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Abstract:
It has been known that McIntyre's local multiplication theory for avalanche photodiodes (APDs) does not fully explain the experimental results for single-carrier InAs APDs, which exhibit excess noise factor values below 2. While it has been established that the inclusion of the dead-space effect in the nonlocal multiplication theory

resolves this discrepancy, no closed-form formulas for the mean gain and excess noise factor have been specialized to InAs APDs in a nonlocal setting. Upon utilizing prior analytical formulation of single-carrier avalanche multiplication based on age-dependent branching theory in conjunction with nonlocal ionization coefficients and thresholds for InAs, closed-form solutions of the mean gain and the excess noise factor for InAs APDs are provided here for the first time. The formulas are validated against published experimental data from InAs APDs across a variety of multiplication region widths and are shown to be applicable for devices with multiplication widths of 500 nm and larger.

SECTION I. Introduction

Infrared (IR) applications operating between 1.55 and 3.5 µm have recently seen an increasing interest, which has led to the research and development of detection systems with greater sensitivity. For photodetection applications, avalanche photodiodes (APDs) provide amplification of the received signal using the process of avalanche multiplication and hence provide improved sensitivity [1], [2]. Accompanying this gain is an increase in noise power, present due to the stochastic nature of the avalanche process, represented by the excess noise factor, F. The use of APDs can be limited in IR applications due to both the excess noise as well as the limiting cutoff wavelength inherent in APDs for use in telecommunication applications. The challenge in designing IR photodetectors, therefore, is to maximize the APD's mean gain, $\langle G\rangle$, up to a point at which excess noise begins to dominate the system noise. To fulfill this need, InAs has been nominated due to its lower noise properties [3], [4].

The first characterization of the mean gain and excess noise factor was done by McIntyre [5], with the formula for the mean gain and excess noise factor, assuming constant ionization coefficients, shown in the following:

$$\begin{aligned} \langle G\rangle = & \quad \frac{1-k}{\exp[-\alpha_{\text{local}}w(1-k)]-k} \\ F = & \quad k\langle G\rangle+(1-k)(2-\frac{1}{\langle G\rangle}) \end{aligned} \tag{1)(2}$$

where α_{local} and β_{local} are the electron and hole ionization coefficients (per unit length), respectively, k is the ionization coefficient ratio (defined as $\beta_{\text{local}}/\alpha_{\text{local}}$) and w is the width of the multiplication region. When $k=0$ and the mean gain approaches infinity, F approaches 2 from above. McIntyre's model failed to give accurate predictions for devices where the multiplication region is smaller than 1 µm [6]. This was attributed to the lack of accommodation for the *dead space*, the distance a carrier needs to travel before it gains enough energy to impact ionize [6], [7]. The dead space mitigates the noise within the device by reducing the stochastic ambiguity in the occurrence of the carrier avalanche [8]. To account for this effect, Hayat *et al.* developed the recursive integral equations, also referred to as the dead space multiplication theory (DSMT), as detailed in [9, eqs. (1)–(21)], for example. The DSMT has been shown to correctly predict the avalanche behavior, such as the mean gain and the excess noise, within thin APDs [7], [10]–[11]. In addition, the DSMT model was also later adopted by McIntyre for formulating history-dependent expressions for mean gain and the excess noise factor [12].

While significant improvement has been made to enhance the gain characteristics of practical APDs, the main challenge in using them remains the high excess noise factor. One way to mitigate the noise is to simplify the design of the detector and ensure that the absorption and multiplication regions are separate, leading to what is called the separate-absorption-multiplication APDs. Apart from this, the excess noise within APD devices can be reduced by choosing materials with ionization coefficients that reduce k. This means that the electron ionization coefficient, α_{local}, and the hole ionization coefficient, β_{local}, are as disparate as possible. Ideally, we would like either α_{local} or β_{local} to be zero, such that the ionization ratio, k , is zero (or infinity in the case of hole injection APDs). In such cases, as can be seen from above in (2), the excess noise approaches 2, which is the limit predicted by the local model developed by McIntyre. However, the model developed by Saleh *et al.* [13], as well as the work of Tan *et al.* [14], which takes the dead space into account, predicted that for such materials, the excess noise can be less than 2.

While $k = 0.02$ for silicon APDs with wide junctions and low electric field, for high-speed CMOS compatible silicon APDs [15]–[16][17][18], k approaches 1 as we reach submicrometer thicknesses [19]. When compared with the case of $k \approx 0$, and for a given thickness of avalanche region, this leads to an increase in the buildup time (defined as the time required for all the impact ionizations to complete, due to additional chain of ionizations from the presence of holes) as well as the tunneling current. Another candidate for single-carrier ionization is HgCdTe [20]–[21][22] but it poses many issues in fabrication [23].

Recently, InAs has been presented as an electron-majority ionization [4], [24], [3], [23], [25] material, which fulfills the requirements of a single-carrier ionization material. InAs then potentially offers reduced noise characteristics and it is a good candidate for fabricating APDs due to its ease of fabrication and availability.

In this paper, closed-form solutions of the mean gain and the excess noise factor for InAs APDs are provided by using the analytical formulation of single-carrier avalanche multiplication based on age-dependent branching theory in conjunction with nonlocal ionization coefficients and thresholds for InAs. We verify analytically, using InAs as an example, that the inclusion of the dead space effect explains excess noise factors that are below 2 in single-carrier devices. The formulations are validated against published experimental data for InAs APDs across a variety of multiplication region widths. In addition, the formulas are compared with the exact numerical method (ENM), which implements the DSMT's recursive integral equations analytically, but is numerically intensive. It is important to note that the DSMT is used here as a reference only; its details are described elsewhere [13], [9] but are not needed to be explained in this paper.

SECTION II. Single Carrier Ionization and Age-Dependent Branching Theory

The behavior of the single-carrier device, such as InAs, will be the subject of study in this paper. For an APD multiplication region of width w, with the avalanche multiplication process initiated by electrons at the edge of the multiplication region, $Z(w)$ is defined as the stochastic total number of electrons produced once the avalanche process settles. The theory of age-dependent branching process dictates that the first and second moments of $Z(w)$ become asymptotically exponential functions of the width. More precisely, the theory developed in [13] dictates that for sufficiently large multiplication widths (determined in the following), the mean gain becomes:

$$\langle G(w) \rangle \approx C_1 e^{bw} \quad (3)$$

whereas the excess noise factor becomes

$$F(w) \approx \frac{C_2}{C_1^2} \triangleq F_\infty \quad (4)$$

with

$$C_1 = \frac{B+1}{2B(DB+D+1)} \quad (5)$$

and

$$C_2 = \frac{C_1^2(B+1)}{1+2B-B^2}. (6)$$

Equations (3)–(6) come directly from [13], here the constants b and $B = b/\alpha_{\mathrm{en}}$ are described as the Malthusian and scaled Malthusian parameters, found by solving the transcendental equations, $2\alpha_{\mathrm{en}} \int_d^{\infty} e^{-by} e^{-\alpha_{\mathrm{en}}(y-d)} dy = 1$ and $2e^{-DB} - B = 1$, respectively. Moreover, $D = \alpha_{\mathrm{en}} d$ is the scaled dead space where the dead space, d, is computed by equating the kinetic energy gained by the ionizing carriers to the threshold energy, whereas α_{en}, termed the enabled electron ionization coefficient, is the ionization coefficient for a carrier that has already traveled the dead space and therefore is capable of impact ionizing. Recent work has shown that the enabled ionization coefficient is given by $\alpha_{\mathrm{en}} = 1/(1/\alpha_{\mathrm{local}} - 2d)$ [26], where αlocal is the experimental ionization coefficients without the dead space effect and found from the literature [3]. The closed-form solution to the mean gain and excess noise factor, (3) and (4), holds provided the width of the device is much greater than the dead space and the practical guidelines for their applicability are discussed later in this section.

To measure the performance of (3) with the dead space effect coming into play, we first consider the case when the scaled dead space may be assumed to be a constant. First is the case of negligible dead space, $D = 0$. In this case, $B = 1$, $b = \alpha_{\mathrm{en}} = \alpha_{\mathrm{local}}$, $C_1 = C_2 = 1$, and the mean gain, $\langle G(w) \rangle = e^{\alpha_{\mathrm{en}} w}$ should then coincide with the gain calculated for negligible dead space using the ENM. In addition, four values of D are modeled, with the results shown in Fig. 1, using two different methodologies: one assuming that D is fixed while the multiplication region width is varied and the other for a fixed width, where αen is varied as a function of the applied electric field in the multiplication region. It was verified using the two methodologies that (3) predicts the mean gain with less than 8% error for $D = 1$ as compared with the ENM. We then use the recursive integral equations to confirm that the noise within the devices approaches the values predicted by (4) with the noise characteristics predicted for $D = 0{,}0.1$ and 0.5 and the results shown in Fig. 2. Thus, we have verified that the approximation works very well for devices with fixed widths and variable ionization coefficients; this is a necessary prerequisite for testing the formulas for the more realistic case of devices with variable widths as well as ionization coefficients, discussed in the following.

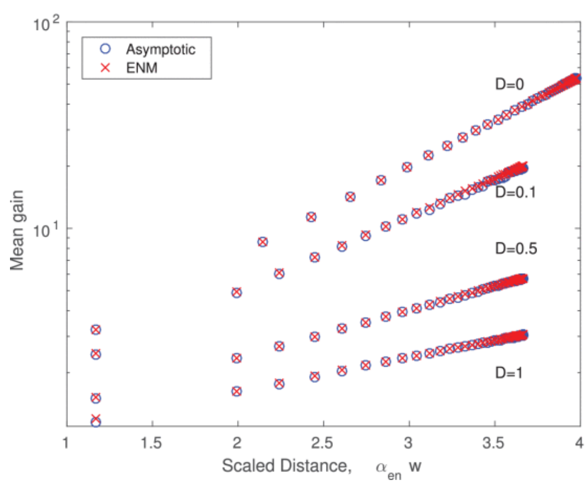

Fig. 1. Mean gain as a function of the scaled distance αenw for different fixed scaled dead spaces D=αend . The asymptotic results closely follow the results from ENM.

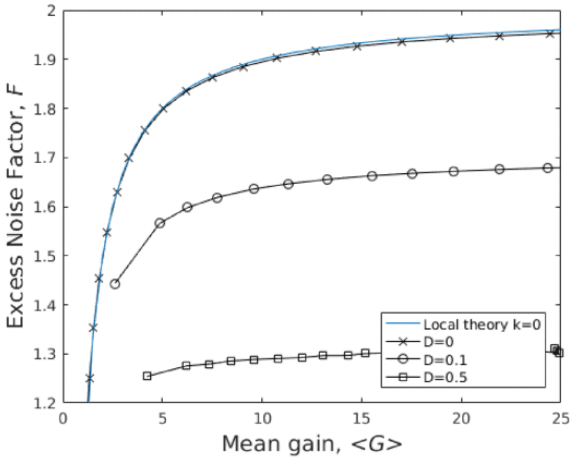

Fig. 2. Excess noise factor as a function of the mean gain for the case of different scaled dead spaces $D = 0{,}0.1$, and 0.5 along with noise trend predicted by local theory. For the special case of $k = 1$, $F = \langle G \rangle$ from the local theory (not shown).

To study the validity of the approximations in (3) and (4) for realistic devices, we must take into account that in such cases, the scaled dead space is variable due to its dependence on the applied electric field in the multiplication region. Here, the equations' performance in predicting the gain and noise for InAs is tested by taking only the multiplication region into account, without considering the absorption region. We choose multiplication widths of 2 and 3.5 µm with the ionization coefficients for InAs at room temperature given as $\alpha_{\text{local}} = 4.62 \times 10^4 \times \exp(-1.39 \times 10^5/E)^{0.378}$ cm−1 [3]. These widths are meaningful and relevant in accordance with their use in fabricating practical InAs devices, such as those in [3] or [24]. For the 2-µm device, the mean gain is calculated using the approximate formula in (3), as well as ENM, and shown as a function of the scaled distance in Fig. 3. Here, the enabled ionization coefficients were found by using the experimental ionization coefficients listed in [3] and the expression from [26] relating them. The approximate excess noise factor was found using the formula in (4) and is shown here as a function of the approximate mean gain in Fig. 4for the two device widths of 2 and 3.5 µm alongside the mean gain and excess noise figures from the experimental device fabricated in [3] for comparison. The scaled dead space, $\alpha_{\text{en}} d$, is dependent on the electric field and has been calculated and stated for the particular values of approximate mean gain and excess noise factor depicted. Here, we note that the approximation performs better at increased gains. This can be attributed to the scaled dead space decreasing as the electric field increases, shown in Fig. 5, which means that the asymptotic equation follows the numerical solution better at higher fields and gains.

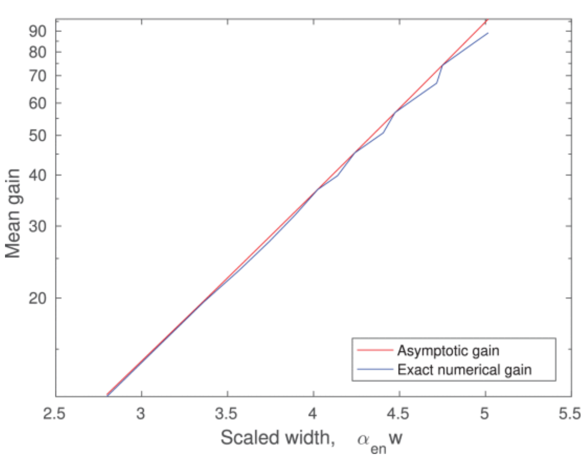

Fig. 3. Mean gain is shown as a function of the scaled distance $\alpha_{en}w$ for the realistic device width of $2\ \mu m$. The asymptotic results closely follow the results from ENM but the variance at high gains may be explained by the numerical errors introduced in gain calculation using ENM. For this case, the scaled dead space $D=\alpha_{en}d$ was found to vary from 0.035 to 0.065.

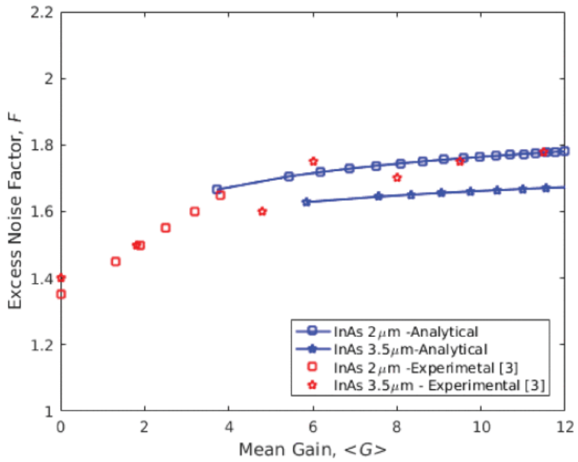

Fig. 4. Excess noise factor as a function of the mean gain for the case of different widths of InAs device, using the approximate formulas. The scaled dead space varies from 0.065 to 0.095 for the 2-μmdevice and from 0.10 to 0.14 for the 3.5-μm device, respectively. When compared with the experimental results from [3], the maximum error in the approximation is less than 10%.

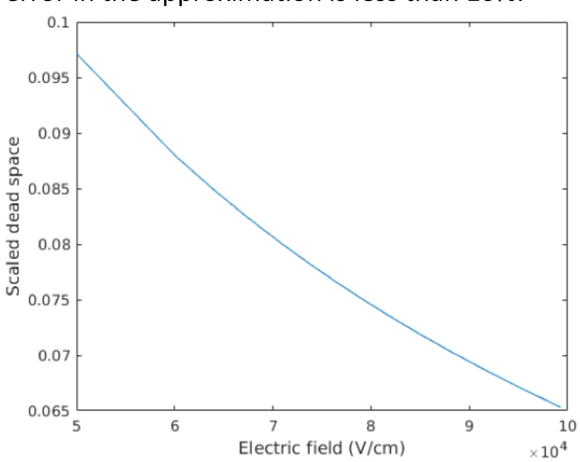

Fig. 5. Scaled dead space is shown as a function of the electric field. As the field increases, the scaled dead space decreases leading to the approximation performing better.

The formulas were modeled for different multiplication widths while considering both the gain as well as the applied bias. At 500 nm, the bias was found to be ≈ 30V for a gain of ≈ 7, which is reasonable for a practical device. Lower widths required much higher biases for similar gains, making them undesirable. Thus, the approximate formulas may be used easily to predict the mean gain and noise for practical InAs APDs.

SECTION III. Conclusion

We have provided closed-form solutions to accurately approximate the mean gain and the excess noise factor of single-carrier ionization InAs devices. We have also looked at the general avalanche properties of single-carrier onization materials and verified the fulfillment of the predictions made by Saleh *et al.*, [13] for such materials. The presence of dead space, and its effect, explains the decrease in the asymptotic excess noise from the value of 2, which is unexplained by the traditional local theory model.

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