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On the Analytical Formulation of Excess Noise In Avalanche Photodiodes With Dead Space

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Abstract

Simple, approximate formulas are developed to calculate the mean gain and excess noise factor for avalanche photodiodes using the dead-space multiplication theory in the regime of small multiplication width and high applied electric field. The accuracy of the approximation is investigated by comparing it to the exact numerical

method using recursive coupled integral equations and it is found that it works for dead spaces up to 15% of the multiplication width, which is substantial. The approximation is also tested for real materials such as GaAs, InP and Si for various multiplication widths, and the results found are accurate within ∼ 15% of the actual noise, which is a significant improvement over the local-theory noise formula. The results obtained for the mean gain also confirm the recently reported relationship between experimentally determined local ionization coefficients and the enabled non-local ionization coefficients.

1. Introduction

Avalanche photodiodes (APDs) play an important role in detecting low-level light due to their greater sensitivity as compared to PIN diodes, and for this reason they are used extensively in many optical systems [1, 2]. The increased sensitivity comes from the APD's gain that is the outcome of the chain of electron/hole impact ionizations in a high-field depletion (multiplication) region. Although the APD's high gain is an advantage, the accompanying excess noise, which results from the stochastic nature of the impact ionization process, is an undesirable effect that undermines the benefits of the gain. For an APD, the dead space is defined as the minimum distance that a newly-generated carrier must travel in order to attain enough energy to be able to impact ionize [3]. When the APD multiplication-region dimension is in submicrons, the dead space becomes an important factor and needs to be included in the calculation of the excess noise [4,5].

One of the first analytical models to calculate the multiplication gain and the excess noise for APDs was developed by McIntyre [6] without taking the dead-space effect into account. This model, also known as the local ionization model, assumed that an electron (hole) at position *x*will impact ionize regardless of its ionization history. Consider a multiplication region extending from $x = 0$ to $x = w$, with an electric field applied in the negative *x*-direction and a photo-generated electron-hole pair at *x* inside the multiplication region. This electron-hole pair will start a chain of ionizations inside the multiplication region, and all electrons [holes] will undergo, on average, $\alpha(x')dx$ [$\beta(x')dx$] impact ionizations per unit distance, dx, where $0 \le x' \le w$. The multiplication factor, $M(x)$, for this device is the average total number of electron-hole pairs generated in the depletion layer from a single electron-hole pair at *x*. The formula for the multiplication factor was derived by McIntyre [6] as

(1) $$M(x) = \frac{\exp(-\int_x^w [\beta(x')-\alpha(x')]dx')}{1-\int_0^w [\beta(x')\exp(-\int_{x'}^w [\beta(x'')-\alpha(x'')]dx'')]dx'}.$$

Here, $M(0)$ is the overall mean gain, labeled $\langle G \rangle$, for a device with electron injection at location $x = 0$. In the special case when the electric field is constant across the multiplication region and the ionization coefficients are equal, we obtain

$$\langle G \rangle = \frac{1}{1-\alpha w}.$$

The excess noise factor, used as a measure of APD's gain fluctuation [7], is denoted as *F*and was found to be [6]

(2) $$F = k \langle G \rangle + (1-k)(2-\frac{1}{\langle G \rangle}),$$

where k is the ionization ratio, β/α. Since this model lacked the inclusion of the dead space, it failed to give an accurate representation of excess noise factor for devices with smaller multiplication regions [4], [8,9].

To account for the dead-space effect in APDs, Hayat *et al*. [3] developed the dead-space multiplication theory (DSMT) where they derived pairs of recurrent coupled integral equations to find the mean gain and excess noise factor. This model, called the non-local model, incorporated the carrier history in its calculations. Once the carriers have traversed the dead space, they are called *enabled*, with enabled ionization coefficients, $\alpha *$ and $\beta *$, for electrons and holes [8], respectively. These recursive integral equations were solved numerically [3], [9], using an iterative approach, referred to in this paper as the exact numerical method (ENM), with results confirmed subsequently by both Monte Carlo simulations [10] as well as experimental data [4], [8,9]. Unlike McIntyre's local-theory model, however, there was a lack of closed-form formulas for the mean gain and excess noise factor using the DSMT. Analytical expressions for mean gain and excess noise factor are useful in calculating other characteristics of the APD such as the signal-to-noise ratio and the error probability in optical receivers [11].

To address the need for analytical expressions for avalanche multiplication in the presence of dead space, Spinelli *et al.* solved the DSMT equations analytically using the first-order expansion of the recursive integral equations. Although their work included the analytical solution for the mean gain [12], it did not handle any excess noise calculations. Hayat *et al*. found an approximate solution to the DSMT equations and obtained closed-form approximate formulas for the mean gain and excess noise factor for the case of unequal ionization coefficients ($k \neq 1$) [13]. This approach has been termed as the characteristic method (CM) [13] and although the formula for the mean gain is relatively simple, the expression for excess noise factor involves the inversion of 9 by 9 matrix.

In this paper, we extend the CM approach and obtain the formulas for the mean gain and excess noise factor from [13] by assuming $k = 1$. This is a valid assumption for APDs where the multiplication width is small and the applied electric field is high. This phenomenon is depicted in **Fig. 1**, where the ionization parameters for Si, InP and GaAs have been plotted as a function of the electric field. It can be seen that as the applied electric field increases to the order of $\sim 10^6$ V/cm, the ionization ratio, k, can be approximated as 1. This approximation is useful in providing us with a simple analytic expression to estimate the mean gain and the excess noise factor in APDs.

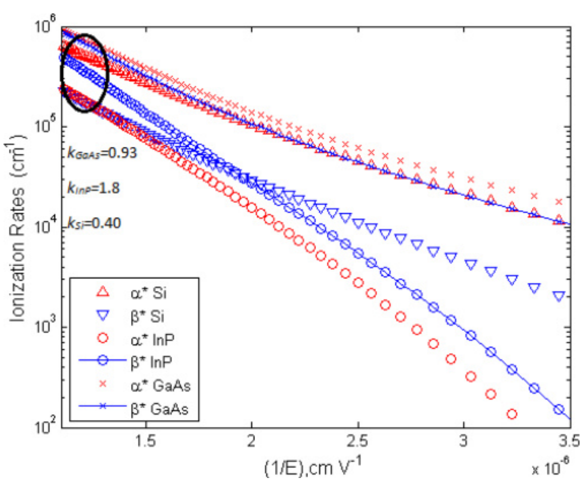

Fig. 1 The enabled ionization parameters, $\alpha *$ and $\beta *$, as a function of the inverse applied electric field for Si, InP and GaAs [14]. The encircled area highlights the ionization coefficients and electric field across the APD devices where the assumption $k \approx 1$ is valid. As an example, for a GaAs APD with the multiplication width = 0.05 − 0.1μm [4], which has $k = 0.86$ and mean gain = 8, the assumption of $k \approx 1$ may be used to get an estimation of the mean gain and the excess noise.

We will also use the formulas derived in this work to confirm the relationship between the enabled electron and hole ionization coefficients, α* and $\beta *$, and the experimental electron and hole ionization coefficients, α and β. This relationship was initially found by Spinelli *et al.* [12] and recently refined by Cheong *et al.* [14] with the inclusion of a physical interpretation. This connection is useful in extracting enabled ionization parameters, which cannot be measured directly, from the experimental ionization parameters, which are readily available in literature [14].

2. Formula for mean gain

We consider an electron (hole), born at location x inside a multiplication region, with a constant electric field applied in the negative *x*-direction. The electron can impact ionize after traveling the dead space, d_e (d_h in case of a parent hole), with enabled ionization coefficients, α^* and β^*, as given in [3]. After the ionization event happens, both the parent electron and secondary electron and hole must travel a dead space d_e (d_h) before they may impact ionize. By applying the CM technique, Hayat *et al*. determined the first and second moments of the random counts $Z(x)$ and $Y(x)$, the total number of carriers generated by an initial electron or hole, respectively, at position *x* in the multiplication region [13]. The random gain is then $G = 0.5(Z(0) + Y(0))$, which can be simplified to $G = 0.5(Z(0) + 1)$ using the initial condition, $Y(0) = 1$ [3]. After determining the first and second moments of the random counts, $z(x) = \langle Z(x) \rangle$, $y(x) = \langle Y(x) \rangle$, $z_2(x) = \langle Z^2(x) \rangle$ and $y_2(x) = \langle Y^2(x) \rangle$, the mean gain and the excess noise factor can be expressed as

(3) $$\langle G \rangle = 0.5(z(0) + 1)$$

and

(4) $$F = \frac{\langle G^2 \rangle}{\langle G \rangle^2} = \frac{(z_0(0) + 4\langle G \rangle - 1)}{4\langle G \rangle^2}.$$

To find the mean gain for the case, $\alpha * = \beta *$, we will solve the DSMT recursive integral equations using a method similar to that used in [13]. We find the mean of the random counts by starting with the differential form of the recurrence equations (1) and (3) from [13],

(5) $$z'(x) - \alpha^*[z(x) - 2z(x + d_e) - y(x + d_e)] = 0$$

and

(6) $$y'(x) + \beta^*[y(x) - 2y(x - d_h) - z(x - d_h)] = 0,$$

with the boundary conditions $z(x) = 1$ if $w - de \leq x \leq w$ and $y(x) = 1\ if\ 0 \leq x \leq d_h$.
Replacing β^* with α^* and assuming that the electron and hole dead spaces are equal ($d_e = d_h = d$), we obtain

(7) $$z'(x) - \alpha^*[z(x) - 2z(x + d) - y(x + d)] = 0$$

and

(8) $$y'(x) + \alpha^*[y(x) - 2y(x - d) - z(x - d)] = 0.$$

Here, to be able to find an analytical solution, we enforce the boundary conditions only at $x = w - d$ for $z(x)$ and at $x = d$ for $y(x)$. This simplification is the reason why, for the CM technique, the formulas obtained are approximate in nature. By applying this assumption, we can now take the general solutions to be $z(x) = c_1 e^{rx}$ and $y(x) = c_2 e^{rx}$, and solve for c_1 and c_2. For a non-zero solution to c_1 and c_2, we arrive at the following characteristic equation:

(9) $$(r - \alpha^* + 2\alpha^* e^{rd})(r + \alpha^* - 2\alpha^* e^{-rd}) + \alpha^{*2} = 0.$$

The solution to this equation gives a double root at $r = 0$, which leads to solutions of the form, $z(x) = c_1 + xc'_1$ and $y(x) = c_2 + xc'_2$. By inserting this solution into Eqs. (5) and (6) and comparing coefficients, we obtain $\alpha^* c'_1 + \alpha^* c'_2 = 0$ and $c'_1 + \alpha^* c_1 + \alpha^* c_2 + 2\alpha^* c'_1 d + \alpha^* c'_2 d = 0$. Next, by applying the boundary conditions, $z(w - d) = 1$ and $y(d) = 1$, and solving for the unknown coefficients, we find $z(0)$. By substituting $z(0)$ in **Eq. (3)**, we finally arrive at the expression for mean gain:

$$\langle G \rangle = \frac{1 + 2\alpha^* d}{1 + 3\alpha^* d - \alpha^* w},$$

which can be rewritten as

(10)

$$\langle G \rangle = \frac{1 + 2\tilde{\alpha}^* d'}{1 + 3\tilde{\alpha}^* d' - \tilde{\alpha}^*},$$

where $\tilde{\alpha}^* = \alpha^* w$ is the normalized enabled ionization coefficient and $d' = d/w$ is the normalized dead space. This formulation for the mean gain also follows directly from the mean gain expression using CM in [13] by applying the limit, $\lim\, \alpha^* \to \beta^* \ \langle G \rangle$, where $\langle G \rangle = \frac{\rho + \exp(rd)}{\rho \exp(r(w-d)) + \exp(rd)}$ and $\rho = \frac{-\alpha^* \exp(rd)}{(r - \alpha^* + 2\alpha^* \exp(rd))}$. On the other hand, by applying the same limit to the analytical mean gain developed by Spinelli *et al.* [12], obtained from applying the first order approximation to the recursive equations, we get

(11) $$\langle G \rangle = \frac{1}{1 + 2\tilde{\alpha}^* d' - \tilde{\alpha}^*},$$

which differs in form and is less accurate than the expression developed in **Eq. (10)**, as can be seen in **Fig. 2**, even for $d' = 0.1$.

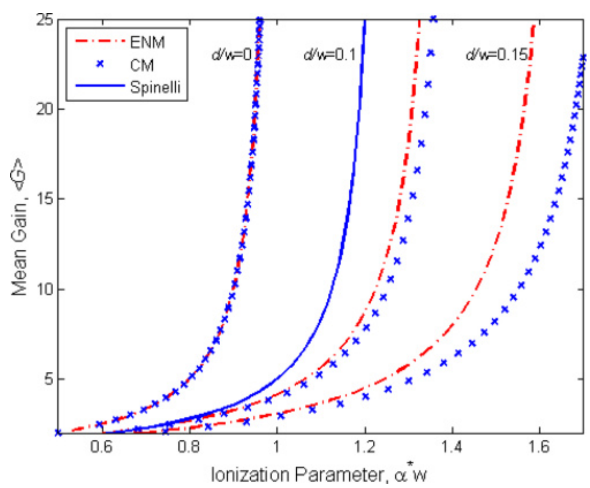

Fig. 2 Mean gain, found from ENM and CM techniques, is shown as a function of the ionization parameter, $\alpha^* w$, for $d' = d/w = 0, 0.1$ and 0.15. These results hold for any avalanche region for which the assumption, $k = 1$ is justified. The mean gain found from Spinelli analytical formulation is also shown for the case of $d' = 0.1$ for comparison.

We can isolate the effect of the dead space on the mean gain by writing **Eq. (10)** in terms of McIntyre's local-theory formula and a correction term, which contains the dead-space effect, and obtain

(12) $$\langle G \rangle = \frac{1}{1 - \tilde{\alpha}^*} + \frac{\tilde{\alpha}^* d' (1 + 2\tilde{\alpha})}{(\tilde{\alpha}^* - 1)(3\tilde{\alpha}^* d' - \tilde{\alpha}^* + 1)}.$$

Clearly, for the special case of negligible normalized dead space ($d' \approx 0$), the expressions for the mean gain from Eqs. (11) and (12) take the well-known form, shown in Eq. (13), and also match the formula from [6]

(13) $\langle G \rangle = \frac{1}{1-\alpha^* w}$.

3. Formula for excess noise factor

We now derive the expression for the excess noise factor for the case, $k = 1$. To do this, we need the second moments of $Z(x)$ and $Y(x)$, $z_2(x)$ and $y_2(x)$, respectively. We start by taking the differential form of the recursive equations (2) and (4) from [13] and substitute $\beta^* = \alpha^*$ to get

(14) $z_2'(x) - \alpha^*[z_2(x) - 2z_2(x+d) - y_2(x+d)] = -2\alpha^* z(x+d)(2y(x+d) + z(x+d))$

and

(15) $y_2'(x) + \alpha^*[y_2(x) - 2y_2(x-d) - z_2(x-d)] = 2\alpha^* y(x-d)(2z(x-d) + y(x-d))$.

The general, homogeneous and particular, solution of such a pair of inhomogeneous differential equations is a superposition of polynomials given by $z_2(x) = p_1 + p_2 x + p_3 x^2 + p_4 x^3 + p_5 x^4$ and $y_2(x) = q_1 + q_2 x + q_3 x^2 + q_4 x^3 + q_5 x^4$. By substituting these proposed solutions in Eqs. (14) and (15), comparing coefficients, and using the boundary conditions, $z_2(w - d) = y_2(d) = 1$, we obtain twelve equations with ten unknowns. By eliminating the redundant equations and solving the remaining independent equations, we find that the higher-order coefficients p_4, p_5, q_4 and q_5 are zero, which makes $z_2(x) = p_1 + p_2 x + p_3 x^2$ and $y_2(x) = q_1 + q_2 x + q_3 x^2$. By solving for p_1, p_2 and p_3, along with q_1, q_2 and q_3, we obtain

$$z_2(0) = \frac{3\alpha^{*3}d^3 + 5\alpha^{*3}d^2 w + \alpha^{*3} d w^2 - \alpha^{*3} w^3 + 7\alpha^{*2} d^2 + 6\alpha^{*2} d w - \alpha^{*2} w^2 + \alpha^* d + 5\alpha^* w + 1}{(3\alpha^* d - \alpha^* w + 1)^3}$$

Next, by substituting $z_2(0)$ and the expression for the mean gain in Eq. (10) into Eq. (4), we finally arrive at the approximate form of the excess noise factor:

$$F = \frac{12\alpha^{*3}d^3 - 4w\alpha^{*3}d^2 + 16\alpha^{*2}d^2 - 4w\alpha^{*2}d + 6\alpha^* d + 1}{(2\alpha^* d + 1)^2(3\alpha^* d - \alpha^* w + 1)},$$

which can be written in terms of the normalized quantities, $\tilde{\alpha}^*$ and d', as

(16) $F = \frac{12\tilde{\alpha}^{*3}d'^3 - 4\tilde{\alpha}^{*3}d'^2 + 16\tilde{\alpha}^{*2}d'^2 - 4\tilde{\alpha}^{*2}d' + 6\tilde{\alpha}^* d' + 1}{(2\tilde{\alpha}^* d' + 1)^2(3\tilde{\alpha}^* d' - \tilde{\alpha}^* + 1)}$.

To isolate the effect of the dead space on the excess noise factor, we rewrite Eq. (16) in terms of McIntyre's local-theory formula and a correction term, which contains the dead-space effect, and obtain

(17) $F = \frac{1}{1-\tilde{\alpha}^*} + f(d')$,

where the correction term, $f(d')$, is

$$\frac{-12\tilde{\alpha}^{*4}d'^3 + 4\tilde{\alpha}^{*4}d'^2 - 16\tilde{\alpha}^{*3}d'^2 + 4\tilde{\alpha}^{*3}d' - 6\tilde{\alpha}^{*2}d' - \tilde{\alpha}^* d'}{d'^3(-12\tilde{\alpha}^{*4} + 12\tilde{\alpha}^{*3}) + d'^2(4\tilde{\alpha}^{*4} - 20\tilde{\alpha}^{*3} + 16\tilde{\alpha}^{*2}) + d'(4\tilde{\alpha}^{*3} - 11\tilde{\alpha}^{*2} + 7\tilde{\alpha}^*) + 1 - 2\tilde{\alpha}^* + \tilde{\alpha}^{*2}}.$$

Again, for the special case of negligible normalized dead space ($d' \approx 0$), the expressions for excess noise factor from Eqs. (16) and (17) take the familiar form of Eq. (13), from [6], as expected.

To check the accuracy of Eq. (17), we computed the excess noise factor from both the CM and ENM techniques, as a function of the mean gain for normalized dead spaces, $d' = 0, 0.1$ and 0.15, as shown in Fig. 3. The effective McIntyre ionization ratio, k_{eff}, stated in Eq. (2), is fitted to the data from the different normalized dead spaces considered and also shown. As the normalized dead space becomes non-negligible, error is introduced in the excess noise factor obtained from the CM technique. For example, for $d' = 0.15$, we observe an error of 15% in the excess noise factor for a mean gain value of 20. Therefore, we can say that there is good agreement between the excess noise factor values found from the CM and ENM techniques up to normalized dead spaces of $d' = 0.15$.

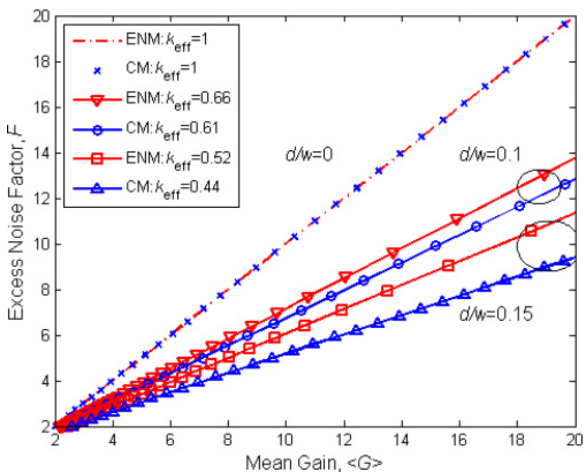

Fig. 3 The excess noise factor, F, as a function of the mean gain, $\langle G \rangle$, is shown for both the ENM and CM techniques. The normalized dead spaces of $d' = d/w = 0,\ 0.1$ and 0.15 are considered for comparison and the effective McIntyre ionization coefficient, k_{eff} is noted for each case and stated in the legend.

We note here that not only is the formula for excess noise factor found using the modified CM much simpler than solving the 9 by 9 matrix in the traditional CM [13], it also matches the ENM results better than the traditional method for cases when k can be approximated as 1, as shown in Fig. 4 for $k = 0.9$. The improvement in the approximation is because the $k = 1$ assumption in the modified CM formula tends to increase F, which, in turn, compensates for the underestimation that the traditional CM approach is known to exhibit. In addition, there are two ways to enforce the $k = 1$ condition in practice: by calculating the electron ionization coefficient and equating it to the hole ionization coefficient, or vice versa. When the ionization parameters for the dominant ionization parameters are chosen, a reduction in the excess noise factor is seen (up to 15%); hence we choose the ionization coefficients for the dominant mechanism in the material.

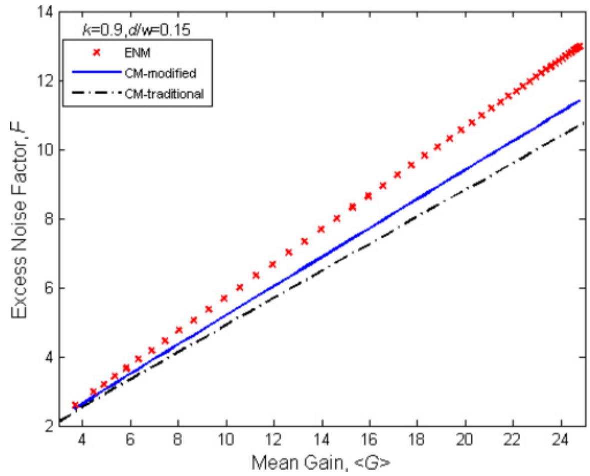

Fig. 4 The excess noise factor as a function of the mean gain is shown for the ENM and traditional CM [13] method for $k = 0.9$ and compared to the modified CM ($k = 1$). The normalized dead space, $d' = d/w$, is taken to be 0.15. It can be seen that the modified CM gives a better approximation than the traditional method.

To see how the formula for F, as shown in Eq. (16) or (17), works for estimating the noise in real devices, we calculate the excess noise factor as a function of the mean gain for different materials. The methodology is as follows: we fix the multiplication width of the device in consideration, use the ionization coefficients of the dominant carrier and assume $k = 1$. The dead spaces are calculated for the dominant carrier as a function of the applied field and Eqs. (10) and (16) are then applied to obtain the approximate mean gain and excess noise factor. This is done for different applied fields and hence the approximate $F\ vs.\ \langle G \rangle$ graph for that particular multiplication width is obtained. This methodology is then repeated for different multiplication widths and we obtain approximate curves for excess noise factor as a function of the mean gain. For comparison, the mean gain and excess noise factor are found for the $k = 1$ case of McIntyre's local-theory model, while for ENM technique we consider the scenario of unequal ionization coefficients ($k \neq 1$). The results are shown in Fig. 5 for the case of GaAs, using the enabled ionization parameters and ionization threshold energies reported in [14], for different multiplication widths, along with experimental data from real GaAs APD devices [15], with multiplication widths of 500 and 800 nm, respectively, for comparison.

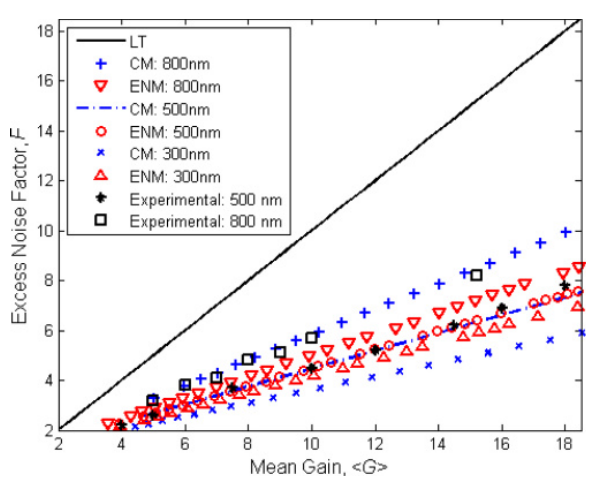

Fig. 5 The excess noise factor, F, shown as a function of the mean gain for various multiplication widths of GaAs. The CM technique predicts the excess noise far better than McIntyre's local-theory (LT) model with equal ionization coefficients assumption. Data is also shown for experimental GaAs APDs of widths 500 and 800 nm, respectively, as reference [15].

For a more accurate analysis, we consider the $k \neq 1$ case for both the ENM and McIntyre's local-theory model and document the relative errors in noise (defined as the difference in the excess noise factor with respect to that from the ENM technique divided by the excess noise factor from ENM) for the CM technique and McIntyre's local-theory model. We do this for GaAs, InP and Si, with results shown in **Fig. 6** for a gain of 22. For smaller multiplication widths (≤ 700nm), the relative error between the McIntyre's local-theory model as compared to the ENM is greater than or equal to 50%, and hence it fails to predict the excess noise factor accurately for smaller multiplication widths of these materials. The CM technique, on the other hand, provides an excess noise value within 15% of the ENM for a range of multiplication widths for GaAs, InP and Si APDs, even though the normalized dead space exceeds 15%.

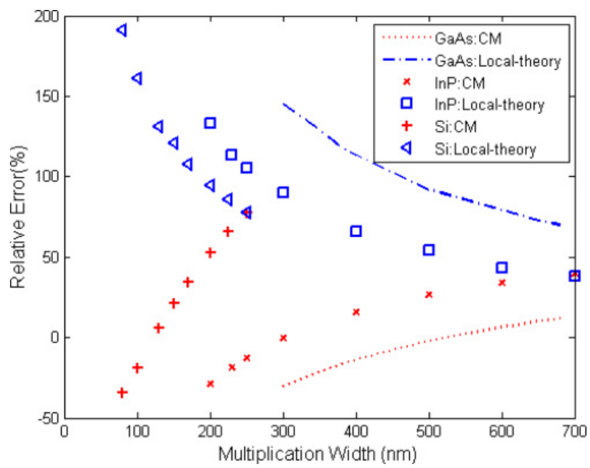

Fig. 6 Relative errors between the excess noise factors, found by comparing the ENM to McIntyre's local-theory model (with $k \neq 1$) and the CM technique, are shown. The errors are plotted as a function of various multiplication widths of GaAs, InP and Si APD devices for a mean gain of 22. We use these values to determine the multiplication widths for which the CM approximation may be used practically.

The expectation, while calculating the mean gain and excess noise, is that the approximation should work well for materials with $k \approx 1$ (such as GaAs), and that we should attain lower multiplication widths using such materials. However, not only are the mean gain and excess noise factor dependent on the set of ionization parameters chosen from literature (and hence differing *k*), they are also sensitive to the d' value at which the calculation is performed. For all materials considered, the minimum multiplication width that gives excess noise within 15% of the ENM is found when d' is no larger than 0.24. The range of materials and multiplication widths for which the CM approximation may be used to predict the mean gain and excess noise factor are listed in **Table 1**. The range of widths listed here are reasonable for thin APD devices such as the silicon CMOS-compatible pn devices developed in [**16**] by Hossain *et al.*

Table 1. Material widths for which the CM techniques predicts noise within 15% of the ENM. The upper limit of *d'* corresponds to the lower limit of the multiplication width and vice versa. From [**14**], the second set of ionization parameters are used for GaAs and Si whereas the third set is used for InP.

Material	Multiplication widths (nm)	d'
GaAs [9]	220 – 475	0.107 – 0.180
GaAs [14]	400 – 680	0.135 – 0.195
InP [9]	137 – 200	0.176 – 0.210
InP [14]	230 – 400	0.142 – 0.200
Si [14]	110 – 140	0.210 – 0.240

Next, for a particular device width, we look at the dependence of the relative error in noise on the mean gain for the CM technique as well as the McIntyre's local-theory model. The results for GaAs, InP and Si are shown in **Fig. 7**. For the CM technique, the relative error becomes constant after a mean gain of 20, and hence, it can predict the excess noise for the APD devices listed in **Table I** for even higher gains without increasing the relative error in the excess noise calculation.

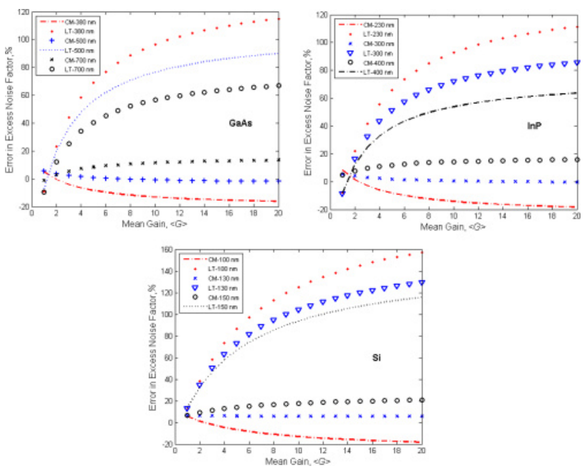

Fig. 7 Relative errors in the excess noise calculated from McIntyre's local-theory (LT) model ($k \neq 1$) and the CM technique, as compared to ENM technique for three different widths of (a) GaAs, (b) InP and (c) Si [14]. As the gain increases, the relative error associated with the CM technique approaches a constant value.

Finally, we summarize the three main factors that govern the accuracy of the reported simplified formula for the excess noise factor in real devices. First, any violation of the $k = 1$ assumption causes the approximate CM formula to overestimate the excess noise factor, F, assuming that the dominant carrier, i.e., the carrier with the higher ionization coefficient, initiates the avalanche process. Second, our choice to set the ionization coefficient of the non-dominant carrier to be equal to that of the dominant carrier makes the effect of dead space more significant (since a smaller field is required to achieve the same ionization coefficient value, which leads to a larger dead space) and, in turn, forces F to decrease. Of course, the opposite choice will lead to an overestimation of F. Third, the increased value of the normalized dead space (e.g., when the width of the multiplication region is reduced by design) also helps underestimate F. Together, these inter-playing factors limit the widths for which the excess noise factor approximation may be successfully used. Consequently, for a given material there exists a range of multiplication-region widths (e.g., as shown in **Table 1**) over which all three competing factors balance out and we obtain a good accuracy in the approximation of the excess noise factor.

4. Relationship between the enabled and experimental ionization parameters

The first attempt at finding the relationship between the enabled ($\alpha *$ and $\beta *$) and experimental (α and β) ionization coefficients was made by Spinelli *et al.* [12], where they equated the multiplication factor found from the first-order approximation of the DSMT and the experimental results. However, they could not explain the physics behind the relationship developed in their findings. Recently, Cheong *et al.* have developed a similar relationship between the two kinds of ionization coefficients by taking into account the physics of the ionization events. This was done by equating the mean ionizing lengths from the DSMT and the local model and comparing them for the same electric field in identical p-i-n structures [14]. Their results are confirmed here, for the special case of $k = 1$. We start with the equation to evaluate mean gain in an APD using the local ionization theory and with the assumption of equal experimental coefficients [6]

(18) $$\langle G \rangle = \frac{1}{1-\alpha w}.$$

Next, we equate Eq. (18) to the mean gain from Eq. (10), and simplify the expression to obtain

(19) $$\alpha = \frac{1-(d/w)}{(\alpha^{*})^{-1}+2d}.$$

Here, α is called α_{device} by Cheong *et al*. [14], and Eq. (19) matches the relationship found in [14].

The device ionization coefficient in Eq. (19) can be used in the traditional formula in Eq. (18) to find a mean gain value that matches the value found through the CM but it fails to predict the excess noise factor correctly, which is as expected. Therefore, to find the excess noise factor in thin APDs with non-negligible normalized dead spaces, we must either use the ENM technique to solve the DSMT recursive integral equations, or the formula given in Eq. (17) for a good approximation for which we require the enabled ionization coefficients.

One way to find the enabled ionization coefficients is by fitting the gain and noise data to the DSMT directly [8, 9]. Using this method, we can search for the values of $\alpha*$ and $\beta*$ (by solving for $\langle G \rangle$ and F after varying $\alpha*$ and $\beta*$) that yield specified gain and excess noise factor. A simpler way to find the enabled ionization parameters is by using the relationship between the enabled and experimental ionization coefficients, found by Cheong *et al.* [14]. Once the enabled ionization coefficients are known, we can easily predict the mean gain and excess noise factor, using Eqs. (10) and (16), respectively.

5. Conclusions

We have found simple approximate formulas to calculate the mean gain and excess noise factor for APDs using the dead-space multiplication theory under the assumption of equal ionization coefficients for electrons and holes. The electric field was assumed to be constant across the multiplication region and the formulas derived require the use of enabled ionization coefficients. The formulas for the excess noise factor, shown in Eqs. (16) or (17), perform very well for a range of multiplication widths and materials (listed in Table 1), yielding errors that are below 15% when compared to the exact values for the excess noise factor. By using the enabled ionization coefficients in the approximate formulas derived in this work, the mean gain and the excess noise factor in APDs can be easily estimated.

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References

1. N. Namekata, S. Adachi, and S. Inoue, “Ultra-low-noise sinusoidally gated avalanche photodiode for high-speed single-photon detection at telecommunication wavelengths,” IEEE Photon. Technol. Lett. **22**(8), 529–531 (2010).

2. A. Tosi, A. Della Frera, A. B. Shehata, and C. Scarcella, “Fully programmable single-photon detection module for InGaAs/InP single-photon avalanche diodes with clean and sub-nanosecond gating transitions,” Rev. Sci. Instrum. **83**(1), 013104 (2012).

3. M. M. Hayat, B. E. A. Saleh, and M. C. Teich, “Effect of dead space on gain and noise of double-carrier-multiplication avalanche photodiodes,” IEEE Trans. Electron. Dev. **39**(3), 546–552 (1992).

4. K. F. Li, D. S. Ong, J. P. R. David, G. J. Rees, R. C. Tozer, P. N. Robson, and R. Grey, “Avalanche multiplication noise characteristics in thin GaAs p+-in+ diodes,” IEEE Trans. Electron. Dev. **45**(10), 2102–2107 (1998).

5. V. Chandramouli, C. M. Maziar, and J. C. Campbell, “Design considerations for high performance avalanche photodiode multiplication layers,” IEEE Trans. Electron. Dev. **41**(5), 648–654 (1994).

6. R. J. McIntyre, “Multiplication noise in uniform avalanche diodes,” IEEE Trans. Electron. Dev. **13**(1), 164–168 (1966).

7. M. C. Teich, K. Matsuo, and B. E. A. Saleh, “Excess noise factors for conventional and superlattice avalanche photodiodes and photomultiplier tubes,” IEEE J. Quantum Electron. **22**(8), 1184–1193 (1986).

8. M. A. Saleh, M. M. Hayat, B. E. A. Saleh, and M. C. Teich, “Dead-space-based theory correctly predicts excess noise factor for thin GaAs and AlGaAs avalanche photodiodes,” IEEE Trans. Electron. Dev. **47**(3), 625–633 (2000).

9. M. A. Saleh, M. M. Hayat, P. P. Sotirelis, A. L. Holmes, J. C. Campbell, B. E. A. Saleh, and M. C. Teich, “Impact-ionization and noise characteristics of thin III–V avalanche photodiodes,” IEEE Trans. Electron. Dev. **48**(12), 2722–2731 (2001).

10. D. S. Ong, K. F. Li, G. J. Rees, G. M. Dunn, J. P. R. David, and P. N. Robson, “A monte carlo investigation of multiplication noise in thin p+-in+ GaAs avalanche photodiodes,” IEEE Trans. Electron. Dev. **45**(8), 1804–1810 (1998).

11. G. P. Agrawal, *Fiber-optic Communication Systems* (John Wiley & Sons, 2012).

12. A. Spinelli and A. L. Lacaita, “Mean gain of avalanche photodiodes in a dead space model,” IEEE Trans. Electron. Dev. **43**(1), 23–30 (1996).

13. M. M. Hayat, Z. Chen, and M. A. Karim, “An analytical approximation for the excess noise factor of avalanche photodiodes with dead space,” IEEE Electron. Dev. Lett. **20**(7), 344–347 (1999).

14. J. S. Cheong, M. M. Hayat, X. Zhou, and J. P. R. David, “Relating the experimental ionization coefficients in semiconductors to the nonlocal ionization coefficients,” IEEE Trans. Electron. Dev. **62**(6), 1946–1952 (2015).

15. K. A. Anselm, P. Yuan, C. Hu, C. Lenox, H. Nie, G. Kinsey, J. C. Campbell, and B. G. Streetman, “Characteristics of GaAs and AlGaAs homojunction avalanche photodiodes with thin multiplication regions,” Appl. Phys. Lett. **20**(26), 3883–3885 (1997).

16. M. M. Hossain, P. Zarkesh-Ha, J. P. R. David, and M. M. Hayat, “Low breakdown voltage CMOS compatible pn junction avalanche photodiode,” in *Proceedings of IEEE Photonics Conference* (IEEE, 2014), pp. 170–171.