The influence of learning and price-level targeting on central bank forward guidance

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The Influence of Learning and Price-Level Targeting on Central Bank Forward Guidance

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Abstract
This paper examines how the effectiveness of central bank forward guidance depends on two key channels: the expectations formation process and the monetary policy regime. The results show that rational expectations relative to an adaptive learning rule amplifies the positive benefits a price-level targeting central bank creates for forward guidance. Specifically, forward guidance generates greater amounts of output and inflation under a price-level than inflation targeting monetary policy regime, but rational expectations overstates these positive benefits compared to adaptive learning. The different responses of expectations between rational expectations and adaptive learning to forward guidance are driving this performance gap. Thus, policymakers should consider how expectations are modeled if forward guidance and price-level targeting are implemented in an economy.
1. Introduction
The unconventional monetary policy of forward guidance—communication to the public about the
future course of policy—has constituted the toolkit of central banks around the world since the
aftermath of the Great Recession. For instance, the U.S. Federal Open Market Committee (FOMC)
conveyed guidance on the future course of the interest rate in their December 2008 statement to the
public when short-term interest rates hit the zero lower bound (ZLB). The Federal Reserve also
implemented forward guidance by communicating information about the path of its long-run inflation
target. In 2012, the Federal Reserve stated that “The Committee judges that inflation at the rate of 2
percent ... is most consistent over the longer run with the Federal Reserve’s statutory mandate”
(Federal Reserve, 2012). In addition, communication about the future course of policy will likely be a
part of the Federal Reserve’s monetary toolkit in the future. Forward guidance could be used again
when ZLB episodes constrain conventional monetary policy. During periods away from the ZLB,
forward guidance can be employed to provide greater clarification and transparency to the public
(see Williams et al., 2013).

The efficacy of forward guidance hinges on two key assumptions—the monetary policy regime and
expectations formation process. As of this writing, the Federal Reserve implements an inflation
targeting regime. However, the U.S. inflation rate has generally been below its Federal Reserve target
of 2% over the period May 2012 through May 2019.1 In response, an alternative framework that has
been debated is a price-level targeting monetary policy (see Bullard, 2018 and Clarida, 2019). Since
inflation and price-level central bank policies target different variables, the type of monetary regime
can influence the efficacy of forward guidance statements about future policy.2 Moreover, the effect a
monetary policy regime has on forward guidance is influenced by the expectations of agents. Since
forward guidance concerns statements about the future course of policy, the way in which
expectations of private sector agents are formed affects the potency of forward guidance. Full-
information rational expectations (FIRE) is the standard way to model agents’ expectations in
macroeconomic models. While a reasonable starting point for analysis, FIRE may not be best suited for
modeling the economy’s expectations as Bernanke (2007), (para. 6) states “The traditional rational-
expectations model of inflation and inflation expectations... is less helpful for thinking about
economies in which (1) the structure of the economy is constantly evolving in ways that are
imperfectly understood by both the public and policymakers.” Therefore, it is natural to investigate the
effectiveness of forward guidance under different central bank policies when FIRE is no longer
assumed.

This paper examines how the effects of central bank forward guidance depend on the monetary policy
regime when an adaptive learning rule replaces the standard FIRE hypothesis. A benchmark New
Keynesian model with a Taylor (1993) rule and time-varying inflation target is augmented with
anticipated or forward guidance shocks on the inflation target. These shocks can be thought of as
communication about the Federal Reserve’s long-run inflation target. The model is solved under both
inflation and price-level targeting central banks. The results are then examined under FIRE and adaptive learning expectations formation processes.

The two types of monetary policy regimes studied in this paper are inflation and price-level targeting. Under both types of central banks, the monetary authority is assumed to adjust its short-term policy rate following a Taylor (1993) rule that depends on lagged interest rates, output gap, and an unanticipated monetary policy shock. However, an inflation targeting central bank includes deviations of the inflation rate from its target in its decision to adjust interest rates. Alternatively, a price-level targeting monetary authority responds to deviations of the price level from its target. This latter central bank regime can be beneficial as it is inherently history dependent. Unlike an inflation targeting regime that lets “bygones be bygones,” a price-level targeting monetary authority reacts to past inflation shocks. If there exists a decrease in the price level and inflation in the past, a price-level targeting central bank will use its policy tool to elevate the price level back to its target causing a period of above average inflation. Any previous decreases in inflation below target are met with subsequent increases in inflation above target so that the growth in the price level achieves its target on average (e.g. 2%). This feature can bring more positive benefits to the economy (see Giannoni (2014)).

The standard New Keynesian model examined in this paper shows that output and inflation depend on expectations of future macroeconomic variables. The conventional method to evaluate expectations is via FIRE. When constructing forecasts about future macroeconomic variables, FIRE assumes agents know the true structure of the economy, the exact relationship between variables (i.e. value of the deep parameters), beliefs of other agents, and distribution of the structural shocks. In contrast, the adaptive learning approach to evaluating expectations does not make as strong assumptions. This approach starts with viewing agents as real-life econometricians (see Evans and Honkapohja, 2013). In particular, when constructing forecasts about future economic variables (e.g. GDP or inflation), he or she might rely on a linear econometric model of the economy. The econometrician would estimate the parameters of this model using Ordinary Least Squares (OLS), and produce forecasts using this estimated version of the economy. He or she would reestimate the model’s parameters and update forecasts when new data about the economy are released. Similarly, adaptive learning agents in this paper proceed in an analogous fashion. They build a linear model based on a VAR(1) in three variables (output gap, inflation, and interest rates) as well as central bank announcements of the inflation target and forward guidance included as additional regressors. As new data arrive, adaptive learning agents estimate the coefficients of their model using OLS and update their beliefs each period. Thus, in the preceding description, it is apparent that both real-life econometricians and adaptive learning agents engage in a similar process of “learning” about the economy.

The results show that rational expectations relative to adaptive learning amplifies the positive benefits a price-level targeting regime creates for the effectiveness of forward guidance. This outcome is shown via the monetary authority communicating forward guidance about its long-run constant inflation target. The benchmark rational expectations results display that the effects of forward guidance on the economy are boosted under a price-level targeting central bank relative to an inflation targeting central bank. However, FIRE overstates these benefits compared to adaptive learning. When an adaptive learning rule replaces the rational expectations assumption, the reactions of output and
inflation to forward guidance under a price-level targeting rule are dampened versus rational expectations. In addition, the different responses of expectations to forward guidance are driving the results. Forecasts of output and inflation react more favorably to forward guidance under rational expectations than under adaptive learning. The reason is that the former type of agents base expectations on the true model of the economy. They know precisely how future statements about the inflation target will affect future variables, and consequently, influence the values of current output and inflation. In contrast, the reaction of expectations to forward guidance under adaptive learning agents is dampened. The reason is that these agents have to estimate the effects of forward guidance statements on the economy, which leads to a performance gap between the two expectations formation processes.

The paper then examines two approaches that influence the performance gap between the two expectations formation processes. If the linear forecasting model of adaptive learning agents separates more from FIRE, the response of macroeconomic variables under adaptive learning is more diminished relative to the baseline scenario. The value of the central bank’s long-run inflation target also influences the performance gap. Higher (lower) values of the inflation target lead to increased (decreased) differences between the responses of rational expectations and adaptive learning to forward guidance announcements under a price-level targeting regime.

Additional extensions are also performed to examine the main results of the paper. If an inflation targeting central bank places more weight on deviations of inflation from its target, the performance gap between rational expectations and adaptive learning agents still exists. However, the better outcomes of a price-level targeting regime relative to inflation targeting regime dampen. The duration of the forward guidance horizon also does not overall influence the baseline results. The length of the forward guidance horizon affects the amount of time rational expectations amplifies the positive effects a price-level targeting regime creates for forward guidance relative to adaptive learning. In addition, the amount of weight in which adaptive learning agents place on new information (i.e. constant gain parameter) is varied. If agents place more weight on recent forecasts errors, output and inflation are overall farther away from their rational expectations counterparts than under the baseline adaptive learning scenario. However, a lower value of the constant gain parameter implies agents’ coefficient estimates do not vary as much from the previous period. Thus, their reactions to forward guidance are closer to their rational expectations counterparts. Moreover, the results of the paper are robust to alternative measures of the forward guidance shocks.

Overall, the results suggest essential takeaways for policymakers. A price-level targeting central bank can increase the effectiveness of forward guidance on the economy relative to inflation targeting. However, FIRE overstates these effects relative to a more realistic adaptive learning rule. Thus, policymakers should consider how expectations are modeled if forward guidance and price-level targeting are implemented in an economy.

1.1. Previous literature
The contribution of the paper to prior research is that it analyzes how the monetary policy regime influences the effectiveness of central bank forward guidance when the expectations formation process varies. It lies at the intersection of three important studies: central bank forward guidance,
price-level targeting, and adaptive learning. This remaining section will describe how the present paper
fits into prior work in those areas.

The effectiveness of central bank forward guidance has been analyzed from other perspectives. For
instance, Eggertsson and Woodford (2003) discuss that management of interest rates far into the
future can be beneficial for an economy at the ZLB. Kiley (2016) show the effects of anticipated
monetary policy (e.g. forward guidance) at the ZLB. McKay et al. (2015) approach the topic from the
complete markets assumption, which can affect the efficacy of forward
guidance. De Graeve et al. (2014) describe that the effectiveness of forward guidance depends on
whether the central bank communicates forecasts on other macroeconomic variables besides the
expected path of interest rates. Carlstrom et al. (2012) and Del Negro et al. (2012) show that standard
macroeconomic models exhibit unusually large responses of the macroeconomic variables to forward
guidance news relative to what the data would indicate. The latter group of authors finds that
augmenting their model with the perpetual youth model
of Blanchard et al. (1985) and Yaari (1965) provides a potential solution to the extreme responses of
the variables to forward guidance. Moreover, the effects of forward guidance on the economy have
been analyzed through the lens of the expectations formation process. Gaus et al. (2015) examines the
effect of forward guidance when the central bank and private sector agents have different forecasting
models. Andrade et al. (2019) incorporate heterogeneous expectations into a New Keynesian model
and show that this heterogeneity in beliefs can influence the effectiveness of forward guidance on the
economy. By utilizing an adaptive learning framework, Ferrero and Secchi (2010) study the effect of
central bank publications of forecasted variables. In particular, when the monetary authority reveals its
expected path of the interest rate, they show that adverse effects on the economy are
possible. Cole (2015) and Cole (2016) discuss that the power of forward guidance is dampened when
FIRE is relaxed in favor of a more realistic adaptive learning rule. Moreover, the current paper adds to
this unconventional monetary policy literature by studying the influence of both the expectations
formation process and type of monetary authority on the effectiveness of forward guidance.

The benefits of a price-level targeting monetary authority relative to an inflation targeting monetary
authority have been indicated in previous literature. Eusepi and Preston (2018) survey the New
Keynesian model literature and discuss that the beneficial features of a price-level targeting regime are
robust to the expectations formation process assumed. Lower fluctuations in inflation can also result
under a price-level targeting regime relative to inflation targeting regime as discussed
by Svensson (1999). Giannoni (2014) explains that the path of optimal monetary policy can result if a
central bank targets the price level. Ball et al. (2005) discuss that optimal policy regards targeting the
price level when demand and productivity shocks hit the economy. Gaspar et al. (2007) explain that
more desirable outcomes are achieved under price-level targeting monetary authority due to
expectations appropriately adjusting to stabilize the economy. Cover and Pecorino (2005) show that
the positive outcomes under price-level targeting occur via the interest rate
channel. Vestin (2006) discusses that the positive benefits occurring under a price-level relative to
inflation targeting central bank are due to the inherent history dependence of the former. In
addition, Bernanke (2017), Amano and Ambler (2008), Billi (2008), and Wolman (2005) explain that a
monetary authority that targets the price level can aid an economy at the ZLB. Billi (2015) shows that a
price-level targeting regime can mitigate the adverse effects of future uncertainty.
Prior research has also analyzed price-level targeting and the expectations formation process. Branch and Evans (2017) examine raising the inflation target under adaptive learning. If agents believe the central bank to be credible, Branch and Evans (2017) find that adverse effects of increasing the inflation target can be mitigated. Hatcher and Minford (2016) analyze both inflation and price-level targeting policies and find that the benefits of the latter depend on the assumption of rational expectations. Williams (2010) and Reifschneider and Roberts (2006) explain that if expectations do not follow FIRE, the benefits of price-level targeting can be dampened at the ZLB. Honkapohja and Mitra (2018) examine price-level targeting and inflation targeting under adaptive learning. They show how the performance of price-level targeting depends on the credibility of the policy. Under adaptive learning and forward guidance, Honkapohja and Mitra (2015) also study price-level targeting with forward guidance. However, forward guidance is implemented only in a price-level targeting rule, whereas the present paper models this unconventional monetary policy in both inflation and price-level targeting rules.

Altogether this paper adds to prior literature by examining a new mechanism through which forward guidance influences the economy: the combination of the expectations formation process and monetary policy regime. The studies described in the previous three paragraphs have focused on other channels through which forward guidance can affect the economy. In addition, the present paper is related to Cole (2018) who examines how the effectiveness of forward guidance changes based on the monetary policy regime. However, his study only analyzed the FIRE case. Cole (2018) finds that a price-level targeting central bank boosts the effectiveness of forward guidance relative to an inflation targeting central bank. What substantially differentiates the present paper from Cole (2018) and prior literature is that I analyze how different expectations formation processes—FIRE and an adaptive learning rule—affect the performance of forward guidance under both inflation and price-level targeting regimes. Specifically, I first examine the effects of forward guidance across different monetary policy rules. I then show that the beneficial effects of a price-level targeting regime for forward guidance are dampened if FIRE is replaced by an adaptive learning rule. Thus, to the best of my knowledge, prior literature has not analyzed forward guidance under different monetary policy regimes and different expectations formation processes. This previous point is significant for policymakers if forward guidance and price-level targeting are to be considered in an economy.

The remaining sections of the paper are organized as follows. Section 2 presents the New Keynesian model augmented with central bank forward guidance. The two expectations formation processes considered in this paper—FIRE and adaptive learning—are detailed in Section 3. Section 4 discusses the main results of the paper. Section 5 considers two approaches—adaptive learning agents’ forecasting model and different values of the inflation target—that could influence the performance gap between rational expectations and adaptive learning. Extensions to the paper are examined in Section 6, which involve a more aggressive inflation targeting central bank, the duration of the forward guidance horizon, the amount that adaptive learning agents discount previous information, and alternative measures of the forward guidance shocks. Section 7 concludes.

2. Model
This paper utilizes the standard New Keynesian model, which is fully derived and analyzed in Woodford (2003a) and Galí (2015). The model includes equations summarizing aggregate demand,
aggregate supply, and the central bank of the economy. The log-linearized equations for the first two sectors are given by

\begin{align}
1) \quad x_t &= \hat{E}_t x_{t+1} - \sigma (i_t - \hat{E}_t \pi_{t+1}) + r^n_t \\
2) \quad \pi_t &= \beta \hat{E}_t \pi_{t+1} + \kappa x_t + \mu_t
\end{align}

Equation (1) represents aggregate demand for an economy. The output gap \((x_t)\) depends on the current period nominal interest rate \((i_t)\), expectations about one-period ahead output gap and inflation \((\hat{E}_t x_{t+1} \text{ and } \hat{E}_t \pi_{t+1}, \text{ respectively})\), and the natural real rate interest shock.\(^4\) The latter can be interpreted as a demand shock (e.g. government spending shock). The parameter \(\sigma > 0\) represents the intertemporal elasticity of substitution of consumption. In addition, the supply side of the economy consists of firms who operate in a monopolistically competitive market where they utilize labor from households to produce their goods and services. Firms are subject to sticky prices a lá Calvo (1983). Specifically, each period a fraction \(0 < \alpha < 1\) of firms cannot optimally adjust their selling price, and instead, set prices to the previous period’s price level. The remaining \(1 - \alpha\) of firms can optimally reset their prices. Consequently, Eq. (2) represents the New Keynesian Phillips Curve (NKPC) showing that current inflation \((\pi_t)\) depends on expected one-period ahead inflation, current output gap, and a cost-push shock \((\mu_t)\). The latter term can be interpreted as changes in input prices that then affect the selling price of the final good or service. The parameter \(0 \leq \beta \leq 1\) defines the household’s discount rate. The slope of the NKPC is defined by the parameter \(\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{(1+\omega\theta)}(\omega + \sigma - 1) > 0\). The parameters \(\omega\) and \(\theta\) constitute the elasticity of real marginal cost with respect to output and elasticity of substitution across differentiated goods, respectively. Moreover, the price level in the economy is defined by the following equation:

\begin{equation}
3) \quad p_t = \pi_t + p_{t-1}
\end{equation}

Finally, the operator \(\hat{E}_t\) represents (potentially) non-rational expectations. \(\hat{E}_t\) is in contrast to mathematical expectations operator \(E_t\), where the latter indicates rational expectations.

The natural real rate interest and cost-push shocks are assumed to follow AR(1) processes given by

\begin{align}
4) \quad r^n_t &= \rho_n r^n_{t-1} + \varepsilon^n_t \\
5) \quad \mu_t &= \rho \mu_{t-1} + \varepsilon^\mu_t
\end{align}

The last terms in Eqs. (4) and (5) are assumed to be formed from the following distributions:

\begin{align}
6) \quad \varepsilon^n_t \sim i.i.d N(0, \sigma^2_n) \\
7) \quad \varepsilon^\mu_t \sim i.i.d N(0, \sigma^2_\mu)
\end{align}

2.1. Central bank

The central bank of the economy follows a monetary policy rule that includes forward guidance and targets either the inflation rate or the price level when adjusting its short-term policy rate \(i_t\). An inflation targeting central bank adjusts the interest rate to itself, inflation in deviation from its target, output gap, and an unanticipated monetary policy shock. A price-level targeting monetary authority alters the interest rate to
changes in itself, price level in deviation from its target, output gap, and an unanticipated monetary policy shock. The central bank also issues the same forward guidance on the inflation target under both inflation and price-level targeting regimes. Moreover, the specification of central bank monetary policy rules under both inflation and price-level targeting are described next.

Inflation and Price-Level Monetary Policy Rules:

A central bank targeting the inflation rate adjusts the interest rate \( i_t \) to itself \( (i_{t-1}) \), deviations of inflation from its target \( (\pi_t - \pi_t^*) \), output gap \( (x_t) \), and an unanticipated monetary policy shock \( (\varepsilon_t^{MP}) \). Thus, the monetary authority behaves according to the following monetary policy rule:

\[
(8) \quad i_t = \rho i_{t-1} + (1 - \rho) \left[ \chi_{\pi} (\pi_t - \pi_t^*) + \chi_x x_t \right] + \varepsilon_t^{MP}
\]

The inflation target is assumed to be time varying and described by the following process:

\[
(9) \quad \pi_t^* = \rho \pi^* \pi_{t-1} + \varepsilon_t^* \pi^*
\]

The structure for Eq. (9) follows from recent literature, such as Milani (2019), Milani and Treadwell (2012), and Del Negro et al. (2014). In addition, the unanticipated monetary policy shock \( (\varepsilon_t^{MP}) \) is assumed to be formed from the following distribution:

\[
(10) \quad \varepsilon_t^{MP} \sim iid N(0, \sigma_{MP}^2)
\]

In contrast, a monetary authority targeting the price level follows a policy rule that adjusts the interest rate to deviations in the price level from its target \( (p_t - \bar{p}_t) \), output gap, and an unanticipated monetary policy shock. Specifically, a price-level targeting central bank changes \( i_t \) according to the following:

\[
(11) \quad i_t = \rho i_{t-1} + (1 - \rho) \left[ \chi_{p} (p_t - \bar{p}_t) + \chi_x x_t \right] + \varepsilon_t^{MP}
\]

where \( \bar{p}_t \) is a deterministic trend and is defined by the following equation:

\[
(12) \quad \bar{p}_t = \bar{p}_{t-1} + \pi_t^*
\]

Thus, the targeted price level is assumed to grow at the rate \( \pi_t^* \), which is defined by Eq. (9). Eq. (11) is similar to Honkapohja and Mitra (2015) and Giannoni (2014). It is also referred to as “Wicksellian” in Woodford (2003a), (chap. 2).

The potential benefits of a price-level targeting regime relative to inflation targeting regard the inherent history dependence of the former. For instance, suppose a negative cost-push shock impacts the economy (i.e. a decrease in \( \mu_t \)). The price level in the economy would then decrease. An inflation targeting regime would continue to pursue the inflation target at the lower price level. It treats the previous negative cost-push shock as a bygone. However, as seen by Eqs. (3) and (11), the price-level targeting central bank would use its monetary policy tool to increase the price level back to its target, which creates a period of elevated inflation. A historical decline in inflation below target would be followed by a period of above target inflation so that inflation achieves its policy defined level on average. Thus, price-level targeting regimes are inherently history dependent while inflation targeting central banks are not. In addition, Giannoni (2014), Woodford (2003a), and Woodford (2003b) explain that history dependence is a desirable feature for the conduct of a central bank.
Adding Forward Guidance: In order to inject central bank forward guidance into the model, the monetary authority is assumed to issue guidance on the future course of the inflation target. Similar to Del Negro et al. (2012), Cole (2015), Cole (2016), and Cole (2018), Eq. (9) is modified and replaced by the following:

\[ \pi_t^* = \rho \pi_t^* \pi_{t-1}^* + \varepsilon_{t}^{FG} + \sum_{l=1}^{L} \varepsilon_{t,l-l}^{FG} \]  

The last term in (13) contains anticipated or forward guidance shocks. Each \( \varepsilon_{t,l-l}^{FG} \) represents a shock known to agents in period \( t - l \), but will not affect \( \pi_t^* \) for another \( l \) periods, i.e. until period \( t \). For all \( l = 1, 2, \ldots, L \), each \( \varepsilon_{t,l-l}^{FG} \) is i.i.d., that is:

\[ \varepsilon_{t,l-l}^{FG} \sim i.i.d N(0, \sigma_{l,l}^{2,FG}), \forall l = 1, 2, \ldots, L \]

\( L \) also denotes the length of the forward guidance horizon. Moreover, \( \sum_{l=1}^{L} \varepsilon_{t,l-l}^{FG} \) represents an additional term derived from the following equations:

\[ v_{1,t} = v_{2,t-1} + \varepsilon_{1,t}^{FG} \]

\[ v_{2,t} = v_{3,t-1} + \varepsilon_{2,t}^{FG} \]

\[ \vdots \]

\[ v_{L,t} = \varepsilon_{L,t}^{FG} \]

which follow from Del Negro et al. (2012) and Laséen and Svensson (2011). Each component of \( v_t = [v_{1,t}, v_{2,t}, \ldots, v_{L,t}] \) contains past central bank communication (i.e. \( v_{2,t-1}, v_{3,t-1}, \ldots, v_{L,t-1} \)) and present central bank communication (i.e. \( \varepsilon_{1,t}^{FG}, \varepsilon_{2,t}^{FG}, \ldots, \varepsilon_{L,t}^{FG} \)) known to agents in period \( t \) that will affect the inflation target 1, 2, \ldots, or \( L \) periods ahead. Moreover, the link between Eqs. (15)–(17) and the last term in Eq. (13) is that the former equations can be simplified to show that \( v_{1,t-1} = \sum_{l=1}^{L} \varepsilon_{t,l-l}^{FG} \).

To summarize, both monetary authority regimes contain the same forward guidance. However, one targets the inflation rate while the other the price level.

This paper models forward guidance via anticipated shocks on the inflation target for the following reasons. First, forward guidance on the inflation target is motivated by the Federal Reserve statement on January 2012 that “The Committee judges that inflation at the rate of 2 percent...is most consistent over the longer run with the Federal Reserve’s statutory mandate” (Federal Reserve, 2012). Thus, as described by Cole (2018), forward guidance on the inflation target can be thought of as clarification and information about what the monetary authority believes is the appropriate future level for the inflation target. Indeed, Bernanke (2015), (para. 5) stated “the Fed’s commitment to 2 percent inflation is providing the public useful information about the FOMC’s likely approach to policy in the year ahead.” In addition, as will be described in the next section, the vector \( v_t \) containing the anticipated shocks can easily be added as an additional regressor to the forecasting model of adaptive learning agents. The effect of forward guidance on the macroeconomic variables (e.g. \( x_t, \pi_t \)) can straightforwardly be modeled and estimated. This setup can be viewed as similar to real-life forecasters who know central bank forward guidance announcements but have to estimate how those statements affect future macroeconomic variables. A constant inflation target can also still be achieved across the forward guidance horizon. Specifically, in Section 4.2, the forward guidance shocks will be
selected such that \( \pi_t^* = \bar{\pi}^* \) across the \( L \) forward guidance periods. Moreover, anticipated shocks have been utilized in the literature to model forward guidance. Laséen and Svensson (2011), Del Negro et al. (2012), Cole (2015), and Cole (2016) utilize anticipated shocks on the nominal interest rate. Honkapohja and Mitra (2015) model forward guidance on a policy variable other than the interest rate as is similar to the present paper. Furthermore, by modeling forward guidance on the inflation target, this paper can directly compare the effects the two different central bank policies have on forward guidance by only changing what the monetary authority targets.

Overall, the equations described above detail either inflation or price-level targeting central bank with forward guidance. The former is described by equations for aggregate demand, New Keynesian Phillips Curve, price level, natural real rate interest shock, cost-push shock, monetary policy rule targeting the inflation rate, time-varying inflation target with forward guidance, and past and present central bank communication affecting the inflation target. Specifically, these are Eqs. (1)–(5), (8), (13), and (15)–(17). When the monetary authority targets the price level, the model is the same but the monetary policy rule targeting the inflation rate is replaced with the monetary policy rule targeting the price level. Specifically, the equations are (1)–(5), (11)-(13), and (15)–(17).

3. Expectations formation processes

The expectations in Eqs. (1) and (2) are evaluated under FIRE and adaptive learning. The former is the standard assumption in macroeconomic models while the latter is a popular alternative. The next two subsections detail each expectations formation process.

3.1. Rational expectations

FIRE assumes agents are endowed with a greater amount of knowledge about the economy than adaptive learning agents. As explained in Evans and Honkapohja (2001) and Evans et al. (2009), rational expectations form forecasts of future macroeconomic variables based on the true model of the economy. Specifically, they know the structure of the model, values of the model’s deep parameters (e.g. \( \beta, \kappa \)), and distribution of the error terms.

To solve the New Keynesian model presented in Section 2 under FIRE, this paper first puts the equations into state space form. For instance, the New Keynesian model with an inflation-targeting regime described in this paper is rewritten in the following way:

\[
\begin{align*}
\bar{\Gamma}_0 \tilde{y}_t &= C + \bar{\Gamma}_1 \tilde{y}_{t-1} + \bar{\Gamma}_2 \tilde{\epsilon}_t + \bar{\Gamma}_3 \zeta_t \\
\end{align*}
\]

where

\[
\begin{align*}
\tilde{y}_t &= [x_t, \pi_t, i_t, p_t, \pi_t^n, \mu_t, v_{1,t}, v_{2,t}, \ldots, v_{L,t}, E_t x_{t+1}, E_t \pi_{t+1}, \pi_t^*]' \\
\tilde{\epsilon}_t &= [\epsilon_t^n, \epsilon_t^\mu, \epsilon_t^{MP}, \epsilon_t^{\pi^*}, \epsilon_t^{R_x}, \epsilon_t^{R_y}, \epsilon_t^{R_z}, \ldots, \epsilon_{L,t}^R]' \\
\end{align*}
\]

where \( E_t \) signifies the usual mathematical operator indicating rational expectations. \( C \) is a vector of constants and \( \zeta_t \) contains the vector of expectations errors defined in the following way:

\[
\begin{align*}
\zeta_t^x &= x_t - E_{t-1} x_t \\
\zeta_t^\pi &= \pi_t - E_{t-1} \pi_t \\
\end{align*}
\]
A standard way to solve this model under FIRE is via the method proposed by Sims (2002). Using the values in Table 1, the solution takes the form:

\[(23) \ddot{Y}_t = \ddot{C} + \xi_1 \ddot{Y}_{t-1} + \xi_2 \ddot{e}_t\]

where the expectations errors are now mapped into the vector of structural shocks, \(\ddot{e}_t\), and \(\ddot{C}, \xi_1,\) and \(\xi_2\) are complicated combinations of the model's deep parameters.8

Table 1. Parameter Values.

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ddot{\tau}) CGL</td>
<td>0.02</td>
</tr>
<tr>
<td>(\sigma) IES</td>
<td>0.15</td>
</tr>
<tr>
<td>(\beta) Discount Factor</td>
<td>0.99</td>
</tr>
<tr>
<td>(\kappa) Function of Price Stickiness</td>
<td>0.001</td>
</tr>
<tr>
<td>(\chi_{\pi}) Feedback Inflation</td>
<td>1.4</td>
</tr>
<tr>
<td>(\chi_X) Feedback Output Gap</td>
<td>1</td>
</tr>
<tr>
<td>(\chi_p) Feedback Price</td>
<td>0.25</td>
</tr>
<tr>
<td>(\rho_n) Autoregressive Demand</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_\mu) Autoregressive Cost-Push</td>
<td>0.5</td>
</tr>
<tr>
<td>(\rho_{n^*}) Autoregressive Inflation Target</td>
<td>0.99</td>
</tr>
<tr>
<td>(\rho) Interest Rate Inertia</td>
<td>0.85</td>
</tr>
<tr>
<td>(L) FG Horizon</td>
<td>12</td>
</tr>
</tbody>
</table>

Note: The standard deviations of the structural shocks are set to 0.0001. FG means forward guidance.

3.2. Adaptive learning

As argued in Evans et al. (2009), Mitra et al. (2013), and Mitra et al. (2017), the FIRE assumption that agents form expectations based on the true model of the economy may be strong. Indeed, Bernanke (2007), (para. 6) stated that “The traditional rational-expectations model of inflation and inflation expectations...is less helpful for thinking about economies in which (1) the structure of the economy is constantly evolving in ways that are imperfectly understood by both the public and policymakers.” In addition, adaptive learning models capture characteristics of a constantly changing economy described in the previous sentence and reconcile with features of the data. For instance, Eusepi and Preston (2018) explain that adaptive learning models are more consistent with the term structure of survey forecasts of macroeconomic variables than rational expectations models. Milani (2007) shows that incorporating adaptive learning into a Dynamic Stochastic General Equilibrium (DSGE) macroeconomic model reconciles better with the data than FIRE. Branch and Evans (2006) explain that adaptive learning algorithms fit well data from the Survey of Professional Forecasters and out-of-sample forecasting. Motivated by these previous reasons, this paper considers adaptive learning for the expectations formation process as an alternative to FIRE.

Adaptive learning agents are assumed to behave according to the cognitive consistency principle (see Evans and Honkapohja, 2013). Under this framework, agents behave like econometricians. If a real-life econometrician was to forecast future macroeconomic variables (e.g. Gross Domestic Product, inflation), he or she might first create a forecasting model. The econometrician would estimate the model’s parameters using standard techniques (e.g. Ordinary Least Squares (OLS)) and then construct expectations of future variables from the estimated model. As new data arrives, he or she would
update his or her expectations accordingly. Thus, it is apparent the econometrician is learning about the economy. Similarly, adaptive learning agents behave as real-life econometricians. In this paper, they construct a VAR(1) model in three variables (output gap, inflation, and interest rates) with central bank announcements of the inflation target and forward guidance included as additional regressors, estimate the parameters using OLS, form forecasts from the estimated model, and update their beliefs as new data arrive.

3.2.1. Forecasting model
Adaptive learning agents utilize a linear model of the economy labeled the “Perceived Law of Motion” (PLM) to construct forecasts. Under both monetary regimes considered in this paper, adaptive learning agents’ PLM is the following:

\[ Y_t = a + b Y_{t-1} + c v_t + d \pi^*_t + \epsilon_t \]

where \( Y_t = [x_t, \pi_t, i_t]' \), \( a, b, c, \) and \( d \) are coefficients of appropriate dimensions to be estimated each period, and \( \epsilon_t \) is the usual regression error term. The vector \( v_t \) can be expressed in the following way:

\[ v_t = \Phi v_{t-1} + \epsilon^{FG}_t \]

where

\[ \Phi = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots \\ 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \]

and \( \epsilon^{FG}_t = [\epsilon^{FG}_{1,t}, \epsilon^{FG}_{2,t}, \cdots, \epsilon^{FG}_{L,t}]' \). Thus, adaptive learning agents build a PLM of endogenous variables based off a VAR(1) model with the inflation target and forward guidance information announced by the central bank as additional regressors. It should also be noted that \( v_{1,t-1} \) is contained in \( \pi^*_t \).

The motivation for the structure of adaptive learning agents’ forecasting model given by Eq. (24) is as follows. A real-life economic forecaster might utilize a data driven VAR(1) with observable macroeconomic variables. He or she would also know forward guidance and inflation target announcements of the central bank (e.g. Federal Reserve). Similarly, adaptive learning agents construct forecasts using a VAR(1) containing output gap, inflation, and interest rates as well as knowledge of forward guidance (\( v_t \)) and time-varying inflation target (\( \pi^*_t \)). Using a PLM based on an empirical model is also motivated by Milani (2019), who estimates a New Keynesian model with adaptive learning and inflation target. It should also be noted that adaptive learning agents’ PLM does not correspond to the Minimum State Variable (MSV) solution under rational expectations. For instance, they do not know the structural shocks (e.g. \( r_{t}^{N}, \mu_t \)). However, Section 5.1 explores the results under different PLM structures.

It is also important to state the informational specifics adaptive learning agents are assumed to possess when constructing their expectations. First, at the beginning of period \( t \), they observe the central bank announcements about forward guidance and the inflation target. Adaptive learning agents use this
new information (i.e. \( v_t \) and \( \pi_t^* \)) and previous period’s coefficient estimates (i.e. \( \phi_{t-1} \)) in their PLM to form their forecasts of future macroeconomic variables. The values of output gap, inflation, and interest rate in period \( t \) are then realized. Finally, agents update their parameter estimates by regressing \( Y_t \) on \( 1, Y_{t-1}, v_t, \text{ and } \pi_t^* \).

Adaptive learning agents update their coefficient estimates every period using OLS. Specifically, \( a, b, c, \text{ and } d \) are updated using the following recursive least squares (RLS) formula:

(27) \[
\phi_t = \phi_{t-1} + \tau_t R_t^{-1} z_t (Y_t - \phi_{t-1} z_t) \]

(28) \[
R_t = R_{t-1} + \tau_t (z_t z_t' - R_{t-1})
\]

where \( \phi = (a, b, c, d)' \) contains the coefficients in the PLM, \( z_t \equiv [1, Y_{t-1}, v_t, \pi_t^*]' \) defines the PLM coefficients, and \( R_t \) is the matrix of second moments of \( z_t \). The last expression in Eq. (27) describes agents’ most recent forecast error (i.e. \( Y_t - \phi_{t-1} z_t \)).

A notable parameter in Eqs. (27) and (28) is \( \tau_t \), which is the “gain” parameter and controls the effect of agents’ most recent forecast error on coefficient estimates. This paper utilizes the discounted or constant gain learning (CGL) case, that is, \( \tau_t = \tau, \) over the decreasing gain or RLS case, that is, \( \tau_t = 1/t, \) for the following reasons. A beneficial and relevant feature of CGL is that recent information is weighted more when agents are updating \( \phi \) as they discount observations geometrically at the rate \( 1 - \tau \) (see Mitra et al., 2017). Thus, as described by Mitra et al. (2017), adaptive learning agents can more quickly track changes in policy than RLS (e.g. recent forward guidance statements about the inflation target).

Under both types of monetary policy regimes, agents utilize their PLM and coefficients to form forecasts. Thus, by forwarding Eq. (24) one period, their expectations are defined by the following:

(29) \[
\hat{E}_t Y_{t+1} = (I_3 + b) a + b^2 Y_{t-1} + (bc + c \Phi) v_t + (bd + d \rho \pi^*) \pi_t^* + dv_{1,t}
\]

The forecasts in Eq. (29) are plugged into aggregate demand and the NKPC, that is, Eqs. (1) and (2) to get the following “Actual Law of Motion” (ALM) of the economy:

(30) \[
\vec{Y}_t = \Gamma_0 (\phi_{t-1}) \vec{Y}_{t-1} + \Gamma_1 (\phi_{t-1}) \vec{Y}_{\pi_t} + \Gamma_2 (\phi_{t-1}) \vec{W}_t + \Gamma_3 (\phi_{t-1}) v_t + \Gamma_4 (\phi_{t-1}) \pi_t^*
\]

where \( \vec{Y}_t = [Y_t, p_t]' = [x_t, \pi_t, i_t, p_t]' \).

An additional comment on the adaptive learning approach utilized in this paper is warranted. Agents are assumed to follow the “Euler Equation” (EE) adaptive learning method when forming expectations. As described above, the model’s equilibrium equations for aggregate demand and NKPC contain one-period ahead expectations. Thus, agents form forecasts of one-period ahead output gap and inflation. The EE approach to learning is in contrast to the “Infinite Horizon” (IH) method. In this setup, Preston (2005) describes that the infinite path of variables (e.g. \( x_t, \pi_t, i_t, r^n_t, \text{ and } \mu_t \)) show up in the equilibrium Eqs. (1) and (2).

The framework presented and analyzed here is the EE method and the reasons are as follows. First, EE learning, which is the more common approach in the literature, is easier and more tractable to implement than IH learning. In addition, Honkapohja et al. (2013) detail that EE and IH learning are
equivalent, valid, and model consistent. The requirement is that agents know or quickly learn the market clearing conditions (e.g. \( y_t = c_t \)). Moreover, the purpose of the paper is to understand how the efficacy of forward guidance under different monetary policy regimes (i.e. inflation or price-level targeting) changes when FIRE is not assumed. By employing the EE approach, FIRE can easily be relaxed as the equations under both expectations formation processes are identical. Thus, IH learning is beyond the purview of the present study, but it would be interesting to examine in a future paper.

4. Results
4.1. Parameterization
The values of the model’s parameters are largely based on existing literature and can be found in Table 1. The parameter concerning aggregate demand, that is, Eq. (1), is the intertemporal elasticity of substitution of consumption \( \sigma \). The value of this parameter is set to 0.15, which closely follows Fuhrer (2000). The parameters involved in the NKPC are \( \beta \) and \( \kappa \). This paper assumes \( \beta = 0.99 \), which is standard in the literature. \( \kappa \) is set to 0.001, which corresponds to a degree of price stickiness (\( \alpha = 0.95 \)) used in Cole (2018) as well as elasticity of real marginal cost with respect to output (\( \omega = 2 \)) and elasticity of substitution across differentiated goods (\( \theta = 11 \)) found in Milani and Treadwell (2012). Moreover, the autoregressive terms on the natural real rate interest and cost-push shocks are not assumed to be highly persistent as \( \rho_n = 0.5 \) and \( \rho_\mu = 0.5 \). The exogenous shocks are also assumed not to be highly dispersed.

The policy side of the economy includes parameters relevant to the monetary policy rules under both central bank regimes. Under an inflation targeting scheme, \( \chi_\pi \) governs the response of interest rates to changes in inflation from its target. Table 1 displays that \( \chi_\pi = 1.4 \), which is near the value in Milani (2007). Section 6.1 will examine the results when an inflation targeting regime is more aggressive, that is, a higher value of \( \chi_\pi \) is employed. This paper assumes \( \chi_p = 0.25 \) for a price-level targeting central bank, which follows Honkapohja and Mitra (2015) and Williams (2010). Both types of monetary policy rules also include interest rate inertia, inflation target, and the output gap. A high degree of interest rate and inflation target inertia is assumed to exist as \( \rho = 0.85 \) and \( \rho_{\pi^*} = 0.99 \). These values closely follow Milani (2019) who estimates an adaptive learning model with habit formation. The value of \( \chi_x \) is set to 1, which follows Williams (2010).\(^{13}\) Finally, the value of the forward guidance horizon is set to 12 motivated by the value used in Cole (2018). However, Section 6.2 will examine the results under shorter and longer horizons as \( L = 12 \) is a somewhat arbitrary value.

A key parameter of interest is the constant gain learning parameter, \( \tilde{\tau} \), which governs the response of the learning coefficients to new information. As a benchmark, this paper utilizes \( \tilde{\tau} = 0.02 \). This value is consistent with prior literature, such as Milani (2019), Milani (2007), Branch and Evans (2006). However, the main results will be analyzed under lower and higher values of \( \tilde{\tau} \), in Section 6.3.

It is also important to examine how the New Keynesian model presented in this paper compares to the data. To accomplish this exercise, the contemporaneous correlations among change in output, inflation, and interest rates (i.e. \( \Delta x_t, \pi_t, \) and \( i_t \)) are compared between rational expectations, adaptive learning (with \( \tilde{\tau} = 0.02 \)), and the data. The data are taken from the FRED database of the Federal Reserve Bank of St. Louis and the relevant acronyms for output, inflation, and interest rates are GDPC1, GDPDEF, and FEDFUNDS. The time period includes 1981:Q3 - 2018:Q4. The results are displayed
The model’s cross-correlations under both expectations formation processes have the same sign as the data. The values of the cross-correlations are relatively in line with the data as well.

Table 2. Cross-Correlations of the Macroeconomic Variables.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Rational Expectations</th>
<th>Adaptive Learning</th>
<th>U.S. Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>corr(Δx, π)</td>
<td>-0.05</td>
<td>-0.04</td>
<td>-0.01</td>
</tr>
<tr>
<td>corr(Δx, i)</td>
<td>0.11</td>
<td>0.12</td>
<td>0.15</td>
</tr>
<tr>
<td>corr(π, i)</td>
<td>0.39</td>
<td>0.49</td>
<td>0.69</td>
</tr>
</tbody>
</table>


4.2. The effects of forward guidance under different monetary policy regimes and expectations formation processes

This section proceeds with the main forward guidance exercise of this paper, that is, implementing a constant inflation target across the forward guidance horizon. This scenario is motivated by the Federal Reserve’s press release in January 2012 stating that “The Committee judges that inflation at the rate of 2 percent...is most consistent over the longer run with the Federal Reserve’s statutory mandate” (Federal Reserve, 2012). Thus, based on the previous quotation, forward guidance on the inflation target can be thought of as clarification and information about what the central bank believes is the appropriate future level for the inflation target.

The constant inflation target exercise is described next and is similar to Del Negro et al. (2012) and Cole (2018). The model is simulated for $T$ periods. In period $T + 1$ the central bank announces that its inflation target will be at a constant target for $L$ periods into the future. This forward guidance statement amounts to anticipated changes by the public that the inflation target will be at a fixed level across the entire forward guidance horizon. Specifically, the central bank chooses the forward guidance shocks $e_t^{FG} = [e_{1t}^{FG}, e_{2t}^{FG}, \ldots, e_{Lt}^{FG}]'$ such that $\pi_t^* = \pi_t^*$ for $L$ periods into the future. In addition, forward guidance shocks are chosen under the assumption of rational expectations. These selected forward guidance shocks are then given to the adaptive learning agents. The results are then compared across inflation and price-level targeting regimes under both rational expectations and adaptive learning.

It is important to clarify the forward guidance shocks that are chosen under rational expectations to produce $\pi_t^* = \pi_t^*$ across the forward guidance horizon. As stated above, these same shocks were also given to adaptive learning agents. Even though the two expectations formation processes are different, the chosen forward guidance shocks produce the same constant inflation target across both rational expectations and adaptive learning. The reason is that the inflation target process is described by Eq. (13), which is not affected by the updated coefficients in $\phi$ and is exogenous to the model’s main equations. Therefore, agents under both expectations formation processes understand the forward guidance announcements that will ensure $\pi_t^* = \pi_t^*$ across the forward guidance horizon. The main difference between rational expectations and adaptive learning arises from the latter type of agents estimating how forward guidance affects the other macroeconomic variables.

The computational specifics of this constant inflation target exercise are also described next before examining the main results of this exercise. First, the shocks that are utilized to simulate the model are drawn from a normal distribution with mean zero and standard deviations stated in Table 1.
shocks are the same in both inflation targeting and price-level targeting simulations across both rational expectations and adaptive learning. In addition, after the forward guidance shocks are chosen in period $T + 1$ to ensure a constant inflation target across the entire horizon, the forward guidance shocks for the remaining time periods are then set to zero. This process is implemented to discern and isolate the effects of forward guidance issued in period $T + 1$. Adaptive learning beliefs are also initialized at their rational expectations counterparts for $\phi$ and the identity matrix for $R$. In addition, as is common in the CGL literature, adaptive learning agents’ beliefs may be explosive due in part to lagged variables included in agents’ PLM (see, for instance, Evans and Honkapohja (2001) and Branch et al. (2014) for more detail on this subject). For this reason, prior research tends to utilize a projection facility to ensure that explosive beliefs are not a part of the updating equations. Specifically, in the present paper, agents do not update their parameter estimates if the following condition is met: the eigenvalues in the $b$ matrix in the PLM have modulus greater than or equal to one. The matrices $\phi$ and $R$ are then returned to their initial values, that is, rational expectations equilibrium and the identity matrix.

The results of this section first show that different monetary policy regimes affect the efficacy of forward guidance. The outcomes are first analyzed in Fig. 1 under the case of rational expectations as it is the standard way to evaluate expectations in the macroeconomic literature. Each line represents the difference between the macroeconomic variable’s value under forward guidance minus the value without forward guidance. The first column displays the results under an inflation targeting central bank while the second shows the outcomes under a price-level targeting central bank. For instance, if the solid line is in positive territory in the first panel of Fig. 1, output is greater with forward guidance than without under an inflation targeting central bank. Under either an inflation or price-level targeting monetary regime, the lines for output and inflation are in positive territory indicating the benefits of forward guidance relative to no forward guidance. The effectiveness of forward guidance is also boosted more under a price-level targeting regime than inflation targeting regime. The values of output and inflation in the upper-right two panels are higher than the values in the upper-left two panels. This result agrees with Cole (2018) showing that the beneficial effects of forward guidance are boosted under a price-level targeting regime compared to inflation targeting regime.

Fig. 1. Effect of Forward Guidance across Monetary Policy Regimes under Rational Expectations

*Note:* IT means inflation targeting and PLT denotes price-level targeting. FG means forward guidance. In addition, each line represents the difference in the variable’s value between forward guidance and a no forward guidance regime under rational expectation. The first column represents an inflation targeting central bank,
while the second denotes a price-level targeting central bank. For instance, if the line in the top-left panel contains positive numbers, the value of output is higher with forward guidance regime than without under rational expectations.

Fig. 2 and the welfare relevant loss function also show a similar result. Each panel in this figure displays the difference between inflation targeting with forward guidance and price-level targeting with forward guidance. The solid line represents rational expectations. For example, if the solid line contains negative values in the top panel, the rational expectations value of output is greater under price-level targeting with forward guidance than inflation targeting with forward guidance. Across the entire forward guidance horizon, price-level targeting monetary policy is shown to boost the effectiveness of forward guidance. The solid line is in negative territory for both output and inflation. The welfare relevant loss function is also calculated as in Galí (2015) over the forward guidance horizon. The values of the loss function are 12.43 and 6.96 for inflation targeting with forward guidance and price-level targeting with forward guidance, respectively. The previous results show price-level targeting boosts the effectiveness of forward guidance relative to inflation targeting. In addition, the reason for more favorable outcomes under price-level targeting regards the reaction of expectations to forward guidance. In Fig. 3, expectations under FIRE is given by the solid line, which shows the value of the macroeconomic variable under inflation targeting with forward guidance minus the value of the macroeconomic variable under price-level targeting with forward guidance. Expectations under a price-level targeting regime react more favorably to forward guidance than under inflation targeting as the solid lines are in negative territory for both expected one-period ahead output and inflation. As described in Cole (2018), agents know that a price-level targeting regime is history dependent, and thus, will target higher levels of the price level in the future when inflation is previously below its target. This targeted increase in prices causes greater amounts of current output and inflation. In addition, expectations reacting more favorably under price-level targeting reconciles with the results of Billi (2008), who describes that a price-level targeting central bank can more appropriately adjust expectations to align with policy.
contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

Fig. 3. Main Results: Evolution of Expectations across Forward Guidance Horizon

Note: Solid Line: Rational Expectations, Dashed Line: Adaptive Learning. IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of expected one-period ahead output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

What result occurs if agents are not assumed to follow FIRE, but rather form forecasts utilizing an adaptive learning rule? The answer is that rational expectations relative to adaptive learning amplifies the positive benefits a price-level targeting regime creates for forward guidance effectiveness. This outcome is displayed in Fig. 2 with the dashed line representing adaptive learning. In the first and second rows, the rational expectations and adaptive learning lines both contain negative values across the forward guidance horizon indicating that the effects of forward guidance are boosted under a price-level targeting central bank relative to inflation targeting central bank. However, rational expectations magnifies these positive benefits of forward guidance under a price-level targeting regime. The solid lines are below the dashed lines throughout the forward guidance horizon showing the amount of output and inflation are overstated under rational expectations relative to adaptive learning with a price-level targeting regime.

What is driving the difference in results between rational expectations and adaptive learning? The answer concerns the reaction of expectations to central bank forward guidance announcements. Fig. 3 displays the responses of $E^*_t x_{t+1}$ and $E^*_t \pi_{t+1}$ under both expectations assumptions. As previously stated, each line represents the difference in the macroeconomic variable between inflation targeting with forward guidance and price-level targeting with forward guidance. The solid line indicates expectations formed under FIRE while the dashed line means expectations formed under adaptive learning. Across the entire forward guidance horizon, expectations of output and inflation react more favorably under price-level targeting than inflation targeting and rational expectations magnifies this difference. The lines are in negative territory and the solid line is below the dashed line. Since rational expectations agents form forecasts based on the true model of the economy, they are able to understand how statements about the future inflation target affect other macroeconomic variables. However, adaptive learning agents base their expectations on an estimated model of the economy. They are not endowed with as much knowledge as rational expectations (e.g.
true values of the coefficients). Adaptive learning agents must continually estimate how forward guidance statements about the inflation target will affect the macroeconomic variables, and thus, their expectations do not react as favorably as their rational expectations counterparts. Therefore, the beneficial features of price-level targeting for forward guidance depend on the expectations formation process.

It is also important to discuss measuring the forward guidance shocks. In this section, a constant inflation target exercise was implemented such that the forward guidance shocks $\epsilon_{FG} = [\epsilon_{1FG}, \epsilon_{2FG}, \cdots, \epsilon_{LFG}]'$ were chosen to ensure $\pi_t^* = \bar{\pi}^*$ across the forward guidance horizon. Some previous literature has measured these shocks in the same way (see Cole, 2018). Alternative ways to measure these shocks have also been implemented. For instance, Del Negro et al. (2012) identified the shocks utilizing descriptive evidences from FOMC statements. Keen et al. (2017) measured forward guidance shocks by comparing moments from the data to those implied by the model. Forward guidance shocks are identified in Bundick and Smith (2019) by examining the unexpected change in futures contract whenever a monetary policy announcement occurred.

This current paper measures the forward guidance shocks with the method of choosing the shocks such that $\pi_t^* = \bar{\pi}^*$ across the entire forward guidance horizon for the following reasons. First, this technique was chosen to correspond to Federal Reserve comments regarding forward guidance on a constant inflation target. For instance, the Federal Reserve issued guidance on the constant inflation target in January 2012: “The Committee judges that inflation at the rate of 2 percent...is most consistent over the longer run with the Federal Reserve’s statutory mandate” (Federal Reserve, 2012). Second, choosing anticipated shocks for a constant target has been implemented in prior literature. As stated above, Cole (2018) chose the forward guidance shocks such that $\pi_t^* = \bar{\pi}^*$ across the entire forward guidance horizon. Del Negro et al. (2012), Cole (2015), and Cole (2016) implemented a policy experiment where forward guidance shocks were chosen such that the policy rate was constant across the entire forward guidance horizon. Furthermore, the scope of the present paper concerns ‘pure’ forward guidance shocks, that is, shocks that would achieve forward guidance objectives (i.e., specific inflation targets). In comparison, prior literature such as Bundick and Smith (2019) identify forward guidance shocks using asset prices, which can reflect not only forward guidance but also possible commitment problems and communication barriers. Since the focus of the current paper solely regards forward guidance shocks that achieve specific inflation targets, the identification method of the present paper seems most warranted. However, in Section 6.4, I follow prior literature on identifying anticipated shocks (e.g., Milani and Rajbhandari, 2020 and Hirose and Kurozumi, 2019) by directly measuring these shocks using estimation and data from the Survey of Professional Forecasters (SPF) from the Federal Reserve Bank of Philadelphia. Section 5.2 also examines a different value of the inflation target, which implies different values of the chosen forward guidance shocks.

Overall, the results of this constant inflation target exercise reveal a primary takeaway: rational expectations relative to adaptive learning amplifies the positive benefits a price-level targeting regimes creates for forward guidance effectiveness. The value of output and inflation is greater under price-level targeting than inflation targeting policy and rational expectations magnifies this difference relative to adaptive learning. A main driver of the results is the reaction of expectations to forward guidance statements. The expectations of rational expectations agents react more favorably to forward
guidance announcements than under adaptive learning. Thus, policymakers should consider how expectations are modeled if forward guidance and price-level targeting are considered for an economy.

5. Explaining the Gap
The results of the previous section showed that the benefits of forward guidance are boosted under a price-level targeting central bank, but FIRE overstates these positive effects relative to adaptive learning. This current section explores in depth the reasons for the performance gap between rational expectations and adaptive learning. Specifically, changes in the forecasting model of adaptive learning agents (i.e. the PLM) and inflation target are explored.

5.1. Examining the PLM
A key difference between rational expectations and adaptive learning that contributed to the main results is the forecasting model. The former type of agents base expectations on the true model of the economy with knowledge of the true value of the coefficients. Adaptive learning agents utilize an estimated model of the economy labeled the PLM to construct expectations about future macroeconomic variables. As described in Section 3.2, this forecasting model was not equivalent to FIRE. It is based on a VAR(1) with central bank announcements of forward guidance and inflation target added as additional regressors. However, a natural question arises: how would the results change if the structure of adaptive learning agents’ forecasting model separates more from FIRE? Would a less rational PLM exacerbate the performance gap between the two expectations formation processes?

This section examines if adaptive learning agents utilize a PLM that moves farther away from the one that would exist under rational expectations. Specifically, the PLM in Eq. (24) is modified to be a VAR(1) plus a constant and is given by:

\[
Y_t = a + bY_{t-1} + \varepsilon_t
\]

where \(Y_t = [x_t, \pi_t, i_t]^\prime\) and \(\varepsilon_t\) is a vector of the usual white noise error terms. The constant inflation target exercise of Section 4.2 is then rerun and compared to the baseline exercises. In addition, Fig. 4 displays the results. As in the benchmark case of Section 4.2, each line represents the difference between inflation targeting with forward guidance and price-level targeting with forward guidance for that specific macroeconomic variable. For instance, if a line is in negative territory in the top panel of Fig. 4, the value of output is higher under a price-level than inflation targeting regime with forward guidance. The solid line represents rational expectations. The dashed line denotes adaptive learning under the benchmark PLM (i.e. Eq. (24)). The dotted line means adaptive learning under a VAR(1) plus a constant PLM (i.e. Eq. (31)).
Fig. 4. Explaining the Gap: Results with Benchmark and Alternative PLMs

Note: Solid Line: Rational Expectations, Dashed Line: Adaptive Learning with Benchmark PLM, Dotted Line: Adaptive Learning with VAR(1) plus a Constant PLM. IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

The results show that the performance gap between the two expectations formation schemes increases if the PLM separates more from FIRE. For instance, Fig. 4 shows the solid line is below the dotted line for output, which supports the baseline results. However, these lines are farther away from each other than are the benchmark adaptive learning and rational expectations case. The distance between the dotted and solid lines is overall greater than the dashed and solid lines the majority of the time for output and inflation in the top two panels of Fig. 4. Since adaptive learning agents lack even less knowledge about how the structure of the economy evolves, their forecasts of future macroeconomic variables diverge more from their rational expectations counterparts. Thus, a less rational PLM exacerbates the performance gap between the two expectations formation processes.

5.2. Inflation target

The benchmark outcomes occurred under a constant inflation target of 1.005. However, a reasonable question regards if the results are influenced by the choice of $\pi^{*}$. Thus, it is natural to examine how the performance gap between rational expectations and adaptive learning to forward guidance reacts when $\pi^{*}_t = \pi^{*}$ changes.

This section examines the model under the constant inflation target exercise of Section 4.2 and a higher value of $\pi^{*}_t = \pi^{*}$. The results are presented in Fig. 5. The black lines represent the benchmark outcomes of Section 4.2 with $\pi^{*}_t = \pi^{*} = 1.005$. For illustrative purposes, the higher value of $\pi^{*}$ is chosen to be 3 and is denoted by the red lines. Under both colored lines, the solid and dashed lines mean rational expectations and adaptive learning, respectively.
Fig. 5. Explaining the Gap: Results with Benchmark and Higher $\pi^*$

*Note:* Solid Line: Rational Expectations, Dashed Line: Adaptive Learning. Black Lines: Benchmark Case ($\pi^* = 1.005$), Red Lines: Higher Inflation Target ($\pi^* = 3$). IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

The results show three takeaways. First, rational expectations relative to adaptive learning amplifies the positive benefits a price-level targeting regime creates for forward guidance effectiveness. For instance, in the middle panel, each line contains negative values and the solid lines are below their counterpart dashed lines. Second, the discrepancies between the two monetary policy regimes enlarge as $\pi^*$ increases. Across all three panels in Fig. 5, the red set of lines is overall below the black pair of lines. The reason is similar to why price-level targeting can bring more benefits than inflation targeting. With a higher inflation target over the forward guidance horizon, a price-level targeting central bank will use its policy tool to increase the price level when it sees inflation is previously below its target. This increase in the price level causes more inflation and overall benefits to the economy. More notably, when central bank forward guidance increases the inflation target to $\pi^* = 3$, the gap between rational expectations and adaptive learning enlarge. The distance between red lines is overall bigger than the distance between the black lines for output and inflation. This feature is due to the process of adaptive learning continually adjusting their expectations, whereas rational expectations forecast the precise reaction of macroeconomic variables to forward guidance. Fig. 6 shows this discrepancy of forecasts between the two expectations formation processes. As before, each line represents the difference in the macroeconomic variable between inflation targeting with forward guidance and price-level targeting with forward guidance. The solid line indicates expectations formed under FIRE while the dashed line means expectations formed under an adaptive learning rule. The black line represents baseline $\pi^* = 1.005$ while the red line $\pi^* = 3$. An increase in inflation target leads to a smaller adjustment in adaptive learning forecasts relative to rational expectations as the former are not able to base their expectations on the true model of the economy. The distance between the red lines is bigger than the distance between the black lines. Thus, the performance gap between rational expectations and adaptive learning is affected by the value of $\pi^*$. 
Fig. 6. Explaining the Gap: Expectations with Benchmark and Higher $\bar{\pi}^*$

Note: Solid Line: Rational Expectations, Dashed Line: Adaptive Learning. Black Lines: Benchmark Case ($\bar{\pi}^* = 1.005$), Red Lines: Higher Inflation Target ($\bar{\pi}^* = 3$). IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of expected one-period ahead output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

Overall, the results of these two subsections indicate reasons why rational expectations produces higher levels of output under forward guidance with price-level targeting relative to adaptive learning. If adaptive learning agents are assumed to be endowed with even less knowledge about the economy, the performance gap between the two expectations formation schemes increases. In addition, a higher value of $\bar{\pi}^*$ magnifies the differences between rational expectations and adaptive learning to forward guidance.

6. Extensions

6.1. Aggressive inflation targeting central bank

The monetary policy rules in Eqs. (8) and (11) contain both similarities and differences. Lagged interest rates, output gap, and an unanticipated monetary policy shock are present in both regimes. However, a main difference is that $\pi_t$ appears in the inflation targeting monetary policy rule and does not in the price-level targeting monetary policy rule. Section 4.1 also stated that the parameter that governs the response of interest rates to deviations in inflation from its target ($\chi_\pi$) was calibrated to 1.4, which followed prior literature. However, it is of interest to examine the results when $\chi_\pi$ increases. Could a more aggressive inflation targeting regime affect the results of the paper?

Fig. 7 displays the outcomes of this exercise. As in Section 4.2, each line displays the difference in the variable between inflation targeting with forward guidance and price-level targeting with forward guidance. To capture an inflation targeting central bank that cares more about changes in inflation, $\chi_\pi = 2$ is utilized and is denoted by the red lines. The benchmark scenario of $\chi_\pi = 1.4$ is represented by the black lines. The solid lines denote rational expectations while the dashed lines denote adaptive learning.
Fig. 7. Extension: Results with Benchmark and More Aggressive Inflation Targeting Central Banks

*Note:* Solid Line: Rational Expectations, Dashed Line: Adaptive Learning. Black Lines: Benchmark Case ($\chi_\pi = 1.4$), Red Lines: Increased Inflation Reaction Parameter ($\chi_\pi = 2$). IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

The results show the gap between adaptive learning and rational expectations still exists, but the effectiveness of price-level targeting relative to inflation targeting dampens. The output and inflation panels in Fig. 7 illustrate this point. The difference between adaptive learning and rational expectations is approximately the same. The solid and dashed lines show about the same distance between each other across both black and red pairs of lines. However, the positive benefits a price-level targeting regime creates for forward guidance lessen relative to inflation targeting as $\chi_\pi$ increases. The red pair of lines are above the black pair of lines. Under an inflation targeting regime, policy can now be more aggressive in responding to deviations of inflation from its target. Thus, this type of central bank creates more positive benefits for the economy than before. An interesting feature of Fig. 7 is that the red dashed line crosses into positive territory in the top panel. This outcome is not surprising as adaptive learning agents are estimating the coefficients of the economy. Notably, it is helpful to examine the beliefs of adaptive learning agents about the sensitivity of output to the inflation target. Fig. 8 plots this belief coefficient (i.e. $d_1$). Each line represents the difference in the estimated value of $d_1$ between inflation and price-level targeting regimes under adaptive learning. For example, if the line contains negative numbers, the estimated value of $d_1$ is higher under a price-level than inflation targeting central bank. The solid line denotes the benchmark case of $\chi_\pi = 1.4$. The dashed line represents the scenario of a more aggressive inflation targeting monetary policy rule (i.e. $\chi_\pi = 2$). When $\chi_\pi$ increases from 1.4 to 2, Fig. 8 displays that the difference between the two regimes regarding the estimate of the $d_1$ parameter decreases. The dashed line is closer to 0 than the solid line. Under an inflation targeting regime with $\chi_\pi = 2$, agents are estimating a higher effect of changes in the inflation target on the economy. Therefore, this result implies fewer discrepancies between inflation and price-level targeting.
6.2. Alternative forward guidance horizons

The value of the forward guidance horizon $L$ was chosen to be twelve periods and was motivated by previous literature. However, it is important to examine whether the results are sensitive to the duration of the forward guidance horizon. Thus, the exercise of Section 4.2 is repeated under shorter and longer forward guidance horizons. Specifically, this subsection will analyze the results when $L = 6$ and $L = 18$.

Figs. 9 and 10 display the results of this exercise. The red pair of lines in Fig. 9 represent a shorter forward guidance horizon (i.e. $L = 6$) under both rational expectations (solid line) and adaptive learning (dashed line). The red pair of lines in Fig. 10 mean a longer forward guidance horizon ($L = 18$) under both rational expectations and adaptive learning. The black pair of lines in both Figs. 9 and 10 denote the benchmark case of $L = 12$ under both rational expectations and adaptive learning. As in Section 4.2, the lines in each panel indicate the difference in the value of the variable between inflation targeting with forward guidance and price-level targeting with forward guidance.
Fig. 9. Extension: Results with Benchmark and Shorter Forward Guidance Horizons

*Note:* Solid Line: Rational Expectations, Dashed Line: Adaptive Learning. Black Lines: Benchmark Forward Guidance Horizon \((L = 12)\), Red Lines: Shorter Forward Guidance Horizon \((L = 6)\). IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

The results of the section produce two takeaways. First, rational expectations relative to adaptive learning still amplifies the positive effects a price-level targeting regime creates for forward guidance regardless of the value of \(L\). Overall, the lines contain negative values for output and inflation as well as the solid lines are below the dashed lines, which confirms the baseline results of Section 4.2. In addition, the effect of the choice of \(L\) regards the length of time that rational expectations amplifies the positive effects a price-level targeting regime creates for forward guidance relative to adaptive learning. Figs. 9 and 10 show that the \(L = 6, L = 12,\) and \(L = 18\) lines closely follow the same path. However, each pair of lines stops when its forward guidance horizon finishes. Thus, the value of \(L\) determines the duration of the performance gap between rational expectations and adaptive learning.

6.3. Alternative constant gains

A key parameter in this paper regards \(\bar{\tau}\), which governs the influence new information from the most recent forecast error of adaptive learning agents has on coefficient estimates. As \(\bar{\tau}\) increases, adaptive learning agents’ new coefficients should vary more relative to the previous period as they place more weight on new information. As \(\bar{\tau}\) decreases, the coefficient estimates do not vary as much from the previous period, and thus, become closer to their rational expectations counterparts. As described in Section 4.1, the value of \(\bar{\tau}\) was set to 0.02. This number was based on prior literature. However, it is important to analyze the outcomes when lower and higher values of \(\bar{\tau}\) are utilized.
The main results of the paper are rerun under lower and higher values of the constant gain parameter and are shown in Fig. 11. To model adaptive learning agents assigning more weight to recent information/forecast error, \( \bar{\tau} \) will be set to 0.03. This scenario is given by the dashed-dotted line. If agents place less weight on recent information/forecast error, the value of the constant gain parameter will be lower. To capture the previous scenario, \( \bar{\tau} \) will be fixed at 0.01. The dotted line represents this case. The benchmark adaptive learning with \( \bar{\tau} = 0.02 \) and rational expectations scenarios are also shown and denoted by the dashed and solid lines, respectively.

Fig. 11. Extension: Results with Benchmark and Alternative Constant Gain Values
Note: Solid Line: Rational Expectations, Dashed Line: Adaptive Learning with \( \bar{\tau} = 0.02 \), Dotted Line: Adaptive Learning with \( \bar{\tau} = 0.01 \), Dashed-Dotted Line: Adaptive Learning with \( \bar{\tau} = 0.03 \). IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

The results display that the value of \( \bar{\tau} \) influences the differences between adaptive learning and rational expectations regarding the positive benefits a price-level targeting regime creates for forward guidance effectiveness. If agents place more weight on recent forecast errors (i.e. \( \bar{\tau} = 0.03 \)), the differences between adaptive learning and rational expectations increase. Adaptive learning agents misvalue more the effects of forward guidance on the inflation target. Overall, in the three panels in Fig. 11, the dashed-dotted line is farther away from the solid line than the dashed line is from the solid line. If agents place less weight on new information (i.e. \( \bar{\tau} = 0.01 \)), their coefficients estimates will vary less from the previous period. Adaptive learning agents do not misvalue forward guidance effects as much as before, and thus, their responses will be closer to their rational expectations counterparts. Overall, the three panels in Fig. 11 show that the dotted line is closer to the solid line than the dashed line is to the solid line.

6.4. Alternative measures of the forward guidance shocks
The benchmark case in Section 4.2 displayed that rational expectations relative to adaptive learning amplifies the positive benefits a price-level targeting monetary authority creates for forward guidance effectiveness. In this baseline exercise, the anticipated/forward guidance shocks (i.e., \( \epsilon^F_{t} = \left[ \epsilon^F_{tL}, \epsilon^F_{tH}, \cdots, \epsilon^F_{tL} \right]' \)) were chosen such that \( \pi^*_t = \bar{\pi}^* \) across the entire forward guidance horizon. This approach was partly implemented to replicate the Federal Reserve issuing forward guidance for a
constant inflation target. However, a natural question regards how alternative methods to measure the forward guidance shocks would influence the results. This section uses Bayesian estimation techniques and the SPF dataset from the Federal Reserve Bank of Philadelphia to measure the anticipated/forward guidance shocks. The data series utilized for the model’s variables for output, inflation, and interest rates are percentage change in U.S. real GDP, percentage change in U.S. GDP deflator, and the federal funds rate. These data are retrieved from the FRED database of the Federal Reserve Bank of St. Louis and the relevant acronyms are GDPC1, GDPDEF, and FEDFUNDS. To measure the anticipated shocks, I follow the literature on identifying anticipated shocks (e.g., Milani and Rajbhandari, 2020 and Hirose and Kurozumi, 2019) by exploiting expectations data from the SPF. Specifically, expectations data on expected one-period ahead (real) output growth and expected one-period ahead inflation are utilized. The relevant acronyms are RGDP and PGDP. In addition, the observation equations are defined as the following:

\[
\begin{bmatrix}
g_{t}^{obs} \\
\pi_{t}^{obs} \\
i_{t}^{obs} \\
E_{t}^{obs} g_{t+1} \\
E_{t}^{obs} \pi_{t+1}
\end{bmatrix} 
= 
\begin{bmatrix}
g_{t} \\
\pi_{t} \\
i_{t} \\
E_{t} g_{t+1} \\
E_{t} \pi_{t+1}
\end{bmatrix} 
+ 
\begin{bmatrix}
\gamma_{g} \\
\gamma_{\pi} \\
\gamma_{r} \\
\gamma_{g1} \\
\gamma_{\pi1}
\end{bmatrix} + 
\begin{bmatrix} 0_{3 \times 2} \end{bmatrix} \begin{bmatrix} o_{g_{t+1}}^{g} \\
o_{\pi_{t+1}}^{\pi} \end{bmatrix}
\]

where \(g_{t}\) is the growth rate of real GDP. As in Cole and Milani (2019) and Cole and Martinez-Garcia (2019), \(i.i.d\) measurement errors (i.e., \(o_{g_{t+1}}^{g}\) and \(o_{\pi_{t+1}}^{\pi}\)) are included for the expectations series.\(^{32}\) Bayesian methods utilized with Dynare (see Adjemian et al. (2011)) are then used to estimate the model.\(^{33}\) In accordance with U.S. data, the model is estimated under the assumption of an inflation-targeting monetary authority. The model is also estimated under the assumption of rational expectations to be consistent with Section 4.2 and for simplicity. Furthermore, Table 3 shows the prior assumptions on the model’s parameters. The means for the main structural parameters are centered over the calibrated values discussed in Section 4.1.\(^{34}\)

| Table 3. Prior & Posterior Estimates of Parameters. |
|---|---|---|---|
| Prior Distr. | Posterior Distribution | Mean | 5% | 95% |
| \(\sigma\) | N(0.15, 0.05) | 0.2457 | 0.1781 | 0.3138 |
| \(\rho\) | B(0.85, 0.10) | 0.7847 | 0.7384 | 0.8304 |
| \(\chi_{\pi}\) | N(1.40, 0.10) | 1.3951 | 1.2389 | 1.5520 |
| \(\chi_{x}\) | N(1, 0.05) | 1.0220 | 0.9416 | 1.1025 |
| \(\rho_{\mu}\) | B(0.50, 0.20) | 0.8363 | 0.8186 | 0.8541 |
| \(\rho_{n}\) | B(0.50, 0.20) | 0.8273 | 0.7582 | 0.8979 |
| \(\sigma_{\mu}\) | IG(0.30, 0.30) | 0.0534 | 0.0469 | 0.0586 |
The results of this section are shown in Table 3 and Fig. 12, Fig. 13, Fig. 14. The estimates of particular importance for this section regard the standard deviations of the forward guidance shocks. These posterior mean estimates are in line with Milani and Treadwell (2012). In addition, to examine how the results would be affected under alternative measurements of the shocks, the exercise of Section 4.2 is implemented with the forward guidance shocks in period $T + 1$ (i.e., $\varepsilon_{T+1}^F = [\varepsilon_{1,T+1}^F, \varepsilon_{2,T+1}^F, \cdots, \varepsilon_{L,T+1}^F]^T$) specified with the estimates of their standard deviations shown in Table 3. This case is given by the red line in Fig. 12. For reference, the black line denotes the forward guidance shocks chosen under the baseline case of Section 4.2. As before, the solid line means the rational expectations case while the dashed the adaptive learning scenario. To illustrate the effects of higher measurements of the forward guidance shocks, I also examine the results when the forward guidance shocks in time period $T + 1$ are chosen to be one and a half times larger than the estimated standard deviation values in Table 3. This case is given by the red lines in Fig. 13. To illustrate the effects of smaller values of the forward guidance shocks, I examine the results when the forward guidance shocks in time period $T + 1$ are chosen to be one-half times smaller than the estimated standard deviation values in Table 3. This case is given by the red lines in Fig. 14. In both Figs. 13 and 14, the black lines denote the case when the standard deviations of the forward guidance shocks are specified with the estimates displayed in Table 3.

| $\sigma_{MP}$ | IG(0.30, 0.30) | 0.2743 | 0.2473 | 0.3010 |
| $\sigma_n$ | IG(0.30, 0.30) | 0.1382 | 0.1020 | 0.1736 |
| $\sigma_{\pi^*}$ | IG(0.04, 0.04) | 0.0384 | 0.0121 | 0.0673 |
| $\sigma_{FG}^1$ | IG(0.04, 0.04) | 0.5923 | 0.4852 | 0.6975 |
| $\sigma_{FG}^2$ | IG(0.04, 0.04) | 0.0419 | 0.0120 | 0.0736 |
| $\sigma_{FG}^3$ | IG(0.04, 0.04) | 0.0427 | 0.0118 | 0.0789 |
| $\sigma_{FG}^4$ | IG(0.04, 0.04) | 0.0408 | 0.0122 | 0.0733 |
| $\sigma_{FG}^5$ | IG(0.04, 0.04) | 0.0425 | 0.0121 | 0.0745 |
| $\sigma_{FG}^6$ | IG(0.04, 0.04) | 0.0815 | 0.0109 | 0.2475 |
| $\sigma_{FG}^7$ | IG(0.04, 0.04) | 0.0747 | 0.0103 | 0.2133 |
| $\sigma_{FG}^8$ | IG(0.04, 0.04) | 0.0480 | 0.0117 | 0.0860 |
| $\sigma_{FG}^9$ | IG(0.04, 0.04) | 0.0397 | 0.0125 | 0.0710 |
| $\sigma_{FG}^{10}$ | IG(0.04, 0.04) | 0.0467 | 0.0115 | 0.0904 |
| $\sigma_{FG}^{11}$ | IG(0.04, 0.04) | 0.0507 | 0.0114 | 0.0979 |
| $\sigma_{FG}^{12}$ | IG(0.04, 0.04) | 0.0449 | 0.0116 | 0.0820 |
| $\sigma_{\gamma}^{me,1}$ | IG(0.30, 0.30) | 0.4325 | 0.3736 | 0.4922 |
| $\sigma_{\pi}^{me,1}$ | IG(0.30, 0.30) | 0.2152 | 0.1977 | 0.2329 |

Note: N: Normal Distribution, B: Beta Distribution, IG: Inverse-Gamma Distribution.
Fig. 12. Extension: Forward Guidance Shocks Measured with SPF Data

Note: Solid Line: Rational Expectations, Dashed Line: Adaptive Learning. Black Lines: Forward Guidance Shocks Under Baseline Exercise of Section 4.2, Red Lines: Forward Guidance Shocks Measured with SPF Data. IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.

Fig. 13. Extension: Forward Guidance Shocks Measured with SPF Data and Larger Forward Guidance Shock Values

Note: Solid Line: Rational Expectations, Dashed Line: Adaptive Learning. Black Lines: Forward Guidance Shocks Measured with SPF Data, Red Lines: Larger Forward Guidance Shocks. IT means inflation targeting and PLT denotes price-level targeting. In addition, each line represents the difference in the variable’s value between inflation and price-level targeting regimes with forward guidance. For instance, if the line in the top panel contains negative numbers, the value of output is higher under a price-level targeting central bank with forward guidance than an inflation targeting central bank with forward guidance.
Figs. 12–14 show that the qualitative results of the paper do not change, but quantitative outcomes can somewhat differ depending on the values of the forward guidance shocks. For instance, when measuring the forward guidance shocks using SPF data, price-level targeting still boosts the effectiveness of forward guidance relative to inflation targeting. Rational expectations relative to adaptive learning also overall amplifies the positive benefits a price-level targeting regime creates for forward guidance effectiveness. In the output and inflation panels in Figs. 12–14, the lines contain negative values and the solid lines are overall below their counterpart dashed lines. However, the quantitative results can change depending on how the forward guidance shocks are measured. Under measuring the shocks with SPF data, rational expectations still outperforms adaptive learning but the size of the gain is somewhat smaller than the benchmark case in which the forward guidance shocks were chosen to achieve a constant inflation target. Fig. 12 shows an overall smaller difference between the red lines than the black lines. The reason is that the values of the shocks are less than the baseline case of Section 4.2. In addition, if the shocks are measured to be larger (i.e., one and a half times larger than the estimated standard deviation values in Table 3), greater differences exist between rational expectations and adaptive learning. In Fig. 13, the overall difference between the red lines for output and inflation are larger than their counterpart black lines. If the value of the forward guidance shock is large, rational expectations agents are able to precisely understand the greater benefits of a larger forward guidance shock relative to adaptive learning agents who use an estimated model of the economy to understand the effects of forward guidance. If the shocks are measured to be at lower values (i.e., one-half times smaller than the estimated standard deviation values in Table 3), Fig. 14 displays that the difference between rational expectations and adaptive learning is not as great. The overall difference between the red lines for output and inflation are not as great relative to the differences between the black lines.

Altogether the results of this section show that the performance gap between rational expectations and adaptive learning to forward guidance under a price-level targeting central bank still exist under different robustness scenarios. A more aggressive inflation-targeting central bank can boost the effectiveness of inflation targeting relative to price-level targeting. However, the gap between the two expectations formation schemes is still present. The length of the forward guidance horizon also affects the duration that rational expectations amplifies the positive benefits a price-level targeting central bank creates for forward guidance relative to adaptive learning. In addition, the constant gain parameter, $\bar{r}$ influences how far or close adaptive learning responses are to their rational expectations counterparts. Finally, the qualitative results of the paper seem to be robust to alternative measures of the forward guidance shocks.
7. Conclusion
The unconventional monetary policy of forward guidance has constituted the toolkit of central banks since the aftermath of the Great Recession. For instance, the Federal Reserve has issued guidance, communication, and clarification on its long-run inflation target. However, the effectiveness of forward guidance depends on the monetary policy regime and expectations formation process. Thus, this paper examines the effects of the monetary policy regime on the efficacy of forward guidance when an adaptive learning rule replaces the standard FIRE.

The results show that rational expectations relative to adaptive learning amplifies the positive benefits a price-level targeting regime creates for the effectiveness of forward guidance. Under rational expectations, the responses of output and inflation to forward guidance are higher under price-level than inflation targeting. However, if FIRE is replaced by an adaptive learning rule, output and inflation do not respond as much to forward guidance under price-level targeting. The main driver of this performance gap between rational expectations and adaptive learning is the reaction of expectations to forward guidance announcements about the future path of the central bank’s inflation target. The forecasts of output and inflation react more favorably under price-level targeting than inflation targeting and rational expectations magnifies this difference. In addition, the forecasting model of adaptive learning agents and the size of the inflation target influence the performance gap. A PLM that separates more from FIRE exacerbates the performance gap between the two expectations formation processes. Higher levels of the inflation target lead to increased differences between rational expectations and adaptive learning. Furthermore, the main results of the paper are robust to additional scenarios. If an inflation targeting central bank is more aggressive at targeting deviations of inflation from its target, the effectiveness of this type of monetary policy regime is increased. However, the performance gap between rational expectations and adaptive learning still exists. The length of the forward guidance horizon affects the duration of the performance gap that exists between the two expectations formation processes. The amount of weight that adaptive learning agents place on new information also influences how alike or dissimilar their responses to forward guidance are relative to rational expectations. Finally, the qualitative results of the paper are robust to alternative measures of the forward guidance shocks. Overall, policymakers should consider how expectations are modeled if forward guidance and price-level targeting are implemented in an economy.

CRediT authorship contribution statement
Stephen J. Cole: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Supervision, Validation, Visualization, Writing - original draft, Writing - review & editing.

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Notes
1The U.S. inflation rate is measured by the annual change in Federal Reserve’s preferred Price Consumption Expenditures index. In this paper, the inflation data are taken from the FRED database of the Federal Reserve Bank of St. Louis. The acronym is PCEPI.
2Indeed, Cole (2018) shows that a price-level targeting regime can boost the effects of forward guidance relative to inflation targeting under the assumption of full-information rational expectations (FIRE).
3Eggertsson and Woodford (2003) discuss optimal monetary policy and history dependence. History dependence incorporated in a monetary policy rule can also play a significant role at the ZLB as illustrated by Bundick (2015).
4The output gap is defined as the difference between current period output and the efficient level of output (see, for instance, Gali, 2015).
5The presence of interest rate inertia in both monetary policy regimes does add a level of history dependence to an inflation targeting regime. However, without \( i_{t-1} \) in Eqs. (8) and (11), a price-level targeting monetary policy would contain history dependence but inflation targeting monetary policy would not.
6The forward guidance shocks are similar to the news shocks of Schmitt-Grohé and Uribe (2012) and Milani and Treadwell (2012). However, the previous authors do not explicitly focus on forward guidance.
7However, forward guidance is included for a price-level targeting regime in Honkapohja and Mitra (2015), but not for an inflation targeting central bank. The present paper also models forward guidance on the inflation target.
The New Keynesian model with a price-level targeting regime can also be solved under FIRE. The vector \( \tilde{Y}_t \) should be augmented with \( \tilde{p}_t \) and \( \tilde{r}_0, \tilde{r}_1, \tilde{r}_2, \) and \( \tilde{r}_3 \) appropriately modified. The same method described by Sims (2002) can then be implemented.

It should be noted that adaptive learning agents believe their forecasts to be optimal each period and do not take into consideration that they will update them in the future. This framework follows from the anticipated utility discussion of Kreps (1998). Thus, the time subscript is left off the coefficients \( a, b, c, \) and \( d \) to emphasize this point. However, as will be detailed in this section, adaptive learning agents will update their coefficients each period.

As is similar to Williams (2010), the same PLM is utilized under both inflation and price-level targeting regimes. This feature is implemented for the following reasons. Utilizing the same learning rule (i.e. Eq. (24)) across regimes helps to understand how changing only the expectations formation assumption affects the benefits of forward guidance for a PLT regime. A real-life forecaster under either inflation or price-level targeting regimes might also build forecasts on a model similar to the PLM presented here.

The ALM in Eq. (30) represents the economy under an inflation targeting central bank. Under a price-level targeting regime, the ALM can easily be found. The resulting vector \( \tilde{Y}_t \) would be augmented with \( \tilde{p}_t \). The same process for evaluating expectations described in this section would be employed and then the coefficient matrices \( \Gamma_0(\phi_{t-1}), \Gamma_1(\phi_{t-1}), \Gamma_2(\phi_{t-1}), \) and \( \Gamma_3(\phi_{t-1}) \) appropriately modified.

The reader is referred to Honkapohja et al. (2013) for further discussion regarding the EE and IH approach to learning.

A relatively higher number is also in line with Rudebusch (2001).

The rational expectations and adaptive learning models correspond to the inflation targeting regime since the data correspond to the U.S. economy and the U.S. has officially not practiced a price-level targeting policy as of the writing of this paper.

The value of \( T \) is chosen to be 258, which corresponds to the number of quarters from 1954:Q3 to 2018:Q4.

This paper uses \( \pi^* = 1.005 \), which follows from Honkapohja and Mitra (2018). Section 5.2 examines the results under a higher value of \( \pi^* \).

As is similar to Park (2018), this assumption seems reasonable as many central banks around the world, who issue forward guidance to the public, utilize DSGE models with the standard FIRE to analyze their economies.

It should be noted that \( \rho_{\pi^*} \) is assumed known by agents along with the autoregressive terms in the other AR(1) processes. This assumption is standard in the literature (see, for example, Branch et al. (2013).

This is similar to Cole (2015), Cole (2016), and Cole (2018).

Branch et al. (2014) describe the intuition for a projection facility as agents holding a belief that the economic system is stable. Agents would then throw out data that does not correspond to a stable economic system.

The projection facility is rarely invoked. For instance, it is induced 0% and 0.17% of the time for inflation and price-level targeting regimes, respectively, for a time period of 10,000. In addition, the projection facility is only implemented for the first \( T \) periods. When the constant inflation
target exercise starts in $T + 1$ through the remaining $L$ periods, the projection facility is not utilized.

The values of the chosen forward guidance shocks that ensure $\pi_t^* = \overline{\pi}^*$ across the entire forward guidance horizon are $\varepsilon_t^{FG} = [\varepsilon_{1,1}^{FG}, \varepsilon_{2,1}^{FG}, \ldots, \varepsilon_{L,1}^{FG}]' = [1.0050, 0.0101, 0.0100, 0.0101, 0.0100, 0.0101, 0.0101, 0.0101, 0.0101, 0.0101]'$. As explained by Galí (2009), these relatively higher values of the chosen anticipated/forward guidance shocks are not uncommon in the literature. In addition, the values of the chosen forward guidance shocks can vary depending on the value of the inflation target $\overline{\pi}^*$. Indeed, Section 5.2 investigates this issue. Section 6.4 also examines alternative methods to measure the forward guidance shocks.

The value of the parameter characterizing the weight on output gap fluctuations (i.e., $\alpha_x$) is set to 0.048, which follows Giannoni (2014).

Under adaptive learning, the values of the welfare relevant loss function are 12.43 and 5.99 for inflation targeting with forward guidance and price-level targeting with forward guidance, respectively. In addition, the (slightly) higher value of the loss function with price-level targeting under FIRE versus adaptive learning occurs due to the greater response of output and inflation with FIRE. The stronger response creates more variation in the macroeconomic variables, and thus, a higher loss function value. Moreover, the main results still hold as seen in Fig. 2. Specifically, the FIRE assumption relative to adaptive learning amplifies the positive benefits a price-level targeting creates for the effects of forward guidance.

His analysis focused exclusively on the rational expectations scenario.

In comparison to the benchmark forecasting model in Eq. (24), the PLM in Eq. (31) noticeably does not contain central bank announcements regarding forward guidance and the inflation target (i.e. $v_t$ and $\pi_t^*$). This forecasting model can be thought of as agents being uncertain about the inflation target and forward guidance.

A notable feature of the top panel in Fig. 4 is that the dotted line crosses into positive value territory. This result occurs because adaptive learning agents with only a VAR(1) plus a constant PLM do not have forward guidance information in their forecasting equation. Thus, the great benefits a PLT regime relative to IT regime can have for forward guidance are muted.

Raising the inflation target has also received attention due to its hypothesized benefits of counteracting negative effects of low equilibrium real interest rates (see, for instance, Bernanke (2017)). Although this paper does not explicitly address this issue, it would be an interesting topic for future studies.

For reference, the chosen forward guidance shocks that ensure a higher inflation target, that is, $\overline{\pi}^* = 3$ are $\varepsilon_t^{FG} = [\varepsilon_{1,1}^{FG}, \varepsilon_{2,1}^{FG}, \ldots, \varepsilon_{L,1}^{FG}]' = [3.0000, 0.0300, 0.0300, 0.0300, 0.0300, 0.0300, 0.0300, 0.0300, 0.0300, 0.0300]'$.

An exception is the red dashed line that goes above the black dashed line for a few periods in the top panel. However, the former is barely above the latter and not for the majority of the time. The main result of this section is also still confirmed: the differences between rational expectations and adaptive learning increase when the inflation target increases.

The $d$ coefficient matrix can be found in the PLM in Eq. (24).

\( \beta \) is fixed to 0.99, while \( \kappa \) is set at 0.001 during estimation. The inflation target persistence parameter \( (\rho_{\pi^*}) \) is fixed at 0.99, which follows from Milani (2019). Eq. (32) also contains constants, which are set to the historical means of their respective data series.

The prior means for the standard deviations of the shocks roughly follow from Milani and Rajbhandari (2020), Milani (2019), and Cole and Milani (2019).

One interesting exception is the estimate of the standard deviation for \( \epsilon_{1,t} \), which is somewhat higher than the others. This may be due to the small scale of the New Keynesian model used in this paper that does not include other types of frictions found in larger scale models (e.g., habit formation in consumption and price indexation). Thus, this higher value may be picking up those other factors.