and width is 50%; the rims diameter is 571.5 mm, and the radius is 508.3 mm. Dimensions of the tires cross section were accurately measured. The WBT consists of five belts, with a specific orientation and width as presented in Table 1. The belt closest to the tires interior was labeled Belt 1, and the one closest to the tread was labeled Belt 5. Table 1 shows the number of reinforcement cords in 10 mm; this information was used to infer the reinforcement spacing. All belts were hosted in the 8.2-mm-thick belt packaged.

Table 1. Belt Width and Reinforcement Orientation

Belt	Width (mm)	Orientation (mm)	Spacing $\frac{\text{cords}}{10}$ mm	Area (mm^2)
Belt 1	315.7	89.0	33	1.370
Belt 2	367.3	18.0	43	0.852
Belt 3	274.8	0.0	49	1.885
Belt 4	343.2	20.0	43	0.811
Belt 5	312.2	18.0	31	1.885
Ply	—	90	39	0.657

The thickness of the inner liner, which is the inner-most tire component, is 2.1 mm. In addition, the distance between the inner liner and the belt package (body ply thickness) is 3.6 mm. Thickness of the tread in the middle of the tire is 26.5 mm, whereas thickness of the crown is 40.4 mm (summation of the inner liner, body ply, belt package, and tread). The shoulder dimension, or the distance from the corner of the outer tread to the inner surface of the tire measured perpendicularly, is 9.9 mm. The tires bead consists of a rectangular array of wires (8×6), each wire is 2.0 mm wide and 1.3 mm long (8×6), 2.0 mm wide and 1.3 mm long each.

Material Properties

The Mooney-Rivlin model was adopted to characterize the behavior of rubber. In this model, the stored strain energy for a hyperelastic incompressible material is given by

(1)

$$W = C_{10}(I_1 - 3) + C_{01}(I_2 - 3)$$

where C_{01} and C_{10} = empirically-determined material constants; I_1 and I_2 = first and second principal invariants of the right Cauchy-Green deformation tensor; and W = strain energy density. Material constants C_{01} and C_{10} for the rubber components considered in this study were provided by the tire manufacturer.

Tire reinforcement consisted of belts, plies, and the bead wire, which were assumed to be linear elastic and were characterized following ASTM D882. In this test, a tensile load was applied to properly clamped reinforcement samples, and the deformation was measured. A typical stress-strain curve showed an initial portion with very low slope as a result of the initial setting of the load. During calculation of the elastic modulus, this portion was discarded, and only the linear part was considered. Five samples were tested for each material, and the average of the five moduli was used as input in the FE model.

Tire-Pavement Friction

Poor and good pavement macrotexture has been examined (Wang et al. 2014) using the slide-velocity-dependent model (Kato and Matsubayashi 1970; Kato et al. 1972) and proper numerical implementation (Oden and Martins 1985). In the present study, the intermediate macrotexture was assumed as representative of most pavements. A static and dynamic friction coefficient of $\mu_s = 0.30$ and $\mu_k = 0.17$, respectively, were used to define the slide-velocity-dependent friction model along with a decay coefficient of $d_c = 0.0002\ \mathrm{s/mm}$.

Arbitrary Lagrangian-Eulerian Formulation

Steady-state transport analysis was used to perform numerical calculations and predict tire-pavement contact stresses (ABAQUS/Simulia). This approach is based on the arbitrary Lagrangian-Eulerian (ALE) formulation, which combines the advantages of the Lagrangian and Eulerian formulation (Belytschko et al. 2000). ALE formulation can transform a dynamic problem, such as steady-state rolling, into a problem where all derivatives are related to space variables. In addition, the mesh refinement needed to handle contact problems can be localized to the region that may be in contact with the rolling surface rather than extending to the whole circumference of the tire.

Analysis Sequence

To take advantage of the various symmetries in the static analysis of the tire, the 3D analysis was divided into two consecutive phases: axisymmetric and 3D model (Fig. 1). In the axisymmetric model, the geometry of the tire's cross section is defined along with the element types and material properties. The main boundary condition identified in this model was the tire-rim contact region. The axisymmetric model was only subjected to tire inflation pressure.

The axisymmetric model was revolved with respect to the tires axis to create the 3D model. In this phase, road and tire were in contact, and the load was applied. Surface-to-surface contact was adopted because it better represents the contact stresses (ABAQUS/Simulia). In addition, fixed boundary conditions at the tire-rim contact region were assigned.

Mesh Configuration

Because of the complexity of the tire structure, special elements were used for tire modeling. A combination of general Cartesian elements in the potential contact region and cylindrical elements in the remaining region provided an efficient balance between accuracy and small computational time. Cylindrical and general Cartesian elements were assigned to different sectors of the tires circumference. Assuming that $\theta = 0^{\circ}$° represents the positive z axis (i.e., perpendicular to the rigid surface) and it is measured clockwise; general elements were located in the sector $150^{\circ} < \theta < 210^{\circ}$, which was assumed to be the potential tire-surface contact sector. Accordingly, the sector $0^{\circ} < \theta < 150^{\circ}$ and $210^{\circ} < \theta < 360^{\circ}$ contained cylindrical elements.

Rubber is an incompressible material and therefore hybrid formulation was chosen for behavior modeling. Conversely, reinforced rubber was modeled using rebar elements. This approach was followed to avoid homogenization of steel reinforcement and surrounding rubber. Each component was considered independently. The material properties obtained in the laboratory for rubber and reinforcement could be directly used in the model definition. Surface elements, embedded in host elements (rubber material), were specified for reinforcement. The reinforcement, or rebar layer, was fully defined by specifying the cross-sectional area, spacing, material properties, and orientation as presented in Table 1.

The most efficient mesh configuration regarding element size was the configuration that provided accurate results with the least amount of elements. Therefore, instead of using a specific response as the indicator of accuracy (e.g., maximum deflection), total strain energy was used. A mesh with strain energy equal to one of the finest mesh $\pm 5\%$ was considered accurate. Various mesh configurations were tested in the axisymmetric and full-tire model until the optimum was found.

Validation

To corroborate the accuracy of the developed tire FE model, validation was implemented. The material properties in the tire structure were slightly modified to match experimentally measured contact area (A_c) and deflection (d) when $P = 44.4\ \text{kN}$ and $S = 690\ \text{MPa}$.

A parametric study was performed as part of the validation. Material constants of each tire component was varied to assess their impact on A_c, d, and vertical contact stresses. The deflection and contact area were primarily affected by the sidewall and tread, respectively. The effect of sidewall on 3D contact stresses and contact area for various rolling conditions was assessed. The maximum percentage change of the $L2$-norm of the vectors storing the output at the contact nodes was 9.3% ($C_{01,\text{sidewall}}$ was changed between 1.5 and 2.25 MPa).

Convergence issues were observed in the bead area because of the great difference in stiffness between the bead wire and bead filler. Homogenization was used to address this issue and, thus, one material was assigned to the region occupied by the bead filler and the bead wire with properties proportional to their corresponding area in the cross section.

During validation, material properties were fixed, and experimental and calculated contact area, deflection, and vertical contact stresses for the other loads ($P = 26.7$, 35.6, and 44.4 kN) and tire inflation pressures ($S = 552$, 690, and 758 kPa) were compared. The difference between measured and calculated deflection, contact area, and maximum vertical contact stresses is presented in Fig. 2. The mean absolute percentage error (MAPE) was used as criteria, which is given by

(2)

$$\mathrm{MAPE} = \frac{100}{m}\sum_{i=1}^{m}\left|\frac{\mathrm{Meas}_i - \mathrm{Calc}_i}{\mathrm{Meas}_i}\right|$$

where m = number of measurements. A static model was used for A_c and d, which better represented the laboratory condition during their measurement. Figs. 2(a and b) show a good agreement for contact area and deflection ($\mathrm{MAPE} = 4.2$ and 8.5%, respectively). It is noted that from a pavement engineering point of view, the contact area is more relevant than deflection.

To compare contact stresses, a free-rolling model was used on a rigid surface with a high friction coefficient. The agreement for maximum vertical contact stress in each rib is not as good as for A_c [Fig. 2(c)]. Ribs 4 and 5 only provided the most reliable contact stresses because of restrictions of the measuring equipment (Hernandez et al. 2014).

Numerical Analysis Matrix

After the tire model was developed and validated, it was used to calculate tire-pavement contact stresses of a tire rolling on an infinitely rigid surface. Three values of load ($P = 26.7$, 35.6, and 44.4 kN), tire inflation pressure ($S = 552$, 690, and 758 kPa), and speed ($V = 8.0$, 65.0, and $115\ \mathrm{km/h}$) were considered. For each speed, not only the free rolling condition was studied, but also braking and traction. Braking and traction conditions were defined using the slip ratio

(3)

$$s_b = 1 - \frac{R_{fr}\omega}{V}$$

(4)

$$s_t = 1 - \frac{V}{R_{fr}\omega}$$

where R_{fr} = free rolling radius; s_b and s_t = slip ratio for braking and traction; V = traveling speed; and ω = angular speed.

The ratios s_b and s_t can vary between 0 and 100%: $s_b = s_t = 0$ indicated free rolling, $s_b = 100\%$ indicated full braking, and $s_t = 100\%$ indicated full traction. Based on the variation of T_y versus ω, $s_b = s_t = 7\%$ was found to represent full braking and traction, respectively. As part of the analysis matrix, four values of s_b and s_t were considered: $s_b = s_t = 1.5$, 3.0, 4.5, and 7.0%. The numerical analysis matrix is summarized in Table 2.

Table 2. Values of Load, Tire Inflation Pressure, Speed, and Rolling Condition Considered

Load (kN)	Pressure (kPa)	Speed (km/h)	Braking (slip %)	Traction (slip %)
$P1 = 26.6$	$S1 = 552$	$V1 = 8$	$B1 = 1.5$	$T1 = 1.5$

$P2 = 35.5$	$S2 = 690$	$V2 = 65$	$B2 = 3.0$	$T2 = 3.0$
$P3 = 44.4$	$S3 = 758$	$V3 = 115$	$B3 = 4.5$	$T3 = 4.5$
—	—	—	$B4 = 7.0$	$T4 = 7.0$

Contact Stresses at Free Rolling

Free rolling, which is characterized by the absence of driving torque, was used as a reference to compare the effect of braking and traction on the 3D contact stresses. The distribution of contact stresses was studied for the loads, tire inflation pressures, and speeds considered. Fig. 3 shows the typical variation of contact stresses in the longitudinal, vertical, and transverse directions when $P = 44.4\ \text{kN}$ and $S = 690\ \text{MPa}$ for various speeds. The behavior for the other loading cases was similar.

The distribution of contact stresses in the three directions was not greatly affected by speed. This was the case with free rolling only, as speed greatly affected some of the stress components in the other rolling conditions.

Load and tire inflation pressure affected differently vertical contact stresses. Inflation pressure primarily modified the peak value along each meridian. Along the third meridian in Rib 5, for instance, at $P = 26.7\ \text{kN}$, $\sigma_{\text{z,max}}$ changed from 0.71 MPa when $S = 552\ \text{kPa}$ to 0.91 MPa when $S = 758\ \text{kPa}$, an increment of 28%. Conversely, the contact length decreased 9.1%. The load remained constant, so the tire balanced the change in inflation pressure by reducing the contact length and increasing the peak vertical contact stress.

The load applied had a reverse effect on vertical contact stresses. When the pressure remained constant and the applied load increased, peak σ_z remained almost constant. The contact length increased 28% when the tire inflation pressure remained constant at 552 kPa, and the load changed from 26.7 to 44.4 kN. As the load increased, the highly compressed zones in the tread reached their load-carrying capacity. Therefore, the tire increased its contact length to compensate the increment in the applied load.

The variation of longitudinal contact stresses along the contact length is defined by the relative deformation of the tread with respect to the contact surface, which is linked to the sliding velocity and its variation along the contact length (Clark 1971; Berger 1959). At the entrance and exit of contact, the tread travels faster than the road, and the direction of this velocity changes once or twice during contact. This was observed in the variation of σ_x along the contact length, where the plots crossed the horizontal axis once or twice.

Regardless of the number of changes in the direction of σ_x, a positive and a negative peak with similar magnitude were observed for the analyzed P - and S -values. The magnitude of these peaks increased because of the applied load and decreased as a result of the tire inflation pressure. This behavior can be explained by the effect of P and S on the contact length. The increase of tire inflation pressure reduced contact length, thereby resulting in a limited distance for relative displacement to build up. Conversely, the applied load increased contact length, thus allowing for higher relative displacements to appear in the contact length and, consequently, for higher longitudinal contact stresses.

Transverse contact stresses are primarily caused by the restriction of tread displacement in the direction perpendicular to traffic. The transfer of load through the tires walls may also influence the distribution of σ_y along the contact length. As shown in Fig. 3(c), a small negative peak was observed at the rear end of the contact length, which might be caused by a combination of tensile longitudinal contact stresses and the influence of the load transferred by the sidewalls in the transverse direction.

Contact Stresses at Braking

Four braking conditions, defined by slip ratios of $s_b = 1.5$, 3.0, 4.5, and 7.0% were studied (7.0% slip was assumed to represent full braking). Fig. 4 shows the variation of contact stresses in the three directions along the third meridian of Rib 5 when $V = 8\ \mathrm{km/h}$, $P = 44.4\ \mathrm{kN}$, and $S = 690\ \mathrm{kPa}$. Based on Fig. 4(a), the main effect on σ_z is seen on the location of its peak values. As s_b increased, the location of $\sigma_{z,\max}$ slightly shifted farther from the center of the contact length. Consequently, the resultant force would not be aligned with the center of the contact length. This phenomenon has implications on the distribution of longitudinal contact stresses and rolling resistance.

As shown in Fig. 4(b), longitudinal contact stresses were greatly affected by the variation in braking condition when compared with free rolling. The variation of σ_x along the contact length during braking results from the superposition of three elements (Clark 1971). The first element affecting the variation of σ_x is distribution of longitudinal contact stresses during free rolling. Second, a resultant torque with respect to the y direction, T_y, creates a reaction during braking in the direction of traffic at the contact between the tire and the rolling surface. This reaction is distributed as longitudinal contact stresses on the tire-surface contact. Third, as the tire rolls, the surface constrains the longitudinal movement of the tread elements, which translates into contact stresses along the traffic direction.

The effect of increasing the slip ratio on the distribution of σ_x along contact length is shown in Fig. 4(b). During free rolling ($s_b = 0\%$), σ_x points in the direction of traffic at the front of the tire and in the opposite direction at the back of the tire. As the slip ratio increases (e.g., $s_b = 1.5\%$), the components mentioned in the previous paragraph begin to accumulate until the limit imposed by the friction coefficient is reached. The points to first attain the limit were located at the rear end of the tire; as the slip ratio increases (e.g., $s_b = 3.0\%$), a greater portion of the contact length matched the maximum friction. At full braking, all the points converged to the limit established by the friction coefficient. The surge in longitudinal contact stresses causes the orientation of the in-plane shear stresses to be predominantly oriented in the direction opposite to traffic. Consequently, the contact stresses in the direction perpendicular to traffic decreased as s_b increased. Fig. 4(c) shows the lowest σ_y for full-braking conditions.

Minimal variability of maximum σ_x was observed as the braking slip ratio increased [Fig. 4(b)]. Consequently, the peak longitudinal contact stress should not be used to assess the potential severity of braking conditions on pavement responses. The total force transferred by the tire through the meridians provided better insight into the effect of braking conditions on contact forces. As expected from the variation of the vertical contact stresses with contact length, the total vertical load carried by each rib did not change with speed or braking condition. A different behavior was noticed for the longitudinal contact forces, where the considered variables affected σ_x differently. The longitudinal force carried by each rib increased as the speed changed from 115 to $8\ \mathrm{km/h}$ at the same braking condition. The friction model adopted in this study considered the effect of sliding speed on friction between the tire and rolling surface. As the speed decreased, the value of the friction coefficient increased, which translated into higher longitudinal contact stresses and forces along and across the tire as speed is reduced.

As the braking slip ratio increased, more points along the contact length reached the limit defined by the friction coefficient, so the role of friction became more important as s_b increased. Because the friction coefficient decreased by decreasing the sliding speed, the reduction of longitudinal contact forces with speed was more pronounced at high s_b values.

A surge in longitudinal contact forces compared with free rolling condition was also noticed. During free rolling, the longitudinal resultant force, F_x, only balanced the moment caused by the offset of the vertical reaction. However, F_x also balanced the braking torque during braking. The braking torque significantly increased F_x. Furthermore, for the three speeds considered, the share of the total longitudinal contact force transferred by the edge ribs increased with the increase of the applied load. At high load values, the sidewalls of the tire transfer a higher fraction of the applied load, thus increasing the contact stresses.

The variation of the total reaction force in the longitudinal direction with speed, load, inflation pressure, and braking slip ratio is presented in Fig. 5. A similar effect of the variables on F_x was observed. First, F_x reduced its magnitude as the moving speed increased; second, the difference between F_x at various P decreased as the braking slip ratio decreased; third, tire inflation pressure barely affected the reaction force in the traffic direction; and, fourth, F_x was minimal at free rolling, but not equal to zero.

Contact Stresses at Traction

Traction in a rolling tire is created when a positive driving torque is applied along the rotation axis; in other words, when the tire rotates at an angular speed higher than the free rolling angular speed. s_t, as defined in Eq. (4), was utilized to characterize traction using four values: $s_t = 1.5$, 3.0, 4.5, and 7.0%. Three aspects were studied: (1) variation of 3D contact stresses along a representative meridian; (2) force distribution carried by each meridian across the tires width; and (3) total resultant in the direction of traffic.

The typical variation of contact stresses in the vertical, longitudinal, and transverse direction when $V = 8.0\ \mathrm{km/h}$, $P = 44.4\ \mathrm{kN}$, and $S = 690\ \mathrm{kPa}$ is shown in Fig. 6. Similar to the braking condition, V did not affect the variation of σ_z, and the location of $\sigma_{z,\max}$ was slightly shifted from the center of the contact length. In this case, the location did not move forward, but rather backwards.

The same three components that generate σ_x in the braking condition apply during traction: Stresses resulting from horizontal reaction, free rolling, and deformation of the tread element. Even though the free rolling condition is the same as in braking, the direction of the other two components changed. As observed in Fig. 6(b), when changing from free rolling to the first traction condition $s_t = 1.5\%$, the negative peak in the rare part of the tire switched to positive. As previously explained, σ_x began to exceed the limit imposed by friction from the rare part of the tire, thus creating a peak value. In some cases, the variation of σ_x along contact length showed two negative peaks.

Even though Fig. 6(b) does not show significant difference between $s_t = 4.5\%$ and $s_t = 10\%$ for the points at the rare region of the contact, a different behavior was observed in the other speeds. If $V \neq 8.0\ \mathrm{km/h}$, σ_x exceeded the friction limit, but full traction showed smaller longitudinal contact stresses than for $s_t = 4.5\%$. This may be caused by the smaller friction coefficient created by the higher slip rate at full traction when compared with $s_t = 4.5\%$.

A clear effect of the degree of traction was observed on the transverse contact stresses. From Fig. 6(c), both peaks in the variation of σ_y along contact length decreased as s_t increased. Transverse contact stresses are primarily caused by the movement restriction created by the ground. As the traction slip ratio increased, the tread element moved in the longitudinal direction rather than the transverse one, causing a reduction in σ_y.

As expected from the unmodified variation in the vertical contact stresses, the total vertical force carried by each meridian did not change with the change in speed or traction.

A similar behavior was also observed with the force in the longitudinal direction. First, the magnitude of longitudinal forces dramatically increased with respect to the free rolling condition as s_t increased. In addition, as speed increased, the force in the longitudinal direction decreased; this can be explained by the reduction of the friction coefficient. It was also noted that under full traction, the force was very similar to full braking, but with an opposite sign, suggesting an antisymmetric behavior between braking and traction with respect to the free rolling condition. Under full braking and full traction, the magnitude of contact forces in the longitudinal direction was controlled by the limit imposed by friction; this limit remained constant regardless of the magnitude of the angular speed and direction of movement.

The variation of the total resultant in the traffic direction, F_x, for the load, tire inflation pressure, and speed cases is summarized in Fig. 7. The behavior at full traction and full braking was similar, but in different directions, as verified by comparing F_x in both cases (Fig. 7 for traction and Fig. 5 for braking). It is also noted that the curves for various tire inflation pressures were as coincidental as in braking, thus signaling a lack of influence of inflation pressure on F_x when the tire was subjected to traction. Finally, as observed in braking, the effect of the applied load in F_x was reduced as s_t decreased.

Regression Analysis

Regression analysis was applied to the variation of vertical contact stresses during free rolling for the speed, load, and tire inflation pressure. For full braking and full traction, not only vertical but also longitudinal contact stresses were fitted. Based on previously presented equations (Guo and Lu 2007), the contact stresses in the vertical and longitudinal direction were assumed to be given by

(5)

$$\sigma_{z,y}(\xi) = \frac{\alpha P}{2ab} c_1 (1 - \xi^{2n})(1 - c_2 \delta \xi)$$

where $a = l/2$ = half contact length (mm); $c_1 = 1 + 1/2n$; $c_2 = [-3(2n + 3)(2n + 1)]$; P = applied tire load (kN); n, α, and δ = fitting parameters; and $\xi = x/a$ = normalized distance along the contact length.

To accurately represent the variation of contact stresses, each rib i was divided into three subribs, and one variation of contact stresses $\sigma_{i,j}$ was assigned to each subrib ($i = 1-8$, and $j = 1-3$). $\sigma_{i,j}$ was calculated considering the share of applied load carried by each subrib (Hernandez et al. 2014). Consequently, 24 equations are needed to fully determine the contact stresses in the vertical or longitudinal direction for a combination of applied load, tire inflation pressure, rolling condition, and speed.

Not only was the coefficient of determination, R^2, but also equilibrium used to verify the quality of the obtained equations. The resultant of the calculated vertical contact stresses from Eq. (5) should be equal to the applied load. The average resultant-to-applied-load ratio and coefficient of determination for all regressions performed were 0.987 and 0.962, respectively.

Braking is particularly relevant in road intersections design, so the information needed to determine vertical and longitudinal contact stresses at full braking and $V = 8\ \mathrm{km/h}$ was provided. Figs. 8 and 9 present the regression coefficients n, α, and δ for the vertical and longitudinal contact stresses, respectively. The horizontal axis indicates each one of the 24 subribs across the tire. Fig. 10 shows the contact length and contact width for the same rolling condition and speed.

Conclusions

A validated FE model for a wide-base truck tire was used to study the 3D contact stresses at various rolling conditions. The model included detailed tire geometry and laboratory-measured material properties of rubber (hyperelastic) and tire reinforcement (linear elastic). In addition, advanced features were included such as rebar elements, cylindrical elements, and sliding-velocity-dependent friction. The analysis matrix included values of applied load, tire inflation pressure, speed, and rolling condition to cover the typical operating conditions of truck tires.

The study discussed the shape of contact stress variation along each direction and the effect of various variables. The vertical contact stresses are unaffected by the traveling speed and rolling condition, but their shape and magnitude would change by applied load and tire inflation pressure. The rolling condition affects the longitudinal contact stresses, where the magnitude greatly increases as the severity of braking and traction becomes more relevant. Longitudinal contact stresses were successfully fitted to a mathematical expression, and regression parameters were provided for the lowest speed at full braking (relevant for road intersection design).

The model presented is being improved to consider rubber materials as linear-hyperviscoelastic. This advancement will allow for the evaluation of the effect of loading rate and tire temperature on contact stresses and energy dissipation (i.e., truck fuel consumption).

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Notation

The following symbols are used in this paper:

A_c	=	contact area;
a	=	half contact length;
b	=	contact width of sub-rib i, j;
C_{10} and C_{01}	=	Mooney-Rivlin constants;
c_1 and c_2	=	fitting parameters;
d	=	tire deflection;
F_x	=	longitudinal resultant force;
I_1 and I_2	=	first and second principal invariant of the deformation tensor;
l	=	contact length;
MAPE	=	mean absolute percentage error;
m	=	number of measurements used in validation;
P	=	applied load;
R_{fr}	=	free-rolling radius;
S	=	tire inflation pressure;
s_t and s_b	=	slip ratio for traction and braking, respectively;
V	=	rolling speed;
W	=	strain energy density;
n, α, and β	=	fitting parameters;
ξ	=	normalized distance along contact length;
$\sigma_{i,j}$	=	contact stresses in rib i subrib j;
$\sigma_{x,y,z}$	=	longitudinal, transverse, and transverse contact stresses and; and
ω	=	angular frequency.

References

Al-Qadi, I. L., and Yoo, P. J. (2007). "Effect of surface tangential contact stresses on flexible pavement response." *J. Assoc. Asphalt Paving Technol.*, 76, 663–692.

Belytschko, T., Lie, W. K., and Moran, B. (2000). *Nonlinear finite elements for continua and structures*, Wiley, Chichester, U.K.

Berger, M. (1959). "Kinematics of a rolling tire and its application to tire performance." *J. Appl. Polymer Sci.*, 2(5), 174–180.

Clark, S. K., ed. (1971). *Mechanics of pneumatic tires*, Department of Commerce, Washington, DC.

Ghoreishy, M. H. R. (2008). "A state of the art review of the finite element modelling of rolling tyres." *Iran. Polym. J.*, 17(8), 571–597.

Ghoreishy, M. H. R., Malekzadeh, M., and Rahimi, H. (2007). "A parametric study on the steady state rolling behaviour of a steel-belted radial tyre." *Iran. Polym. J.*, 16(8), 539–548.

Gruber, P., and Sharp, R. S. (2012). "Shear forces in the contact patch of a braked-racing tyre." *Veh. Syst. Dyn.*, 50(12), 1761–1778.

Gruber, P., Sharp, R. S., and Crocombe, A. D. (2012). "Normal and shear forces in the contact patch of a braked racing tyre. 1: Results from a finite-element model." *Veh. Syst. Dyn.*, 50(2), 323–337.

Guo, K., and Lu, D. (2007). "UniTire: Unified tire model for vehicle dynamic simulation." *Veh. Syst. Dyn.: Int. J. Veh. Mech. Mobility*, 45(S1), 79–99.

Hernandez, J., Gamez, A., Al-Qadi, I., and De Beer, M. (2014). "Introducing an analytical approach for predicting 3d tire-pavement contact load." Transportation Research Record 2456, Transportation Research Board, Washington, DC, 75–84.

Kato, S., and Matsubayashi, T. (1970). "On the dynamic behavior of machine tool slideway: 1st report, characteristics of static friction in stick-slip motion." *Bull. JSME*, 13(55), 170–179.

Kato, S., Sato, N., and Matsubayashi, T. (1972). "Some considerations on characteristics of static friction of machine tool slideway." *J. Tribol.*, 94(3), 234–247.

NAPA. (2013). "Perpetual pavement." 〈http://www.asphaltpavement.org〉 (Jan. 13, 2015).

Oden, J. T., and Martins, J. A. C. (1985). "Computational physics models and computational methods for dynamic friction phenomena." *Comput. Methods Appl. Mech. Eng.*, 52(1–3), 527–634.

Wang, H., Al-Qadi, I. L., and Stanciulescu, I. (2012). "Simulation of tyre-pavement interaction for predicting contact stresses at static and various rolling conditions." *Int. J. Pavement Eng.*, 13(4), 310–321.

Wang, H., Al-Qadi, I. L., and Stanciulescu, I. (2014). "Effect of surface friction on tire-pavement contact stresses during vehicle maneuvering." *J. Eng. Mech.*, 10.1061/(ASCE)EM.1943-7889.0000691, 04014001.

Wang, Y. S., and Wu, J. (2009). "Numerical analysis on the steady-state rolling of load-carrying tire." 2nd Int. Conf. on Intelligent Computation Technology and Automation, IEEE Computer Society, New York, NY, 566–569.