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An efficient cost function for the optimization of an *n*-layered isotropic cloaked cylinder

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Abstract

In this paper, we present an efficient cost function for optimizing *n*-layered isotropic cloaked cylinders. Cost function efficiency is achieved by extracting the expression for the angle independent scatterer contribution of an associated Green's function. Therefore, since this cost function is not a function of angle, accounting for every bistatic angle is not necessary and thus more efficient than other cost functions. With this general and efficient cost function, isotropic

cloaked cylinders can be optimized for many layers and material parameters. To demonstrate this, optimized cloaked cylinders made of 10, 20 and 30 equal thickness layers are presented for TE and TM incidence. Furthermore, we study the effect layer thickness has on optimized cloaks by optimizing a 10 layer cloaked cylinder over the material parameters and individual layer thicknesses.

The optimized material parameters in this effort do not exhibit the dual nature that is evident in the ideal transformation optics design. This indicates that the inevitable field penetration and subsequent PEC boundary condition at the cylinder must be taken into account for an optimal cloaked cylinder design. Furthermore, a more effective cloaked cylinder can be designed by optimizing both layer thickness and material parameters than by additional layers alone.

1. Introduction

Since the transformation optics (TO) cloaked cylinder was presented in,[1] many efforts have been made to approximate the ideal material requirements. These efforts include manipulating the required anisotropic cloak material parameters[2–7] and still other efforts use effective medium isotropic material approximations.[8–10] The problem these references address is that the material parameters required for the TO cloak are not physically realizable and thus must be approximated. This problem is then compounded by the fact that any approximation deviates from the ideal design and allows incident energy to penetrate the cloak and impinge upon the object to be cloaked, a cylinder in our case. Since the TO method does not account for this boundary condition, approximating the ideal TO parameters inevitably results in suboptimal non-ideal cloak performance.

Other cloak design methods are based on a ray optics approach implementing conformal mapping.[11,12] This method uses index of refraction to force fields around an object and results in a cloak coating that requires a material parameter gradient. While this method gives a general recipe for a cloak design, it does not provide a relationship between cloaking effectiveness and a particular refractive index profile. This relationship is needed to solve the engineering problem of, as Leonhardt stated, 'finding the most practical design'.[11]

If one could derive a mathematical relationship between a cloak design and its effectiveness, then an optimization approach can be used to mitigate the necessity of deviating from the ideal cloak material requirements.[13–16] The cost function in these efforts is made of an expression for the scattered fields. This expression is derived for the given stratification profile by expanding the fields in the structure and enforcing boundary conditions; of particular interest is the PEC boundary condition at the cylinder-cloak interface. However, simply minimizing the scattered fields at a single bistatic angle does not necessarily result in cloaking. A reduction in the scattered field could also be caused by absorption (loss) or deflection. Therefore, the scattered fields for every bistatic angle must be taken into account to ensure scattering reduction occurs for all observation angles. The cost functions in[14–16] attain angle independency essentially by averaging over all bistatic angles. While this is one way to render a cost function angle independent, accounting for every bistatic angle is computationally expensive which is only exacerbated during optimization. Additionally, these methods are based on a static stratification profile and therefore are not general.

In this paper, we propose an efficient cost function for an isotropic, n-layered cloaked cylinder. Both the TO and ray optics approach require a graded material parameter profile.[1,11] While fabrication of a material parameter gradient is possible, a discretely layered geometry simplifies the design and fabrication of a manufacturable cloak. The cost function we propose is derived from a Green's function of an n-layered coated cylinder and is not a function of angle (incident or observation). Green's function provides the unitary response of the system. This contains the response due to the source and the response due to the scatterer as observed at a given location. The benefit of using a Green's function is that the response due to the scatterer can be isolated. Using the scatterer response as the cost function ensures a cloaking mechanism for radar echo width (REW) reduction is required and therefore accounting for every bistatic angle is not necessary. This is because eliminating the response due to the scatterer within Green's function requires that an observer, regardless of position (outside the cloak), would only observe the response due to the source without any evidence of a scatterer.

With a general and efficient cost function, cloaked cylinders can be optimized over large sets of parameters with varied numbers of layers. To demonstrate this, we optimize the isotropic material parameters of 10, 20 and 30 equal thickness layer cloaks with a fixed outer radius. We also present results from optimizing a 10 layer cloak where the material parameters, individual layer thicknesses and outer radius of the cloak layer are optimized.

The results show that while the ideal anisotropic TO cloak material parameters for the TM and TE cases exhibit a dual nature (i.e. $\epsilon_Z^{TM} = \mu_Z^{TE}$) as evidenced by (4), the parameters for the optimized cloaks do not exhibit this type of behaviour. This indicates that the fundamental difference in TM and TE boundary conditions at the PEC cylinder must be taken into account to optimize non-ideal cloaked cylinders. The data also show that optimizing for layer thicknesses and material parameters is more effective than material parameters alone or additional layers. Throughout this paper we use the shortened TM to mean TMz ($\vec{E}$ is only z directed) and TE to mean TEz ($\vec{H}$ is only z directed) where z is along the height of the infinite cylinder.

2. Cost function development

We use Green's function for an n-layered cylinder presented in.[8] The interested reader may find the full derivation in McGuirk's dissertation.[17] The relevant geometry is shown in figure 1.

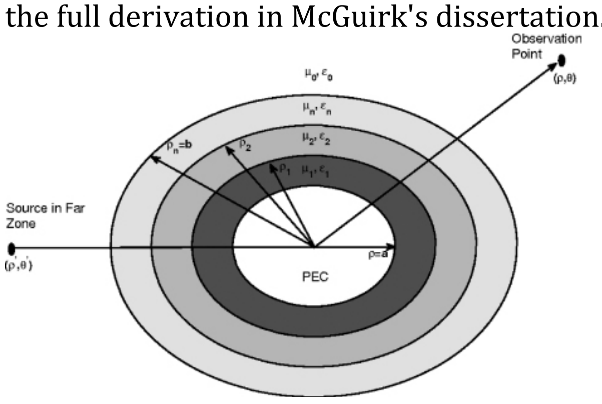

Figure 1. n-layered dielectric coated PEC cylinder geometry.

Green's function is given as

$$G = -\frac{j}{4}\sum_{\nu=0}^{\infty}\frac{\epsilon_\nu}{A_\nu^{n+1}}\cos[\nu(\theta-\theta')]\left[A_\nu^{n+1}J_\nu(k_i\rho) + B_\nu^{n+1}H_\nu^{(2)}(k_i\rho)\right]H_\nu^{(2)}(k_0\rho'), \tag{1}$$

$$A_\nu^{n+1} = 1,$$

$$B_\nu^1 = -A_\nu^1 K_\nu,$$

where ϵ_ν is the Neumann number, $K_\nu = J_\nu'\left(\frac{(k_1\alpha)}{H_\nu'^{(2)}(k_1\alpha)}\right)$ for TE incidence, $K_\nu = J_\nu\left(\frac{(k_1\alpha)}{H_\nu^{(2)}(k_1\alpha)}\right)$ for TM incidence and ' denotes differentiation with respect to the Bessel function argument. The A_ν^{n+1}, B_ν^{n+1} variables correspond with the free space region and are solved through enforcing tangential boundary conditions at the layer interfaces. This results in a system of equations which can be solved as a matrix equation[17] or algebraically.[18]

Green's function in (1) can be expressed as

$$G_{total} = G_{source} + G_{scattered} \tag{2}$$

where $G_{\text{scattered}}$ is the response due to the scatterer and $G_{source}$is the response due to the source. If we can make the contribution by the scatterer $G_{scattered} = 0$, then an observer will only observe the contribution of the source with no evidence of a scatterer and therefore the object is cloaked. The scattered contribution of Green's function can be written as

$$G_{scattered} = -\frac{j}{4}\sum_{\nu}^{\infty}\epsilon_\nu\cos[\nu(\theta-\theta')]\times B_\nu^{n+1}H_\nu^{(2)}(k_i\rho)H_\nu^{(2)}(k_0\rho') \tag{3}$$

From (3), it is apparent the coefficient B_ν^{n+1} must be minimized in order to create a cloak. Each mode must be minimized and not simply the summation of modes, therefore the cost function is $\sum_{\nu=0}^{\infty}|B_\nu^{n+1}|$. We use the same summation truncation criteria as in.[8]

To calculate the Bessel functions, the numerical method presented in[19] is implemented and can be used for the cases of complex mode and complex argument. Unlike the asymptotic methods for calculating the Bessel functions, this numerical method is valid everywhere and is more accurate.[19]

The Nelder–Mead simplex search method implemented in MATLAB is used to perform an unconstrained optimization of the material parameters assuming equal thickness layers in section 3.2. This method requires an initial guess and the results are dependent upon this guess. The TO effective medium approximated isotropic values serve as the initial guess.

In section 3.3, we use a constrained active-set optimization algorithm implemented in MATLAB to ensure valid layer thicknesses (i.e. $\rho_1 < \rho_2$). The initial guess in this section is made of the optimized results and geometry of section 3.2. Regardless of the number of parameters, the optimization algorithm for all test cases runs until the change in the cost function is less than 10^{-6} which is quite small considering the cost function for the bare PEC cylinder with TE incidence is

$$\sum_{\nu=0}^{\infty} |B_\nu^{n+1}| \approx 4.4$$

.

3. Results

In this section, the results for TE and TM incidence are presented. We compare the results of the simulated ideal anisotropic TO, 10 layer TO effective medium isotropic approximation and our optimized cloak. To verify our design method, we used COMSOL, a FEM-based commercial Computational ElectroMagnetic (CEM) code.

The discretization and quantization inherent in numerical simulation require the determination of a numerical 'noise floor' in assessing the relative performance of the various cloak designs. In particular, we need to establish the REW of the empty simulation space. In figure 2, we show the REW of the uncoated PEC cylinder, the ideal anisotropic TO cloak and the empty simulation. Since we want to focus on the cylinder simulations, further figures omit the empty simulation data.

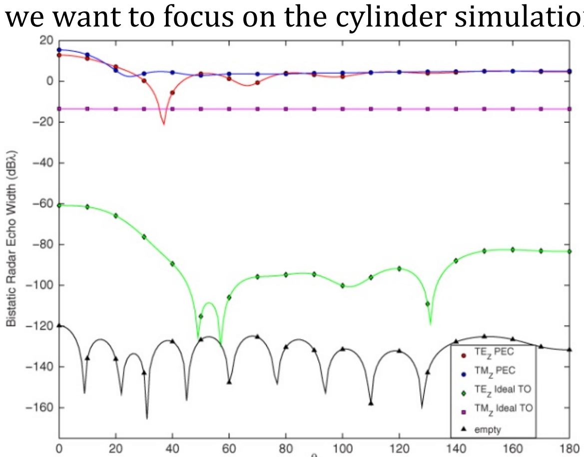

Figure 2. Simulation TEz and TMz REW comparison between uncoated PEC cylinder, the ideal TO cloaked cylinder and the empty simulation space.

At this point it is important to mention that, as was noted in,[5,20] the computer simulations of the ideal anisotropic TM and TE cloaks are less than ideal due to the discretization required and the material parameter singularities at the PEC-cloak interface. The simulated ideal TM and TE anisotropic cloaks implemented identical meshes with a maximum element length (MEL) of 0.05λ. Finer meshes were studied but rendered similar results. Even with identical meshes, the difference between the incident polarizations of the simulated ideal cloaks is stark. Not only does the simulated ideal TE cloak have much lower scattering than the TM case, but the shape of the REW is quite different. This could be due to the fact that waves are impinging the PEC in the simulation since the boundary condition at the PEC is polarization dependent. Additionally, TE and TM impingement upon a coated PEC cylinder excite very different azimuthal surface waves which do affect the REW.[21]

3.1. TO effective medium approximated isotropic cloak

Since the optimization algorithms in this section require an initial guess, we use the effective medium isotropic approximation of the ideal TO cloak. The ideal cloak uses the TO approach to define the spatially varying anisotropic material parameters as

$$\mu_\rho = \frac{\rho - \alpha}{\rho} \mu_\theta = \frac{\rho}{\rho - \alpha} \mu_z = \frac{\rho - \alpha}{\rho} \left(\frac{b}{b - \alpha} \right)^2 ,$$
$$\epsilon_\rho = \frac{\rho - \alpha}{\rho} \epsilon_\theta = \frac{\rho}{\rho - \alpha} \epsilon_z = \frac{\rho - \alpha}{\rho} \left(\frac{b}{b - \alpha} \right)^2 , \tag{4}$$

where a is the radius of the PEC cylinder and b is the outer radius of the cloak layer.[7] We can model the TO anisotropic cloak as a series of isotropic layers using effective medium theory for the TE case with

$$\epsilon_\theta = \frac{\epsilon_1 + \eta \epsilon_2}{1 + n}$$
$$\frac{1}{\epsilon_\rho} = \frac{1}{1 + \eta} \left(\frac{1}{\epsilon_1} + \frac{\eta}{\epsilon_2} \right) \tag{5}$$

$$\eta = \frac{t_2}{t_1}$$

where t_1, t_2 are the layer thicknesses.[22] For the TM case, replace ϵ with μ.

3.2. Optimizing material parameters

In this section, the PEC cylinder radius is $a = 1\lambda$. The outer radius of the entire structure is $b = 2\lambda$; all of the layers are of equal thickness. The REW simulation data can be seen in figures 3 and 4. It is evident that optimizing a cloak of larger numbers of layers does not change the REW appreciably. We also note that the 10 layer optimized TM cloak can achieve better cloaking performance than the simulated TM ideal anisotropic TO cloak, but the same is not true for the TE case. This will be discussed further in section 3.4. The final cost function values can be seen in table 1.

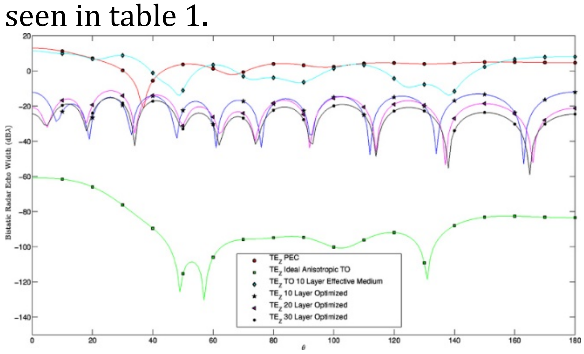

Figure 3. REW comparison with TEz incidence.

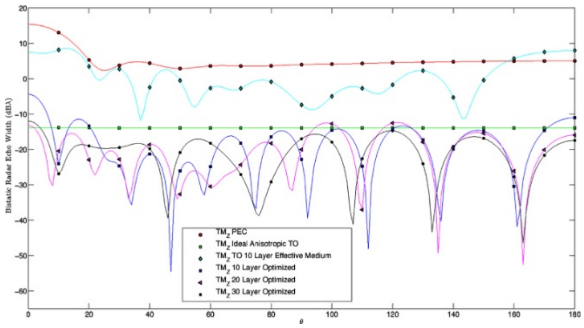

Figure 4. REW comparison with TMz incidence.

Table 1. Final cost function values.

	TE	TM
PEC	4.4361	5.1329
10 layer effective medium	4.5617	3.5023
10 layer optimized	0.3908	0.3898
20 layer optimized	0.2284	0.2571
30 layer optimized	0.1455	0.1732

3.3. Optimizing material parameters and layer thicknesses

In this next test case, the effects of optimizing material parameters in conjunction with layer thicknesses are studied. The PEC cylinder radius is maintained at $a = 1\lambda$ and the individual layer material parameters and thicknesses are optimized. The sum of the layer thicknesses dictates the outer radius, b. The initial guess for the test structures in this section is made of the 10 layer geometry and optimized material parameters of section 3.2. The final cost function values for this test case are 0.0093 and 0.024 for TE and TM incidence, respectively.

The optimization results in tables 2 and 3 show that the geometry and material parameters for optimal cloaking are quite different between the TE and TM cases. The material values in the previous section were omitted for brevity but also exhibited this difference between incident polarization. The simulation field patterns can be seen in figure 5 and the corresponding REW plots can be seen in figures 6 and 7. These results indicate that optimizing both material parameters and layer thicknesses renders more effective cloaks than through just material parameters alone.

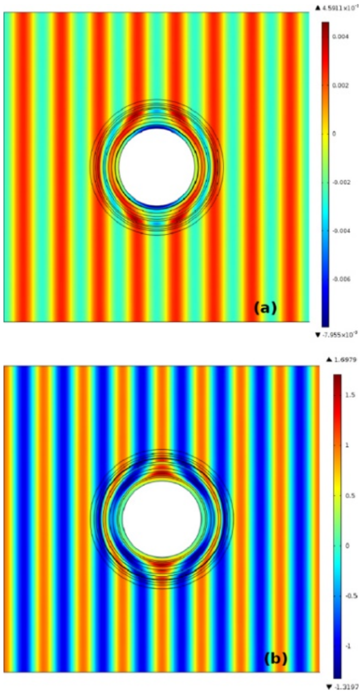

Figure 5. $\grave{z}$ directed fields for (a) TEz and (b) TMz cloaks with optimal material parameters and layer thicknesses. Power flow is from left to right.

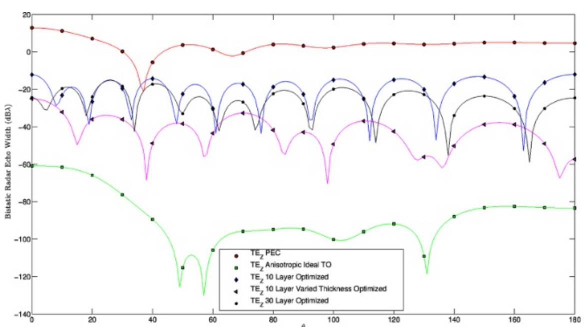

Figure 6. REW Comparison between fixed geometry optimized cloaks and a 10 layer cloak with optimized material parameters and geometry for TEz incidence.

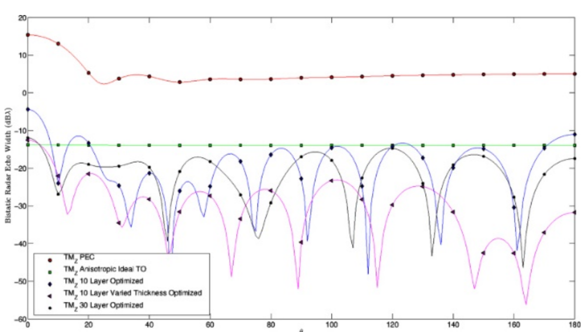

Figure 7. REW Comparison between fixed geometry optimized cloaks and a 10 layer cloak with optimized material parameters and geometry for TEz incidence.

Table 2. Optimized material parameters and layer thicknesses for a 10 layer isotropic cloak with TEz incidence and fixed inner radius $a = 1\lambda$. The outer radius is $b = \rho_{10} = 1.6492\lambda$.

Layer	ϵ_r	μ_r	$\rho(\lambda)$

1	13.2151	0.3580	1.0809
2	0.0165	0.5227	1.0948
3	7.6057	0.5209	1.1826
4	0.1134	2.0364	1.2721
5	5.2840	1.6437	1.3499
6	0.1119	1.7774	1.3973
7	5.3018	1.6098	1.5120
8	0.1134	1.5120	1.5434
9	3.4234	2.0896	1.6009
10	0.5826	0.8274	1.6492

Table 3. Optimized material parameters and layer thicknesses for a 10 layer isotropic cloak with TMz incidence and fixed inner radius $a = 1\lambda$. The outer radius is $b = \rho_{10} = 1.7166\lambda$.

Layer	ϵ_r	μ_r	$\rho(\lambda)$
1	1.0028	6.6475	1.1002
2	1.1884	0.0226	1.1392
3	0.9212	8.6933	1.2481
4	0.9577	0.1493	1.3287
5	1.6500	4.3149	1.4287
6	2.7566	0.0745	1.4908
7	3.0754	4.2868	1.5786
8	1.2093	0.0410	1.5941
9	2.9656	3.1757	1.6625
10	0.2619	0.6178	1.7166

This is expected when we examine the form of the continuity boundary conditions that are used to solve for the coefficients of Green's function.[17,18] For convenience, the expressions that enforce continuity of the tangential fields between the second and third layer can be seen in (6).

$$A_v^2 jJ_v(k_2\rho_2) + B_v^2 H_v^{(2)}(k_2\rho_2) = A_v^3 J_v(k_3\rho_2) + B_v^3 H_v^{(2)}(k_3\rho_2),$$
$$\frac{\kappa_3}{k_2}\left(A_v^2 J_v'(k_2\rho_2) + B_v^2 H_v'^{(2)}(k_2\rho_2)\right) = \frac{\kappa_2}{k_3}\left(A_v^3 J_v'(k_3\rho_2) + B_v^3 H_v'^{(2)}(k_3\rho_2)\right).$$
$$\kappa = \begin{cases} \epsilon_r = \dfrac{\epsilon}{\epsilon_0}\ TE, \\ \mu_r = \dfrac{\mu}{\mu_0}\ TM. \end{cases} \tag{6}$$

When only the material parameters are optimized, we are limited to only a portion of the Bessel function argument and along with the term $\frac{\kappa_{i+1}}{k_i}$. However, when layer thickness is added as a free variable, the Bessel function argument can be manipulated somewhat independently to the term $\frac{\kappa_{i+1}}{k_i}$. It is this additional fine tuning to the Bessel function argument that results in more optimal cloaks.

3.4. Further discussion

A few observations can be made from the results of the optimized cloaks. First, as noted before, the optimized material parameters do not exhibit the dual nature that is dictated by TO cloak design. This is due to the fact that the cost function takes into account the difference between TM and TE impingement upon a PEC cylinder. When the cost function is minimized, the interaction with the PEC boundary is used in conjunction with the cloak layers and material parameters to provide all angle reduction of REW. In contrast, the ideal TO cloak operates purely through the graded material parameters due to its foundation in a coordinate transform.

Lastly, it is noted that the TE cloaks are more effective than the TM cloaks for most of the cases simulated in this paper. This is probably due to the difference in boundary conditions at the PEC cylinder. It stands to reason that as the main contribution of scattering is minimized with the cost function that the excited azimuthally travelling surface waves will have a greater effect on the REW. It has been shown that these excited surface waves do affect the REW of a coated cylinder.[21] The propagation constants of the excited surface waves correspond to the poles of our cost function.[18] Furthermore, the propagation constants of excited azimuthal surface waves are highly dependent upon the stratification profile, material parameters and incident polarization.[23–26] As such, further study of these waves may provide more insight into the scattering of non-ideal cloaks. It may be possible that an optimization process which takes this into account could surpass the results presented here.

3.5. Design considerations

This optimization approach could be used in two ways to design a manufacturable cloak. The first method is to use an optimization algorithm to dictate the optimal material parameters then attempt to find the best materials to fit these requirements. Metamaterials could be used to fill these requirements since they allow some degree of material parameter tailoring based on the metamaterial geometries. Although, if metamaterials are used, the thickness of the individual layers will need to be designed in such a way as not to violate the size constraints.[27]
In contrast, an alternative method is to begin with a discrete set of material parameters that can be manufactured. In this situation, a constrained optimization algorithm can be used to search this discrete set for an optimal solution. This approach was used in Paul's dissertation where the discrete set of manufacturable material parameters was generated by different metamaterial cells.[28] In this method, the optimal solution is guaranteed to be manufacturable. However, there is no guarantee that the optimal solution will actually cloak. It stands to reason that the larger and more diverse a discrete set, the more likely an effective cloak design can be found.

4. Conclusion

In this paper, we presented an efficient cost function for a cloaked cylinder which is general to any stratification profile. With an efficient cost function we can optimize large sets of parameters. To demonstrate this, we optimized multiple test cases where 20 to 60 material and geometric parameters of a cloaked cylinder were optimized. Optimized cloaked cylinders made of 10, 20 and 30 isotropic equal thickness layers were presented along with a 10 layer case in which the material parameters and layer thicknesses were optimized. The data show that optimizing for material parameters and layer thicknesses improves the cloaking effectiveness significantly as compared to optimizing material parameters alone. The results also indicate that the inherent difference between TM and TE incidence plays a role in the performance of these non-ideal cloaks.

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