Learning About Modeling in Teacher Preparation Programs

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This study explores opportunities that secondary mathematics teacher preparation programs provide to learn about modeling in algebra. Forty-eight course instructors and ten focus groups at five universities were interviewed to answer questions related to modeling. With the analysis of the interview transcripts and related course materials, we found few opportunities for PSTs to engage with the full modeling cycle. Examples of opportunities to learn about algebraic modeling and the participants’ perspectives on the opportunities can contribute to the study of modeling and algebra in teacher education.

Keywords: Modeling, Teacher Education-Preservice, Algebra and Algebraic Thinking

Mathematical modeling is a critical component in school mathematics as it supports students in developing a way of thinking and interacting with the world that will be necessary later in life. Through modeling, students develop skills necessary to successfully make sense of and interact with complex mathematical systems (Lesh & Doerr, 2003). To teach mathematical modeling, teachers must learn new ways of thinking about and interacting with mathematics. Anhalt and Cortez (2015) argued, “Mathematical modeling problems present unique challenges for teachers, who are not typically required to take courses on modeling as part of their preparation” (p. 2).

In response to new guidance from Common Core State Standards for Mathematics (CCSSM) related to modeling (National Governor’s Association Center for Best Practices [NGA] & Council of Chief State School Officers [CCSSO], 2010), there is a need for research exploring whether and how teacher preparation programs have made changes in their curricula to prepare secondary pre-service teachers (PSTs) to learn about modeling based on CCSSM. CCSSM included mathematical modeling as a mathematical practice across preK-12 grades and as a mathematical content area for High School (NGA & CCSSO, 2010). The CCSSM modeling cycle is described as a complex, iterative process in which assumptions are made, tested mathematically, interpreted and validated, and then potentially revised multiple times. Beginning teachers should be familiar with all steps of the mathematical modeling process as described in the CCSSM, should have had opportunities to develop their own expertise, and should be aware of the importance of meta-cognitive reflection throughout the modeling process. We report findings from research focused around modeling, conducted as part of a collaborative NSF research project, Preparing to Teach Algebra, which investigated opportunities secondary mathematics teacher preparation programs provided to learn about algebra. In this paper, we aim to answer a question, “What opportunities do secondary mathematics teacher preparation programs provide to learn about algebra related to the modeling standards described in CCSSM?”

Methods

To investigate PSTs’ opportunities to learn algebra and modeling, we conducted interviews with secondary PSTs and instructors of required courses, and collected instructional materials (e.g., syllabi, project descriptions) used in the courses at five universities. We call these universities: Great Lakes University (GLU), Midwestern Research University (MRU), Midwestern Urban University (MUU), Southeastern Research University (SRU), and Western Urban University (WUU). Different
from other universities, WUU only admits PSTs who have already completed a Bachelor’s degree in mathematics.

Among all the courses required in the secondary mathematics preparation program at the universities, we selected mathematics courses that include algebra content (e.g., Linear Algebra, Geometry, Statistics and Probability). If the teacher education program required mathematics for teachers courses (e.g., Algebra for Teachers, Geometry for Teachers), we interviewed an instructor from each course. We also interviewed an instructor from each of the mathematics education courses (e.g., Secondary Mathematics Methods) and general education courses (e.g., Teaching in a Diverse Society) that may include opportunity to learn about equity in algebra. Table 1 summarizes the course types and the number of instructors interviewed at each university.

<table>
<thead>
<tr>
<th>Course Type</th>
<th>GLU</th>
<th>MRU</th>
<th>MUU</th>
<th>SRU</th>
<th>WUU</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mathematics (M)</td>
<td>5</td>
<td>4</td>
<td>6</td>
<td>5</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Mathematics for Teachers (MftT)</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>3</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>Mathematics Education (ME)</td>
<td>3</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>1</td>
<td>16</td>
</tr>
<tr>
<td>General Education (GE)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>6</td>
</tr>
</tbody>
</table>

Additionally, two groups of 3-4 PSTs from each university were interviewed. We designed interview protocols in parallel for instructors and focus groups. We asked instructors and students about modeling generally first. Then we asked specifically about modeling process as described in CCSSM. In addition to instructor interviews, we collected corresponding instructional materials.

To analyze the interview transcripts and course materials, we developed six codes, named for each element of modeling process described in CCSSM: a) identifying and selecting variables, b) formulating a model, c) analyzing and performing operations, d) interpreting the results, e) validating the conclusions, and f) reporting on the conclusions. Two pairs of us independently coded each transcript based on these nodes using NVivo 10 and we then compared our coding and resolved any discrepancies.

Findings

We address our research question by describing how each group of participants responded to the interview questions related to the opportunities for PSTs to learn about modeling in algebra. The number in Table 2 represents the number of opportunities (e.g., tasks, discussions, lecture, problem) that instructors or PSTs reported related to each element of modeling process at the five institutions. Note that no General Education instructors at any university described opportunities to learn modeling, so we do not include them below.

Mathematics Instructors. Mathematics instructors reported several opportunities for PSTs to learn about “formulating a model by creating and selecting appropriate representations” and “validating the conclusions.” One example of “formulating a model” is presented by Probability and Statistics instructor from GLU: “like my take-home final, they were doing a lot of this [formulating a model] because they had data, and they had to analyze it.” In terms of “validating the conclusions,” Linear Algebra instructor at GLU reported that he provided a unique solution to a traffic flow problem that led PSTs to validate conclusions. Other instructors described how PSTs identified variables, performed operations, and interpreted the results, but no one reported an example of “reporting on the conclusions.”
Table 2: Opportunities to learn about modeling in different types of courses

<table>
<thead>
<tr>
<th>Elements of Modeling Process</th>
<th>M (20)</th>
<th>MfT (6)</th>
<th>ME (16)</th>
<th>GE (6)</th>
<th>FG (10)</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Identifying and selecting variables</td>
<td>2</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>11</td>
</tr>
<tr>
<td>b) Formulating a model</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>c) Analyzing and performing operations</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>7</td>
</tr>
<tr>
<td>d) Interpreting the results</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>0</td>
<td>7</td>
<td>13</td>
</tr>
<tr>
<td>e) Validating the conclusions,</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>0</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>f) Reporting on the conclusions</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>TOTAL</td>
<td>11</td>
<td>18</td>
<td>21</td>
<td>0</td>
<td>24</td>
<td>74</td>
</tr>
</tbody>
</table>

Note. Number of instructors (or number of focus groups) interviewed in parentheses.

Mathematics for Teachers Instructors. Mathematics for Teachers instructors reported a number of opportunities even though they were few (6 of 48 instructors). One example illustrates how they asked PSTs to engage with modeling and to use the activity for teaching their own students: PSTs identified “functions to fit the data given movie money data” and considered “how to use this activity in future teaching.”

These instructors also provided more opportunities related to the modeling process “identify and select variable” compared to the other participants. A following task introduced by Secondary Math Connection 1 instructor at SRU presented such modeling process:

Ana is sitting in the bucket of a Ferris wheel. She is exactly 46.7 feet from the center and is at the 3 o’clock position as the Ferris wheel starts turning. Sketch graphs that represents Ana’s different location on the Ferris wheel.

We see in this task that variables were not given in the problem statement. When PSTs sketched graphs, they decided which quantity would be labeled as independent or dependent variables.

One modeling process reported by not Mathematics instructors, but Mathematics for Teachers instructors instead, was “reporting on the conclusions.” A Secondary Mathematics Connections 1 instructor at SRU said, “validating conclusions, reporting on conclusions, if the mindset’s done, then this all gets wrapped up into one. You’re always interpreting, validating and reporting within that process.” This instructor connected several modeling process: interpreting, validating, and reporting the results.

Mathematics Education Instructors. As a group, Mathematics Education instructors provided more opportunities related to “validating the conclusions” than the other instructors. An activity provided by the Secondary Mathematics Methods instructor from WUU shows that PSTs had the opportunity to compare their answers with others during the modeling process. Mathematics Education instructors also provided several opportunities for PSTs to formulate a model. A Secondary Math Methods instructor at MUU described PSTs opportunity to discuss the meaning of variables in context and generate a model using representations when solving for the number of border tiles in a pool. The discussion led by this instructor might help PSTs make connections between the real-world situation (e.g., border tiles for a pool) and mathematics.

Focus Groups. While instructors most often reported opportunities to “formulating a model by creating and selecting appropriate representations,” PSTs most often described the element, “interpreting the results.” Specifically, they described a process of interpreting the results in the given context: “it’s a lot of word problems to where when you’re given an equation, it’s not just okay x=3 it’s like, in the problem we wrote this equation to model this situation. What does that variable mean?” [Focus Group from SRU]. As described, PSTs considered the meaning of each variable in
the equation that represents the problem context. PSTs also mentioned that it would be helpful to have more time validating and refining their process of solving a problem. A PST from MRU said,

I feel like there is more of a “This is what you need, get really good at this skill” but there’s not like a “Explore it and refine what you've seen.”

This example shows that PSTs considered the value of exploring the problem on their own and refine the solutions, but they thought such opportunity was not provided much in their program.

Discussion and Conclusions

Findings show that both instructors and PSTs reported examples of opportunities that addressed few elements of modeling process. More opportunities to learn about modeling might be revealed from teacher education programs if our study is extended to mathematical modeling, rather than algebraic modeling. Also, if we had observed classroom practice, we may have been able to find more opportunities addressing the full modeling cycle. Nonetheless, the findings from this study show several opportunities for PSTs to learn about algebraic modeling, but also show that not all opportunities involved the full modeling cycle recommended by CCSSM. This result shows that PSTs need more opportunities to explore the complete modeling process (Anhalt & Cortez, 2015; Pollack & Garfunkel, 2013).

Certain modeling elements were emphasized by PSTs and instructors. PSTs reported the value of validating and refining the process of solving a modeling problem. Some PSTs remembered how a mathematics education instructor emphasized reporting and justifying answers throughout the entire modeling process. A mathematics for teachers course instructor also provided an example of reporting conclusions, saying that this process can be connected with interpreting and validating the results. Despite interviewing more mathematics instructors (20 instructors) than mathematics education instructors (16 instructors), more opportunities to learn about modeling described in CCSSM were reported by mathematics education instructors. Mathematics instructors, who did not provide any example related to the last element of modeling process (reporting on the conclusions), need to consider specific ways of helping PSTs report conclusions throughout modeling process.

Overall, this study incorporates the large data including interview transcripts of 48 course instructors and 10 focus groups from five universities, and corresponding course materials, which enabled us to present the overview of learning opportunities reported by instructors and future teachers. We also presented the extent and element of modeling process addressed in different courses and found that few opportunities involving the entire modeling cycle. This result supports the need for future teachers to learn about more modeling activities that address all the modeling processes.

References


