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Real Estate and Relative Risk Aversion with Generalized Recursive Preferences

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Abstract
This paper investigates how real estate wealth affects the household’s attitude toward risk, and derives the closed-form expressions for risk aversion with generalized recursive preferences. We find three channels through which real estate wealth affects risk aversion, and these channels are absent in the traditional measure of relative risk aversion as in Arrow (1965) and Pratt (1964). First, illiquidity and fluctuations in real estate value increase consumption risk, thereby increasing risk aversion. Second, real estate as an asset provides a cushion for absorbing negative shocks to households, reducing risk aversion. Third, an increase in real estate prices lowers the profit of the firm that uses real estate as a factor of production, induces a decline in the real wage,
and causes a rise in consumption risk. This channel increases risk aversion. We study how these channels as a whole determine relative risk aversion using a basic real business cycle model with generalized recursive preferences and compare the results with the case of expected utility preferences. Finally, we explore the implications of the firm’s and the household’s real estate holdings and illiquidity of real estate on the risk premiums for equity and real estate.

Keywords
Risk aversion, Real estate risk premium, Real estate prices, Generalized recursive preferences

1. Introduction

Macroeconomic literature emphasizes the real estate market as an important source of business cycle fluctuations (e.g., Iacoviello and Neri, 2010, Liu et al., 2013). The interaction of the real estate market with the macroeconomy has received substantial attention since the financial crisis of 2007–2009. Disruptions in real estate prices significantly alter the household’s optimal choices, such as those for consumption, saving, and labor. In the finance literature, as pointed out by Piazzesi and Schneider (2016), real estate has two characteristics: a consumption good and an asset in household portfolios. Real estate is undoubtedly the largest share of household portfolios (Eiling et al., 2019). Despite this fact, it is surprising that real estate has received much less attention than stocks in the asset pricing literature.

The objective of this paper is to investigate how real estate affects the household’s attitude toward risk in a production economy when households are endowed with generalized recursive preference. To do so, we rigorously derive the closed-form expressions of risk aversion with period utility including real estate. Specifically, the assumption of additive separability between consumption and real estate is not imposed in our derivation, and therefore the expressions are generally applicable in many types of utility specifications.

We find three channels through which real estate affects the household’s attitude toward risk. First, compared to other assets, holding and trading real estate entail large transaction and repairing costs that are proportional to real estate prices. These costs are likely to reduce trading volume of real estate, making it illiquid. Fluctuations in real estate value also impose additional risk on real estate owners. Our closed-form expressions reveal that these risks increase the household’s relative risk aversion. We refer to this channel as the real estate risk channel. It predicts a positive relationship between relative risk aversion and real estate prices since both the size of the gamble and the costs increase with real estate prices. Second, households are able to absorb negative shocks to the economy by liquidating real estate wealth. The cushioning channel, in which real estate acts as a cushion to negative shocks, reduces relative risk aversion. This channel implies a negative relationship between relative risk aversion and real estate prices. Finally, the third channel is associated with competition between firms and households to purchase real estate, which consists of land or the buildings on it. As in Liu et al. (2013), our model implies that a rise in the share of real estate owned by households leads to a decline in the remaining share for firms. A rise in real estate demand by households drives up real estate prices, reducing firms’ real estate holdings. The profit of firms declines since real estate as a factor of production becomes expensive. A decline in profit lowers the real wage, consumption, and production. We find that this crowding-out channel, associated with general equilibrium effects, increases relative risk aversion as it generates consumption risk, and the impact on relative risk aversion rises with the curvature parameter of generalized recursive preferences. This channel thus predicts a positive link between relative risk aversion and real estate prices. We also find that the crowding-out channel disappears when households are characterized by expected utility preferences. It is worth emphasizing that these channels are not captured by the conventional measure of risk aversion by Arrow (1965) and Pratt (1964). Our measure of relative risk aversion implies that the conventional measure could be misleading if real estate is ignored.
In order to comprehensively understand the effects of real estate on relative risk aversion, we use a simple real business cycle model. In our baseline calibration, the crowding-out channel and the real estate risk channel are more important than the cushioning channel. Accordingly, the model predicts a positive relationship between real estate prices and relative risk aversion. Interestingly, risk aversion is still positively related to real estate prices even when households are endowed with expected utility preferences. This is because the real estate risk channel dominates the cushioning channel.

To explore the implications of real estate on risk aversion in light of asset pricing, we examine the effect of real estate on both the equity premium and the real estate risk premium. The real business cycle model with generalized recursive preferences implies that the household’s and the firm’s holding of real estate significantly affects not only relative risk aversion but also the risk premia on equity and real estate. Generalized recursive preferences are essential for the model to generate sizable risk premia.

Arrow (1965) and Pratt (1964) define the coefficient of relative risk aversion using a static model that abstracts labor and real estate. Swanson (2012) shows that a household’s attitude toward risk can be different when labor is included in period utility. Swanson (2018) extends his previous study with generalized recursive preferences. These papers show that labor adjustments to a negative income shock reduce relative risk aversion. A main difference between our paper and these studies is that our model incorporates real estate in period utility as well as labor. While Swanson, 2012, Swanson, 2018 studies the implications of labor adjustments on relative risk aversion, this paper studies how real estate, in its role as an asset, a consumption good, and as a production factor affects relative risk aversion. Zanetti (2014) derives the close-form expression of relative risk aversion when households receive utility from real estate. He finds that real estate wealth can reduce risk aversion due to the cushioning channel. This paper differs from Zanetti (2014) in at least four respects. First, Zanetti (2014) is necessarily silent about the remaining two channels since his model assumes that the real price of real estate is constant and households are characterized by expected utility preferences instead of generalized recursive preferences. Second, Zanetti (2014) treats real estate as liquid assets, while this paper does not. Third, Zanetti (2014) considers an endowment economy, while this paper incorporates a production economy. As shown in this paper, the production sector has a substantial impact on risk aversion and risk premiums. Finally, we analyze the impact of real estate on the equity and real estate risk premiums, whereas Zanetti (2014) does not study the implications of real estate on risk premiums.

Piazzesi et al. (2007) studies issues related to returns on stocks and housing using a consumption-based asset pricing model with housing and expected utility preferences. Their model shows that introducing housing into the model increases the equity premium and that the relative share of housing in the consumption basket helps forecast excess stock returns. Jaccard (2011) uses a production-based asset pricing model with habit formation and building restrictions to account for the equity premium and the housing risk premium. This paper is different from these papers in that our main objectives are to derive the coefficient of relative risk aversion and then to study how the firm’s and the household’s real estate holding influences relative risk aversion. Our model also differs from Piazzesi et al. (2007) and Jaccard (2011) in that our model assumes firms use real estate for production, firms and households face liquidity risk of real estate, and the household has generalized recursive preferences. We show that real estate as a factor of production has a substantial impact on relative risk aversion and the risk premiums. Illiquidity of real estate is also an important determinant of relative risk aversion and risk premiums.

The organization of the paper is as follows. Section 2 describes the household’s problem. Section 3 shows derivation of risk aversion with generalized recursive preferences. Section 4 studies how risk aversion, the equity premium, and the real estate premium are determined in a simple real business cycle model. Section 5 concludes.
2. The model
This section describes the representative household’s problem. Time is discrete and continues forever. Each period, the household chooses consumption, \( c_t \), the real estate stock, \( h_t \), the quantity of real risk-free bonds, \( b_t \), and labor, \( l_t \). The flow budget constraint of the household is given by

\[
b_t = (1 + r_t)b_{t-1} + w_t l_t + d_t - c_t - q^h_t h_t + q^h_t (1 - \tau - \kappa) h_{t-1},
\]

where \( r_t \) is the real risk-free return, \( w_t \) is the real wage, \( q^h_t \) is the real price of real estate, \( d_t \) is net transfer payments, \( \tau \) is the maintenance fee rate of the real estate stock, and \( \kappa \) denotes the real estate commission fee rate which captures illiquidity of real estate stocks. The commission fee rate is about 5 to 6 percent in the United States (Delcoure and Miller, 2002, Barwick and Wong, 2019).2

Each household is assumed to have generalized recursive preferences as in Epstein and Zin (1989), Weil (1989), and Swanson (2018):

\[
V(b_{t-1}, h_{t-1}; \Theta_t) = \max_{c_t, h_t, l_t \in \Gamma} u(c_t^*, h_t^*, l_t^*) + \beta(E_t V(b_t^*, h_t^*; \Theta_{t+1}))^{1-\alpha},
\]

where \( V(b_{t-1}, h_{t-1}; \Theta_t) \) is the household’s generalized value function, \( \Gamma \) is the choice set for \( c_t, h_t \), and \( l_t \), \( u \) is the period utility function, \( \beta \in (0,1) \) is the household’s time discount factor, \( \alpha \) measures the additional curvature of generalized recursive preferences, and the state of the aggregate economy, \( \Theta_t \), controls the processes for \( w_t, r_t, q_t^h \), and \( d_t \). We also assume that \( c_t^* = c^*(b_{t-1}, h_{t-1}; \Theta_t), h_t^* = h^*(b_{t-1}, h_{t-1}; \Theta_t), \) and \( l_t^* = l^*(b_{t-1}, h_{t-1}; \Theta_t) \) are the optimal interior solutions for consumption, real estate, and labor, respectively.

3. Risk aversion and real estate wealth
In this section, we derive the closed-form expression for risk aversion with real estate and generalized recursive preferences. Our goal is to understand how real estate in period utility affects an economic agent’s attitude toward risk at the nonstochastic steady state. We investigate whether real estate wealth increases risk avoidance or provides space for taking additional risk.

3.1. The coefficient of absolute risk aversion
We consider the household’s aversion to a hypothetical one-shot gamble in period \( t \) in the budget constraint:

\[
b_t^* = (1 + r_t)b_{t-1} + w_t l_t^* + d_t - c_t^* - q^h_t h_t^* + q^h_t (1 - \tau - \kappa) h_{t-1} + \sigma \epsilon_{t+1},
\]

where \( \epsilon_{t+1} \in [\xi, \Xi] \) is a random variable representing the gamble. The \( \epsilon \) is dated \( t + 1 \) to indicate that the outcome of the gamble is not known in period \( t \). It is also assumed that \( E_t \epsilon_{t+1} = 0 \) and \( E_t \epsilon_{t+1}^2 = 1 \). The standard deviation \( \sigma \) represents the size of the gamble.

We also consider how much the household would be willing to pay to avoid the gamble. If the household pays a one-time fee, \( \mu \), to avoid the gamble, its budget constraint is given by

\[
b_t^* = (1 + r_t)b_{t-1} + w_t l_t^* + d_t - c_t^* - q^h_t h_t^* + q^h_t (1 - \tau - \kappa) h_{t-1} - \mu.
\]
**Proposition 1** Absolute Risk Aversion

Let \((b, h; \Theta)\) be an interior point of \(X\) where \(X\) is the domain of \((b, h; \Theta)\). Then, the coefficient of absolute risk aversion, \(R^a(b, h; \Theta)\), exists and

\[
R^a(b, h; \Theta) = -\frac{V_{11}(b, h; \Theta)}{V_1(b, h; \Theta)} + \alpha \frac{V_1(b, h; \Theta)}{V(b, h; \Theta)}
\]

where \(V_1\) and \(V_{11}\) are the first and second partial derivatives of the household’s value function with respect to the first element.  

Sketch of the Proof

As shown in Swanson (2012), the household’s welfare loss due to paying \(\mu_t\) to avoid the gamble can be written as

\[
-\frac{d\mu_t}{1 + r_t} V_1(b_{t-1}, h_{t-1}; \Theta_t).
\]

Notice that the first-order Taylor series expansion of the value function delivers

\[
V\left(b_{t-1} - \frac{d\mu_t}{1 + r_t}, h_{t-1}; \Theta_t\right) 
\approx V(b_{t-1}, h_{t-1}; \Theta_t) - \frac{d\mu_t}{1 + r_t} V_1(b_{t-1}, h_{t-1}; \Theta_t)
\]

where \(d\mu = \mu - 0\). The fee is paid at the end of period \(t\) right before the gamble, while the value function \(V_1(b_{t-1}, h_{t-1}; \Theta_t)\) is evaluated at the beginning of period \(t\). In this respect, the infinitesimal fee \(d\mu\) is discounted by the interest rate. Thus, the welfare loss from the discounted fee to avoid the gamble is approximated to be \(\frac{d\mu_t}{1 + r_t} V_1(b_{t-1}, h_{t-1}; \Theta_t)\).

The welfare loss from the gamble can be also derived using (2), (3), and the optimality conditions for consumption, real estate, and labor. The household’s first-order conditions with respect to \(c_t^*, h_t^*, l_t^*\) are

\[
\begin{align*}
  u_1(c_t^*, h_t^*, l_t^*) &= E_t M_{t+1} V_1(b_t^*, h_t^*; \Theta_{t+1}), \\
  u_2(c_t^*, h_t^*, l_t^*) &= E_t M_{t+1} \left[q_t V_1(b_t^*, h_t^*; \Theta_{t+1}) - V_2(b_t^*, h_t^*; \Theta_{t+1})\right], \\
  u_3(c_t^*, h_t^*, l_t^*) &= -w_t E_t M_{t+1} V_1(b_t^*, h_t^*; \Theta_{t+1}),
\end{align*}
\]

respectively. The term \(M_{t+1}\) is defined as \(M_{t+1} = \beta(E_{t} V(b_t^*, h_t^*; \Theta_{t+1})^{\alpha/(1-\alpha)} - V(b_t^*, h_t^*; \Theta_{t+1})^{-\alpha})\), and \(u_1, u_2,\) and \(u_3\) denote the corresponding partial derivatives of period utility.

The first-order impact of a change in \(\sigma\) on the household’s welfare or value function is zero. The second-order effect of a change in \(\sigma\) on household welfare can be summarized as:

\[
\begin{align*}
\frac{d\sigma^2}{2} \left[-\beta \alpha (E_t V_{t+1}^{1-\alpha})^{\alpha/(1-\alpha)} E_t V_{t+1}^{\alpha-1} V_{1t+1}^2 + \beta (E_t V_{t+1}^{1-\alpha})^{\alpha/(1-\alpha)} E_t V_{t+1}^{\alpha} V_{11t+1}\right]
\end{align*}
\]
Following Arrow (1965), Pratt (1964), and Swanson (2012), we define absolute risk aversion as

\[
R^a(b_{t-1}, h_{t-1}; \theta_t) = \lim_{\sigma \to 0} \frac{\mu(b_{t-1}, h_{t-1}; \theta_t; \sigma)}{\sigma^2/2}
\]

where \(\mu(b_{t-1}, h_{t-1}; \theta_t; \sigma)\) denotes willingness of the household to pay the fee to avoid the gamble with the size \(\sigma\).\(^7\) (10) implies that risk aversion rises as the willingness to pay \(\mu\) in response to an infinitesimal change in \(\sigma^2\) increases. (6) is the welfare loss from paying the fee to avoid the gamble, while (9) is the welfare loss arising from the uncertainty of the gamble. Equating (6) to (9) delivers absolute risk aversion at the nonstochastic steady state, which is given by:

\[
R^a(b, h; \theta) = -\frac{V_{11}(b, h; \theta)}{V_{1}(b, h; \theta)} + \alpha \frac{V_{1}(b, h; \theta)}{V(b, h; \theta)}
\]

which is exactly the same expression as in Swanson (2018) except for the fact that the real estate stock \(h\) is considered in the value function. Our work can be viewed as an extension of his work by incorporating real estate in the utility function.

**Proposition 2 Closed-form Expression of Absolute Risk Aversion**

The household’s coefficient of absolute risk aversion at the nonstochastic steady state is given by

\[
R^a(b, h; \theta) = \frac{\lambda^h u_{12} + \lambda^l u_{13} - u_{11}}{u_1} \frac{r(1+r)}{(1+w\lambda^l)(1+r) + (\tau + \kappa - r)q^h\lambda^h + \alpha \frac{ru_1}{u}}
\]

where \(u_i\) and \(u_{ij}\) for \(i \in [1,2,3]\) and \(j \in [1,2,3]\) denote the corresponding partial derivatives of the household’s period utility at the nonstochastic steady state. The parameters \(\lambda^h\) and \(\lambda^l\) are defined as

\[
\lambda^h = \frac{u_1(u_{12}u_{33} - u_{13}u_{23}) + u_2(u_{13}^2 - u_{11}u_{33}) + u_3(u_{11}u_{23} - u_{12}u_{13})}{u_1(u_{22}u_{33} - u_{23}^2) + u_2(u_{13}u_{23} - u_{12}u_{33}) + u_3(u_{12}u_{23} - u_{13}u_{22})}
\]

and

\[
\lambda^l = \frac{u_1(u_{12}u_{23} - u_{13}u_{22}) + u_2(u_{12}u_{13} - u_{11}u_{23}) + u_3(u_{11}u_{22} - u_{12}u_{13})}{u_1(u_{23}^2 - u_{22}u_{33}) + u_2(u_{12}u_{33} - u_{13}u_{23}) + u_3(u_{13}u_{22} - u_{12}u_{23})}
\]

**Sketch of the Proof**

(11) can be written as (12) using the optimality conditions (8), the budget constraint (1), and the first order conditions of the value function (2) with respect to the state variables, \(b_{t-1}\) and \(h_{t-1}\). See Supplemental Appendix for proof.

3.2. The coefficient of relative risk aversion

We derive relative risk aversion by taking into account the household’s wealth at time \(t\) as the hypothetical gamble.\(^8\)
**Definition 1 Relative Risk Aversion When Wealth Includes Both Consumption and the Value of Real Estate**

Let \((b, h; \theta)\) be an interior point of \(X\). The household’s coefficient of relative risk aversion, \(R^{ch}(b, h; \theta)\), is given by \(A^{ch} R^a(b, h; \theta)\) where \(A^{ch}\) is the nonstochastic steady state value of the discounted sum of present and future wealth, \(A^{ch} \equiv (1 + r_t)^{-1}E_t \sum_{j=0}^{\infty} m_{t+j} \left(c^*_{t+j} + q^h_{t+j}(h^*_t - h^*_t) + (\tau + \kappa)h^*_t\right)\). The stochastic discount factor of the household is denoted as \(m_{t+1} = \frac{u_1}{u_1} M_{t+1}\), and the discount factor \((1 + r_t)^{-1}\) indicates that wealth is evaluated in beginning of period \(t\).

**Proposition 3 Closed-form Expression of Relative Risk Aversion When Wealth Includes Both Consumption and the Value of Real Estate**

The coefficient of relative risk aversion at the nonstochastic steady state is given by

\[
R^{ch}(b, h; \theta) = A^{ch} R^a(b, h; \theta) = \frac{\lambda^h u_{12} + \lambda^l u_{13} - u_{11}}{u_1} \left(1 + r\right) \left(c + (\tau + \kappa)q^h h\right) \frac{(1 + w\lambda^l)(1 + r) + (\tau + \kappa - r)q^h \lambda^h + \alpha(c + (\tau + \kappa)q^h h)}{u_1}
\]

since \(A^{ch} = \frac{c + (\tau + \kappa)q^h h}{r}\) in the nonstochastic steady state.

**4. Numerical example**

In order to illustrate the impact of real estate on risk aversion and risk premium, we describe a real business cycle (RBC) model with generalized recursive preferences.

**4.1. The basic neoclassical model with real estate**

There is a unit continuum of identical households. Each household chooses consumption, real estate, labor, and real risk-free bond to maximize (2) subject to the budget constraint (1). Period utility is given by

\[
log \varphi_t = (1 - \rho_\varphi) \varphi + \rho_\varphi log \varphi_{t-1} + \varepsilon_t^\varphi,
\]

where \(\varphi_t\) denotes the real estate preference shock, \(\eta > 0\) is the relative weight on labor, and \(\chi > 0\) denotes the inverse Frisch elasticity of the labor supply. The preference shock \(\varphi_t\), follows an exogenous AR(1) process:

\[
log \varphi_t = (1 - \rho_\varphi) \varphi + \rho_\varphi log \varphi_{t-1} + \varepsilon_t^\varphi,
\]

where \(\rho_\varphi \in (-1,1)\) and \(\varepsilon_t^\varphi \sim i.i.d. N(0, \sigma_\varphi^2)\). The steady state value of the preference shock, \(\varphi\), represents the relative utility weight of real estate. The utility function is assumed to be additively separable, which makes derivation of the coefficient of relative risk aversion more tractable and interpretable. The modification for non-separability does not affect the key results in this paper.

The first order necessary conditions for bond, real estate, and labor are given by

\[
1 = E_t m_{t+1}(1 + r_{t+1}),
\]
\[ q_t^h = E_t m_{t+1} (1 - \tau - \kappa) q_{t+1}^h + \varphi_t \frac{c_t}{h_t}, \text{and} \]

and

\[ \eta t^\chi c_t = w_t, \]

where \( m_{t+1} = \frac{c_t}{c_{t+1}} M_{t+1} \) denotes the stochastic discount factor of the household. The dynamics of real estate prices are determined by (19), and the intratemporal Euler equation (20) determines labor supply.9

There is a unit continuum of perfectly competitive firms producing goods using physical capital, labor, and real estate as inputs. The production function is given by

\[ y_t = A_t (k_t^{1-\omega} h_{f,t}^{\omega})^{1-\theta} t^\theta, \]

where \( k_t \) is physical capital, \( h_{f,t} \) denotes real estate stock held by producers, \( \omega \in (0,1) \) and \( \theta \in (0,1) \) are parameters that determine the relative shares of real estate and labor, respectively, and \( A_t \) is an exogenous aggregate productivity following an AR(1) process:

\[ y_t = A_t (k_t^{1-\omega} h_{f,t}^{\omega})^{1-\theta} t^\theta, \]

where \( \rho_A \in (0,1) \) is a parameter capturing persistence of technology, and \( \epsilon_t^A \) is an i.i.d. white noise process with mean zero and variance \( \sigma_A^2 \). The law of motion for physical capital evolves according to

\[ k_{t+1} = (1 - \delta) k_t + \zeta_t i_t, \]

where it denotes investment, \( \delta \) is the capital depreciation rate, and \( \zeta_t \) denotes the investment shock following an exogenous AR(1) process:

\[ \log \zeta_t = \rho_\zeta \log \zeta_{t-1} + \epsilon_t^\zeta, \]

where \( \rho_\zeta \in (-1,1) \), and \( \epsilon_t^\zeta \sim i.i.d. N(0, \sigma_\zeta^2) \). The firm chooses capital, real estate, and labor to maximize the present discounted value of net revenues. Solving the profit maximization problem yields the first order necessary conditions for capital, real estate, and labor:

\[ q_t^k = E_t m_{t+1} \left( (1 - \omega)(1 - \theta) \frac{y_{t+1}^r}{k_{t+1}} + (1 - \delta) q_{t+1}^k \right). \]
\[ q_t^h = E_t m_{t+1} \left( \omega (1 - \theta) \frac{y_{t+1}}{h_{f,t+1}} + (1 - \tau - \kappa) q_{t+1}^h \right), \]

(27)

\[ w_t = \frac{\theta y_t}{l_t}, \]

where \( q_t^k \) denotes capital price.

There are a continuum of representative capital producers who supply new capital exploiting the final output. Capital producers are subject to the convex cost of adjusting investment. They sell capital to firms at price \( q_t^k \). The profit maximization problem is given by:

(28)

\[ \max_{i_t} E_t \sum_{j=0}^{\infty} m_{t+j} \left( (q_{t+j}^k \xi_{t+j} i_{t+j} - i_{t+j}) \right) - \frac{\xi}{2} \left( \frac{i_{t+j}}{i_{t+j-1}} - 1 \right) i_{t+j}, \]

where \( \xi \) governs the cost of adjusting investment. The first order necessary condition with respect to new capital yields

(29)

\[ \xi q_t^k = 1 + \frac{\xi}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right) + \xi \left( \frac{i_t}{i_{t-1}} - 1 \right) \left( \frac{i_t}{i_{t-1}} - 1 \right) - E_t m_{t+1} \xi \left( \frac{i_{t+1}}{i_t} - 1 \right) \left( \frac{i_{t+1}}{i_t} - 1 \right), \]

which determines the supply of investment.

Finally, in a competitive equilibrium, all markets clear. The goods market clearing condition leads to

(30)

\[ y_t = c_t + \left( 1 + \frac{\xi}{2} \left( \frac{i_t}{i_{t-1}} - 1 \right) \right) i_t + q_t^h (\tau + \kappa) (h_t + h_{f,t}). \]

The real estate market clearing condition leads to

(31)

\[ h_t + h_{f,t} = H \]

where \( H \) denotes the total supply of real estate and it is normalized to unity as is common in the literature (e.g., Iacoviello, 2005, Liu et al., 2013). Lastly, the labor market clears so that labor supply equals labor demand.

### 4.2. Relative risk aversion

Given period utility (16), the coefficients of relative risk aversion, (15), can be expressed as

(32)

\[ R_{ch}^h (b, h; \theta) = \left( 1 + (\tau + \kappa) \frac{q_t^h h_t}{c} \right) \left[ \frac{1}{1 + \frac{\eta}{\chi} + \frac{\phi}{r + \tau + \kappa}} + \frac{1}{u} \right], \]
when the nonstochastic steady state labor is normalized at unity, \( l = 1 \), by setting the relative weight on labor \( \eta \) appropriately for simplicity. When real estate is dropped from the model, the coefficients of relative risk aversion can be written as \( R^{ch}(b, h; \theta) = \frac{1}{1 + \chi} + \frac{\alpha}{u} \). This closed-form expression is isomorphic to the relative risk aversion derived in Swanson (2018). A decline in disutility from labor reduces the relative risk aversion since households are better able to protect themselves against a negative income shock by increasing labor. Therefore, given \( \eta > 0 \) the risk aversion declines when the parameter \( \chi \) decreases (Swanson, 2012, Swanson, 2018).\(^{10}\)

There are three main channels through which real estate affects the coefficient of relative risk aversion. First, the coefficient of relative risk aversion is larger when transaction and maintenance costs are embedded into the model. Compared to other assets, larger holding and selling costs of real estate prevent active trading, making real estate illiquid. In particular, transaction costs prevent the household from offsetting shocks to income by varying their real estate stocks. Thus, larger transaction costs imply a higher level of relative risk aversion. The coefficient of relative risk aversion is positively related to the size of the hypothetical gamble, \( q^h h \), which is related to transaction and maintenance costs of real estate.\(^{11}\) This channel associated with these costs and real estate wealth is referred to as the real estate risk channel.

Second, the relative risk aversion coefficient, \( R^{ch} \), declines when the household derives utility from real estate. When \( \varphi = 0 \), the household may reduce consumption drastically in response to a large negative income shock. In the case of \( \varphi > 0 \), total spending on both real estate and consumption can also decline remarkably in response to the same shock, while spending on real estate or consumption goods can decrease relatively moderately. Given the concave utility function, the decline in period utility is lower when \( \varphi > 0 \) than when \( \varphi = 0 \). Thus, the household faces a lower risk associated with period utility when utility is driven from not only consumption but also real estate services. Consequently, as shown in (32), the deviation of \( \varphi \) from zero lowers the relative risk aversion. When the parameter \( \varphi \) increases, the household derives more utility from real estate, leading to a higher demand for it. The real estate price therefore rises with the parameter \( \varphi \). Notice that we hold \( q^h = \frac{\varphi}{1 - \beta(1 - \tau - \kappa)} h \) at the nonstochastic steady state due to (19). Given a relatively high price of real estate, the household is able to smooth consumption by slightly reducing real estate stock, making utility less volatile in the face of a negative income shock. As a result, the coefficient of relative risk aversion declines with the parameter \( \varphi \).\(^{12}\) We call this channel the cushioning channel.

Finally, as discussed above, a rise in the parameter \( \varphi \) induces a higher demand for real estate by households. The increased demand causes real estate prices to rise, lowering the amount of real estate owned by firms. A decline in the firm’s share of real estate leads to a fall in the marginal product of labor. Notice that real estate is an important input for production. Accordingly, the household’s income declines, and a fall in expenditure on goods and real estate services lowers the level of utility, \( u \).\(^{13}\) As shown in (32), the decreased utility drives up the coefficient of relative risk aversion. We call this channel the crowding-out channel.

4.3. Quantitative analysis

To better understand the relation between real estate and the coefficient of relative risk aversion, we provide a quantitative analysis. Table 1 shows the parameter values selected for the model. There are 18 parameters. Six parameters are related to real estate, while the remaining twelve are not. Most parameter values are fairly standard. The discount factor, \( \beta \), is set to 0.99, implying an annual real interest rate of 4 percent. The share of
labor, $\theta$, is fixed at 0.7, and the depreciation rate of capital is set to 0.025. These parameter values are broadly in line with King and Rebelo (1999). We choose a value of 0.92 for the relative utility weight of labor, $\eta$, to set nonstochastic steady state value of labor to unity for simplicity. The inverse Frisch elasticity of labor supply, $\chi$, is set to 3 following Del Negro et al. (2015). The elasticity of investment adjustment cost, $\xi$, is equal to 0.3, the persistence of an investment shock, $\rho_c$, is set to 0.7, and the standard deviation of investment shock, $\sigma_c$, is set to 0.01. These values that help match the volatility of investment are in the range of estimates in the literature (e.g., Christensen and Dib, 2008, Justiniano et al., 2011, Liu et al., 2013).

Table 1. Benchmark calibration.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Descriptions</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount rate</td>
<td>0.99</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Epstein–Zin parameter</td>
<td>6</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Relative utility weight of real estate</td>
<td>0.05</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Relative utility weight of labor</td>
<td>0.92</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Inverse Frisch elasticity of labor supply</td>
<td>3</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Maintenance fee rate</td>
<td>0.0035</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Realtor commission fee</td>
<td>0.05</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Degree of leverage</td>
<td>3</td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elasticity of output to labor</td>
<td>0.7</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Elasticity of output to real estate</td>
<td>0.15</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.025</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Elasticity of investment adjustment cost</td>
<td>0.3</td>
</tr>
<tr>
<td><strong>Shock process</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_A$</td>
<td>Persistence of technology</td>
<td>1</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Standard deviation of technology shock</td>
<td>0.005</td>
</tr>
<tr>
<td>$\tilde{\rho}_c$</td>
<td>Persistence of investment shock</td>
<td>0.7</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Standard deviation of investment shock</td>
<td>0.01</td>
</tr>
<tr>
<td>$\rho_\varphi$</td>
<td>Persistence of real estate preference shock</td>
<td>0.98</td>
</tr>
<tr>
<td>$\sigma_\varphi$</td>
<td>Standard deviation of real estate preference shock</td>
<td>0.01</td>
</tr>
</tbody>
</table>

The Epstein–Zin parameter, $\alpha$, is fixed to 6. This value implies that the coefficient of relative risk aversion, $R^{ch}$, is about 79. As shown in the literature, a large relative risk aversion coefficient is necessary to match the observed risk premiums on equities and bonds. For example, Rudebusch and Swanson (2012) set the coefficient of relative risk aversion to 110 to explain the bond premium. Swanson (2019) choose 60 to match the risk premiums for bonds and equities. The persistence of a technology shock is set to 1 as in Tallarini (2000). This high persistence is essential for a sizable risk premium (e.g., Swanson, 2019). The degree of leverage is fixed at 3 in line with the finance literature (e.g., Abel, 1999, Gourio, 2012, Campbell et al., 2014, Swanson, 2019).

Our choices for the real estate-related parameter values are as follows. The relative utility weight of real estate, $\varphi$, is set to 0.05, while the firm’s real estate share, $\omega$, is set to 0.15. The standard deviation and persistence of real estate preference shock are set to 0.98 and 0.01, respectively. These values are chosen to match the volatility and autocorrelation of real estate prices. These parameter values are similar to estimates of Iacoviello (2005) and Liu et al. (2013). The maintenance fee rate, $\tau$, cannot be observed directly from the data, so the observed house depreciation rate is used as a proxy. As pointed out by Piazzesi and Schneider (2016), “all
depreciation is “essential maintenance” without which the house is uninhabitable”. The parameter $\tau$ is set to 0.0035, indicating that the annual cost is 1.4 percent of real estate value. This is consistent with the estimate of Davis and Heathcote (2005). The parameter $\kappa$ is set to a value of 0.05, consistent with a commission fee of 5 percent of the real estate price. Delcoure and Miller (2002), Piazzesi and Schneider (2016), and Barwick and Wong (2019) report the commission fee rate is about 5 to 6 percent in the United States. As we show in Table 2, the model with these parameter values does a reasonable job in accounting for the volatility of key variables. In subsequent sections, we also discuss how our results vary with alternative values of the key real estate-related parameters, $\varphi$, $\omega$, and $\kappa$.

Table 2 reports the empirical moments of the variables of interest and the model-implied moments. The quarterly US macroeconomic data is from 1985:Q1 to 2008:Q4. Real GDP, consumption, investment, and the 3-month Treasury bill rate are taken from the FRED database of the St. Louis Fed, and real estate prices are from Davis and Heathcote (2005). The equity premium is from Mehra and Prescott (2003), while the real estate (housing) risk premium is from Piazzesi et al. (2007). All series are HP filtered. Panel A shows that the model does a reasonable job in matching the observed standard deviations of the key macroeconomic variables. Panel B reveals that the model-implied risk premia on equity ($\psi^e_t, \psi^e_{lev, t}$) and real estate ($\psi^h_t$) are quite close to those from the data. $\psi^e_t$ and $\psi^e_{lev, t}$ denote the equity premium when stocks are modeled as a claim on the firm’s profit and a levered claim on consumption stream, respectively. $\psi^h_t$ denotes the real estate risk premium, which can be defined using a claim on rent. Since the model explains both the macroeconomic moments and the risk premia well, we use the model to investigate the relationship between real estate wealth and the household’s attitude toward risk.

<table>
<thead>
<tr>
<th>Panel A: Standard deviations</th>
<th>US data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta y)$</td>
<td>0.52</td>
<td>0.52</td>
</tr>
<tr>
<td>$\sigma(\Delta c)$</td>
<td>0.46</td>
<td>0.61</td>
</tr>
<tr>
<td>$\sigma(\Delta i)$</td>
<td>2.54</td>
<td>2.39</td>
</tr>
<tr>
<td>$\sigma(\Delta q^h)$</td>
<td>0.71</td>
<td>0.72</td>
</tr>
<tr>
<td>$\sigma(r)$</td>
<td>1.02</td>
<td>0.82</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: Risk premium</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi^e_t$</td>
<td>8.06</td>
<td>7.54</td>
</tr>
<tr>
<td>$\psi^e_{lev, t}$</td>
<td>–</td>
<td>3.16</td>
</tr>
<tr>
<td>$\psi^h_t$</td>
<td>1.77</td>
<td>1.78</td>
</tr>
</tbody>
</table>

Note: This table presents unconditional moments of key macroeconomic variables from the data and from the model. $\Delta x_t$ denotes the first difference of variable $x_t$. $\psi^e_t$ and $\psi^e_{lev, t}$ denote the equity premium when stocks are modeled as a claim on the firm’s profit and a levered claim on a consumption stream, respectively. $\psi^h_t$ denotes the real estate risk premium, which is defined using a claim on rent as shown in Section 4.4.

The steady state values of $c$, $h$, $q^h$, and $u$ are required to compute the relative risk aversion coefficients. Table 3 reports how the nonstochastic steady state values of the key variables vary with the relative weight on real estate, $\varphi$, in the utility function. The parameter $\varphi$ ranges from 0.0 to 0.09. This range is chosen since the estimate of $\varphi$ is about 0.05 in the literature (e.g., Liu et al., 2013).

Table 3. Real estate preferences and steady state values.
The table shows how the nonstochastic steady state values of macroeconomic variables change as the relative weight on real estate, $\varphi$, in the utility function increases.

The table shows that consumption, output, and period utility ($c$, $y$, and $u$) decline as the parameter $\varphi$ increases.

The economic intuition behind these results is as follows. The demand for real estate rises with $\varphi$ since the household derives more utility from real estate. The increased demand pushes up the real estate price, $q^h$, reducing the firm’s share of real estate, $h_f$. Firms use less real estate for production, which in turn decreases the marginal product of labor and the wage. A decline in wages lowers consumption and output. Given a budget constraint, expensive real estate prices reduce consumption further. Thus, the decline in consumption lowers period utility, $u$.

The steady state values have important implications on relative risk aversion. The crowding-out channel suggests that an increase in $\varphi$ causes the relative risk aversion coefficient to rise due to the decline in $u$. A rise in $\varphi$ raises the real estate wealth to consumption ratio, $\frac{q^h h}{c}$, driving up the relative risk aversion coefficient as shown in (32). The cushioning channel implies that a rise in the real estate price, $q^h$, helps households hedge against a negative income shock since real estate can be liquidated.

We here study how model specifications regarding preferences and costs of transaction and maintenance change the coefficient of relative risk aversion. Table 4 presents the coefficients of relative risk aversion under each model specification. With zero transaction and maintenance costs and expected utility preferences, the coefficient of relative risk aversion, $R_{ach}$, depends on the parameters associated with labor and the relative weight on real estate in the utility function. It shows that the relative risk aversion varies inversely with $\varphi$ due to the cushioning channel of real estate. Once the household is endowed with generalized recursive preferences and transaction and maintenance costs are abstracted from the model, the coefficient of relative risk aversion, $R_{ach}$, is determined by not only the cushioning channel but also the crowding-out channel. As discussed before, there is a negative relationship between $\varphi$ and $u$. A rise in $\varphi$ causes $u$ to decline, increasing the coefficient of relative risk aversion. When the household has expected utility preferences and transaction and maintenance costs are incorporated into the model, the coefficient of relative risk aversion, $R_{ach}$, is affected by both the cushioning channel and the real estate risk channel. The latter suggests that larger costs of transaction and maintenance imply lower levels of relative risk aversion. Finally, $R_{ach}$ includes all three channels. These relative risk aversion coefficients help us determine the effect of these channels on the relative risk aversion.

\[
\begin{array}{ccccccc}
\varphi & c & h & h_f & q^h & y & u \\
0.00 & 1.53 & 0.00 & 0.99 & 1.39 & 1.97 & 0.20 \\
0.01 & 1.51 & 0.14 & 0.85 & 1.62 & 1.95 & 0.16 \\
0.03 & 1.46 & 0.33 & 0.66 & 2.05 & 1.92 & 0.11 \\
0.05 & 1.42 & 0.45 & 0.54 & 2.47 & 1.90 & 0.07 \\
0.07 & 1.38 & 0.53 & 0.46 & 2.87 & 1.88 & 0.04 \\
0.09 & 1.35 & 0.59 & 0.40 & 3.25 & 1.86 & 0.01 \\
\end{array}
\]

Note: This table shows how the nonstochastic steady state values of macroeconomic variables change as the relative weight on real estate, $\varphi$, in the utility function increases.
Table 4. The coefficients of relative risk aversion.

<table>
<thead>
<tr>
<th></th>
<th>Expected utility preferences: $\alpha = 0$</th>
<th>Generalized recursive preferences: $\alpha &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau = \kappa = 0$</td>
<td>$R^{ch}_a = \frac{1}{1 + \eta \frac{\tau}{\chi} + \varphi}$</td>
<td>$R^{ch}_b = \frac{1}{1 + \eta \frac{\tau}{\chi} + \varphi} + \frac{1}{u}$</td>
</tr>
<tr>
<td>$\tau &gt; 0, \kappa &gt; 0$</td>
<td>$R^{ch}_c = \left(1 + (\tau + \kappa) \frac{q^h h}{c} \right) \left(1 + \eta \frac{\tau}{\chi} + \varphi \frac{r - \tau - \kappa}{r + \tau + \kappa} \right)$</td>
<td>$R^{ch} = \left(1 + (\tau + \kappa) \frac{q^h h}{c} \right) \left(1 + \eta \frac{\tau}{\chi} + \varphi \frac{r - \tau - \kappa}{r + \tau + \kappa} + \frac{1}{u} \right)$</td>
</tr>
</tbody>
</table>
Fig. 1 plots the relative risk aversion coefficients \( R^c_{\alpha} , R^c_{b}, R^c_{\gamma}, \) and \( R^c_{\chi} \) against the parameter \( \varphi \) to see the relationship between relative risk aversion and the relative weight on real estate in the utility function. The left panel of Fig. 1. A shows values of \( R^c_{\chi} \) in the function of \( \varphi \). As the parameter \( \varphi \) rises from 0 to 0.09, the relative risk aversion declines from 0.765 to 0.716 due to the cushioning channel.\(^{15}\) \( R^c_{\chi} \) differs from the conventional relative risk aversion measure of Arrow (1965) and Pratt (1964), \( -\frac{cu''(c)}{u'(c)} \), in which only a consumption good affects the household’s utility. For the case of logarithmic utility function, the Arrow–Pratt measure of relative risk aversion is one. The right panel of Fig. 1. A for \( R^c_{\chi} \) shows that the cushioning effect is reversed with the presence of transaction and maintenance costs. This result shows that the real estate risk channel is more important than the cushioning channel in determining the relative risk aversion, \( R^c_{\chi} \). Our closed-form expressions show that ignoring transaction and maintenance costs of real estate could lead to inaccurate measure of relative risk aversion.

A. Expected Utility Preferences (\( \alpha = 0 \))

Panel B of Fig. 1 shows values of the coefficients of relative risk aversion, \( R^c_{\alpha} \) and \( R^c_{\gamma} \), against \( \varphi \) with respect to the curvature parameter of generalized recursive preferences, \( \alpha \in [1,2,\ldots,6] \). We find that replacing expected utility preferences with generalized recursive preferences significantly increases relative risk aversion regardless of the value of \( \alpha \), and relative risk aversion is highly sensitive to this parameter. Comparing \( R^c_{\alpha} \) and \( R^c_{\gamma} \) (or \( R^c_{\chi} \) and \( R^c_{\chi} \)) reveals that relative risk aversion dramatically increases due to the term \( \alpha \frac{1}{u} \), which is related to the crowding-out channel. Below, we show that the difference between \( R^c_{\alpha} \) and \( R^c_{\gamma} \) (or between \( R^c_{\chi} \) and \( R^c_{\chi} \)) is mostly attributable to the crowding-out channel. Notice that even when the parameter \( \varphi \) is set to zero, the relative risk aversion coefficient, \( R^c_{\chi} \), still has the term \( \alpha \frac{1}{u} \), implying that it does not purely capture the crowding out channel.

Fig. 1. The coefficients of relative risk aversion and the relative weight on real estate (\( \varphi \)). Note: The x-axis represents the relative weight on real estate, \( \varphi \), in the utility function.
We here study the size of the crowding channel and its sensitivity to the parameter $\alpha$ and $\varphi$. To this end, we plot the difference between $R_{bc}^{ch}$ and $R_{bc}^{ch}|_{u=u_0}$ against $\varphi$ with respect to the curvature parameter $\alpha$ ranging from one to six and present it in the left panel of Fig. 2. $R_{bc}^{ch}|_{u=u_0}$ denotes the value of $R_{bc}^{ch}$ when $u$ is fixed at $u_0$, where $u_0$ is the level of utility when firms own all real estate in the economy for production so that there is no crowding-out effect. The difference between $R_{bc}^{ch}$ and $R_{bc}^{ch}|_{u=u_0}$ is $\alpha \left(\frac{1}{u} - \frac{1}{u_0}\right)$, which captures the size of the crowding-out impact on the relative risk aversion. The figure shows that the effect of the crowding-out channel on relative risk aversion exponentially increases as the parameter $\varphi$ rises. It is also greatly affected by the parameter $\alpha$. When $\varphi = 0.05$, $R_{bc}^{ch} - R_{bc}^{ch}|_{u=u_0}$ changes from 7.7 to 46.4 as the parameter $\alpha$ increases from 1 to 6. The impact of this channel to the relative risk aversion becomes larger than those of the other channels as the parameter $\alpha$ increases.

![Graph](image)

Fig. 2. The effect of crowding-out channel on relative risk aversion. Note: The x-axis represents the relative weight on real estate, $\varphi$, in the utility function.

The right panel of Fig. 2 illustrates how the difference between $R_{bc}^{ch}$ and $R_{bc}^{ch}|_{u=u_0}$ varies with the parameter $\varphi$ when the household is subject to transaction and maintenance costs of real estate. The difference between $R_{bc}^{ch}$ and $R_{bc}^{ch}|_{u=u_0}$ is $\alpha \left(\frac{1}{u} - \frac{1}{u_0}\right) + (\tau + \kappa) \frac{q^h c}{\alpha} \left[\frac{1}{u} - \frac{1}{u_0}\right]$, which captures the crowding-out channel and its interaction with the real estate risk channel. Accordingly, the difference between the left and right panel of Fig. 2 arises from the interaction term of the channels.

Overall, the figures reveal that the most influential channel in determining the coefficient of relative risk aversion is the crowding-out channel, whereas the impact of the cushioning channel and the real estate risk channel is relatively smaller. The role of the cushioning channel in determining the coefficient of relative risk aversion is likely to be more limited when introducing occasionally binding collateral constraints into the model. In our model, real estate stocks can be liquidated when a negative shock hits the economy. Introducing occasionally binding collateral constraints can limit the hedging role of real estate against a negative shock, reducing the impact of the cushioning channel on relative risk aversion.
4.4. Simple exercises: Asset pricing implications

This subsection conducts simple exercises to investigate the impact of the parameter $\varphi$ and $\alpha$ on the equity premium and the real estate risk premium.

4.4.1. The equity premium

In order to price equity, we follow conventional asset pricing theory based on the stochastic discount factor (e.g., Mehra and Prescott, 1985, Cochrane, 2009). The price of an equity security in equilibrium is given by

$$p_t^e = E_t(m_{t+1}(d_{t+1} + p_{t+1}^e)),$$

where $p_t^e$ denotes the ex-dividend price of an equity at time $t$, and $d_{t+1}$ is a real dividend at time $t + 1$. We define dividend in two different ways. The dividend can be defined as the real profit from the firm as follows:

$$d_{t+1} = y_{t+1} - w_{t+1}l_{t+1} - q_i^k_{t+1} - q_i^b_{t+1}(h_{f,t+1} - (1 - \tau - \kappa)h_{f,t}).$$

Alternatively, the dividend can be considered as a levered claim on aggregate consumption following Abel (1999), Gourio (2012), Campbell et al. (2014), and Swanson (2019). So, $d_{t+1} = c_{t+1}^v$, where the parameter $v$ can be interpreted as capturing broad leverage in the economy, including operational and financial leverage.\footnote{We set $v$ to 3 as in Abel (1999).}

The ex-post gross return on equity is defined as

$$R_{t+1}^e = \frac{d_{t+1} + p_{t+1}^e}{p_t^e}.$$ Then, (33) can be written as

$$1 = E_t^e(m_{t+1}R_{t+1}^e),$$

which is the same form as the intertemporal Euler equation. The equity premium $\psi_t^e$ is defined as the difference between the expected real return to equity and the risk-free rate,

$$\psi_t^e \equiv E_t^e(R_{t+1}^e - (1 + r_{t+1}).$$

The equity premium is denoted as $\psi_t^e$ when real profit is exploited for a dividend, while it is denoted as $\psi_{t,\text{lev}}^e$ when the levered consumption claim is used as a dividend.

Fig. 3 illustrates the relation between the equity premiums and the relative weight on real estate, $\varphi$, in the utility function. It is assumed that the household has generalized recursive preferences. The vertical axis is the equity premium in annualized percentage points, while the horizontal axis is the value of the parameter $\varphi$ which ranges from 0.00 to 0.09.
The figure shows that the equity premiums are positively associated with the magnitude of utility that the household obtains from real estate. Economic agents require more compensation as relative risk aversion increases with the parameter $\phi$. For the case in which the dividend is defined as the firm’s profit, the equity premium $\psi^e_t$ increases from 1.46 to 194.86 percent as the parameter $\phi$ rises from 0.00 to 0.09 (left panel). On the other hand, the levered consumption claim makes the equity premium relatively less sensitive to the value of $\phi$ (right panel). The equity premium $\psi^{lev,e}_t$ increases from 1.24 to 17.10 percent for the rise of $\phi$ from 0.00 to 0.09. The figures show that the equity premiums rise faster as the parameter $\phi$ increases. Notice that this pattern is also observed in the coefficient of relative risk aversion.

4.4.2. The real estate risk premium

This subsection examines the relationship between the real estate risk premium and the parameter $\phi$. The real estate risk premium is defined as the difference between the return on real estate and the risk-free rate. By recursively substituting out the future real estate price into (19), the real estate price can be expressed as the sum of current and discounted future marginal rates of substitution between real estate and consumption as follows:

$$q_t^h = E_t \sum_{j=0}^{\infty} m_{t,t+j}(1 - \tau - \kappa)^j \phi_{t+j} c_{t+j}/r_{t+j}$$

where $m_{t,t+j} \equiv \prod_{i=1}^{j} m_{t+i}$. This equation implies that the presence of large transaction costs could substantially lower the price of real estate. The marginal rate of substitution between real estate and consumption can be interpreted as dividend or rent (e.g., Han, 2013). The gross return on real estate at time $t + 1$ is defined as $R^h_{t+1} \equiv \frac{q^h_{t+1} \phi_{t+1} c^h_{t+1}}{q^h_t r^h_{t+1}}$, and it yields the real estate risk premium given by

$$\psi^{lev,e}_t$$
\[ \psi_t^h \equiv E_t R_{t+1}^h - (1 + r_{t+1}). \]

Fig. 4 shows the relation between the parameter \( \varphi \) and the real estate risk premium. We assume that the household has generalized recursive preferences. We also find that a rise in the parameter \( \varphi \) drives up the real estate risk premium as it increases relative risk aversion. The real estate risk premium rises from 0.53 to 30.50 percent as the parameter \( \varphi \) increases from 0 to 0.09. As shown in Panel B in Table 2, the model-implied real estate risk premium is 1.78 percent at our baseline calibration (\( \varphi = 0.05 \)). The empirical estimate of the real estate risk premium by Piazzesi et al. (2007) is 1.77 percent. Our results imply that an increase in relative risk aversion due to real estate may have a substantial contribution to the real estate risk premium.

![Real Estate Risk Premium, \( \psi^h \)](image)

Fig. 4. The real estate risk premium and the relative weight on real estate (\( \varphi \)). Note: The model-implied equity premium is in annualized percentage points. The x-axis represents the relative weight on real estate, \( \varphi \), in the utility function. The Epstein–Zin parameter \( \alpha \) is fixed at 6.

4.5. Realtor commission fees and relative risk aversion

This subsection studies how the relative risk aversion coefficient varies with transaction costs. The parameter \( \kappa \) can be interpreted as capturing not only realtor commission fees but also additional costs such as transfer and property taxes, home preparations, attorney fees, and real estate fees like escrow fees, title insurance, and HOA transfers. According to Zillow, the most popular real estate website that has about 36 million visitors per month as of 2020, average costs for sellers “range from 8% to 10% of the home’s sale price” in the United States.20

In order to investigate to what extent the coefficient of relative risk aversion varies with the parameter \( \kappa \), we first present the steady state values of the variables of interest with respect to the parameter \( \kappa \). Table 5 shows that a rise in the parameter \( \kappa \) leads to a decline in consumption, utility, and real estate price. Among them, the most dramatic change is related to the price of real estate. As the parameter \( \kappa \) rises from 0 to 0.1, the steady state price of real estate declines from 11.88 to 1.37. However, the firm’s (or household’s) share of real estate does not change much in response to a change in the parameter \( \kappa \) because the costs of transaction occur regardless of type of agents. Accordingly, there is no substantial change in output.
Fig. 5 plots the coefficient of relative risk aversion against the parameter $\kappa$ when the household is endowed with expected utility preferences (Panel A) and with generalized recursive preferences (Panel B). When the household has expected utility preferences, the coefficient of relative risk aversion rises by 7 percent, from 0.769 to 0.822, as $\kappa$ increases from 0 to 0.1. This change in the relative risk aversion is not negligible. Recall that the Arrow–Pratt measure of relative risk aversion is 1 for the logarithmic utility function. With generalized recursive preferences, the relative risk aversion substantially increases from 39.9 to 89.9 as the parameter $\kappa$ rises from 0 to 0.10. Our results imply that large transaction costs of real estate could play a crucial role in determining the household’s attitudes toward risk.

A. Expected Utility Preferences: $\alpha = 0$  
B. Generalized Recursive Preferences: $\alpha = 6$

![Fig. 5. The coefficients of relative risk aversion and realtor commission fee ($\kappa$). Note: The x-axis represents the realtor commission fee, $\kappa$.](image)

Table 5. Realtor commission fee and the steady state values.

<table>
<thead>
<tr>
<th>$\kappa$</th>
<th>c</th>
<th>h</th>
<th>$h_f$</th>
<th>$q^h$</th>
<th>y</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.51</td>
<td>0.47</td>
<td>0.52</td>
<td>11.88</td>
<td>1.89</td>
<td>0.15</td>
</tr>
<tr>
<td>0.02</td>
<td>1.44</td>
<td>0.46</td>
<td>0.53</td>
<td>4.71</td>
<td>1.89</td>
<td>0.09</td>
</tr>
<tr>
<td>0.04</td>
<td>1.42</td>
<td>0.45</td>
<td>0.54</td>
<td>2.93</td>
<td>1.89</td>
<td>0.08</td>
</tr>
<tr>
<td>0.06</td>
<td>1.41</td>
<td>0.45</td>
<td>0.54</td>
<td>2.13</td>
<td>1.90</td>
<td>0.07</td>
</tr>
<tr>
<td>0.08</td>
<td>1.41</td>
<td>0.45</td>
<td>0.54</td>
<td>1.67</td>
<td>1.90</td>
<td>0.07</td>
</tr>
<tr>
<td>0.10</td>
<td>1.41</td>
<td>0.45</td>
<td>0.54</td>
<td>1.38</td>
<td>1.90</td>
<td>0.07</td>
</tr>
</tbody>
</table>

Table 6. The real estate demand of firms and the steady state values.

<table>
<thead>
<tr>
<th>$\omega$</th>
<th>c</th>
<th>h</th>
<th>$h_f$</th>
<th>$q^h$</th>
<th>y</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>1.89</td>
<td>0.99</td>
<td>0.00</td>
<td>1.50</td>
<td>2.50</td>
<td>0.40</td>
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<tr>
<td>0.03</td>
<td>1.75</td>
<td>0.80</td>
<td>0.19</td>
<td>1.72</td>
<td>2.33</td>
<td>0.32</td>
</tr>
<tr>
<td>0.06</td>
<td>1.65</td>
<td>0.67</td>
<td>0.32</td>
<td>1.93</td>
<td>2.20</td>
<td>0.25</td>
</tr>
<tr>
<td>0.09</td>
<td>1.57</td>
<td>0.58</td>
<td>0.41</td>
<td>2.13</td>
<td>2.09</td>
<td>0.19</td>
</tr>
<tr>
<td>0.12</td>
<td>1.49</td>
<td>0.51</td>
<td>0.48</td>
<td>2.31</td>
<td>1.99</td>
<td>0.13</td>
</tr>
<tr>
<td>0.15</td>
<td>1.42</td>
<td>0.45</td>
<td>0.54</td>
<td>2.47</td>
<td>1.90</td>
<td>0.07</td>
</tr>
<tr>
<td>0.18</td>
<td>1.35</td>
<td>0.41</td>
<td>0.58</td>
<td>2.62</td>
<td>1.81</td>
<td>0.02</td>
</tr>
</tbody>
</table>
The relationship of the equity and real estate risk premiums with the parameter $\kappa$ is summarized in Fig. 6, holding all other parameters fixed at their baseline calibration as in Table 1. The curvature parameter of generalized recursive preferences is assumed to be 6. When dividend is defined as the firm’s profit, the equity premium, $\psi^e$, increases from 2.35 to 9.45 percent as the parameter $\kappa$ rises from 0 to 0.10. For the case that the dividend is based on a levered consumption claim, the alternative equity premium measure, $\psi^e_{lev}$, also rises from 1.64 to 3.56 percent. We also find the real estate risk premium, $\psi^h$, increases from 0.51 to 2.97 percent. These results show that large transaction costs matter in determining the additional rewards required by agents to hold risky assets.

**Fig. 6. Risk premia and realtor commission fee ($\kappa$).** Note: The model-implied equity and real estate premiums in annualized percentage points. The x-axis represents the realtor commission fee, $\kappa$. The Epstein–Zin parameter $\alpha$ is fixed at 6.

### 4.6. Real estate as a production factor and relative risk aversion

In a previous subsection, we investigated how the parameter $\varphi$ affects the relative real estate share of the household, $h$, in determining relative risk aversion. This subsection studies the relationship between the parameter $\omega$ and relative risk aversion. Recall that the parameter $\omega$ determines the relative real estate share to capital in the production function so that it plays a crucial role in accounting for the relative real estate share of the firm, $h_F$.

In order to explore how the parameter $\omega$ influences relative risk aversion, we first compute the steady state values with $\omega$ increasing from 0 to 0.18 by 0.03 and report them in Table 6. It shows that the share of real estate by the household declines as the parameter $\omega$ increases. On the other hand, the firm’s share increases with the parameter $\omega$. A higher demand for real estate by the firm drives up the price of real estate. Accordingly, consumption declines as expenditure of the household on real estate rises. The fall in consumption could lead to a decline in output and utility in the steady state. When the real estate share of firms increases from 0 to 0.18, the household’s utility declines from 0.40 to 0.02 due to a drastic decline in consumption.

**Fig. 7** illustrates the relationship between relative risk aversion and the parameter $\omega$. Panel A shows the results when the parameter $\alpha$ is set to zero. We again see that there is a negative relationship between relative risk aversion and the parameter $\omega$. As shown in Table 6, real estate wealth of the household, $h_q^h$, declines from 1.48 to 1.07 as the relative estate share to capital, $\omega$, in production rises from 0 to 0.18. Therefore, there is a negative relationship between relative risk aversion and real estate wealth. Panel B is generated under the assumption that the household has generalized recursive preferences. It shows that the coefficient of relative risk aversion increases substantially as the parameter $\omega$ increases from 0 to 0.18. Notice that the real estate
price increases as the firm’s demand for real estate rises. A higher demand for real estate by the firm leads to a rise in its price and a decline in consumption and utility, yielding a surge in relative risk aversion due to the crowding-out channel. Finally, the large difference between the relative risk aversions in Panel A and B is mostly explained by the crowding out channel, in which the interaction between utility and the curvature of generalized recursive preferences plays a crucial role in determining relative risk aversion. The importance of this channel heavily depends on the size of the Epstein–Zin parameter $\alpha$.

A. Expected Utility Preferences: $\alpha = 0$  
B. Generalized Recursive Preferences: $\alpha = 6$

Fig. 7. The coefficients of relative risk aversion and relative real estate share to capital in production ($\omega$). Note: The x-axis represents the relative real estate share to physical capital, $\omega$, in the firm’s production function.

Fig. 8 presents the equity premiums, $\psi^e$ and $\psi^e_{lev}$, and the real estate risk premium, $\psi^h$ when the household has generalized recursive preferences. As the relative real estate share to capital, $\omega$, rises from 0 to 0.18, the equity premium computed using the real profit of the firm rises from about 0.59 to 54.07 percent. The equity premium on the basis of the levered consumption claim increases from 0.7 to 8.82 percent. The equity premium is a bit sensitive to the parameter $\omega$, as is the coefficient of relative risk aversion. These results show that the firm’s real estate holding matters in determining not only the dividend but also the equity premium. The figure also shows that the real estate risk premium also rises with the parameter $\omega$. A rise in the parameter $\omega$ from 0 to 0.18 causes the real estate premium to rise from 0.21 to about 9.52 percent. Overall, we find that a higher risk aversion leads to a larger risk premium.

Fig. 8. Risk premia and relative real estate share to capital in production ($\omega$). Note: The x-axis represents the relative real estate share to physical capital, $\omega$, in the firm’s production function.
5. Conclusion
Real estate is an important asset not only for households but also for firms. As pointed out by Eiling et al. (2019), housing explains half of the median-wealth household portfolio. Undoubtedly, real estate is an essential element for firms' production. This paper derives the household’s relative risk aversion when real estate is owned by the household for utility and consumption smoothing and by the firms for production. We find three channels through which real estate affects relative risk aversion. Both the real estate wealth risk channel and the crowding out channel imply that relative risk aversion increases with real estate price, while the cushioning channel suggests that the former decreases with the latter. The relationship between risk aversion and real estate price depends on whether the household is endowed with generalized recursive preferences or expected utility preferences. The basic real business cycle model with generalized recursive (expected utility) preferences predicts that relative risk aversion is positively (negatively) related to real estate price at our baseline calibration.

CRediT authorship contribution statement
Sungjun Huh: Conceptualization, Methodology, Investigation, Software, Visualization, Writing - original draft, Writing - review & editing. Insu Kim: Conceptualization, Methodology, Investigation, Visualization, Writing - original draft, Writing - review & editing.

References
Swanson, 2019. Swanson Eric T. A Macroeconomic Model of Equities and Real, Nominal, and Defaultable Debt. unpublished manuscript University of California, Irvine (2019)

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In this paper, and all errors and omissions, are our own and are not necessarily those of the individuals above. An online appendix for this article is available at *Journal of Macroeconomics* online.

1. In the U.S., realtor commission fees are about 5 percent of real estate price.
2. On average, it takes 3 months to sell houses according to Zillow (https://www.zillow.com/sellers-guide/costs-to-sell-a-house/). Since the national price-to-(annual) rent ratio is about 18 as of 2020, the cost of the time to sell a house, measured with respect to rent, is about 1.4 percent of house price. This cost is much less than real estate commission fee. We can also capture this cost by increasing the value of $\kappa$ by 1.4 percent.
3. In this paper, we derive risk aversion using wealth-gamble approach (e.g., Stiglitz, 1969, Constantinides, 1990, Swanson, 2012). Alternatively, risk aversion can be measured by household’s utility function over consumption and consumption bundle (e.g., Kihlstrom and Mirman, 1974, Kihlstrom and Mirman, 1981).
4. We rigorously derive the closed-form expressions for risk aversion with real estate in line with Swanson, 2012, Swanson, 2018. To derive (5), we assume that $u' > 0$ and $u'' < 0$. The absolute risk aversion has the same functional form as Swanson (2018) derives except for the presence of real estate.
5. See Supplemental Appendix for details.
6. We show both the first-order and the second-order impacts in Supplemental Appendix.
7. Notice that $\mu(b_{t-1}, h_{t-1}; \Theta_t; \sigma)$ depends on the economic state so that the absolute risk aversion depends the economic state as well.
8. The difference between absolute and relative risk aversion, accordingly, is the household’s wealth level.
9. As shown in (36), transaction and maintenance costs reduce real estate prices.
10. This paper focuses on the implications of real estate on relative risk aversion, while Swanson, 2012, Swanson, 2018 focuses on how labor affects relative risk aversion.
11. Note that $q_{hh}$ appears in the measure of relative risk aversion since the discounted sum of present and future wealth, $A_c$, is multiplied to the coefficient of absolute risk aversion, $R_a$, as shown in Proposition 3.
12. Although Swanson, 2012, Swanson, 2018 do not consider real estate, they derive a closed-form solution of risk aversion not only with consumption but also with labor and document that labor margin provides a cushion for negative shocks. Increasing labor to a negative income shock lowers the coefficient of relative risk aversion. In our paper, we show that real estate also provide an additional cushioning effect.
13. The relative weight on real estate is estimated to be 0.045 in Liu et al. (2013), indicating that utility heavily depends on non-real estate consumption goods.
14. We provide a detailed discussion for the definitions of the equity and real estate premiums in Section 4.4.
15. The labor margin emphasized by Swanson (2012) lowers the relative risk aversion from 1 to 0.765. Allowing the household to derive (dis)utility from real estate and labor reduces relative risk aversion since the household is able to adjust the stock of real estate and hours worked in response to a negative shock.
16. We did not introduce occasionally binding collateral constraints into the model due to technical issues in deriving relative risk aversion.
17. The degree of leverage is positively associated with fixed production costs (Gourio, 2012, Campbell et al., 2014, Swanson, 2019).
18. We also studied the case of expected utility preferences. The equity premium varies inversely with the parameter $\phi$ but the model-implied equity premium is very small. Results are available upon request.
19. The main objective of this paper is to derive the coefficient of relative risk aversion and to study how it is related to the equity premium and the real estate premium. Matching the risk premiums is not our focus. However, we find that the model does well in matching the data.
The relative real estate wealth to consumption, $hqc$, does not change much as the parameter $\omega$ increases from 0 to 0.19. This term that appears in (32) drives up relative risk aversion, but its impact is small. However, a decline in utility leads to a surge in the coefficient of relative risk aversion.