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Corrigendum to "Taxonomies of Model-theoretically Defined Topological Properties"

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CORRIGENDUM TO "TAXONOMIES OF MODEL-THEORETICALLY DEFINED TOPOLOGICAL PROPERTIES"

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Abstract. An error has been found in the cited paper; namely, Theorem 3.1 is false.

1. I would like to correct a simple, but serious, error in [1]; namely Theorem 3.1 therein is quite false: It *can* happen that there are compact Hausdorff spaces X and Y with $X \equiv Y$ (indeed $X \doteqdot Y$) but $X \not\equiv_{\mathbf{T}_t} Y$. I am most grateful to Lutz Heindorf for communicating [3] the following straightforward example: Let X and Y be any two Boolean spaces with infinite dense sets of isolated points. Then $B(X)$ and $B(Y)$ are Wallman bases for X and Y respectively, are infinite atomic Boolean algebras, and hence, by the Tarski invariants theorem, are elementarily equivalent. Thus $X \doteqdot Y$. However, one can easily pick X and Y as above so that $X \not\equiv_{\mathbf{T}_t} Y$; e.g., let X and Y be the ordinal spaces $\omega + 1$ and $\omega^2 + 1$ respectively. Then Y has a point of Cantor-Bendixson derivative 2, while X does not. This fact can be expressed in a sentence of Φ_t.

The faulty inference in the proof of Theorem 3.1 of [1] occurs in the penultimate sentence: If W and Z are two Tichonov spaces with Wallman bases that are lattice-isomorphic, it does *not* generally follow that W and Z are homeomorphic. (We could make the inference if either W and Z were both compact or the Wallman bases contained all the singletons, but in our case W and Z are topological ultrapowers and neither condition holds.)

2. In Professor Heindorf's communication [3], there were some further interesting facts that enrich the content of [1].

2.1. The 3-cell $\mathscr{I}^3$ is characterized by $\mathbf{T}_F$ in {metrizable} [2]. (This augments Theorem 1.2 in [1].)

2.2. There is a complete description of the spaces that are (finitely) characterized by certain taxonomies in {metrizable Boolean}. Let $\mathscr{R}$ be the class of R. S. Pierce's "compact 0-dimensional metric spaces of finite type" [7].

THEOREM [5]. *For any metrizable Boolean space X, the following are equivalent:*
(i) $X \in \mathscr{R}$.

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(ii) *X is finitely characterized by* $\mathbf{T}_F$ *in* {*metrizable Boolean*}.

(iii) *X is finitely characterized by* $\mathbf{T}_t$ *in* {*metrizable Boolean*}. ∎

(This result addresses issue (**I**2) in [1].)

2.3. THEOREM [6]. *There are* c $\mathbf{T}_t$*-taxa (hence* c $\mathbf{T}_F$*-taxa) in* {*metrizable Boolean*}. ∎

(This result addresses issue (**I**3) in [1], and answers a question raised in the penultimate paragraph on p. 592 therein. See also the paragraphs following the proof of Theorem 2.10.)

2.4. THEOREM [4]. {*metrizable Boolean*} *is dense in* {*Boolean*}, *relative to* $\mathbf{T}_t$. ∎

(This result addresses issue (**I**6) in [1].)

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