10-2021

Using Reflective Journals to Characterize Pre-Service Teacher Professional Noticing Skills

Leigh A. Van den Kieboom

Teacher Educator, Vol. 86, No. 4 (October/December 2021): 347-371. DOI.

Using Reflective Journals to Characterize Pre-Service Teacher Professional Noticing Skills

Leigh A. van den Kieboom
Marquette University

ABSTRACT

This study examined professional teacher noticing in the context of written responses 12 pre-service teachers (PSTs) provided in a reflect- ive journal after posing addition and subtraction problems for students in an after-school tutoring program. Professional teacher noticing skills, attending, interpreting, and deciding, were situated within the well-defined mathematics content and associated trajectory of student strategies for the meaning of addition and subtraction. A three-point rubric was designed and utilized to analyze PST attending, interpreting, and deciding responses. Results characterize what PSTs "could do" relative to each skill as well as development of noticing skills over time. Results also highlight how PSTs included a rationale and next steps in deciding responses that related to sup- porting student understanding of the meaning of addition and subtraction, providing additional practice, and teaching solution strategies. Implications for teacher education are discussed.

Introduction

Supporting students in learning mathematics with understanding by making sense of mathematical concepts and procedures has been a fundamental component of K-12 mathematics education reform initiatives (Kilpatrick et al., [17]; National Council of Teachers of Mathematics, [22], [23]). Teachers who support students in learning mathematics with understanding use student thinking to guide their instruction (Franke et al., [10]). To help teachers, including pre-service teachers (PSTs), to learn how to use student thinking to guide instruction, mathematics education researchers have begun to characterize the skills involved in the professional noticing of children's mathematical thinking (Jacobs et al., [15]). This set of interrelated skills include: (1) attending to student strategies, (2) interpreting student understanding, and (3) deciding how to respond based on student understanding. Mathematics teacher educators argue that teacher education programs must provide PSTs opportunities to develop professional noticing skills early in their program (Star et al., [30]; van Es, [34]).

The purpose of this paper is to present work and research related to PSTS and professional teacher noticing. More specifically, the work and research reported in this paper utilized a rubric to characterize the professional noticing skills, attending, interpreting, and deciding, PSTs recorded in a reflective journal after planning and implementing instruction for a small group of students over the course of an eight week after-school tutoring program. Two frameworks guided analysis: (1) professional teacher noticing of children's mathematical thinking (Jacobs et al., [15]) and, (2) research on mathematics content and related student thinking about the meaning of addition and subtraction (Carpenter et al., [4]; Dougherty, [8]; National Governor's Association [NGA] & Council of Chief State School Offices [CSSO] Common Core State Standards in Mathematics, [CCSSM], 2010; National Research Council, [NRC], [24]).

Literature review

Professional teacher noticing

The research around professional teacher noticing builds on Mason's ([19]) idea that competent professionals who are experts in their field are aware of particularly relevant events and know how to foreground such events when making informed decisions. Mathematics teacher educators involved in this work generally agree that professional teacher noticing includes recognizing and making sense of significant events during classroom instruction (Sherin & van Es, [29]; Star et al., [30]; Star & Strickland, [31]; van Es & Sherin, [35]). Jacobs et al. ([15]) developed a conceptual framework for professional teacher noticing based on their analysis of how 131 practicing and PSTs responded to written prompts after viewing video clips of students solving whole-number problems. Their framework utilized a rubric to describe what experts "could do" (p. 180) and includes three interrelated skills. The first skill, attending, refers to the ways in which teachers focus on the mathematical details involved in strategies students use to solve problems. Interpreting, the second skill, requires teachers to use evidence from student mathematical thinking to make inferences about what students know about mathematics and how they know it. The third skill, deciding, describes how teachers use their reasoning about student mathematical understanding to make in-the-moment decisions during instruction. The framework makes the discrete noticing skills explicit while positioning them as an integrated set that supports teachers in making instructional decisions based on student understanding.

Professional teacher noticing and PSTs

Early research on professional teacher noticing and PSTs relied heavily on the use of video-recordings of classroom instruction (Castro Superfine et al., [5]; Males, [18]; McDuffie et al., [21]; Schack et al., [25]; Star et al., [30]; Star & Strickland, [31]). Results of this research uncovered PST proclivity to attend to insignificant events such as classroom management (Star & Strickland, [31]), prompting teacher educators to develop guidelines, tools, or scaffolding, to enhance PST noticing. For instance, Star et al. ([30]) and Males ([18]) engaged their secondary PSTs in guided viewing of video-recordings to help them attend to noteworthy features of instruction such as the mathematical content involved in a lesson. Guided viewing supported secondary PSTs in Star et al's. ([30]) study in attending to classroom environment and management. Secondary PSTs in Males ([18]) study identified communication and mathematics content as noteworthy components of video-recorded lessons.

In a similar fashion, McDuffie et al. ([21]) created a tool to help PSTs analyze video-recordings through the perspective of four lenses: (1) teaching, (2) learning, (3) task, and (4) power and participation. The tool supported PSTs in moving from a simple retelling of events to discussing significant interactions related to mathematics. Castro Superfine et al. ([5]) utilized teacher educator scaffolding to support PST analysis of video-recordings of classroom instruction. They found that video-recordings must be carefully selected to maximize PST learning. They also uncovered the interrelated nature of teacher noticing skills, maintaining that teacher educators should first help PSTs to learn how to interpret classroom events as a first step in developing all three skills.

Other researchers have situated professional teacher noticing within the context of a mathematics domain that includes student learning trajectories. Schack et al. ([25]), for instance, asked PSTs to observe a video-recording of a mathematics teacher educator conducting a one-on-one diagnostic interview to assess elementary student stages of early arithmetic learning. PSTs then conducted their own diagnostic interview in a field experience, using their noticing skills to identify stages of early arithmetic learning. PSTs improved all three skills. Schack et al. ([26]) used the context of one-on-one diagnostic interviews in a field experience, intentionally pairing professional noticing with student thinking about the equal sign and equality. Results indicated that PSTs noticed the strategies students used to solve a task without necessarily focusing on student thinking about the equal sign and equality, suggesting that PSTs needed more scaffolding in learning how to notice this aspect of student thinking. Ivars et al. ([14]) situated professional teacher noticing in the context of the part-whole meaning of fractions. They found that the student learning trajectory related to part-whole fractions supported PSTs in learning how to notice student understanding. Fernández et al. ([9]) coupled professional teacher noticing with student thinking about proportional and non-proportional reasoning. Results from their study, in which PSTs examined student solutions to proportional reasoning problems, were used to create a trajectory to characterize the development of PST noticing skills. The four-level trajectory categorized PSTs who distinguished proportional situations (level 2) from those who did not (levels 1) and identified PSTs who used mathematically significant details to interpret student thinking (level 4) from those who did not (level 3).

The meaning of addition and subtraction: Mathematics content and student thinking

A substantial portion of the elementary mathematics curriculum focuses on the meaning of addition and subtraction (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]; NRC, [24]). If elementary teachers are to use student thinking about addition and subtraction to drive instruction, it is critical that they develop a deep knowledge of: (1) concepts and procedures involved in adding and subtracting, (2) conceptual categories of problem types that foster student understanding of the meaning of addition and subtraction, and, (3) strategies students use when solving addition and subtraction problems.

Carpenter and colleagues (Carpenter et al., [3]) were the first to argue that elementary students develop an understanding of the meaning of addition and subtraction when they encounter and solve problems in their everyday lives. The ideas behind their research-based model, Cognitively Guided Instruction (CGI), are included in curricular documents and research summaries that guide elementary teachers in their work (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]; NRC, [24]). As Figure 1 illustrates, the taxonomy of addition and subtraction problems can be conceptualized as: (1) Add To and Take From situations in which the result, change, or start quantity is unknown, (2) Put Together/Take Apart situations in which numbers are composed or decomposed, and, (3) Compare situations in which the differences between quantities are examined. While Add To and Take From problem types involve explicit action, Put Together and Compare problem types comprise static relationships.

Figure 1. Taxonomy of addition and subtraction problem types.

	Result Unknown	**Change Unknown**	**Start Unknown**
Add To	Martin had 9 pencils. His brother gave him 2 more pencils. How many pencils does Martin have now?	Martin had 7 pencils. His brother gave him some more pencils. Now Martin has 9 pencils. How many pencils did his brother give him?	Martin had some pencils. His brother gave him 2 more pencils. Now Martin has 9 pencils. How many pencils did Martin have to start?
Take From	Martin had 9 pencils. He gave 5 pencils to Jasmine. How many pencils does Martin have now?	Martin had 9 pencils. He gave some pencils to Jasmine. Now Martin has 5 pencils. How many pencils did Martin give to Jasmine?	Martin had some pencils. He gave 4 pencils to Jasmine. Then he had 5 pencils left. How many pencils did Martin have to start?
	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put Together/ Take Apart	Martin has 2 red pencils and 9 blue pencils. How many pencils does Martin have?	Martin has 9 pencils. Two are red and the rest are blue. How many pencils are blue?	Martin has 9 pencils. How many can he put in his yellow pencil case and how many in his purple pencil case?
	Difference Unknown	**Bigger Unknown**	**Smaller Unknown**
Compare	Martin has 7 pencils. Jasmine has 9 pencils. How many more pencils does Jasmine have than Martin?	Jasmine has two more pencils than Martin. Martin has 7 pencils. How many pencils does Jasmine have?	Jasmine has two more pencils than Martin. Martin has 9 pencils. How many pencils does Jasmine have?

Research summaries and curricular documents (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]; NRC, [24]) also use learning trajectories to describe how elementary students develop increasingly sophisticated strategies to solve addition and subtraction problems. Over the course of several years, elementary students progress through stages as they begin to make sense of and solve addition and subtraction problems: (1) Direct Modeling; using physical objects to show the action found in a problem; (2) Counting; counting forward or backward to find the total or unknown quantity in a problem, (3) Derived Facts; using a known fact to solve a problem, and, (4) Recalled Facts; retrieving a fact from memory to solve a problem (Figure 2).

Figure 2. Taxonomy of student solution strategies. Adapted from Carpenter et al., [4]; CCSSM, [24], Dougherty, [8]; NRC, [24].

Solution Strategy	Problem Type	Example
Direct Modeling: Represent and solve a problem situation using objects, drawings, or fingers to model the explicit action in the problem.	**Add To Result Unknown:** Martin had 9 pencils. His brother gave him 2 more pencils. How many pencils does Martin have now?	**Student Strategy:** A set of 9 objects and 2 objects is constructed. The two sets are joined and the total of the two sets is counted.
Counting: Represent and solve a problem situation by counting forward or backward to find the total or unknown quantity in a problem.	**Compare Difference Unknown:** Martin has 7 pencils. Jasmine has 9 pencils. How many more pencils does Jasmine have than Martin?	**Student Strategy:** Backward counting sequence is initiated from 9. Sequence continues for 2 more counts. The last number in the counting sequence is the answer.
Derived Facts: Represent and solve a problem by using a known fact to solve the problem.	**Compare Smaller Unknown:** Jasmine has two more pencils than Martin. Martin has 9 pencils. How many pencils does Jasmine have?	**Student Strategy:** A known number fact is used to compose or decompose the numbers in the problem. "I know that IO plus 2 is 12 and 9 is one less than 10 so the answer would be 11.
Recalled Facts: Represent and solve a problem by stating the answer.	**Put Together/Take Apart:** Martin has 9 pencils. Two are red and the rest are blue. How many pencils are blue?	**Student Strategy:** The student states the answer, 7, retrieving the number fact from memory.

Research shows that posing these types of addition and subtraction problems and asking students to communicate the strategies they use to solve them is effective in several ways. Teachers who use this approach in their practice create a more productive classroom environment for students, improve student achievement, and strengthen their own content and pedagogical knowledge (Carpenter et al., [3]; Clarke, 2008; Franke et al., [10]).

Framing the study

The research presented in this paper utilized Jacob et al's. ([15]) construct of professional teacher noticing. The study reported here is similar to others in that PSTs were provided opportunities to learn about and develop all three noticing skills while enrolled in a teacher preparation program. Moreover, as similar to other investigations (Fernández et al., [9]; Ivars et al., [14]; Schack et al., [25], [26]), in this study, professional teacher noticing was situated in the context of a specific mathematics domain, the meaning of addition and subtraction, which includes a well-developed trajectory of student thinking.

On the other hand, this study is different because PSTs autonomously planned and implemented lessons for a small group of students in an after-school tutoring program and then recorded their professional teacher noticing skill in a reflective journal. To date, most studies about PSTs and professional teacher noticing have relied on video-recordings of mathematics instruction (Castro Superfine et al., [5]; Males, [18]; McDuffie et al., [21]; Schack et al., [25]; Star et al., [30]; Star & Strickland, [31]) or artifacts of student solutions (Fernández et al., [9]; Ivars et al., [14]) as proxies for authentic work with students.

Research goal and research questions

Mathematics education reform initiatives create a vision in which all students learn mathematics with understanding while developing mathematical practices commonly used in the discipline (CCSSM, 2010; Kilpatrick et al., [17]; NCTM, [22], [23]). Teachers who support their students in developing this kind of understanding know how to use student thinking to inform their instruction. Supporting PSTs in learning how to notice student thinking plays a key role in preparing them to implement this vision into their own practice.

This study was designed as an instructional intervention in an integrated mathematics content and methods course (with a concurrent field experience) with the goal of preparing PSTs for a future practice anchored in mathematics education reform efforts focused on student thinking (CCSSM, 2010; Kilpatrick et al., [17]; NCTM, [22], [23]). The purpose of the intervention was to engage PSTs in learning about and enacting professional teacher noticing (Jacobs et al., [15]) in the context of the well-defined mathematical content and associated student thinking about the meaning of addition and subtraction (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]; NRC, [24]). The study examined the following questions:

When recording professional teacher noticing skills in a reflective journal -

1. Attending; What strategies do PSTs identify in elementary student solutions to addition and subtraction problems and what mathematical details do PSTs provide when identifying these strategies?
2. Interpreting; What evidence do PSTs use to interpret elementary student understanding of addition and subtraction?
3. Deciding; What evidence of elementary student understanding of addition and subtraction do PSTs use in deciding what to do next?

Method

Context and participants

An elementary number and operations mathematics content course and concurrent mathematics methods course with a field experience (College of Education after-school tutoring program) provided the context for this study. Taught jointly by faculty from the Department of Mathematics and the College of Education (author), the integrated courses drew on situated cognition learning theory (Brown et al., [2]; Greeno, [11]) to support PSTs in learning how to teach mathematics with a view toward student thinking. Accordingly, PST learning was situated in the context of authentic activity, teaching mathematics in an after-school tutoring program. Thus, PSTs learned how to utilize professional teacher noticing as a conceptual tool to develop a vision for their beginning practice.

Participants included 12 PSTs, 83% females, 17% males, 75% Caucasian, and 25% African American students, enrolled in the courses at a mid-sized private university in the Midwestern U.S. At the time of the study, the participants, who were seeking a license to teach in grades one through eight and were one semester away from their student-teaching experience.

Mathematics content course

The mathematics content course met two days a week for 75-minutes each session throughout a 16-week semester and addressed topics in elementary number and operations. During the first four weeks of the course and prior to work with elementary students in the after-school tutoring program, PSTs engaged in a unit of study about operations with whole numbers to increase their knowledge of how and why the procedures for operating on single and multi-digit numbers work. As part of this unit, PSTs learned about the taxonomy of problem types and the increasingly sophisticated solution strategies, practiced identifying and writing problems, and engaged in discussions about how different problems types supported students in developing an understanding of the meaning of addition and subtraction.

Mathematics methods course and after-School tutoring program

The concurrent mathematics methods course was taught in the College of Education by the author and involved eight weeks of university-based instruction followed by eight weeks of work with elementary students in the College of Education's after-school tutoring program. During the eight weeks of university-based instruction, and prior to work with elementary students in the after-school tutoring program, PSTs learned about professional teacher noticing by reading about the construct (Thomas et al., [33]) and engaging in classroom discussions about the interrelated set of skills. They also rehearsed, with each other, how to: (1) pose addition and subtraction problems, (2) attend to strategies students used to solve problems, (3) interpret student understanding, and, (4) make instructional decisions based on student understanding. In addition, to prepare for the work of attending, interpreting and deciding in the after-school tutoring program, PSTs analyzed video-recordings of classroom instruction in which elementary students solved addition and subtraction problems.

Throughout the eight weeks in the after-school tutoring program, each PST worked with a small group (3) of second grade students who were recommended for participation by their regular classroom teacher based on their low performance in the mathematics. PSTs planned and implemented thirty-minutes of instruction two times per week for eight weeks. A regular part of this instruction included posing addition and subtraction problems for students to solve.

While working with students in the after-school tutoring program, each PST used a reflective journal to chronicle one student's approach (the same student over the course of the eight weeks) to solving addition or subtraction problems. Questions in the journal followed Schön's ([27]) model for reflection, asking PSTs to use their experiential knowledge when looking back on their teaching to engage in reflection-on-action. More specifically, questions in the reflective journal asked PSTs to draw on their knowledge of student understanding of the meaning of addition and subtraction and professional teacher noticing to record: (A) What problem did you pose for your student? (B) Attending – What strategy did your student use to solve the problem you posed? (C) Interpreting – What did you learn about your student's understanding of the meaning of addition or subtraction as he/she solve the problem? (D) Deciding – Based on what you learned about your student's understanding of the meaning of addition or subtraction what will you do next? In summary, over the course of the semester, PSTs (N = 12) completed 16 journal entries that detailed their professional noticing skills related to the problems (n = 191)[1] they posed for one elementary student (N = 12).

Data sources

Data for this study included 16 journal entries PSTs completed over the course of eight weeks. The content PSTs included in their response to each of the reflective journal questions served as the unit of analysis. PST responses to journal questions differed in terms of word count with a mean word count of 634 words, a minimum word count of 175 words and a maximum word count of 1278 words.

Data analysis

Data for this study were analyzed in two phases. First, a qualitative approach (Strauss & Corbin, [32]) was used to analyze the content (Berg, [1]) of PST journal responses. To do so, the author and graduate research assistant developed a three-point rubric to assess responses related to attending, interpreting, and deciding. The rubric was developed with a view toward Jacobs et al's. ([15]) framework for professional teacher noticing and the literature on the mathematics content and associated student thinking about the meaning of addition and subtraction (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]; NRC, [24]). Second, quantitative analyses were conducted to determine a mean score for each skill and compare PST responses over time. A one-way repeated measure analysis of variance (ANOVA) were conducted to compare the means of the three noticing skills and a z-test and t-test performed to explore PST noticing skills over time.

Phase one: Analyzing PST responses

Attending.

The three-point rubric was first used to analyze responses PSTs provided for reflective journal question B: What strategy did your student use to solve the problem you posed? Thus, an attending response was evaluated as (3), *Proficient* if the PST identified the strategy their student used to solve the problem and provided "significant mathematical details" (Jacobs et al., [15], p. 170), related to the meaning of addition or subtraction, to specifically describe how their student used the strategy to solve the problem. Several examples follow to illustrate significant mathematical details in the context of the meaning of addition and subtraction. For instance, a PST who provided significant mathematical details when attending to a direct modeling strategy, explained how their student used cubes to show the action in a particular problem, detailing, for example, how their student created: (a) a set of cubes and removed cubes to model subtraction, (b) two sets of cubes and joined the sets to model addition, or, (c) two sets of cubes and compared sets to find the difference. A PST who provided significant mathematical details when attending to a counting strategy noted the numbers their student used in a counting sequence as well as whether their student counted up, counted down, or counted on from the larger number. A PST who included significant mathematical details when attending to a derived fact strategy chronicled the chain of thinking their student provided when using a known fact to solve a problem (i.e., "I know that 10 + 10 is 20 and 10 + 9 is one less than 10 + 10 so the answer would be 19.").

In contrast, an attending response was assessed as (2), *Developing* if the PST identified the strategy their student used to solve the problem but provided limited mathematical details to generally describe how their student used the strategy. Responses including limited mathematical details simply named a direct modeling, counting, or derived fact strategy without specifically describing how their student employed the strategy to solve the problem.

Finally, an attending response was evaluated as (1), *Needs Improvement* if the PST failed to identify the strategy their student used to solve the problem. In these responses, PSTs simply reported that their student solved the problem without identifying the strategy used.

Interpreting.
Next, the three-point rubric was utilized to assess PST responses to journal question C: What did you learn about your student's understanding of addition or subtraction as he/she solved the problem? Again, as with Jacobs et al's. ([15]) rubric, interpreting responses were evaluated in terms of evidence PSTs used to make inferences about student understanding. Accordingly, an interpreting response was evaluated as (3), *Proficient* if the PST used significant mathematical details (see previous examples) from the strategy their student used to solve the problem as well as details about how the student conceptualized the meaning of addition or subtraction in the problem, as evidence to make inferences about their student's understanding of addition or subtraction. On the other hand, a response was evaluated as (2), *Developing* if the preservice teacher used limited mathematical details from the strategy their student used to solve the problem, and generally referenced how their student conceptualized the meaning of addition or subtraction in the problem, as evidence to make inferences about their student's understanding of addition or subtraction. PSTs whose interpreting responses were appraised as (1), *Needs Improvement* failed to interpret their student's understanding of addition or subtraction. In these responses, PSTs neglected to use mathematical details as evidence to make inferences about their student's understanding of addition or subtraction.

Deciding.
Lastly, the three-point rubric was used to consider PST responses to journal question D: Based on what you learned about your student's understanding of addition or subtraction, what will you do next? As consistent with Jacobs et al's. ([15]) rubric, deciding responses were analyzed in terms of evidence used and rationale included but did not assess "execution of the response" (p. 173). Thus, a deciding response was evaluated as (3), *Proficient* if the PST made an instructional decision using evidence that specifically connected to their student's understanding of addition and subtraction and provided a rationale for next steps related to problem type or solution strategy. In contrast, a PST deciding response was categorized as (2), *Developing* if the PST made an instructional decision using evidence that generally connected their student's understanding of addition or subtraction and provided a

rationale for next steps related to problem type or solution strategy. Finally, a deciding response was evaluated as (1), *Needs Improvement* when a PST failed to make an instructional decision connected to their student's understanding of addition or subtraction. Additional analysis of deciding response were conducted utilizing open qualitative coding methods (Strauss & Corbin, [32]) to further examine the rationale PSTs included in their responses.

To establish trustworthiness of the qualitative analysis process related to the three-point rubric, Cohen's kappa coefficient was used to measure inter-reliability for qualitative items. Interrater reliability on the written content included in the reflective journals was acceptable as indexed by Cohen's Kappa coefficient (k = 0.949; p = < 0.001). The differences between raters were compared and resolved through joint analysis to reach 100% agreement.

Phase two: Quantifying PST responses

After using the three-point rubric to assess PST responses for attending, interpreting, and deciding, average ratings were calculated across the entire collection of responses to produce a mean score for each skill. In addition, one-way repeated-measures analysis of variance (ANOVA) were conducted to compare the means of the three noticing skills, attending, interpreting, and deciding. Lastly, average scores for responses to questions B (attending), C (interpreting), and D (deciding) were calculated and a z-test and t-test performed to compare PST responses over time. Quantitative analyses were conducted using SPSS version 26.

Results

This study utilized a three-point rubric to examine PST professional teacher noticing skills, attending, interpreting, and deciding, in the context of written response PSTs recorded in a reflective journal after posing addition and subtraction problems for students in an after-school tutoring program. The results related to PST attending are presented first followed by interpreting and deciding. The findings further characterize PST professional noticing skills. The results presented are limited by the specific context in which the study was conducted, including a small number of study participants, the absence of a comparison group, and within the context of one teacher preparation program. Pseudonyms are used throughout presentation of results when discussing students in the after-school tutoring program.

Attending

Examples of PST attending responses rated as (3), *Proficient* (2), *Developing* and (1), *Needs Improvement* are presented next to characterize what PSTs "could do" (Jacobs et al., [15], p. 180) when attending to strategies students used to solve addition and subtraction problems. PST #6's response is discussed first, followed by PST #12's and PST #9's response.

PST #6 posed a Put Together/Total Unknown problem for her student: Aja and Alexis want to take their friends to the park. Aja has 5 friends and Alexis has 9 friends. How many friends do Aja and Alexis take to the park? PST #6 provided the following response to journal question B – What strategy did your student use to solve the problem you posed?: "Alexis used a direct modeling strategy. She made a set of 5 cubes and a set of 9 cubes, then joined the two sets together, recounted the cubes in the new set and said that there were 14 friends."

In general, as PST #6's example shows, PSTs in this study proficiently (score 3) identified the strategy their student used to solve a problem and included significant mathematical details to describe how their student used the strategy to solve the problem. Moreover, in these responses, PSTs made direct connections between the strategy their student used and the conceptualization of the meaning of addition or subtraction in the problem type. For example, as indicative of the majority of attending responses, PST #6, identified the strategy, in this case, a Direct Modeling strategy, her student used to solve the Put Together/Total Unknown problem. In addition, PST #6 provided significant mathematical details to specify how her student used the strategy in the context of a Put Together/Total Unknown problem, explaining in her response how her student "made a set of 5 cubes and a set of 9 cubes, then joined the two sets together." PST #6 continued, describing how her student "recounted the cubes in the new set and said that there were 14 friends" to determine the unknown answer. Attending in this way would position PST #6 to go on to interpret her student's understanding of the meaning of addition in the context of the Put Together/Total Unknown problem: That her student conceptualized the addition in this problem as joining two sets. Attending to this level of detail would also help PST #6 to assess her student's progression on the taxonomy of strategies by noting that her student counted and recounted sets to solve the problem.

In contrast, in a smaller number of attending responses, PSTs identified the strategy their student used to solve a problem but provided limited mathematical details. In these responses, describing how students used a strategy was disconnected from the meaning of addition or subtraction in the problem type. PST #12's response in journal #14 is typical of this kind of response. PST #12 posed a Take From/Result Unknown Problem: Karla had 11 cupcakes. She gave 7 to Devan. How many cupcakes does Carla have now? After her student solved the problem, PST #12 provided the following attending response: "Jessenia used unifex cubes and a direct modeling strategy to solve the problem."

In her journal, PST #12 reported that her student used "unifex cubes" and a "direct modeling strategy" to solve the Take From problem. In this response, PST #12 identified the strategy (i.e., Direct Modeling) her student used to solve the problem. In this example, however, PST #12 neglected to fully explain how her student employed the Direct Modeling strategy in the context of the Take From problem. Did her student, create a group of eleven cubes and take seven cubes from the original group? Did her student create a group of eleven cubes and a group of seven cubes and compare the two groups to determine the difference? Without this level of detail, it would be difficult for PST #12 to interpret her student's understanding of the meaning of subtraction in the context of the Take From problem. The lack of detail in attending would also hinder PST #12 in determining her student's progression on the trajectory of strategies.

Finally, a limited number of PST attending responses were evaluated as (1), *Needs Improvement*. PST #9's example from journal #5 characterizes these responses. In journal #5 PST #9 mislabeled the Take From/Change Unknown problem she posed for her student as Take From/Part Unknown: Megan had 8 crayons. She gave some crayons to Jeff. Then she had 5 crayons left. How many crayons did Megan give to Jeff? Mislabeling the problem may have contributed to PST #9's difficulty in attending to the strategy her student used to solve the problem. In her attending response, PST #9, simply reported that her student "solved the problem correctly". As indicative of this type of attending response, PST #9 failed to identify the strategy and provide details about how her student used the strategy to solve the problem. Failure to attend to the strategy a student uses to solve a problem makes it difficult for a teacher to interpret how a student conceptualizes the meaning of addition or subtraction. A lack of attending also makes it hard for a teacher to assess student progress on the taxonomy of solution strategies.

Attending patterns

Table 1 provides a summary of PST attending scores. Results included in the table reveal two patterns. First, across all responses, PSTs most often received (3), *Proficient* scores for attending. Second, comparing the scores PSTs received for attending across the different student strategies revealed an important distinction. In highlighting these distinctions, note that students select a strategy to solve a problem. Nonetheless, as Table 1 shows, PSTs received a majority of (3), *Proficient* scores and very few (1), *Needs Improvement* scores when attending to the significant mathematical details included in Direct Modeling, Counting, and Recalled Facts strategies. In contrast, PSTs received almost an equal number of (3), *Proficient*, (2), *Developing*, and (1), *Need Improvement* scores when attending to the significant mathematical details included in the Derived Facts strategy students used to solve addition and subtraction problems.

Table 1. PST attending skills.

Solution Strategy	#, %	Needs Improv. (1)	Developing (2)	Proficient (3)
Direct Modeling	120 (63%)	1 (1%)	14 (11%)	105 (88%)
Counting	46 (24%)	4 (9%)	5 (11%)	37 (80%)
Derived Facts	11 (6%)	4 (36%)	3 (28%)	4 (36%)
Recalled Facts	14 (7%)	0 (0%)	2 (14%)	12 (86%)
Total	191 (100%)	9 (5%)	24 (12%)	158 (83%)

Interpreting

Results related to PST interpreting responses considered (3), *Proficient* (2), *Developing* and (1), *Needs Improvement* are presented next, beginning with PST #3. PST #3's response highlights the features of interpreting response assessed as (3), *Proficient*. As characteristic of these responses, PSTs linked the strategy their student used to solve a problem, with the way in which the meaning of addition or subtraction was conceptualized in the problem, to interpret student understanding. PST #3 wrote in journal #13 that she posed an Add To/Start Unknown problem for her student: Mike had some tennis balls. He lost 14 of them. Now he has 21. How many tennis balls did Mike start with? After her student solve the problem, PST #3 provided the following interpreting response in her journal:

> Matt could not make sense of what was happening in this problem. I learned that he does not understand this type of problem with the start unknown. He knew that he had to do something with the two numbers given, but he just guessed that he had to subtract. He kept writing an equation to subtract 14 from 21. He didn't understand that if Mike lost 14 tennis balls and had 21 tennis balls that his answer of 7 wouldn't make sense because if Mike added the lost tennis balls to what he had. The total number of tennis balls would be larger than 21 (35).

PST #3 used significant mathematical details from her student's failed attempt at solving the Add To/Start Unknown problem, which included a description of her student's difficulty in conceptualizing the meaning of addition, to interpret her student's lack of understanding. PST #3 noted that her student "could not make sense of what was happening in this problem." She went on to detail how her student "guessed that he had to subtract" and "kept subtracting 14 from 21" to attempt solve the problem. PST #3 utilized these details as evidence to make inferences about her student's difficulty in conceptualizing the meaning of addition in the problem: "He didn't understand that if Mike lost 14 tennis balls and had 21 tennis balls that his answer of 7 wouldn't make sense if Mike added the lost tennis balls to what he had. The total number of tennis balls would be larger than 21 (35)." This level of detail would position PST #3 to make an instructional decision in terms of what to do next to support her student in understanding of the meaning of addition in the context of an Add to/Start Unknown problem.

PST #4's journal excerpt typifies interpreting responses assessed as (2), *Developing*. These responses were characterized by a disconnection between the strategy used and the meaning of addition or subtraction entailed in a problem. PST #4 posed a Compare//Bigger Unknown problem: Kenzie has 14 erasers. Noah has 7 more erasers than Kenzie. How many erasers does Noah have? PST #4's interpreting response in journal #16 reported that, "Tony understands a compare problem and used direct modeling to solve it."

While PST #4 used limited mathematical details to report that his student "understands a compare problem" and "used direct modeling" to solve the problem, he neglected to provide evidence to fully articulate how his student used a Direct Modeling strategy to conceptualize the comparison in the problem. Did PST #4's student use a Direct Modeling strategy to create a group of 14 counters for Kenzie, a group of 14 counters for Noah, add 7 counters to Noah's group and compare the two groups to find the (bigger) unknown total? It appeared instead that PST #4 relied on his student's correct answer as evidence of his student's understanding of the meaning of addition or subtraction in the compare problem. A correct answer, however, does not reveal student thinking or understanding.

Finally, PST #7's example from journal #4 provides an account of the attributes of PST interpreting responses rated as (1), *Needs Improvement*. PST #7 posed a Compare/Bigger Unknown problem: There were 18 sandwiches and 13 hamburgers at the party. How many more sandwiches were there? To interpret her student's understanding of the meaning of addition and subtraction involved in this problem PST #7 reported: "I learned that Jessenia is very organized. She color-coded the cubes and stacked them instead of sloppily putting them into a pile. She clearly understands how to do this." In her response, PST #7 communicated that her student "clearly understands how to do this problem" but failed to make inferences about how her student understood the meaning of addition or subtraction in the context of the Compare/Bigger Unknown problem. PST #7 also failed to provide specific mathematical details about the strategy her student used to solve the problem. Instead, PST #7 included irrelevant information about her student's organizational abilities and use of a color-coding scheme in her interpreting response.

Interpreting patterns

Table 2 provides an overview of the scores PSTs received for interpreting responses in relationship to the type of addition or subtraction problem posed and the strategies students used. Data analysis revealed an interesting pattern related to the type of problems PSTs posed to elicit student understanding. First, in the overwhelming majority of interpreting responses, PSTs made inferences about student understanding of the meaning of addition and subtraction in the context of Add To and Take From problems (77% of problems posed). Each of the Add To and Take From problems involve explicit action. In comparison, very few interpreting responses made inferences about student understanding of the meaning of addition and subtraction in the context of different Put Together or Compare problems (23% of problems posed). Each of these problem types contain a static relationship. Simply stated, over the course of eight weeks in the after-school program, PSTs did not pose the full array of problem types to interpret student understanding of the meaning of addition and subtraction. No discernable patterns were found between PST interpreting responses and student strategies.

Table 2. PST interpreting skills.

Problem Type & Solution Strategy	Problem Type Frequency	Needs Improv. (1)	Developing (2)	Proficient (3)
Add To/Result Unknown	22			
Direct Modeling		0	5	6
Counting		2	1	3
Derived Facts		1	3	0
Recalled Facts		0	0	1
Add To/Change Unknown	19			
Direct Modeling		2	8	6
Counting		1	0	2
Derived Facts		0	0	0
Recalled Facts		0	0	0
Add to/Start Unknown	27			
Direct Modeling		0	8	4
Counting		1	3	5
Derived Facts		0	1	1
Recalled Facts		1	2	1
Take From/Result Unknown	16			
Direct Modeling		0	4	5
Counting		0	2	1
Derived Facts		2	1	0
Recalled Facts		0	1	0
Take From/Change Unknown	38			
Direct Modeling		1	16	9
Counting		2	2	3
Derived Facts		1	0	0
Recalled Facts		0	1	3
Take From/Start Unknown	26			
Direct Modeling		1	7	6
Counting		2	4	2
Derived Facts		0	0	0
Recalled Facts		0	2	2

Put Together/Take Apart Total Unknown	12			
Direct Modeling		0	9	3
Counting		0	0	0
Derived Facts		0	0	0
Recalled Facts		0	0	0
Put Together/Take Apart Addend Unknown	8			
Direct Modeling		1	3	2
Counting		0	1	0
Derived Facts		0	1	0
Recalled Facts		0	0	0
Put Together/Take Apart Both Addends Unknown	1			
Direct Modeling		0	0	0
Counting		0	0	0
Derived Facts		1	0	0
Recalled Facts		0	0	0
Compare Difference Unknown	15			
Direct Modeling		0	3	6
Counting		0	3	3
Derived Facts		0	0	0
Recalled Facts		0	0	0
Compare Bigger Unknown	5			
Direct Modeling		0	1	1
Counting		1	0	2
Derived Facts		0	0	0
Recalled Facts		0	0	0
Compare Small Unknown	2			
Direct Modeling		0	2	0
Counting		0	0	0
Derived Facts		0	0	0
Recalled Facts		0	0	0
Total	191	20	94	77

Deciding

To highlight PST deciding skills assessed as (3), *Proficient* (2), *Developing* and (1), *Needs Improvement*, PST #8's response is discussed first followed by PST #1's and PST #2's response. PST #8 wrote in journal #4 that he posed a Put Together/Total Unknown problem for his student: Emilio has 7 red cars and 12 blue cars. How many cars does Emilio have? As consistent with deciding responses evaluated as

(3), *Proficient*, PST #8 used evidence to detail his student's understanding and provide a rationale for his decision:

> Maya has a good understanding of how to combine two groups to find the total. She was very good at solving this type of problem with these kinds of numbers. She used the direct modeling strategy efficiently. So next time I am planning on posing easy problems like Add To or Take From problems with the result unknown that I know Maya can solve. I will also use smaller numbers for these problems. I want to help Maya move away from direct modeling. I think she can use counting strategies with easier problem types and smaller numbers.

When recording his instructional decision, PST #8 used evidence to detail that his student understood how to "combine two groups to find the total." He also noted that his student used the "direct modeling strategy efficiently" to solve the Put Together/Total Unknown problem. In providing this rationale, PST #8 specifically connected his student's understanding of the meaning of addition in the Put Together/Total Unknown problem to his decision. PST #8 explained that he would pose "easy problems like Add To or Take From with the result unknown" in conjunction with "smaller numbers" to "help" his student "move away from direct modeling" and toward "counting strategies". Ultimately, PST #8 decided to pose problems to support his student in moving along the trajectory of solution strategies.

In comparison, PST #1's deciding response, assessed as (2), *Developing*, like the majority of these responses, made her decision using evidence generally connected to her student's understanding of the meaning of addition. PST #1 reported in journal #6 that she posed an Add To/Change Unknown problem: James has 4 hats. He gets some more hats. Now he has 7 hats. How many hats did James get? After her student solved the problem, PST #1 recorded her deciding response: "I will pose another Add To/Change Unknown problem with more difficult numbers to challenge him."

PST #1 intimated in her response that her student understood the problem and communicated that since her student solved the problem, she would pose another "with more difficult numbers". While this may have been an appropriate decision, including evidence and specifically connecting this evidence to her student's conceptualization of addition in the Add To/Change Unknown problem, would have created a stronger rationale for her decision.

Finally, PST #2's deciding response was evaluated as (1), *Needs Improvement*. As is the case for this kind of response, PST #2 failed to use evidence from her student's understanding and a rationale to inform her decision. PST #2 posed a Take From/Start Unknown Problem: Ian had some baseball cards. He left 10 on the bus. Now he has 8. How many baseball cards did he start with? In journal #4, PST #2 wrote, "I will keep working with my student." The response, which was not connected to her student's understanding of the meaning of subtraction in the context of the Take From/Start Unknown problem, lacked a rationale that might help to explain what problem PST #2 decided to pose next and why. Simply put, while PST #2's response indicated continued work with her student, it failed to qualify as an instructional decision informed by student understanding.

Deciding patterns

The results presented in Table 3 provide an overview of the scores PSTs received for their deciding responses. As the table shows, 159 of the 191 (83%) deciding responses were evaluated as (2), *Developing* or (3), *Proficient.*

Table 3. PST deciding skills.

Noticing Skill	Needs Improv. (1)	Developing (2)	Proficient (3)
Deciding	32 (17%)	112 (59%)	47 (24%)

As previously discussed, the rubric used in this study was adapted from Jacobs et al. ([15]), who assessed deciding skills in terms of: (1) evidence used in making decisions, (2) rationale explaining reasoning, and (3) next steps for instruction. In the majority of deciding responses (Table 3), PSTs made an instructional decision using evidence specifically (proficient) or generally (developing) connected to student understanding of the meaning of addition or subtraction. Therefore, additional analysis of PST deciding responses evaluated as (2), *Developing* and (3), *Proficient* was conducted to further explore PST deciding skills. Data analysis revealed three patterns related to deciding rationale and next steps: (1) supporting student understanding of the meaning of addition and subtraction, (2) providing students more practice with a particular problem type, and (3) teaching students a strategy to support movement along the trajectory of strategies.

Supporting Student Understanding of the Meaning of Addition and Subtraction.

In 50% (80/159) of deciding responses assessed as (2), *Developing* or (3), *Proficient*, PSTs included a rationale for their decision relative to supporting student understanding of the meaning of addition or subtraction, and described the next steps they would take for instruction. For example, PST #3 reasoned that she would pose a "Take From/Start Unknown" problem to provide her student the opportunity to make sense of subtraction in the context of this problem type. PSTs #11 and #5, on the other hand, explained that they would pose a problem they perceived as "easier" (PST #11) or "more difficult" (PST #5) to help their student understand the meaning of addition or subtraction.

Providing Students More Practice with a Particular Problem Type.

In a smaller percentage of deciding responses (35%) evaluated as (2), *Developing* and (3), *Proficient*, PSTs included a rationale for their decision related to providing their student more practice with a problem type. In these responses, PSTs referenced posing the same problem type with smaller (i.e., easier) or harder numbers. PST #2, for example, explained that she would "pose more Take From/Result Unknown problems with smaller numbers" to provide her student additional practice. PST #1, on the other hand, wrote that she would ask her student to "do more Take From/Start Unknown problems...but use harder numbers" to give her student "more practice".

Teaching a Strategy to Support Movement Along the Trajectory of Strategies.

Finally, in 15% of deciding responses appraised as (2), *Developing* and (3), *Proficient*, PSTs explained that they would pose a problem type in order to teach their student a specific strategy. PST #12, for instance, indicated that she would pose a "Take From/Result Unknown problem next time because I want to teach Jessenia the counting down strategy."

Distinguishing and comparing PST attending, interpreting, and deciding skills

Descriptive statistics are presented in Table 4 to provide a summary of PST professional noticing skills. Overall, PSTs' mean score for attending was highest followed by mean scores for interpreting and deciding. To further explore whether differences existed between the means for attending, interpreting, and deciding, post hoc analyses, including a one-way repeated measures of variance (ANOVA) were conducted. Results of these analyses revealed statistically significant pairwise comparisons for the individual noticing skills, however, while interpreting the results, Mauchly's test indicated that the assumption of sphericity had been violated $\chi2$ (2) = 32.94, p =.0001. Epsillon (ε) was 0.862. Therefore, Greenhouse-Geiser corrected tests are reported, and the degrees of freedom have been adjusted accordingly. Thus, study variables (attending, interpreting, and deciding) had a significant effect beyond that expected by chance alone $F(1.7, 327.5) = 162.6$, $p < 0.001$, omega squared, ω^2 =.329. In addition, post hoc analyses with a Bonferroni-adjustment revealed statistically significant differences with a 95% confidence level between all bivariate correlations of the mean scores for attending, interpreting, and deciding.

Table 4. Descriptive statistics for PST noticing skills.

Noticing Skill	Needs Improv. (1)	Developing (2)	Proficient (3)	M	SD
Attending	9 (5%)	24 (13%)	158 (82%)	2.78	.52
Attending Journal #1	1 (8%)	2 (17%)	9 (75%)	2.67	.65
Attending Journal #16	0 (0%)	0 (0%)	12 (100%)	3.00	.00
Interpreting	20 (11%)	94 (49%)	77 (40%)	2.30	.65
Interpreting Journal #1	1 (8%)	7 (58%)	4 (34%)	2.25	.62
Interpreting Journal #16	0 (0%)	7 (58%)	5 (42%)	2.42	.52
Deciding	32 (17%)	112 (50%)	47 (24%)	2.08	.64
Deciding Journal #1	2 (17%)	9 (75%)	1 (8%)	1.92	.52
Deciding Journal #16	2 (17%)	7 (58%)	3 (25%)	2.08	.70

1 Note. N = 191. The Mean and SD are based upon ratings between 1 and 3 for Needs.

2 Improvement (1), Developing (2), and Proficient (3).

Table 4 also shows the number of (3), *Proficient* (2), *Developing* and (1) *Needs Improvement* scores as well as mean scores PSTs received for attending, interpreting, and deciding in journal #1 and #16. Results show that overall, the number of (3), *Proficient* ratings PSTs received increased between journal #1 and journal #16. The table also shows that the mean score for each skill, attending, interpreting, and deciding, improved over time. Results of z-tests for proportions, however, indicated that the increase in the number of (3), *Proficient* ratings were not statistically significant. In addition, results of a t-test revealed that improvement in mean scores over the course of the eight weeks (journal #1 to journal #16) were not statistically significant. Examined together, these findings suggest that PSTs noticing skills, attending, interpreting, and deciding, did not improve over time.

Discussion

This study investigated professional teacher noticing, attending, interpreting, and deciding (Jacobs et al., [15]), in the context of written responses PSTs provided in a reflective journal after planning and implementing instruction for a small group of students in an after-school tutoring program over the course of eight weeks. As consistent with several studies (van den Kieboom et al., [36]; Fernández et al., [9]; Ivars et al., [14]; Schack et al., [25]), this study, situated professional teacher noticing within the well-defined mathematics content domain and accompanying student learning trajectory for the meaning of addition and subtraction (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]; NRC, [24]). Several aspects of the study are discussed to characterize PST professional noticing skills.

Building on the learning trajectory to characterize PST noticing skills

Jacobs et al. ([15]) utilized a rubric to paint a picture of what pre-service and practicing teachers "could do" (p. 180) when developing a construct for professional teacher noticing. Their rubric incorporated language that described how experts include mathematically significant details and robust evidence to attend to strategies students use to solve problems, interpret student understanding, and decide how to respond based on student understanding. In a similar fashion, Fernández et al. ([9]) captured an initial trajectory of the development of PST noticing skills in the context of proportional and non-proportional reasoning. Like Jacobs et al. ([15]), the amount of evidence used when noticing was a key element in developing this trajectory. Fernández et al. ([9]) concluded that since their work related to a specific topic, more research is needed to investigate what PSTs notice in different mathematics domains and how the development of PST noticing skills can be characterized relative to these different domains. The results of this study, which utilized a rubric to assess noticing skills PSTs recorded in a reflective journal, builds on the work of Jacobs et al. ([15]) and Fernández et al. ([9]). More specifically, results of this study provide another way to characterize what PSTs "could do" when attending to strategies students used to solve addition and subtraction problems, interpreting student understanding of the meaning of addition and subtraction, and deciding what to do next based on student understanding.

Characterizing PST attending skills

Results related to the first research question, which examined PST attending skills, are promising and provide important insights for teacher educators involved in the work of professional teacher noticing. Overall, PSTs in this study were near proficient (score 3) in attending, consistently including significant mathematical details related to the meaning of addition or subtraction in their reflective journals to chronicle strategies students used to solve addition and subtraction problems. Additional analysis of PST attending responses, however, revealed that PSTs received fewer (3), *Proficient* scores when attending to Derived Fact strategies as compared to Direct Modeling, Counting, and Recalled Fact strategies. These results need to be interpreted with caution given that students select a solution strategy when solving addition or subtraction problems. Nonetheless, the results may suggest that it was more difficult for PSTs to attend to the significant details involved in the Derived Facts strategy. On the other hand, these results might simply indicate that PSTs had less experience attending to the significant mathematical details in the Derived Fact strategy because students used this strategy infrequently. Either way, mathematics teacher educators will want to ensure that PSTs can proficiently attend to any strategy students use to solve an addition or subtraction problem. More research is needed to explore PST proficiency in attending to the Derived Fact strategy.

Results related to the first research question suggest that situating professional teacher noticing within the well-defined mathematics domain and associated student learning trajectory related to the meaning of addition and subtraction provided scaffolding that supported PSTs in learning how to attend to the strategies students use to solve addition and subtraction problems. These results can also be used in conjunction with several other studies (Fernández et al., [9]; Ivars et al., [14]; Schack et al., [25]) to guide teacher educators in thinking about how to situate teacher noticing within the context of other well-defined mathematics domains such as student thinking about the equal sign (Harbour et al., [13]; Matthews et al., [20]), fraction representations (Kara & Incikabi, 2018; Shahbari & Peled, [28]), or geometric thinking (Denizli & Erdogan, [7]). Like the meaning of addition and subtraction, each of these domains articulates concepts and procedures integral to student understanding, identifies a variety of approaches that foster student understanding, details a trajectory of student learning, and specifies thinking strategies students use to approach and solve problems. Situating professional teacher noticing within a well-defined content domain provides a viable avenue to support PSTs in simultaneously learning about mathematics content while developing attending skills.

Characterizing PST interpreting and deciding skills

The second and third research questions in this study investigated the evidence PSTs used and the rationale they provided when interpreting student understanding of the meaning of addition or subtraction and deciding next steps. Overall, the findings indicate that PSTs were developing (score 2) their interpreting and deciding skills. In addition, results of quantitative analysis, which compared overall mean scores for attending, interpreting, and deciding, uncovered significant differences between each of the means. These results reinforce the findings of several other studies (Castro Superfine et al., [5]; Fernández et al., [9]; Ivars et al., [14]; Schack et al., [25]), which suggest that interpreting and deciding skills are more difficult for PSTs than attending skills. Revisiting the research literature related to professional teacher noticing and PSTs sheds light on why interpreting and deciding may be more difficult.

PST interpreting skills

Fernández et al. ([9]) determined that PSTs with inadequate knowledge of multiplicative and additive reasoning had difficulty interpreting student understanding. The results of this study propose that in contrast to Fernández et al.'s findings the link between inadequate knowledge and interpreting skills is more nuanced. For example, as previously discussed, PSTs in this study had adequate knowledge of strategies students used to solve addition and subtraction problems as evidenced by their prolific ([3]), *Proficient* scores for attending. The majority of responses for interpreting, however, were developing (score 2). In other words, PSTs used limited mathematical details from the strategy their student used to solve a problem, and generally referenced how their student conceptualized the meaning of addition or subtraction in the problem, as evidence to interpret student understanding. As a result, PSTs failed to make direct links between strategies students used and the way in which the meaning of addition and subtraction was conceptualized in a problem type to interpret student understanding.

Findings related to interpreting response also revealed that PSTs did not utilize the full collection of problem types over the course of eight weeks, thus limiting the potential of using problems to interpret student understanding of the meaning of addition and subtraction. Instead, PSTs predominantly posed Add To and Take From problems, all of which involve explicit action that students can more easily solve via Direct Modeling and Counting strategies. Examined together, these results indicate that although PSTs had adequate knowledge of solution strategies, they displayed inadequate knowledge of problem types including knowledge of how to explicitly link problem types *and* solution strategies to foster student understanding of the meaning of addition and subtraction.

Castro Superfine et al's. ([5]) research provides one perspective to use in thinking about results related to PST interpreting. They contend that attending and interpreting are "inextricably linked" (p. 422) and therefore must be addressed together when supporting PSTs in learning how to notice. The results of this study confirm and extend Castro Superfine et al's research. In this study, which was situated in the domain of the meaning of addition and subtraction and associated student thinking, not only are attending and interpreting inextricably linked, but so too are the conceptual categories of problem types and taxonomy of strategies.

PST deciding skills

Overall, PST deciding responses were characterized as developing (score 2). Thus, while PSTs made instructional decisions using evidence generally connected to student understanding of addition or subtraction, they included a rationale to explain their next steps. In fact, additional analysis of PST deciding responses uncovered that half of all responses included a rationale to explain their decisions relative to: (1) supporting student understanding of the meaning of addition and subtraction, (2) providing students more practice with a particular problem type, and (3) teaching students solution strategies to help them move along the trajectory of learning. These findings are promising, and as consistent with Ivars et al. ([14]), indicate that PSTs who use a student learning trajectory as a scaffolding mechanism to develop their deciding skills can choose appropriate next activities for their students. Furthermore, these results highlight how PSTs were beginning to use their knowledge of the meaning of addition and subtraction and related student thinking in action to make appropriate instructional decisions for their students.

Implications for teacher preparation

Given that the goal of teacher preparation is to support PSTs in using noticing skills in their future practice, mathematics teacher educators need to find ways to move PSTs along the trajectory of learning for professional teacher noticing. Jacobs et al's. ([15]) construct for professional teacher noticing describes what experts "could do" in terms of attending, interpreting, and deciding. PSTs, like the ones involved in this study, are novices who are learning about and developing professional noticing skills. Reflecting on lessons learned in the form of missed opportunities related to this study provide additional insights about how to further support PST learning.

Schack et al. ([25]) used pedagogies of enactment (Grossman et al., [12]) to afford PSTs opportunities to examine, decompose, and rehearse professional teacher noticing. PSTs in their study improved all three skills. Several changes have been made to the current version of the mathematics methods course and field experience (after-school tutoring program), using pedagogies of enactment, to create additional opportunities for PSTs to rehearse and reflect on their noticing skills. These changes build on the approach Schack et al. ([25]) took and target making connections between the three noticing skills as well as linking the conceptual categories of problem types for addition and subtraction and student strategies.

First, in the revised version of the mathematics methods course, PSTs are provided opportunities to examine the practice of interpreting and deciding. As Schack et al. ([25]) did, the course instructor now models for PSTs how to interpret student understanding and decide next steps using examples form the after-school tutoring program. Instructor modeling creates a window through which PSTs gain access to teacher thinking as a more knowledgeable expert explains the steps involved in each skill. Showing PSTs how to link evidence of student thinking, gathered while attending to student strategies to interpret student understanding, is an integral component of this modeling (Castro Superfine et al., [5]).

Second, PSTs now spend time in the methods course decomposing the discrete skills involved in interpreting and deciding. Like Schack et al. ([25]), video clips of students solving addition and subtraction problems are shown to help PSTs focus on these discrete skills. PSTs first use the taxonomy of student strategies (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]) to identify the problem type and solution approach observed in a video clip. Then, PSTs discuss in small groups, how to use evidence gathered while attending, to interpret student understanding. This strategy provides scaffolding for PSTs as they build on their proficient attending skills to learn how to interpret student understanding. The instructor and PSTs also draw on curricular documents and research summaries (Carpenter et al., [4]; CCSSM, 2010; Dougherty, [8]) to generate a decision tree that details the actions a teacher might take in response to student understanding. In developing this decision tree, PSTs are explicitly prompted to provide a rationale for their next steps.

Third, in the current version of the mathematics course, work with students in the after-school tutoring program is used as an opportunity to rehearse interpreting and deciding skills. For example, PSTs now use the rubric, developed for this study, in conjunction with video-recordings of their work with students, to provide each other feedback on interpreting and deciding skills. Moreover, the course instructor observes PSTs in real time in the after-school tutoring program, utilizing the three-point rubric in a post observation conference to provide PSTs substantive feedback about their interpreting and deciding skills. Opportunities to rehearse in combination with substantive feedback from peers and the course instructor create the context for PSTs to improve their interpreting and deciding skills over time.

Final remarks

Reform efforts in K-12 mathematics education in the U.S. call for teachers who support students in learning mathematics with understanding (Kilpatrick et al., [17]; NCTM, [22], [23]). Teachers who respond to this challenge utilize professional teacher noticing skills (Jacobs et al., [15]) to achieve this goal. Research related to professional teacher noticing and PSTs provides evidence that PSTs can learn about and develop noticing skills while enrolled in a teacher preparation program (van den Kieboom et al., [36]; Castro Superfine et al., [5]; Males, [18]; McDuffie et al., [21]; Schack et al., [25]; Star et al., [30]; Star & Strickland, [31]). But how can teacher educators maximize learning so that PSTs are prepared to effectively enact their professional noticing skills on the first day of their practice? The results of this study can be used to guide teacher educators in thinking about how to support PSTs in learning about, developing, and enacting professional noticing skills. Doing so positions PSTs to help their future students learn mathematics with understanding.

Notes

[1] *One student was absent one day over the course of the eight-week tutoring program, therefore the PSTs posed 191 addition or subtraction problems.*

References

Berg, B. L. (2007). *Qualitative research methods for social sciences*. Pearson.

Brown, J. S., Collins, A., & Duguid, S. (1989). Situated cognition and the culture of learning. *Educational Researcher*, 18 (1), 32 – 42. https://0-doi-org.libus.csd.mu.edu/10.3102/0013189X018001032

Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97 (1), 3 – 20. https://0-doi-org.libus.csd.mu.edu/10.1086/461846

Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2014). *Children's mathematics: Cognitively guide instruction.* Heinemann.

Castro Superfine, A., Fisher, A., Bragelman, J., & Amador, J. M. (2017). Shifting perspectives on preservice teachers' noticing of children's mathematical thinking. In E. Schack, M. Fisher, & J.A. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts and frameworks* (pp. 409 – 426). Springer International Publishing.

Clarke, B. (2008). A framework of growth points as a powerful teacher development tool. In D. Tirosh & T. Wood (Eds.), *International handbook of mathematics teacher education: Vol. 2. Tools and processes in mathematics teacher education* (pp. 235 – 256). Sense Publishers.

Denizli, Z. A., & Erdogan, A. (2018). Development of a three dimensional geometric thinking test for early graders. *Journal on Mathematics Education*, 9 (2), 213 – 226. https://0-doi-org.libus.csd.mu.edu/10.22342/jme.9.2.5741.213-226

Dougherty, B. J. (2014). *Putting essential understandings into practice: Pre-k-2*. National Council of Teachers of Mathematics.

Fernández, C., Llinares, S., & Valls, J. (2013). Primary school teacher's noticing of students' mathematical thinking in problem solving. *The Mathematics Enthusiast*, 10 (1), 19.

Franke, M. L., & Kazemi, E., & Battey, D. (2007). Mathematics teaching and classroom practice. In Lester, F. K. Jr. (Ed.) *Second handbook of research on mathematics teaching and learning* (pp. 225 – 256). Charlotte, NC: Information Age Publishing.

Greeno, J. (1997). Theories and practices of thinking and learning to think. *American Journal of Education*, 106 (1), 85 – 126. https://0-doi-org.libus.csd.mu.edu/10.1086/444177

Grossman, P., Hammerness, K., & McDonald, M. (2009). Redefining teaching, re-imagining teacher education. *Teachers and Teaching*, 15 (2), 273 – 289. https://0-doi-org.libus.csd.mu.edu/10.1080/13540600902875340

Harbour, K. E., Karp, K. S., & Lingo, A. S. (2016). Inquiry into action: Diagnosing and Addressing students' relational thinking about the equal sign. *Teaching Exceptional Children*, 49 (2), 126 – 133. https://0-doi-org.libus.csd.mu.edu/10.1177/0040059916673310

Ivars, P., Fernández, C., & Llinares, S. (2019). A learning trajectory as a scaffold for pre-service teachers' noticing of students' mathematical understanding. *International Journal of Science and Mathematics Education*, 18, 529–548. https://0-doi-org.libus.csd.mu.edu/10.1007/s10763-019-09973-4

Jacobs, V. R., Lamb, L. L., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41 (2), 169 – 202. https://0-doi-org.libus.csd.mu.edu/10.5951/jresematheduc.41.2.0169

Kara, F., & Incikabi, L. (2018). Sixth grade students' skills of using multiple representations in addition and subtraction operations in fractions. *International Journal Electronic Journal of Elementary Education*, 10 (4), 463 – 474.

Kilpatrick, J., Swafford, J., & Findell, B. (2001). *Adding it up: Helping children learn mathematics*. National Academy Press.

Males, L. (2017). Using video of peer teaching to examine grades 6-12 preservice teachers' noticing. In E. Schack, M. Fisher, & J. A. Wilhelm (Eds.), *Teacher noticing: Bridging and broadening perspectives, contexts and frameworks* (pp. 91 – 109). Springer International Publishing.

Mason, J. (2002). *Researching your own practice. The discipline of noticing*. Routledge-Falmer. https://0-doi-org.libus.csd.mu.edu/10.4324/9780203471876

Matthews, P., Rittle-Johnson, B., McEldoon, K., & Taylor, R. (2012). Measure for measure: What combining diverse measures reveals about children's understanding of the equal sign as an indicator of mathematical equality. *Journal for Research in Mathematics Education*, 43 (3), 316 – 350. https://0-doi-org.libus.csd.mu.edu/10.5951/jresematheduc.43.3.0316

McDuffie, A. R., Foote, M. Q., Bolson, C., Turner, E. E., Aguirre, J. M., Bartell, T. G., Drake, C., & Land, T. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education*, 17 (3), 245 – 270. https://0-doi-org.libus.csd.mu.edu/10.1007/s10857-013-9257-0

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. NCTM.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. NCTM.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common core standards for mathematics*. National Governors Association Center for Best Practices & Council of Chief State School Officers.

National Research Council. (2009). *Mathematics learning in early childhood: Paths toward excellence and equity.* C. T. Cross, T. A. Woods, & H. E. Schweingruber (Eds.). Committee on Early Childhood Mathematics, Center for Education, Division of Behavioral and Social Sciences and Education, The National Academy Press.

Schack, E. O., Fisher, M. H., & Wilhelm, J. A. (Eds.). (2017). *Teacher noticing: Bridging and broadening perspectives, contexts and frameworks.* Springer International Publishing.

Schack, E. O., Fisher, M. H., Thomas, J. N., Eisenhardt, S., Tassell, J., & Yoder, M. (2013). Prospective elementary school teachers' professional noticing of children's early numeracy. *Journal of Mathematics Teacher Education*, 16 (5), 379 – 397. https://0-doi-org.libus.csd.mu.edu/10.1007/s10857-013-9240-9

Schön, D. A. (1983). *The reflective practitioner: How professionals think in action*. Basic Books.

Shahbari, J. A., & Peled, I. (2017). Modelling in primary school: Constructing conceptual Models and making sense of fractions. *International Journal of Science and Mathematics Education*, 15 (2), 371 – 391. DOI https://0-doi-org.libus.csd.mu.edu/10.1007/s10763-015-9701-x

Sherin, M. G., & van Es, E. A. (2005). Using video to support teachers' ability to notice classroom interactions. *Journal of Technology and Teacher Education*, 13, 475 – 491.

Star, J. R., & Strickland, S. K. (2008). Learning to observe: Using video to improve preservice mathematics teachers' ability to notice. *Journal of Mathematics Teacher Education*, 11 (2), 107 – 125. https://0-doi-org.libus.csd.mu.edu/10.1007/s10857-007-9063-7

Star, J. R., Lynch, K., & Perova, N. (2011). Using a video to improve preservice mathematics teachers' abilities to attend to classroom features. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 117 – 133). Routledge.

Strauss, A., & Corbin, J. (1998). *Basics of qualitative research: Procedures and techniques for developing grounded theory.* Sage.

Thomas, J. N., Eisenhardt, S., Fisher, M. H., Schack, E. O., Tassell, J., & Yoder, M. (2014). Professional noticing: Developing responsive mathematics teaching. *Teaching Children Mathematics*, 21 (5), 294 – 301. https://0-doi-org.libus.csd.mu.edu/10.5951/teacchilmath.21.5.0294

van Es, E. A. (2011). A framework for learning to notice student thinking. In M. G. Sherin, V. R. Jacobs, & R. A. Philipp (Eds.), *Mathematics teacher noticing: Seeing through teachers' eyes* (pp. 164 – 181). Taylor and Francis.

van den Kieboom, L. A., Magier, M. T., & Moyer, J. M. (2017). Teacher noticing: Bridging and broadening perspectives, contexts and frameworks. In E. O. Schack, M. H. Fisher, & J. A. Wilhelm (Eds.), Cham, Switzerland: Springer International Publishing.

van Es, E. A., & Sherin, M. G. (2002). Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. *Journal of Technology and Teacher Education*, 10 (4), 571 – 596.