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# Aristotle, The Pythagoreans, and Structural Realism

Owen Goldin

Marquette University, [owen.goldin@marquette.edu](mailto:owen.goldin@marquette.edu)

# Aristotle, the Pythagoreans, and Structural Realism

Owen Goldin

*Department of Philosophy, Marquette University  
Milwaukee, WI*

## **Abstract**

[...]iron has to have the feature of rigidity for it to serve as the subject of a saw,"" and the human tissues must have the features that they have, if they are to be the subject of a human form. [...]the matter of a saw cannot be too soft, but, at least for certain varieties of sawing, it cannot be too hard either.

## **I**

Two basic questions of physics are: what is the world made of, and why do these constituents do the things they do? These two questions are closely related. If certain kinds of things are the ultimate constituents of the world, it can only be because their characteristics explain what we observe to be the case. Further, if it were possible to grasp laws or necessary truths that explain absolutely everything, there would be no need to appeal to any kind that underlies those truths or laws; such a move would do no theoretical work.

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Nonetheless, in the West most scientists have offered explanations that appeal to both what the basic things or stuff are, and the fundamental features they possess or laws they obey.

This approach goes back to Aristotle, who insisted that explanation in physics requires identifying both a material substrate and a formal basis for what is to be explained. He argued that this holds in regard to explaining both the existence of substances and the fact that they bear certain attributes. In order to explain both why a substance exists and why a substance does what it does, one must appeal to its essence, as expressed in a definition that includes both matter and form.<sup>1</sup> An event or attribute (such as anger) is to be accounted for both by pointing to the persisting substrate (the blood around the heart) and the form it takes on (a kind of boiling that is the result of perceived anger).<sup>2</sup>

*Metaphysics* 1 tells the story of how, in fits and starts and to varying degrees, earlier thinkers came to realize that explanation demands identifying all four causes, including both the material and formal causes. Aristotle's earliest philosophical predecessors, the Milesians, are credited with offering explanations on the basis of matter.<sup>3</sup> Aristotle criticizes their accounts as radically incomplete on the grounds that matter alone cannot account for all things and their characteristics. For example, it cannot account for goodness or beauty.<sup>4</sup>

While the Milesians are said to have identified certain kinds of stuff as basic, Aristotle takes other predecessors, the so-called Pythagoreans, to have done the same for number. Aristotle associates them with the Milesians, insofar as he understands them to give at least some numbers the status of matter. The explanatory strategy of explaining derivative kinds on the basis of the characteristics of the basic kinds is the same; their dispute with the Milesians concerns what basic characteristics are most explanatory.<sup>5</sup> Aristotle is dismissive of the Pythagorean notion of numbers that are not quantities of substances: "[A] number, whatever it is, is always a number of certain things, either of fire or earth or of units."<sup>6</sup> Aristotle dismisses the ontology of the Pythagoreans by indicating the confusion of positing as basic what must inhere in a substrate. Perhaps it is a similar reaction to the notion of a number ontology that has led some contemporary

scholars<sup>7</sup> to deny that Philolaus (Aristotle's likely source for Pythagorean number ontology) could have possibly thought that all things are made of number.

Aristotle takes the Milesian explanatory strategy to be one of accounting for things on the basis of their being made up of certain stuffs, which, if not identical with the perceived constituents of familiar objects, are at least conceivable along the same lines. Today there are few, if any, neo-Milesians. As Planck wrote: "the physical world has become progressively more and more abstract; purely formal mathematical operations play a growing part, while qualitative differences tend to be explained more and more by means of quantitative differences."<sup>8</sup> Mathematical features are formal. Nonetheless, Planck himself did not follow the path of Aristotle's Pythagoreans. He felt compelled to posit a kind of substrate to physical reality, even though he cautioned that imagining that substrate as like the stuffs or particles familiar from sensation can be highly misleading.

In contrast, the Pythagorean strategy has new adherents, and not only because, in this physics, "purely formal mathematical operations play a growing part." Within recent years, a number of philosophers and computer scientists have suggested a kind of return to an ontology not too different from that of the Pythagoreans as described by Aristotle. This view has been called "structural realism": "realism" because, on this account, science describes the world as it really is; "structural" because the realities identified are not kinds of stuff or objects, but structures.

For this reason, it is time to again take up the Aristotelian objection, that such an ontology is incoherent since any explanation of physical reality must be implicitly committed to a material substrate. I do so in the present paper, first by reconsidering the evidence, in order to become clear on exactly what Aristotle's objection is, before then showing how contemporary structural realists posit an ontology much like that of Aristotle's "Pythagoreans." Both take the objects of knowledge to be structure, not what is structured. I discuss both how pancomputationalists such as Edward Fredkin approach the Pythagorean account insofar as on their account all reality can in principle be expressed as one (very big) number, made up of discrete units, and how even more moderate varieties of structural realism, like

that of Floridi, share with pancomputationalism the aspect of Pythagorean ontology that Aristotle finds so objectionable: positing structure or form with no substrate. I conclude by arguing that Aristotle himself is drawn to something close or identical to a structural realist ontology in book 7 of the *Metaphysics*. He himself comes to see that those aspects of the world that are real, and as such intelligible, are formal. He would agree with Saunders, who writes, "I believe that objects are structures; I see no reason to suppose that there are ultimate constituents of the world, which are not themselves to be understood in structural terms. So far as I am concerned, it is turtles all the way down."<sup>9</sup> Such an account confirms the main lines of Pythagorean ontology.

## II

Aristotle discusses several ontologies which posit mathematical entities as basic. Much of books 13 and 14 of the *Metaphysics* is devoted to what Aristotle takes to be the incoherence of the ontology of Plato and other members of the Academy who posited quantities as principles separate from the things of which they are principles.<sup>10</sup> *Metaphysics* 1.5 focuses on the distinctly mathematical character of the ontology of thinkers who are referred to as "the Pythagoreans." Aristotle writes:

[T]he so-called Pythagoreans who were the first to put their hand to mathematics both advanced it and, having been brought up with it, thought that its principles are the principles of everything. Since among such principles, numbers are by nature primary, and they thought that they saw among them many "resemblances" (ὁμοιώματα), among beings and among events (οὐσι καὶ γιγνομένοις), for than one does in fire and air and water—for example, the attribute (πάθος) of number in this thing here is justice and in that, soul and intellect, and something else is good timing (καιρός) and, so to speak, this is how things are for each of the other things—and further, since they saw in numbers attributes and ratios of scales (τῶν ἁρμονιῶν) — since, then, other things seemed to have their entire nature modeled on numbers, and since, of all nature, numbers are primary, they took the elements of numbers to be the elements of all things, and the whole heaven to be a scale and a number."<sup>11</sup>

The context is a review of the views of Aristotle's predecessors concerning the principles or sources of all things.<sup>12</sup> Who does Aristotle have in mind here, and does he understand them correctly? Some earlier theorists had an intense interest in mathematical research, and investigated correspondences between numbers and other entities, but there is no evidence that they built on their research in order to reach general ontological or cosmological conclusions.<sup>13</sup> It was Philolaus who was the first to employ mathematics to further the sorts of investigations into the constitution of the cosmos that were first pursued by the Milesians.<sup>14</sup> So, Aristotle is generally thought to have here had Philolaus in mind, and to have interpreted him as positing number as constitutive of things. Huffman has recently argued that this is not supported by the primary evidence, fragment 4: "And indeed all things that are known have number. For it is not possible that anything whatsoever be understood or known without this." Huffman argues that this direct evidence supports attributing to Philolaus only the more modest claim that things are *known* by means of number.<sup>15</sup> But Aristotle interprets them as presenting a mathematical ontology,<sup>16</sup> and then proceeds to argue against the *Metaphysics* physical cogency of that reconstruction.

Aristotle relates that Pythagorean speculation concerning the mathematical basis of reality has its origin in how mathematical objects and attributes manifest themselves outside the realm of the mathematical as such, as resemblances to mathematical entities. What are these resemblances? Elsewhere Aristotle tells us that Eurytus determined the number of a horse by seeing how many pebbles need to be assembled in order to describe its shape;<sup>17</sup> perhaps the Pythagoreans also had in mind how the sums, differences, and the like of numbers considered as such (that is, not as the numbers of *things*) are reflected in the sums, differences, and so forth of common objects like apples. The mathematical characteristics that are basic are seen in the phenomenal characteristics of what they cause. However, the one example that Aristotle here presents, how certain ratios (*logoi*) are manifest in the musical scale<sup>18</sup> does not easily conform to this model. At *Metaphysics* 14.3.1090a23-25 Aristotle says that in this and similar cases the attributes (*pathē*) of number are present in the derivative things. The notes of the scale, however, or the character of harmonies do not resemble the underlying mathematical characteristics in any overt way; there is nothing in the phenomenological characteristics of

hearing an octave that suggests the ratio of a double. Perhaps Aristotle is assuming that any intelligible form of causality involves the transmission or manifestation of the cause in the effect, even in cases in which the nature of this connection is not immediately apparent.<sup>19</sup> Alternatively, or in addition, Aristotle may have in mind the mode of causal explanation employed by other Pythagoreans, that of the Table of Opposites, according to which items in one column are thought to carry with them other items in the same column.<sup>20</sup> Perhaps items in the same column are thereby thought to resemble each other. I suspect that the term "resemblance" is not Aristotle's own, and that he is at a loss to clearly describe the mode of causality to which the Pythagoreans are appealing. For Aristotle speculates that the Pythagoreans took numerical relationships and attributes to be material causes of nonnumerical relations and attributes.<sup>21</sup>

Aristotle does not say that these Pythagoreans identified numbers as the elements of things; rather, he says that, on this account, it is the elements of numbers that are the elements of things, including the totality of things, which Aristotle refers to as the whole heaven. We may speculate that according to Aristotle's reconstruction, these elements would serve as the ultimate material cause of all things, and numbers would serve as a higher level of matter. The elements of number are said to be the even and the odd, which are in turn associated with the unlimited and the limit.<sup>22</sup> The same passage coordinates them with the limit and the unlimited, which are at the very least somehow associated with the principles of limiters and unlimiteds posited by Philolaus, but Aristotle does not spend a great deal of time worrying about the ontological status of an independently existing odd or even. For from an Aristotelian perspective, the notion of something being even that is not a specific even number is no more perplexing than the notion of a number that is not the number of a collection of nonquantitative units.

Aristotle's fundamental objection to the Pythagorean account concerns the order of ontological dependence, as laid out in the *Categories*. Positing numbers as a substrate reverses the relation of dependence between number and that which is numbered. Aristotle directs his objection to attempts, like that of Plato's *Timaeus*, to construct bodies out of geometrical simples. "And, in general, conclusions contrary alike to the truth and to the usual views follow, if

one supposes the objects of mathematics to exist thus as separate entities. For if they exist thus they must be prior to sensible spatial magnitudes, but in truth they must be posterior; for the incomplete spatial magnitude is in the order of generation prior, but in the order of substance posterior, as the lifeless is to the living."<sup>23</sup> Aristotle asserts that his argument also bears on attempts, such as that imputed to the Pythagoreans, to generate bodies out of numbers.<sup>24</sup> But are not quantitative terms present in the definitions of substances (as when an animal is defined as two-footed)? Aristotle would agree with Philolaus's assertion in fragment 4, that numbers are principles of our knowledge of things, but would insist that one need not be misled by the presence of quantitative terms in the *logos* by which a substance is defined, for priority in *logos* is not equivalent to ontological priority.

"For the things that are prior in regard to substance are those that, taken by themselves, exceed in regard to being (τῆ μὲν γὰρ οὐσία πρότερα ὅσα χωριζόμενα τῷ εἶναι ὑπερβάλλει), but those that are prior in regard to *logos* are those that are prior to those whose *logoi* come from their *logoi*, and these are not coextensive. For if there are not attributes, such as a "moving" or "pale," apart from substance, then pale is prior to pale man in respect to *logos*, but not in respect to substance."<sup>24</sup>

As in the *Categories*, the ontological primacy of substance over nonsubstances is asserted but not argued for; an example is meant to suffice. A quantity, such as a number, is an attribute and as such can be independent of the numbered no more than a quality, like pale, can be independent of that which is qualified. What is not a substance requires a substance in which to inhere, while a substance does not require that which is not substantial.<sup>25</sup> Aristotle's account is problematic: though one might grant that there cannot be a "red" which is not the red of an object, one can similarly say of many substances that they cannot exist without being of some color or another (red, blue, or another).<sup>26</sup> The fundamental difference between substances and nonsubstances lies in what Aristotle in the *Categories* identifies as a unique feature of a primary substance: as a substrate, it remains the same as it undergoes change.<sup>27</sup> The same apple can turn from green to red, but the same red cannot turn from an apple to a pear. In regard to quantitative features, among which numbers are found, the same substance can change in respect to quantity, but the



same quantity cannot change in respect to the substrate of which it is a quantity. This is what it means to "exceed in being."<sup>28</sup>

The Pythagoreans are positing numbers, or the elements of numbers, as the fundamental beings or elements out of which all things are constituted. Numbers, or their elements, play the role in Pythagorean *Metaphysics* that substances play in Aristotelian *Metaphysics*: they are the ultimate substrate of things. The Aristotelian response here is a bald assertion that this cannot be so. The Pythagoreans need not be without rejoinder. Aristotle himself grants that there can be substantial change while a nonsubstantial feature remains the same. One stuff can become another (as wine becomes vinegar) while the place (the inside of a bottle) remains constant."<sup>29</sup> Aristotle has the theoretical resources for dealing with this, insofar as he posits a material substrate that underlies both wine and vinegar.<sup>30</sup> But, as he is well aware from his study of the *Timaeus*, one could also posit space or place as that which stays the same while the characteristics that occupy place change, and that place might well be understood quantitatively. Positing a quantity as that which remains the same through a process of change is not nonsense on the face of it.

Elsewhere Aristotle presents a more fundamental objection against a Pythagorean mathematical-physical ontology. Physical objects are those with a nature (*phusis*) which is a kind of principle of motion and change.<sup>31</sup> The principles posited by the Pythagoreans in question apply to all beings, both those that are capable of motion and those that are not. For this reason, the principles that the Pythagoreans posit as responsible for physical reality are more appropriate (*ἀρμολογίας*) for accounts that are "higher" (that is, more general) than those concerning nature.<sup>32</sup> Motion in general can be neither derived from nor explained by the principles they posit, namely, limit and unlimited, even and odd, and number.<sup>33</sup> A fortiori such accounts say nothing about the most basic natural motions (upward and downward) proper to light and heavy bodies.<sup>34</sup> "Insofar as they make natural bodies, which have lightness and weight, out of numbers, which are without lightness and weight, they seem to speak of another heaven and other bodies, not of those that are sensible."<sup>35</sup>

Aristotle is here asking about the source of the nonmathematical characteristics of bodies.<sup>36</sup> His example is weight (and its counterpart, lightness). Weight is for us a quantity, identified with a measurement read off of a scale, whether real or hypothetical. But for Aristotle, weight is a potentiality to move downward;<sup>37</sup> although the motion can be described quantitatively, the motion is not itself a quantity. How is it, Aristotle wonders, that quantities alone can generate regular natural motions?<sup>38</sup> In an implicit appeal to the Parmenidean principle that nothing comes from nothing,<sup>39</sup> Aristotle argues that weight can only come from what has weight, and numbers are not the sort of thing that has weight. On the face of it, this is an odd argument for Aristotle to make. For why is it any less conceivable for quantities to have weight than it is for a substance, like a living thing, to have a color? On Aristotle's account, living things are essentially bodies, bodies by their essence have surfaces, and surfaces are (usually) colored. Likewise, the Pythagoreans can be understood as saying that numbers are simply the sorts of things as to have weight (sometimes?). Aristotle would likely respond to this objection by appealing to his distinction between potentiality and actuality. Even though color may well not be part of their essence, living things have this color or that because their essence entails the possession of certain potentialities, which either include or entail the potentiality to have a color. In contrast, the essences of numbers are wholly mathematical. They have no potentialities, outside of the "powers" by which they stand in certain properly arithmetical relations to other numbers. This is why nonmathematical features cannot be inferred from mathematical definitions.<sup>40</sup> Considered in themselves, they neither include nor entail the potentiality to have nonnumerical features. To meet Aristotle's objection, the Pythagoreans would have to show how numbers include more than the mathematical features that are posited by the arithmeticians who study them; they also include certain nonmathematical potentialities. Such numbers would not be numbers as studied by mathematics. So on Aristotle's account, quantities require a nonquantitative substrate, and if one truly grasps the essence of the substrate, one would understand why it has the essential quantitative features that it has (for example, by grasping the essence of a horse, one would understand why it has the shape that it has, and why its size lies within a certain range). But note that the two accounts are structurally parallel.

On the Pythagorean account, nonquantities require a quantitative substrate, and if one truly grasps the essence of the substrate, one would understand why the substrate has the nonquantitative features that it has. For example, a study of the ratios holding among numbers would explain why musical concords sound as they do.

We have seen that Aristotle faults the Pythagoreans on the grounds that they try to explain all things on the basis of mathematical objects. Not all facts of the world can be made intelligible on the basis of mathematical principles; only mathematical truths can. Nonmathematical truths can be explained only on the basis of nonmathematical principles. In nearly all cases, these principles will be the essences of the substances in which mathematical entities inhere. It turns out that in some cases (especially in biology), explanation will be grounded not in the essence of the substance, but in the essence of the material kind in which the substance inheres. In either such case, the Pythagoreans are to be faulted with grounding explanations in formal characteristics (such as mathematical features) alone, to the exclusion of the kinds that are their subjects. Both must be known. Both exist, and certain formal characteristics, such as the mathematical ones, must be recognized as inherent in more basic principles.

### **III**

Some of Aristotle's criticisms of the Pythagoreans are less persuasive to us now than they would have been to his contemporaries. Aristotle presumes that there is no way to derive most of the natural characteristics of things (qualities such as color and temperature, as well as the natural motions proper to certain kinds) from mathematical characteristics alone. Modern science casts doubt on this. That is not to say that there are not still major difficulties in accounting for the phenomenological aspects of color, sound, and the like. This remains a major mystery to which those who adopt computational models of cognition (which includes most structural realists) have little to say. But today's Aristotelians would echo Aristotle's main criticism of Pythagorean number ontology: formal

structures and quantities inhere in more basic subjects.<sup>41</sup> In order to know things, one must know both the structures and the subjects.

Both of these points are denied by "ontic" structural realism, which has been defined as "a realism towards physical structures in the sense of networks of concrete physical relations, without these relations being dependent on fundamental physical objects that possess an intrinsic identity as their relata."<sup>42</sup> This view has a number of philosophical motivations, of which the main one is epistemological. Structural realism is seen as a way of bypassing the traditional debate in the philosophy of science between realism and antirealism. Realists have insisted that scientific theories are offered as describing the world as it really is. The realist account of the semantics of scientific propositions is supported by the answer one usually receives when one asks scientists themselves what they are saying: that their accounts are in fact offering at least a provisional account of how things are in the world. To this antirealists point out that scientific theories are in a constant state of revision, and that there is no way to be sure that some future theory might come to supplant a current one. So, the argument goes, even the best of our scientific theories are likely someday to be condemned as false. It would follow that scientific theories, on the realist understanding, are attempts to do what might well be undoable. But any philosophy of science that would cast scientific theories to the flames has something amiss. So antirealists deny that successful scientific theories are to be understood as explaining the real constitution of things. Their success lies elsewhere. Different kinds of antirealists have different views as to what this something else is. For example, instrumentalists take scientific propositions to be parts of theories, which themselves are to be regarded as instruments enabling one to make predictions concerning what will be observed under certain situations, an ability of vital importance for technology.

Structural realism is presented as a way to solve the problem. On that account, a scientific theory is not telling us about things, that is, the subjects of certain formal characteristics. Rather, it is telling us about formal characteristics alone: "What differentiates the resulting form of structural realism from standard scientific realism is that the latter regards the mind-independent modal relations between phenomena as supervenient on the properties of unobservable objects

and the external relations between them, rather than this structure being ontologically basic."<sup>43</sup> The structural characteristics of formerly accepted theories are isomorphic to, or mappable onto, the structure of the theories that have superseded them. The new theories are better not because they indicate what is real whereas the old theories did not, but because have the advantage of saying *more* about the very structure that was recognized all along.<sup>44</sup> Structural realists say that in identifying basic structures science does describe the world as it is, but makes no claims about the nature of anything that underlies those structures.<sup>45</sup>

Structural realism has some support in innovations in computer science. Understanding the human mind as arising from computations grounded in the brain allows for an answer to a major problem with realism: how exactly is it that the mind is thought to be able to apprehend an extramental reality? The classic Aristotelian answer is that the human being is so constituted as to enable the knowable aspect of a thing to be present in the knowing subject. Thus, the sensible form red, by virtue of which I know that an object (such as an apple) is red, becomes somehow present in the sense organ, which is part of the whole ensouled perceptive animal.<sup>46</sup> When one knows the essence of a substance such as an apple tree, one in a sense becomes identical with the form of the apple tree, a process that, according to Aristotle, requires an immaterial soul.<sup>47</sup> The structural realist can make a similar move without compromising the materialist presuppositions of much of cognitive science. The structures in the world are information; a computational system, such as the human mind, can access that very information at various "levels of abstraction" and internally represent and manipulate it in certain ways that allow for various interactions with the world.<sup>48</sup>

A third advantage of structural realism is the virtue of economy. Structural realism allows for a minimum of ontological commitments. We recall that the Milesians, as Aristotle interpreted them, took there to be at bottom one kind of thing, the nature of which would explain the multiplicity of phenomena. Aristotle argues that this scheme is unworkable, for adequate explanations require a multiplicity of formal elements, which in turn requires various kinds of matter in which to be instantiated. So, on the Aristotelian scheme, any adequate ontology requires two kinds of beings. There are formal elements, and there are

the subjects in which those elements inhere. In a contemporary context, these two can be understood as the natural laws that govern basic things or particles, on the one hand, and those basic things or particles themselves, on the other. Traditional scientific realism is committed to both of these, even though the content of its theories concern only the former — as theories say nothing about strings or quarks beyond the mathematical accounts of their features and behavior. Traditional realism, then, is committed to positing the existence of that which is knowable only as placeholder for its formal characteristics. But what if all of these formal elements are united as being "information"?<sup>49</sup> What if all there is is information, and physical reality does not require a physical substrate for that information? Structural realism avoids commitment to an unknowable principle. Like the Milesians on Aristotle's understanding, structural realists posit only one kind of thing in principle able to ground explanations of everything.

There is a possible fourth advantage to structural realism. If taken in a certain direction, it has potential to answer some very big questions concerning the nature of causality and of time and space. In the guise of pancomputationalism, structural realism goes beyond the thesis that physical processes, as governed by scientific laws, are a matter of regularities in how certain arrays of information determine others, to a view concerning how exactly this occurs: the cosmos is a vast computer.<sup>50</sup> Physical reality is constituted by information of multidimensional space and time, laid out, somehow, in some sort of a grid (which can have as few as one dimension).<sup>51</sup> Physical laws are the software, according to which information at one point in space/time is determined as information at another point. Such a thesis not only gives us a unified account of what the world is made of, but also allows the processes that govern the world to have some element of intelligibility. We might not be able make sense of light being both a wave and a particle, or of causal action at a distance, but the notion of a universal Turing machine is one that is familiar and intelligible. It provides a model by which we can make sense of the workings of all physical reality.<sup>52</sup> But the thesis that the cosmos is a computer has been widely rejected among structural realists.<sup>53</sup> This is not only because it seems fanciful and too open to quasi-theological speculations concerning the origins and the purpose of the system. It is also because it rests on two assumptions not shared by most

physicists. First, this thesis requires that reality be understood as discrete, not continuous (for computers work through manipulation of determinate symbols, all of which can in principle be encoded in a binary way, as a sequence of 0s and 1s). There is no evidence for this, except extrapolation from the historical record of the history of science.<sup>54</sup> Second, the thesis is deterministic. Leaving aside familiar philosophical objections to determinism, the thesis counters the standard (Copenhagen) view of the indeterminacy of events at the quantum level.<sup>55</sup> In both cases, Fredkin is confident that future developments in physics will confirm his hunches.

In both varieties of structural realism there are deep parallels to the thought of the Pythagoreans as Aristotle describes them. Yes, the Pythagoreans restrict their ontology to numbers, while contemporary structural realists are open to structures of all kinds. But numbers, taken either individually or in relation to one another, are examples of structure. Not all structures can be expressed numerically, and for the Pythagoreans as for all ancient Greeks, numbers would be integers, for which reason continuous structures would not be accepted in Pythagorean ontology. But if one accepts the thesis of pancomputationalism, the correspondence would be more exact, as all Turing machines manipulate discrete data, and all discrete data can be encoded as a series of binary elements, 0s and 1s, which together can be understood as constituting numbers (in base 2).

According to structural realism of whatever variety, things are known by virtue of knowing structures. In ontic structural realism, these structures are the objects in question—there is no substrate of the numbers or structures posited as an unknowable surd. (This is not strictly speaking the case for pancomputationalism. For on this view the universe is the result of a cosmic computer. Both software and data are informational. But what of the hardware? This is unknown and unknowable. Fredkin calls it "the Other." He treats it as noumenal and waxes theological concerning its source.<sup>56</sup> Perhaps this hardware can be considered the substrate for the structures involved — but only in an extended sense, as the structures have their status as information only in the context of a whole system of other structures. Suppose that we have a series of 0s and 1s as instantiated in a series of physical switches. The switches themselves are the proper subject not for the information, but only for the attributes being open or closed, all



of which is irrelevant to the computational system considered as such. For Fredkin, the information that is constitutive of physical reality is encoded in something, but that something is not itself part of or an aspect of the physical reality that is known. So although there is some sort of unknowable substrate, it would not be a substrate in the Aristotelian sense. It is not the case that the substrate has special potentialities for the kind of information instantiated in it — rather, like Aristotle's passive intellect, it can serve as a substrate for any information whatsoever. As such it is of a different ontological order than the information within it. The reality that the hypothesis is meant to account for is the reality of the structural information that is computed, not the system that is itself responsible for the computations.)

We see that structural realism, like Aristotelian realism, recognizes the need to posit an ontological grounding for the adequacy of scientific theories. But it does so without assuming that there is a certain ontological substrate for the formal structures by which we know things. Here it is in agreement with Pythagorean ontology and epistemology as described by Aristotle. Aristotle rejects such a scheme as unintelligible. He asserts without argument that numbers and other quantities must be quantities of some more basic thing. In the final part of this paper I suggest that, in his most probing speculations concerning substance, he himself resorts to Pythagorean ways of understanding the ontology of form.

## IV

In *Metaphysics* 7.3 Aristotle pursues the ultimate implications of his view that the foundational realities or elements of things, which he calls *ousiai* (substances), are the ultimate subjects. What can we say about such substrates, considered in themselves? Aristotle conducts a mental experiment by which he examines such a substrate, from which we have mentally subtracted all actual features as well as the potentialities for them. He is left with a something that is nothing, a substrate that has no characteristics at all,<sup>57</sup> an Aristotelian analogue to the "bare particular" of the early analytic tradition.<sup>58</sup> Such a thing is inconceivable, hence impossible. Exactly what conclusion Aristotle draws from this is unclear. To a large extent this issue determines the



path one will take in interpreting Aristotle's larger account of substance in the central books of the *Metaphysics*. Perhaps Aristotle drops ultimate subjecthood as a criterion for substantiality. Perhaps Aristotle maintains that criterion, and comes to affirm that it is not indeterminate matter but form or the form/matter composite that serves as ultimate subject, and hence substance.

One moral of the mental experiment is clear: one ought not to follow the Milesians in trying to explain by identifying basic substrates, without an account of the formal characteristics of those substrates that make them what they are. It is on Aristotle's own account inconceivable, and hence impossible, for there to be a featureless substrate, and features are formal characteristics distinct, at least in *logos*, from the subjects in which they inhere.

The subjects Aristotle posits are not featureless. They have essences. Substances are subjects with essences that make them the kind of substance they are. Substances come to be insofar as their form (or essence) is actualized in a more basic subject, which is matter. This matter itself has certain features, which are responsible for the potentialities to take on certain substantial forms. Thus iron has to have the feature of rigidity for it to serve as the subject of a saw, and the human tissues must have the features that they have, if they are to be the subject of a human form. These features are themselves formal; they are determinations of an indeterminate qualitative expanse. Thus the matter of a saw cannot be too soft, but, at least for certain varieties of sawing, it cannot be too hard either. Flesh cannot be too hard or too soft. For Aristotle, these determinate states of rigidity or heat are not quantitative. But they are not substantial either.

We recall that Aristotle's core criticism against the Pythagoreans was that they were positing a nonsubstance as a subject. Aristotle does the same in regard to matter. Matter, considered simply as a substrate, is unintelligible. The formal features of the kind of matter are what give it ontological standing and some degree of intelligibility. Likewise, substances serve as ontological subjects, and are basic in scientific explanation of other aspects of reality, only insofar as they have certain definitional features, which are also formal.

We have seen that structural realists are compelled to accept one of two options. Either a subject of information is posited (as a kind of hardware and basis for digital memory) — about which nothing can be known and which stands outside of the reality in which one lives — or one dispenses altogether with an ontological subject for information. These positions are similar to those between which Aristotle must choose: positing as ultimate subject an indeterminate unknowable subject, and positing as ultimate subject what is constituted by one or more formal characteristics. While certain structural realists entertain the possibility that it is structure "all the way down," Aristotle, like the Pythagoreans he so readily dismisses, and like pancomputationalists, would insist on a bottom level of form, structure, or information in which all else inheres.<sup>60</sup>

## Footnotes

1Aristotle, *Metaphysics* 7.10, 7.17.

2Aristotle, *De anima* 1.1.403a24-b9.

3Aristotle, *Metaphysics* 1.3.

4Aristotle, *Metaphysics* 1.3.984b8-22. Following Harold F. Cherniss, *Aristotle's Criticism of Presocratic Philosophy* (New York: Octagon Books, 1964), Aristotle's criticisms are often regarded as anachronistic. The distinction between matter and form may be fundamentally inapplicable to the Presocratic worldview. For example, for Heraclitus fire is not merely a substrate; it is also *logos*, a structural and regulative principle. Perhaps Heraclitus would reject the need to distinguish the two aspects of a physical or cosmological principle. Identifying a single principle responsible for both has the virtue of economy. We shall see that such motivations of economy underlie attempts, like that of contemporary structural realism, to regard all explanatory principles as formal.

5Aristotle, *Metaphysics* 1.5.986a27-32.

6Aristotle, *Metaphysics* 14.5.1092b19-20. Except where noted, all translations from Aristotle are from *Complete Works of Aristotle: The Revised Oxford Translation*, 2 vols., ed. Jonathan Barnes (Princeton, N.J.: Princeton University Press, 1984).

7Most notable is Carl A. Huffman, *Philolaus of Croton: Pythagorean and Presocratic: A Commentary on the Fragments and Testimonia with Interpretive Essays* (Cambridge: Cambridge University Press, 2006).

8Max Planck, *The Universe in Light of Modern Physics*, trans. Walter Henry Johnson (London: George Allen and Unwin, 1931), 14.

- 9 Simon Saunders, "Structural Realism, Again," *Synthese* 136, no. 1 (2003): 129. See Luciano Floridi, *The Philosophy of Information* (Oxford: Clarendon Press, 2011), who concludes, "it might be Russian dolls (informational objects) all the way in" (471). He is not leaving open the possibility of a bottom level of noninformational reality; rather, his point is that though all levels of reality are informational, there may or may not be a fundamental level of information.
- 10 For example, forms are identified with numbers; it is unclear whether this identification holds for all forms or only some. It is also unclear what it means to call forms numbers, as they are distinguished from the sorts of numbers with which the arithmetician deals (*Metaphysics* 13.6.1080b11-14). Further, both forms and numbers were apparently thought to have been derived from more basic principles such as the One and "the Great and the Small" (*Metaphysics* 14.1.1087b4-9). Aristotle's discussion focuses on the conceptual difficulties raised by these complexities.
- 11 Aristotle, *Metaphysics* 1.5.985b23-986a3 (my translation).
- 12 Aristotle, *Metaphysics* 1.3.983b1-6.
- 13 It is unclear what relationship those earlier Pythagoreans with mathematical interests have to the *mathêmatici*: Walter Burkert. *Lore and Science in Ancient Pythaoreanism* trans. Edwin Minar, Jr. (Cambridge, Mass.: Harvard University Press, 1972), has shown that Aristotle is the source for the very distinction of them as a group distinct from the *acusmatici* (196). A concise review of what can be safely attributed to Pythagorean thinkers other than Philolaus can be found in Carl A. Huffman, "Pythagoreanism," *The Stanford Encyclopedia of Philosophy* (Spring 2015), ed. Edward N. Zalta, <http://plato.stanford.edu/archives/spr2015/entries/pythagoreanism/>.
- 14 See especially Burkert, *Lore and Science in Ancient Pythagoreanism*, tr. Edwin Minar (Cambridge, MA: Harvard University Press), 1984; also Geoffrey Kirk, John E. Raven, and Malcolm Schofield, *The Presocratic Philosophers: A Critical History with a Selection of Texts*, 2nd. ed. (Cambridge: Cambridge University Press), 42-3.
- 15 Huffman, *Philolaus of Croton: Pythagorean and Presocratic* (Cambridge: Cambridge University Press), 1993, 57-74. The translation of Fr. 4 is Huffman's..
- 16 Cf. Huffman, *Philolaus*, p. 59: "My thesis is that the doctrine that all things are numbers was not stated in any of Aristotle's sources, including Philolaus, whom I believe to be Aristotle's main source. Instead the doctrine represents Aristotle's own succinct formulation of the Pythagorean outlook. He is saying that what Pythagorean philosophy amounts to is the doctrine that all things are numbers."
- 17 Aristotle, *Metaphysics* 14.5.1092b8-13.

- 18The extent to which earlier Pythagoreans were aware of the numerical basis of musical intervals, and the extent to which they were responsible for the discovery of this relation, are controversial, but fragment 6a shows that Philolaus was both aware and keenly interested in these correlations. See Huffman, *Philolaus*, 145-65.
- 19On Aristotle's use of what Anthony Charles Lloyd has called the "Transmission Theory of Causation," see A. C. Lloyd, "The Principle That the Cause Is Greater than Its Effect," *Phronesis* 21, no. 2 (1976): 146-56. See also Alexander P. D. Mourelatos. "Aristotle's Rationalist Account of Qualitative Interaction," *Phronesis* 29, no. 1 (1984): 1-16.
- 20On the logic behind this kind of archaic explanation, see Owen Goldin, "The Pythagorean Table of Opposites, Symbolic Classification, and Aristotle," *Science in Context* 28, no. 2 (2015): 171-93.
- 21"Evidently, then, these thinkers also consider that number is the principle both as matter for things and as forming their modifications and states" (*Metaphysics* 1.1.986a16-7). Aristotle proceeds to identify even and odd as the principles of number, and a little later asserts that these Pythagoreans identified not numbers but these principles of number as material causes, just as a different group of Pythagoreans identified certain opposites, arranged in the columns of the Table of Opposites, as material causes. "But how these principles can be brought together under the causes we have named has not been clearly and articulately stated by them; they seem, however, to range the elements under the head of matter; for out of these as immanent parts they say substance is composed and moulded." (*Metaphysics* 1.1.986b4-8, see also 13.6.1080b16-21. and 3.3.1090a20-3.) At *Metaphysics* 14.5.1092b8-13 Aristotle recognizes that the Pythagorean Eurytus took numbers to constitute spatial boundaries responsible for things, which suggests that he may have taken numbers to be formal causes, but Aristotle considers this suggestion too silly to consider further. He then turns to Pythagoreans whom he takes to have been more serious, in positing numbers as principles; presumably Philolaus is to be included in this group. They are aware of the causal importance of ratios, which suggests that they recognized formal causation. Ratios however, are ratios of numbers; these numbers were considered matter (1092b13-23).
- 22*Metaphysics* 1.5.986a17-24. Because Aristotle speaks of "the limited and the unlimited" and not "the limit and the unlimited" or "limiters and unlimited," he has in mind neither the ontology of Plato's *Philebus* (which would be suggested by the first phrase) nor that of Philolaus (which would be suggested by the second). I suggest that Aristotle is understanding odd and limited, on the one hand, and even and unlimited, on the other, as associated in the manner of the Table of

Opposites of the other Pythagoreans, discussed at 986a23-24. According to such a scheme, "odd and limited" and "even and unlimited" are correlative opposites, so that the presence of one is associated with the presence of the other. Neither has ontological or explanatory priority. See Goldin, "The Pythagorean Table of Opposites." In contrast to my interpretation of Aristotle's strategy here, Huffman (*Philolaus*, 47 n. 1) attempts to understand Philolaus in isolation from the Pythagoreans that posited the Table of Opposites. Although Aristotle distinguishes the two groups of Pythagoreans, there is no reason to think that he would insist that the thought of each group must be understood without any appeal to the thought of the other.

23 Aristotle, *Metaphysics* 13.2.1077a15-19.

24 Aristotle, *Metaphysics* 13.2.1077b2-7. (The translation is mine.)

25 Aristotle, *Categories* 5.2a34-b6).

26 John L. Ackrill, *Aristotle: Categories and de Interpretatione* (Oxford: Clarendon Press, 1978), 82-3.

27 Aristotle, *Categories* 5.4a10-21.

28 Thus the Oxford translation "continue to exist" and Julia Annas, *Aristotle's Metaphysics Books M and N*, (Oxford: Clarendon Press, 1988), 94, "go on existing."

29 Aristotle, *Physics*, 4.1 208b1-8

30 Aristotle analyzes this change as a reversion of wine into its matter, which is water, which in turn has a potentiality for certain changes, which lead to vinegar. *Metaphysics* 9.5.1024b29-1025a6.

31 Aristotle, *Physics* 2.1.192b12-23

32 Aristotle, *Metaphysics* 1.8.990a5-8, see also 14.3.1091a13-8

33 Aristotle, *Metaphysics* 1.8.990a8-10

34 Aristotle, *Metaphysics* 1.8.990a12-14

35 Aristotle, *Metaphysics* 14.3.1090a32-5

36 Cf. Aristotle, *De Caelo* 3.1.299b14-5.

37 Aristotle, *De Caelo* 4.1.

38 Insofar as the elements of the cosmos in Plato's *Timaeus* are quantitative (geometrical figures), the accounts of weight and lightness offered in this dialogue could be understood as attempts to meet Aristotle's objection. But even here it is not the geometrical features of the elemental solids, alone, that is responsible for perceived motions of bodies: appeal is made to the primeval disorderly motion and the orderly motions of soul, as what Aristotle would call efficient causes, as well as to the plan of the Demiurge, as a final cause.

39 Parmenides fr. 8.7-9.

- 40 Thus Aristotle insists that nonmathematical premises, like “beautiful” cannot be inferred on the basis of mathematical definitions; see *APO*. 1.7.
- 41 Thus James Franklin, *An Aristotelian Realist Philosophy of Mathematics: Mathematics as the Science of Quantity and Structure* (New York: Palgrave Macmillan) 2014, 56, appealing to David Armstrong, defines “structural” as a characteristic of properties that belong to particulars.
- 42 Michael Esfeld and Vincent Lam, “Ontic structural realism as a *Metaphysics*physics of objects,” in Alisa and Peter Bokulich, eds., *Scientific Structuralism* (Dordrecht: Springer) 2010, 143.
- 43 James Ladyman, “Science, *Metaphysics*physics, and Structural Realism,” *Philosophica* 67.1 (2001), 73.
- 44 John Worrall, “Structural Realism: The Best of Both Worlds?,” *Dialectica* 43 (1989) 99-124.
- 45 Structure itself is to be taken as “ontologically subsistent.” See James Ladyman, “What is Structural Realism?” *Studies in History and Philosophy of Science* 29.3 (1998), 420. We note in passing that Philolaus’ Fr. 4 may be interpreted as advocating structural realism. Perhaps in saying that nothing can be known without number, Philolaus is saying that number is real. On such an interpretation, Philolaus might admit that there might be more to reality but number, but he would nonetheless say that that whatever we know of thing is numerical. What is not numerical is unknowable. But this is speculative. Such an interpretation could be justly charged with presupposing an anachronistic awareness of certain epistemological disputes.
- 46 Aristotle, *De Anima*, 2.5
- 47 Aristotle, *De Anima*, 3.4
- 48 A full account of how this might work is offered by Floridi 2011.
- 49 Not all data is information, just as not every array constitutes structure. For an attempt to distinguish information from noninformational data, see Floridi, 80-107.
- 50 The most prominent exponent of this view is Edward Fredkin. “Finite Nature implies that the basic substrate of physics operates in a manner similar to the workings of certain specialized computers called cellular automata.” Edward Fredkin, “A New Cosmogony” 1992 <http://www.leptonica.com/cachedpages/fredkin-cosmogony.html>, accessed August 13, 2014. An informal account of Fredkin’s ideas can be found in Robert Wright, *Three Scientists and their Gods* (New York: Times Books), 1988, 1-80.
- 51 Edward Fredkin, “A New Cosmogony”: “There is no need for a space with three dimensions: computation can do just fine in spaces of any number of dimensions!”



- 52 Note that this thesis is different from the much discussed proposal of Bostrum that the universe is a simulation. (Nick Bostrom, "Are You Living in a Computer Simulation?" *Philosophical Quarterly* 53.211 (2003), 243-55. A simulation requires a cognizing subject outside of the simulation for whom the simulated reality appears. Pancomputationalism proposes that all cognizing subjects are themselves part of the whole computational system.
- 53 See for example Floridi, 316-38.
- 54 Edward Fredkin, "Finite Nature" 1992, *Proceedings of the XXVIIth Rencontre de Moriond* and [http://52.7.130.124/wp-content/uploads/2015/07/finite\\_nature.pdf](http://52.7.130.124/wp-content/uploads/2015/07/finite_nature.pdf) accessed August 13, 2015. "[W]hat is interesting is that so many concepts once thought of as continuous are now accepted as discrete. Finite Nature assumes that that historical process will continue to a logical conclusion where, at the bottom, everything will turn out to be atomic or discrete, including space and time."
- 55 Fredkin, "Finite Nature": "Uncertainty is at the heart of quantum mechanics. Finite Nature requires that we rule out true, locally generated randomness because such numbers would not, in this context, be considered finite. The reason is that there is no way to create, within a computer, a truly random number that is orthogonal to everything in the computer."
- 56 Fredkin, "A New Cosmogony."
- 58 See Daniel Graham, *Aristotle's Two Systems* (Oxford: Clarendon Press) 1988, 207-32.
- 59 Aristotle, *Physics*, 2.9
- 60 A previous version of this paper was presented to 2012 meeting of the *Metaphysics* physical Society of America, at the University of Georgia. I am indebted to that audience for its comments and questions, and to Mark Schulz, for editorial assistance.