A Continuum Damage Approach to Spallation and the Role of Microinertia

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ABSTRACT

Spall failure is of interest due to its prevalence in high strain rate problems in which the spallation is driven by the interaction of release waves. In this article, a porosity-based damage model that includes microinertial effects is used to examine spall failure. The model is successfully calibrated to plate impact-driven experiments and then used to evaluate experimental conditions producing more extreme strain rate conditions, such as those in laser-driven experiments. The incorporation of microinertia allows us to better understand the increase in apparent macroscopic spall strength seen at high strain rates. Correspondingly, we conclude that the incorporation of microinertial effects improves the model’s predictive capabilities. Microinertial effects result in more severe local tensile stresses that affect the damage evolution, and microinertia can play a significant role in the approach to the material’s ideal strength at extreme loading rates. A preliminary parametric study is also carried out to investigate the role of microstructural aspects such as nucleation volume fraction and initial pore radius. One counter-intuitive result from the microinertial effects is that, for a given nucleation site volume fraction, having larger initial pore nucleation sites can lead to an improved macroscopic spall strength.

I. INTRODUCTION

Spallation is a process in which dynamic loading leads to a zone of tensile stress within a material due to the interaction of rarefaction waves, resulting in partial or complete failure of the material. Following the first observation of spall failure by Hopkinson,1 it has been an area of avid interest, with research activities driven by a variety of applications involving dynamic loading.

Both experimental and numerical approaches have contributed to understanding the processes involved in creating spall failure. In addition to the damage mechanisms associated directly with spall, understanding spall failure requires understanding associated phenomena for shock-wave propagation2 and high strain rate plasticity.3–5 Although it is commonly understood that spall failure in ductile materials is governed by the nucleation, growth, and coalescence of voids/porosity,6–8 the modeling of these processes remains an area of active interest. The importance of post-mortem analysis of spall experiments to better understand the role of porosity in the failure mechanism has been demonstrated by Jones et al.9 Work has been done using direct numerical simulations (DNS) to explicitly model voids and their behavior with relevance to dynamically loaded conditions,8–10 highlighting complexities in porosity-mediated failure. Other work using molecular dynamics (MD) simulations has provided insights for smaller length scales and correspondingly higher strain rates.11–14 These studies have lent valuable insights, especially into the nucleation and growth stages of porosity evolution. However, while such DNS and MD simulation work helps us understand what is happening at the micromechanical scale, it does not enable a direct connection to component-scale response.

Homogenized continuum scale damage models allow for simulation at the component-scale and for connection to a range of...
experimental results, and fidelity enhancements allowing damage models to capture detailed subscale response are of interest. A variety of failure models have been used in the literature to simulate spall failure. These include critical stress/strain models like Johnson-Cook and Grady failure criteria and more advanced models such as the Cochran–Banner and Gurson-based formulations for void growth. Critical stress/strain criteria have fallen out of favor in recent literature as they estimate failure as a single event, instead of as a dissipative process, which causes both numerical inconsistencies and poor predictive capability. Coupled models that degrade the material’s effective behavior with the nucleation, growth, and coalescence of porosity have been demonstrated to have better predictive capability in simulating spall failure.

The rapid growth of laser technology in the past two decades has evolved the study of the spallation process past the limits of plate impact and explosive-driven experiments. Evolution of the technology has raised questions that may not have been significant for experiments probing material response at lower strain rates, especially as we push the boundaries toward the ideal strength of materials. Among these questions is the role of micro-scale inertia. The presence of a void and its growth represents a moving boundary and the associated outward motion of material involves micro-scale inertial. Preliminary studies examining microscopically homogeneous deformation have shown that this effect grows significantly with increasing strain rate and becomes particularly significant around strain rates on the order of $10^7$/s. These and higher strain rates are being accessed by nano-second laser pulse experiments while other experiments have demonstrated even shorter pulse times, which may lead to even higher strain rates. With the effect of microinertia being directly related to the size, spacing, and shape of voids, it stands to reason that a robust damage model that incorporates porosity growth and microinertial effects is desirable in modeling spallation at these higher strain rates. Since the nucleation, growth, and coalescence of voids are also heavily dependent on microstructural aspects such as nucleation sites, grain structure, and crystal lattice orientation, it is also of interest to understand how these aspects couple with microinertia and whether they show any first-order influence on spallation. Overall, these topics are of significant interest as the scientific community considers investments to probe the dynamic material response at flagship experimental facilities.

In this work, a Cocks–Ashby-based damage model is used in conjunction with a rate-dependent plasticity model and a treatment of microinertial effects to simulate spall across a range of strain rates, with application to spall in both plate impact and laser-driven experimental platforms. Specifically, we examine results for tantalum, comparing against existing data from laser-driven experiments with pulse durations on the order of nanoseconds and from a suite of plate impact experiments. This allows the analysis of the transferability of such models across different strain rate regimes and provides a framework for modeling of spall failure for larger specimens and components. The contribution of microinertia to spall strength is analyzed, as are the implications for high-rate spall problems. Parametric studies are also carried out to qualitatively assess the influence of certain microstructural aspects that could lead to improvement in spall strength by taking advantage of microinertial effects.

II. MODEL FORMULATION

The formulation employed is computationally efficient and robust enough for general use simulations of component-scale response, while including features that allow us to explore the effects of rate-dependent material response and the role of microinertia. Here, we focus on the porosity-mechanics-related aspects of the model. The remaining aspects of the constitutive model are described in more detail elsewhere with the form specialized for body-centered cubic metals such as tantalum and with the flow strength of the material based on an evolving dislocation density. The model was originally formulated and calibrated based on sub-scale modeling focused on strain rates above $10^7$/s and was subsequently adapted to better capture experimental data at lower strain rates. For the subset of parameters that are calibrated to experimental data, the values in this work are based on results for a well-characterized lot of tantalum from H.C. Starck GmbH. The equation of the state follows a common Mie-Grüneisen form and parameterization and the shear modulus behavior vs density is based on density functional theory calculations with thermal effects captured using a function of homologous temperature, as in Eq. (9) in Ref. The Appendix provides a summary of the strength model and the specific parameters used for the work here.

Assuming spherical pores, the kinematics associated with porosity evolution are relatively simple. We track a scalar volume fraction of porosity, $f$, with an associated multiplicative relative volume factor $f_d = \frac{1 - f_n}{1 - f}$, in which $f_n$ is the volume fraction associated with porosity nucleation. The total porosity includes the nucleated porosity, so that $f \geq f_n$ and $f_d \geq 1$. In this idealization, $f_n$ accounts for nucleation from sites, as in the cracking or decohesion of precipitates, such that the nucleation process itself does not change the volume of the material.

The growth of pores and, thus, the evolution of $f$ are accommodated by a plastic deformation of the “matrix” material surrounding the pores. Additionally, the porosity degrades the resistance of the material to both volumetric and deviatoric (shear) deformation. Overall, a semi-implicit time integration of the constitutive model requires the solution to a coupled system of non-linear equations. We solve concurrently for the evolution of porosity and for the effective plastic strain rate.

Accounting for large volume changes and nucleation effects, the average pore radius is

$$r^3 = \left( \frac{f N}{\frac{1}{2} \pi N} \right) \left( \frac{1 - f_n}{1 - f} \right),$$

where $N$ is the number density of pores in a reference configuration and $f$ is a uniform volume change of the porous ensemble such that changes in $f$ do not produce changes in $f$. Motivated by the work of Cocks and Ashby for capturing rate-dependent effects in porosity modeling, the dominant term in the porosity evolution
is tied to the equivalent plastic strain rate $\dot{\varepsilon}_p$ and takes the form

$$
\dot{f}_c = c_1 \sinh \left[ c_2 \left( n - \frac{1}{2} \right) \frac{\sigma_m}{\tau} \right] \left[ \frac{1}{1 - (1 - f)^n} - (1 - f) \right] \dot{\varepsilon}_p \tag{2}
$$

in which $\sigma_m$, $\tau$, and $n$ are associated with pressure, strength, and strain rate sensitivity, respectively, along with calibration parameters, $c_1$ and $c_2$, as described in Ref. 34. The driving force for porosity evolution is $\sigma_m = -P_m - P_i - \sigma_{mi}$, accounting for the matrix pressure $P_m$, Laplace pressure $P_i$, associated with surface tension, and microinertial contributions $\sigma_{mi}$. For convenience, we define $\sigma_{mi} = -P_{mi}$ and note that the homogenization of the micro-scale momentum balance to derive the microinertial contribution is such that $\sigma_{mi}$ takes the role of the macro-scale value. Through this homogenization, the driving force $\sigma_m$ is modulated by micro-scale inertial effects without the need for spatial discretization resolving the acceleration of the material around each evolving pore.

The matrix strength $\tau$ is evaluated at an effective plastic strain rate $\dot{\varepsilon}_p$ that includes contributions from plasticity associated with pore growth. The rate sensitivity coefficient $n$ can be taken as a constant or computed from

$$
n = \left( \frac{\dot{\varepsilon}_p}{\tau} \frac{\partial \tau}{\partial \dot{\varepsilon}_p} \right)^{-1}, \tag{3}
$$

with the derivative taken at a fixed state.

As described elsewhere, the matrix material flow strength includes contributions associated with both thermally activated and phonon-drag-limited motion of dislocations. If the strain rate is high enough that the dislocation velocity transitions into the phonon-drag regime and the drag-related contribution to the flow strength dominates, then $\tau$ tends toward unity for linear phonon-drag, significantly modulating the porosity kinetics. See Eq. (A7) in the Appendix, where the drag contribution is roughly linear in the dislocation velocity when the dislocation velocity is well below the shear wave speed $c_0$, and Fig. 1 in Ref. 3. This behavior of the model naturally captures effects at elevated strain rate that have motivated previous work on specialized porosity growth rate models. Under some deformation conditions, the flow strength is dominated by a Taylor hardening contribution that scales with the shear modulus and the square root of the dislocation density. This hardening contribution is treated as being rate-independent, and when it is dominant, the overall $n$ in Eq. (3) becomes much larger than unity. Thus, the flow strength model can produce a wide range of $n$ values for use in the porosity kinetics, with $n$ depending on the current conditions and, through the evolved dislocation density, the previous history of the material.

As in Ref. 34, the overall porosity evolution rate includes an additional term that allows for growth even when $\dot{\varepsilon}_p$ is zero, but that contribution is negligible for results shown here. For additional information, see Refs. 34 and 46.

The model for nucleation is similar to the one employed in Refs. 20 and 47 and produces nucleated porosity, $f_n$, up to a volume fraction $f_n^*$. The maximum $\sigma_m$ experienced by a given portion of material. Eq. (4) gives the functional form of $f_n$, where parameters $\sigma_m$ and $\sigma_5$ are the mean and standard deviation of the $\sigma_m$ values over which nucleation sites are activated,

$$
f_n = \frac{f_n^*}{2} \left[ 1 + \text{erf} \left( \frac{\sigma_m - \sigma_5}{\sigma_5} \right) \right]. \tag{4}
$$

The local number density of pores evolves with the nucleation based on the volume fraction nucleated and the specified size of the nucleation sites. As part of exploring the dependence of microinertial effects on the microstructure, we will explore variation with the radius of the nucleation sites ($r_n$) and the nucleated volume fraction. For a given $f_n^*$, the number density of available nucleation sites, and correspondingly the length scale in the microstructure, varies as $r_n^{-3}$.

Microinertial contributions to the driving force naturally involve the mass density of the matrix material and depend strongly on the current pore radius. Following the approach of Molinari and coworkers, we employ a form

$$
\sigma_{mi} = r^2 \rho_n \int f_m^* \left[ D_f \left( f_{mi}^* - f_{mi}^{*2/3} \right) + D_k \left( 3 f_{mi}^* - 5 f_{mi}^{*2/3} - \frac{1}{2} f_{mi}^* \right) \right]. \tag{5}
$$

where $\rho_n$ is the reference mass density, $r$ is the average pore radius, and $D_k$ is the mean volumetric plastic strain rate associated with pore growth. To avoid numerical problems and to be consistent with the length scale implied by the overall population of nucleation sites, we use $f_{mi} = f + f_n - f_n^*$. Given that $f$ includes the porosity associated with both nucleation and growth, this definition results in $f_{mi} \geq f_n^*$ at all times. We also note that for the applications described here the model parameters are such that the distribution underlying Eq. (4) is narrow and most of the nucleation sites at a given material location are activated over a narrow time interval.

Overall, volumetric softening due to porosity is captured by the dependence of the porosity growth rate [Eq. (2)] on current porosity, and as in Ref. 46, deviatoric response is degraded by a factor $1 - \tanh (\alpha_n f)$ that scales the effective strength of the material.

III. METHODOLOGY

Simulations are conducted using the arbitrary Lagrangian–Eulerian simulation code ALE3D. Results of these simulations are compared to experimental results for the rear surface velocity of the target and post-mortem porosity measurements in the target. Wave propagation effects allow the central portion of material to be modeled in one dimension. That is, the lateral extent is large enough that lateral effects do not influence the experimentally observed rear surface velocity measurements during the time interval of interest. Simulations of the plate impact experiments employ impactor and target thicknesses correspond to those in the experiments. Simulations of the laser-driven spall experiments utilize a similar quasi-1D approach but with a pressure pulse boundary.
condition on the incident face rather than an impactor. Figure 1 shows schematics for the problem setups.

For simulations of the laser-driven experiments, the pressure pulse duration is kept the same as that used in experiments (10 ns) and the pressure magnitude is adjusted to match the peak velocity in the velocity trace measurements. A polystyrene ablator was utilized in the experiments and the effect of the ablation process is roughly approximated using a decreasing triangular pressure pulse shape, which can be seen in Fig. 2. While the fidelity in treating the drive condition could be improved through the simulation of the process of laser-induced ablation of the target, the use of this approximate pressure pulse profile produces satisfactory results for the study conducted here, leading to spallation and rear surface velocity results consistent with those in experiments.

The nominal spall strength and nominal strain rate are calculated based on the pullback velocity\(^2,5\) according to the following equations:

\[
\sigma_{\text{spall}} \approx \frac{\rho_0 C_L}{1 + \frac{C_B}{C_L}} \Delta u_{\text{release}},
\]

\[
\dot{\epsilon} \approx \frac{1}{2C_B} \frac{\Delta u_{\text{release}}}{\Delta t_{\text{release}}},
\]

where \(u\) is the rear surface velocity, \(\rho_0\) is the initial density, and \(C_B\) and \(C_L\) are the bulk and longitudinal sound speed, respectively. The experimentally measurable quantities associated with Eqs. (6) and (7) are labeled in Fig. 3.

These measures are convenient indicators of nominal macroscopic spall strength and nominal strain rate. However, it should be noted that these nominal quantities may differ significantly from the microscopic stress and strain rate levels that are seen within the material, as will be discussed below.

A. Model calibration

The model form has been used in simulations of a variety of plate impact-based experiments, and here, it is specifically calibrated against data collected at Los Alamos National Laboratory\(^3\) for an experiment with an impact velocity of 246 m/s. The flyer and target are made of the same tantalum material and have thicknesses of 2.083 and 4.267 mm, respectively. The velocity trace is obtained from velocimetry measurements on the free surface, and post-mortem examination of the target provides a porosity profile.\(^3\) Simulation results are compared against these experimental data in Figs. 3 and 4. The model is calibrated against the velocity trace and the post-mortem porosity data, through the calibration process.
The results show excellent agreement with the experimental observations for both the pullback signal and the width of the porosity distribution. The velocity trace captures the overall behavior of the material quite well. Notable exceptions are the amplitude of the elastic precursor, which can display complex behavior and is affected by the initial condition of the material, and the absence of anelastic effects in the release portion of the simulated velocity trace. However, these effects are not the emphasis of the current analysis and should not affect the conclusions drawn here. The key measurements here are those related to the peak and trough region of the release and the post-mortem porosity distribution. Late-time portions of the velocity trace after the pullback feature can be influenced by lateral effects and are not examined here. In Fig. 4, we see that the width of the porosity distribution is somewhat narrower in the simulation than in the experiment. Simulations with crystal-mechanics-based models that resolve subgrain response, such as those in Ref. 29, might better capture the width of the porosity distribution given that they resolve additional microstructural heterogeneity. However, here we focus on microinertial effects without introducing the additional complexities associated with crystal-mechanics-based modeling.

IV. RESULTS AND DISCUSSION

With confidence in the model based on the performance for plate impact simulations, 1D calculations are performed to mimic laser-driven spall experiments reported by Remington et al. for polycrystalline tantalum with a thickness of 250 μm. Velocity trace comparisons for the two experiments examined here are shown in Fig. 5. The two experiments were conducted with the same pulse duration but with different laser intensities and, thus, different drive pressures, and results for these specific experiments are drawn from Fig. 9 in Remington et al. All subsequent laser-driven simulations are conducted for the same dimensions and pulse conditions, unless otherwise stated. The experiment with a larger drive pressure produces a larger peak velocity on the back surface of the target. Given the simplifications employed in simulating the experimental drive conditions, Fig. 5 shows good agreement of the simulations with the experimental results.

As described in Sec. III, nominal spall strength and strain rate on release can be computed from the simulated velocity traces. The results of simulations that include microinertial effects show nominal spall strengths of 7.2 and 6.4 GPa for simulations of the experiments with higher and lower drive pressures, respectively. The corresponding nominal strain rates are calculated to be $7.6 \times 10^6$/s and $4.6 \times 10^6$/s. For the higher-pressure laser-driven spall problem, Fig. 6 compares results with and without microinertial effects in the calculation and, thus, the influence on the nominal spall strength. A difference of 170 MPa is seen in the nominal spall strength without incorporation of microinertia. The results show that failing to
account for microinertia produces a significant error in the prediction of spall strength. As we will see from discussion below, the differences induced by microinertia are even more pronounced when examining the stress states experienced by the material inside of the target.

Kanel et al. have shown that a material’s spall strength can increase sharply with increases in strain rate.58 This classical result has been observed across multiple materials. Idealized homogeneous deformation (or “material point”) calculations can be used as a simple means of systematically exploring the effects of various aspects of a model across a range of strain rates, and Fig. 7 shows idealized spall strength vs strain rate with and without incorporation of microinertial effects. To roughly mimic the loading conditions that produce spallation, in these calculations, the material is first compressed and then expanded in one direction while being constrained in the other two lateral directions. The idealized spall strength is simply the maximum $\sigma_m$ attained. Kinetics in both the strength model and in the porosity growth model play a role in capturing the experimentally observed behavior of sharply increasing spall strength at extremely high strain rates. Moreover, the simple calculations suggest that microinertial effects would become significant at strain rates on the order of $10^6$/s. This specific observation is for the reference nucleation parameters provided in Table II, and the threshold would change with microstructural details that influence the nucleation behavior. As shown above, laser-driven spall problems access these elevated strain rates at which microinertial effects become significant. This trend in Fig. 7 of increasing influence of microinertial effects with increasing strain rate manifests in Fig. 8, which contains the collected results for the plate impact and laser-driven spall simulations. In Fig. 8, the $\sigma_m$ values that are shown are the peak values experienced by the material in the immediate vicinity of the spall plane.

Results at the highest strain rate shown in Fig. 8 are from simulations corresponding to laser-driven spall conditions that would access even higher strain rates than those shown in Fig. 6. Experiments in the literature have utilized thinner targets and shorter pulses to achieve higher strain rates.59 The rarefaction waves fan out less in these thinner targets as they propagate over shorter distances, producing higher strain rates as the material goes into tension. To test the hypothesis that microinertial effects would become more significant in such experiments, a similar laser-induced spall problem is carried out with a target 50 $\mu$m thick and with a pulse duration of 1 ns while keeping all other conditions, such as the pressure magnitude, the same as those for the simulation in Fig. 5 with the lower peak pressure. Velocity traces for this test case are shown in Fig. 9 and the contribution to nominal spall strength from microinertial effects increases by almost an order of magnitude. From Fig. 8, we see that the increase in the $\sigma_m$ interior to the material at this highest strain rate is even more pronounced than the increase in the nominal spall strength, $\sigma_{s\text{panl}}$, computed from the free surface velocity trace. This effect is discussed in more detail in Sec. IV A.

### A. Results for stress states reached inside the target

Recent continuum scale studies60 and DNS studies9,10 have emphasized the difference between the nominal spall strength estimated using the velocity trace [using Eq. (6)] and the maximum tensile hydrostatic stress levels ($\sigma_m$) actually simulated interior to the body. This is true even for plate impact problems. This phenomenon is also apparent in the simulations conducted here, both with and without microinertial effects in the model. The discrepancy between the two values is enhanced with the incorporation of microinertia. During initial pore growth, the microinertial contribution ($\sigma_{mi}$) to the driving force suppresses porosity growth. This results in the material experiencing a far greater tensile stress ($\sigma_m$) in order to drive porosity growth. Behavior of the model is such that as the strain rate is further increased this discrepancy widens, and the use of the nominal spall strength from the velocity trace...
can be a misleading representation of the actual conditions experienced by the material. The difference between the $\sigma_m$ interior to the material and the $\sigma_{spall}$ from Eq. (6) may be due to the evolution of the wave structure as it propagates from the interior of the body to the free surface at which the velocity trace is measured.

In addition to higher stress levels in the vicinity of the spall plane, the DNS results also show that the neighborhood of the void can actually experience transient localized compressive stress states due to the microinertial effects.9 These two effects manifest in the current calculations, as demonstrated by Fig. 10, which shows simulation results for the material location experiencing maximum porosity growth in the spall plane for the higher-pressure laser-driven shot in Fig. 5. The initial hydrostatic stress required for porosity growth is significantly higher with the incorporation of microinertial stress. Fluctuations in the microinertial stress, which enter the driving force for porosity growth, induce fluctuations in the far-field $\sigma_m$ that enters into the macroscopic solution of the balance of linear momentum. We believe these fluctuations in the $\sigma_m$ are related to the transient localized compressive stress states observed in the DNS studies.

B. Microinertial considerations in material selection

The influence of microinertia presents an appealing proposition: can the apparent increase in strength due to microinertial effects be harnessed to improve material performance? Traditional wisdom has been to use ultra-pure materials for such applications as they present improved strength due to their being nucleation-starved, as shown in Fig. 12. If this microinertial effect can be harnessed, then this presents an exciting opportunity for the development of new engineered materials with better performance under spallation. Following this vein of thought, a parametric study is carried out to examine the possible avenues of research. The main contribution to the microinertial stress arises from two factors, strain rate and void size, with the strain rate being influenced by the loading conditions and the void size for a given volume fraction being related to the characteristic microstructural length scale associated with pore spacing. However, since the microinertial stress scales with the square of the void radius, can the void size be manipulated to improve the spall strength? Figure 11 shows the effect of different initial void radii on the velocity trace for a laser-driven spall problem, while holding constant other quantities, including the total volume fraction available for nucleation.
The larger initial void sizes lead to larger contributions from microinertia and, thus, to a significant increase in apparent macroscopic spall strength. This is borne out by the plot of $\sigma_m$ in Fig. 13, which shows that for the initially larger void radius, the material experiences larger tensile stress as the porosity driving-force contribution from microinertia increasingly impedes void growth. Looking back at Eq. (5), this effect occurs due to the $r^2$ term in the microinertial stress. This $r^2$ term captures the fact that for a given porosity volume fraction the characteristic spacing among pores increases when $r$ is larger and the number density of pores is correspondingly smaller. That is, the characteristic length scale has increased and there is more mass and, thus, more microinertia associated with the acceleration of material to accommodate pore growth. This observation is in line with other studies of pore size and length scale effects associated with microinertia, which show slower growth rates for larger voids.61

This indicates that aspects of the microstructure remain important in high-rate loading scenarios. While in quasi-static applications it may be desirable to have nucleation sites that are smaller and more finely distributed, the observations here suggest that, for a given volume fraction of nuclei, in high-rate applications it may be advantageous to have larger sites that are fewer in number. This counter-intuitive result due to the presence of significant microinertial stress opens up interesting opportunities for engineering materials for greater spall resistance. However, care is warranted in pursuing this idea. If larger nucleation sites produce
longer characteristic length scales due to microinertial effects, then the continuum assumptions employed here may break down. A DNS study would be the first step to ensure that continuum modeling of such problems is valid. DNS studies would also lend credibility to the trends shown here and shed light on other factors such as interaction of larger voids and void spacings to see if they play a significant role. Such studies are the object of ongoing research, which could further inform design strategies and optimization for material performance against spall.

In the foregoing, we assume that the sites nucleate regardless of their size and that nearly all nucleation sites are activated due to the high driving forces. In practical engineering materials, interfacial character and other aspects influencing nucleation could change with site size, complicating the overall considerations. Similarly, the assumption of constant volume fraction for nucleation may or may not be possible depending on the development of processing techniques. With such high tensile stresses, there may be activation of homogenous nucleation sites as well, such as those from vacancies as suggested by Wilkerson and Ramesh. There may nevertheless be a useful trade space to explore in the composition or processing of a given class of materials, particularly with the advent of advanced manufacturing processes that can produce graded microstructures with varying levels of initial porosity and defects.

V. CONCLUSIONS

A Cocks–Ashby-based porosity model that incorporates effects of microinertia was used to model plate impact and laser-driven spall experiments. It was found that the model performed well at predicting the macroscopic response of the material along with the porosity distribution at these high strain rate regimes. Incorporation of rate sensitivity and inertial effects in the homogenized model has benefits for capturing results across a range of conditions, improving confidence in component-scale simulations while retaining a computationally efficient approach.

Incorporation of micro-scale inertia provided a better understanding of the increase in apparent macroscopic spall strength typically seen with increasing strain rates. The contribution to the spall strength became particularly significant in the simulations of laser-driven spall conditions and the trend shows that this effect would amplify as the strain rates are further increased. This is an important consideration, as omitting microinertia could lead to inaccurate representation of the physics in high rate problems and lead to poor predictions of porosity kinetics and of associated failure behavior. The presence of microinertial stress leads to higher hydrostatic stresses being experienced by the material to drive growth of porosity and stress levels could approach the ideal strength of the material. Such large stress magnitudes could lead to activation of void nucleation mechanisms not commonly seen in experiments at lower strain rates, such as nucleation at impurity sites that do not favor nucleation at lower stress levels or nucleation by vacancy clustering as discussed by Wilkerson and Ramesh. When interpreting observations of spall, it is important to distinguish between the apparent macroscopic spall strength, which includes microinertial contributions, and the inherent strength of the material. As seen in this study, the microinertial stress can affect micro-scale peak stresses and lead to micro-scale stress fluctuations. There are also significant implications for the interpretation of experimental results, as modeling high rate problems without incorporating microinertia can lead to exaggerated calibration parameters for plasticity and porosity models. Such parameterizations would fail to represent the true physical behavior and would not transfer well to other loading conditions.

Preliminary parametric studies have also been carried out to explore the effect of microstructural aspects on the microinertial contribution to spall strength. Traditionally, high-purity materials have been used to access improved spall strength due to their nucleation-starved microstructure which requires larger stress to drive porosity growth. In a similar vein, the clustering of any impurities that can contribute to porosity nucleation into larger sites that are fewer in number could lead to improvement in the macroscopic spall strength due to an increase in the effects of microinertia. These ideas point to interesting avenues for further research in the engineering of materials for resistance to spallation.

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AUTHOR DECLARATIONS

Conflict of Interest

The authors have no conflicts to disclose.

DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding author upon reasonable request.

APPENDIX: STRENGTH MODEL AND PARAMETERS

Given that the specific model parameters utilized here have not yet been published elsewhere, we include the parameters in Table III and, for completeness, a summary of the relevant equations. See Refs. 3 and 4 for additional details. In Table III, parameter values in bold are specific to the calibration to the tantalum material from H. C. Starck GmbH.

The flow strength is given by

\[
\sigma = M \left( \tau (\nu, p_m, \Theta) \rho_p(p_m) + \tilde{\tau}(\rho_p, p_m, \Theta) + \tau_a \right),
\]

with \( M \) being a Taylor factor, \( \tilde{\tau} \) and \( \tau_a \) being athermal contributions, \( \tau_p = \{ \tau_p(1 + C_{p2} \frac{p_m}{G_0}) \} \) being the pressure-dependent Peterls stress, and \( \tau' = (\tau_0 + \tau_d^{1/q})^{1/q} \) capturing rate effects from thermally activated and drag-limited dislocation motion.
Strengthening from dislocation density follows a standard Taylor hardening form with
\[ \tau(\rho_\perp, \rho_m, \theta) = \zeta b G(\rho_m, \theta) \sqrt{\rho_\perp}, \] (A2)
with hardening coefficient \( \zeta \), Burgers vector magnitude \( b \), temperature \( \theta \), and dislocation density \( \rho_\perp \).

The plastic strain rate \( \dot{\varepsilon}_p \) and dislocation density directly provide the average dislocation velocity,
\[ \dot{v} = \frac{\dot{\varepsilon}_p M}{\eta \rho \cdot b}, \] (A3)
and its modulated value
\[ \ddot{v} = \frac{v_m}{v_0} = \left( \frac{\left( v - v_0 \right)}{\left( v - v_0 \right)} \right)^{1/\chi} \] (A4)
for use in the equations for thermally activated and drag-limited contributions,
\[ \ddot{\tau}_0(v, \theta) = \alpha_0 \exp\left( \frac{\theta}{\alpha_0} \right) \left( \exp[\beta \ln(v + v_0)] - \exp[\beta \ln(v_0)] \right), \] (A5)
\[ \beta(\theta) = \beta_0 \left[ 1 - \exp\left( -\frac{\theta}{\beta_0} \right) \right], \] (A6)

### Table III. Material parameters.

<table>
<thead>
<tr>
<th>( M )</th>
<th>( \eta )</th>
<th>( b ) (Å)</th>
<th>( \tau_k ) (MPa)</th>
<th>( \tau_{p0} ) (MPa)</th>
<th>( G_0 ) (GPa)</th>
<th>( G_{p3} ) (GPa)</th>
<th>( C_0 ) (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.08</td>
<td>2.0</td>
<td>2.86</td>
<td>525</td>
<td>69</td>
<td>1.0005</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>( X_0 )</td>
<td>( X_1 )</td>
<td>37.166</td>
<td>0.030</td>
<td>75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( R_0 ) (cm²)</td>
<td>( \varepsilon_{R0} )</td>
<td>( n_R )</td>
<td>( \rho_{1L0} ) (cm²)</td>
<td>( \varepsilon_{S0} )</td>
<td>( n_S )</td>
<td>( \chi )</td>
<td></td>
</tr>
<tr>
<td>11.2×10¹⁴</td>
<td>5×10⁻¹²</td>
<td>0.37</td>
<td>5×10¹⁴</td>
<td>10⁻¹⁰</td>
<td>0.5</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>( \rho_{1L0} ) (μm⁻¹)</td>
<td>( \varepsilon_L )</td>
<td>( Q_L ) (K)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25×10¹⁰</td>
<td>1</td>
<td>0</td>
<td>1462</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \dot{\tau}_D(v, \rho_m, \theta) = \max \left( \frac{\chi_0 (\frac{\varepsilon}{\rho} - \chi_1)}{1 - \left( \frac{\varepsilon}{\rho} \right)^2}, 0 \right), \] (A7)

Dislocation density evolution follows a form that tends to saturation at steady deformation conditions
\[ \dot{\rho}_\perp = R \left( 1 - \frac{\rho_\perp}{\rho_{\perp sat}} \right) \dot{\varepsilon}_p, \] (A8)
with evolution rate prefactor
\[ R = R_0 \left( \frac{\dot{\varepsilon}_p}{\dot{\varepsilon}_s} + \varepsilon_{B0} \right)^{n_b}, \] (A9)
in which
\[ \dot{\varepsilon}_s = \frac{v_{m0} G_0}{\chi_{\rho \cdot b}} \] (A10)
is akin to a scaled dislocation mobility and the saturation density has high-rate and low-rate branches,
\[ \rho_{\perp sat} = \max \left( \rho_{\perp satH}, \rho_{\perp satL} \right), \] (A11)
\[ \rho_{\perp satH} = \rho_{\perp L0} \exp \left( -\frac{\theta}{Q_L} \right), \] (A12)
\[ \rho_{\perp satL} = \rho_{\perp L0} \exp \left( -\frac{\theta}{Q_L} \right), \] (A13)

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