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Performance Resilience Analysis of Dynamic Feedback Controllers for both Multiplicative and Additive Gain Perturbations

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Abstract: In this work, a procedure is presented for performance resilience analysis of continuous-time systems controlled by full-order dynamic feedback compensators. The resilience property is defined in terms of defining maximum perturbations on both the controller and observer gains that will maintain controller and observer eigenvalues in disjoint regions in the complex plane so that the closed loop system will sustain certain performance characteristics. The desired performance is characterized by the response speed and magnitude bounds on the response. Therefore, the intersection of two regions are chosen, vertical strips and sector regions, to guarantee upper and lower bounds on the settling (or response) time and an upper bound on the percent overshoot simultaneously. Maximum allowable gain perturbations are obtained based on the designer's choices of closed loop eigenvalues and the regions in which eigenvalues remain when the gains are perturbed. The linear matrix inequality technique is used throughout the analysis process. Illustrative examples are included to demonstrate the effectiveness of the proposed methodology.

Keywords: Robustness and resilience analysis, dynamic feedback controller, linear matrix inequalities

1. INTRODUCTION

In this work, an analysis procedure is presented to evaluate the performance resilience property of a dynamic feedback controller for continuous-time systems. After designing the controller, the designers are able to determine upper bounds on the allowable deviations from nominal that the controller and observer gains can have while still maintaining the performance specifications required by the designer, specified by eigenvalue locations. These bounds are determined using linear matrix inequality (LMI) techniques (Boyd, et al., 1994).

A controller design is said to be resilient if no significant performance deterioration occurs due to a small perturbation in the controller or observer gains (Keel and Bhattacharyya, 1997). Resilient control problems became a popular research topic in the 1990s. Some design methods including LMI have been used in resilient control in the work of Jadbabaie, et al. (1994), Dorato (1998) and Haddad and Corrado (1998). Some other more recent representative work on resilient control or "non-fragile" systems can also be found in the following. Peaucelle, et al. (1998) focuses on resilient state-feedback with H_∞ using LMI. Stochastic resilient observer design is addressed in Yaz, et al. (2006). A resilient design of a large class of uncertain nonlinear systems is proposed in Feng, et al. (2013) and Feng, et al. (2014). Resilient control theory is also applied to some cutting-edge technologies in practice in the past several years: Rieger (2010) gives an overview of some examples and benchmark aspects, Jin, et al. (2013) integrates resilient control into nuclear power plants which requires significant precision and Hu, et al. (2013) uses resilient control in serial manufacturing networks.

Most of the cited work above discussed conditions for the resilience of the controller against perturbations which may destabilize the closed-loop system. However, to our best knowledge, the study of performance resilience is rarely addressed in the literature. Other than achieving system stabilization, performance resilience analysis guarantees certain system performance including settling time, overshot and rise time (Goodwin and Sin, 1984).

It should be emphasized that the present work is on the analysis rather than design for resilience, unlike the majority of the previous literature. This work is an extension of Feng, et al. (2015a, b) in the following aspects: accommodating both multiplicative and additive perturbations; a more comprehensive definition of perturbation bound; multiple eigenvalue regions combined to achieve several performance objectives simultaneously and improved results that are shown in examples to be less conservative. Using the LMI results of this work will ensure a quick and convenient approach to obtain bounds on the perturbations without going through extensive system simulation so that the required performances are still achieved.

The following notation is used in this work: $x \in R^n$ denotes an n-dimensional vector with real elements with the norm $\|x\| = (x^T x)^{1/2}$ where $(\cdot)^T$ represents the transpose. For a square matrix, A^{-1} is the inverse of matrix A; $A > 0$ ($A<0$) implies that A is a positive (negative) definite matrix; $\lambda_{\max}(A)$ ($\lambda_{\min}(A)$) are the maximum (minimum) eigenvalue of the symmetric A matrix; I is an identity matrix of appropriate dimension.

The following results are used in this paper:

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Lemma 1. Schur complement.

The following are equivalent:

(a) $\begin{bmatrix} A & B \\ B^T & C \end{bmatrix} > 0$.

(b) $C > 0$ & $A - BC^{-1}B^T > 0$

(c) $A > 0$ & $C - B^T A^{-1} B > 0$.

Lemma 2.

$-(A^T M^{-1} A + B^T MB) \le A^T B + B^T A \le A^T M^{-1} A + B^T MB$ is true for any $M = M^T > 0$.

Lemma 3. $\lambda_{\max}(A^T A) = \lambda_{\max}(AA^T)$ for any matrix A.

2. PROBLEM FORMULATION

Let us consider a continuous time linear system,

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx + Du \end{aligned} \tag{1}$$

where $x \in R^n$ is the state, $u \in R^m$ is the input and y is the measured output.

A linear state estimate feedback control is used in this system,

$$u = \tilde{K}\hat{x} \tag{2}$$

where $\hat{x}$ is the state estimate. The nominal feedback gain K may be perturbed due to the reasons cited in the introduction as

$$\tilde{K} = \Delta_c K \Delta_b + \Delta_a \tag{3}$$

both additively and multiplicatively. However, in the following, the perturbation

$$\tilde{K} = K\Delta_b + \Delta_a \tag{4}$$

will be used without loss of generality. The validation of the use of this alternative form is given in the Appendix 1 in Feng, et al. (2016).

A Luenberger (identity) observer is used in which both the output y and input u are known,

$$\dot{\hat{x}} = A\hat{x} + Bu + \tilde{L}(y - C\hat{x} - Du) \tag{5}$$

where the observer gain L may also be perturbed as

$$\tilde{L} = \Delta_e L + \Delta_d \tag{6}$$

We assume that the perturbations on both the feedback and observer gains are bounded as,

$$\begin{aligned} (K\Delta_b + \Delta_a - K)^T M_0 (K\Delta_b + \Delta_a - K) &\le M_1 \\ (\Delta_e L + \Delta_d - L)^T N_0 (\Delta_e L + \Delta_d - L) &\le N_1 \end{aligned} \tag{7}$$

where M_0, M_1, N_0 and N_1 are symmetric positive definite matrices with appropriate dimension.

For the error of the state estimation defined as $e = x - \hat{x}$, we obtain the error update equation by subtracting (5) from (1) as

$$\dot{e} = [A - \tilde{L}C]e \tag{8}$$

By introducing an augmented state variable X=[x ; e], the dynamics of the closed loop system will be

$$\dot{X} = \begin{bmatrix} \dot{x} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} A + B\tilde{K} & -B\tilde{K} \\ 0 & A - \tilde{L}C \end{bmatrix} \begin{bmatrix} x \\ e \end{bmatrix} \tag{9}$$

As stated in the introduction, our goal is to analyze the dynamic feedback design to determine how large the perturbation can be so that the performance of the system with perturbed controller and observer gains remains acceptable. In addition to achieving stability, we will also identify acceptable controller and observer performances in terms of regions in the complex plane within which the eigenvalues of the perturbed controller and observer must remain. Eigenvalue positions result in different system performances such as settling time, percent overshoot and rise time, etc. As shown in Figure 1, for this work the eigenvalues of the closed loop system are chosen to lie within the intersection of a vertical strip between -σ_a and -σ_b and a sector region with angle φ_c for the controller, and a vertical strip between -σ_c and -σ_d and a sector region with angle φ_o for the observer to guarantee upper and lower bounds on the settling (or response) time and upper bound on percent overshoot simultaneously.

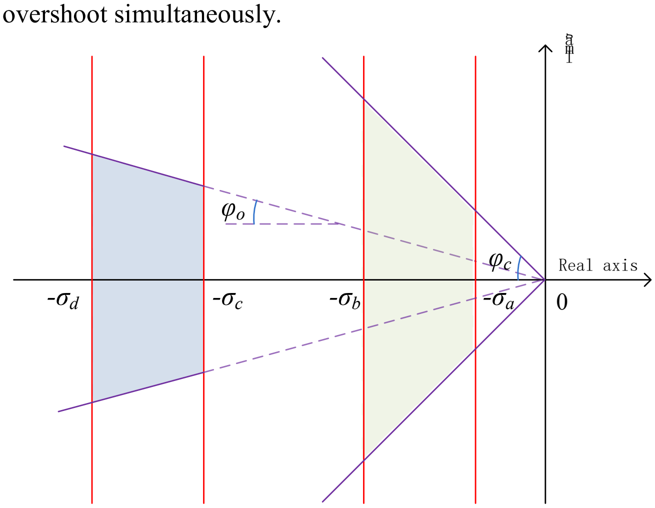

Fig. 1. Desired controller and observer eigenvalues regions

The resilience analysis starts with Lyapunov energy approach and based on the decoupled nature of the state and observer dynamics in (9). We have the following conditions for the controller: (Duan and Yu, 2013)

$$(A + B\tilde{K} + \sigma_a I)^T P_c + P_c(A + B\tilde{K} + \sigma_a I) < 0 \tag{10}$$

$$(A + B\tilde{K} + \sigma_b I)^T P_c + P_c(A + B\tilde{K} + \sigma_b I) > 0 \tag{11}$$

$$\begin{bmatrix} [(A+B\tilde{K})^T P_c + P_c(A+B\tilde{K})]\sin\varphi_c & [(A+B\tilde{K})^T P_c - P_c(A+B\tilde{K})]\cos\varphi_c \\ [P_c(A+B\tilde{K}) - (A+B\tilde{K})^T P_c]\cos\varphi_c & [(A+B\tilde{K})^T P_c + P_c(A+B\tilde{K})]\sin\varphi_c \end{bmatrix} < 0 \tag{12}$$

and similar conditions for the observers:

$$(A - \tilde{L}C + \sigma_c I)^T P_o + P_o(A - \tilde{L}C + \sigma_c I) < 0 \tag{13}$$

$$(A - \tilde{L}C + \sigma_d I)^T P_o + P_o(A - \tilde{L}C + \sigma_d I) > 0 \tag{14}$$

$$\begin{bmatrix}[(A-\tilde{L}C)^T P_o + P_o(A-\tilde{L}C)]\sin\varphi_o & [(A-\tilde{L}C)^T P_o - P_o(A-\tilde{L}C)]\cos\varphi_o \\ [P_o(A-\tilde{L}C)-(A-\tilde{L}C)^T P_o]\cos\varphi_o & [(A-\tilde{L}C)^T P_o + P_o(A-\tilde{L}C)]\sin\varphi_o\end{bmatrix} < 0 \quad (15)$$

3. MAIN RESULTS

Theorem 1. Dynamic feedback controller design is stability and performance resilient in the sense of maintaining the eigenvalues within the regions shown in Fig. 1 for perturbed controller and observer gains (7) for the system described by (9), if the following LMIs are feasible for some P_c, P_o, M_0, M_1, N_0, N_1>0.

$$\begin{bmatrix}-A_c^T P_c - P_c A_c - 2\sigma_a P_c - M_1 & P_c B \\ B^T P_c & M_0\end{bmatrix} > 0 \quad (16)$$

$$\begin{bmatrix}A_c^T P_c + P_c A_c + 2\sigma_b P_c - M_1 & P_c B \\ B^T P_c & M_0\end{bmatrix} > 0 \quad (17)$$

$$\begin{bmatrix}(-A_c^T P_c - P_c A_c)\sin\varphi_c - M_1 & (P_c A_c - A_c^T P_c)\cos\varphi_c & P_c B\sin\varphi_c & -P_c B\cos\varphi_c \\ (A_c^T P_c - P_c A_c)\cos\varphi_c & (-A_c^T P_c - P_c A_c)\sin\varphi_c - M_1 & P_c B\cos\varphi_c & P_c B\sin\varphi_c \\ B^T P_c\sin\varphi_c & B^T P_c\cos\varphi_c & M_0 & 0 \\ -B^T P_c\cos\varphi_c & B^T P_c\sin\varphi_c & 0 & M_0\end{bmatrix} > 0 \quad (18)$$

$$\begin{bmatrix}-A_o^T P_o - P_o A_o - 2\sigma_c P_o - C^T N_1 C & P_o \\ P_o & N_0\end{bmatrix} > 0 \quad (19)$$

$$\begin{bmatrix}A_o^T P_o + P_o A_o + 2\sigma_d P_o - C^T N_1 C & P_o \\ P_o & N_0\end{bmatrix} > 0 \quad (20)$$

$$\begin{bmatrix}(-A_o^T P_o - P_o A_o)\sin\varphi_o - C^T N_1 C & (P_o A_o - A_o^T P_o)\cos\varphi_o & P_o\sin\varphi_o & -P_o\cos\varphi_o \\ (A_o^T P_o - P_o A_o)\cos\varphi_o & (-A_o^T P_o - P_o A_o)\sin\varphi_o - C^T N_1 C & P_o\cos\varphi_o & P_o\sin\varphi_o \\ P_o\sin\varphi_o & P_o\cos\varphi_o & N_0 & 0 \\ P_o\cos\varphi_o & P_o\sin\varphi_o & 0 & N_0\end{bmatrix} > 0 \quad (21)$$

Comments: In LMI (16)-(21), we define $A_c = A+BK$ and $A_o = A\text{-}LC$, where A, B, C, K, L and σ_a, σ_b, σ_c, σ_d, φ_c, φ_o are all known. The unknowns are P_c, P_o, M_0, M_1, N_0 and N_1. All unknowns are in linear form in the above LMIs. The proof is given in the following.

3.1 Controller eigenvalue right bound condition

From (10), by adding and subtracting the same term BK in the inequality, we have

$$\begin{aligned}&-(A+B(K\Delta_b+\Delta_a)+\sigma_a I+BK-BK)^T P_c \\ &\quad - P_c(A+B(K\Delta_b+\Delta_a)+\sigma_a I+BK-BK) > 0\end{aligned} \quad (22)$$

Rearranging and moving the terms with "Δ" to the right hand side of the inequality, we have

$$\begin{aligned}&-(A+BK)^T P_c - P_c(A+BK) - 2\sigma_a P_c \\ &> (K\Delta_b+\Delta_a-K)^T B^T P_c + P_c B(K\Delta_b+\Delta_a-K)\end{aligned} \quad (23)$$

Applying Lemma 2 to the right hand side of (23) and define $\Delta_K = K\Delta_b + \Delta_a - K$, we have

$$\Delta_K{}^T B^T P_c + P_c B\Delta_K \le \Delta_K{}^T M_0 \Delta_K + P_c B M_0{}^{-1} B^T P_c \quad (24)$$

By using (24) and (7), we obtain the following sufficient condition for (23),

$$-A_c{}^T P_c - P_c A_c - 2\sigma_a P_c > M_1 + P_c B M_0{}^{-1} B^T P_c \quad (25)$$

Applying lemma 1 to (25), we obtain (16).

3.2 Controller eigenvalue left bound condition

From (11), and similar to (22), adding and subtracting the same term BK, we have the following left bound condition,

$$\begin{aligned}&-(A+B(K\Delta_b+\Delta_a)+\sigma_b I+BK-BK)^T P_c \\ &\quad - P_c(A+B(K\Delta_b+\Delta_a)+\sigma_b I+BK-BK) < 0\end{aligned} \quad (26)$$

Rearranging (26), we have

$$\begin{aligned}&(A+BK)^T P_c + P_c(A+BK) + 2\sigma_b P_c \\ &+\Delta_K{}^T B^T P_c + P_c B\Delta_K > 0\end{aligned} \quad (27)$$

Applying Lemma 2 for the second line of (27), we have

$$\Delta_K{}^T B^T P_c + P_c B\Delta_K \ge -\Delta_K{}^T M_0 \Delta_K - P_c B M_0{}^{-1} B^T P_c \quad (28)$$

By using (28) and (7), we obtain the following sufficient condition for (27),

$$A_c{}^T P_c + P_c A_c + 2\sigma_b P_c - M_1 - P_c B M_0{}^{-1} B^T P_c > 0 \quad (29)$$

Applying lemma 1 to (29), we obtain (17).

3.3 Controller eigenvalue sector condition

Inequality (12) describes a sector region with an angle φ_c. Rearranging (12) and moving the terms with "Δ" to the right hand side of the inequality, we have

$$\begin{aligned}&\begin{bmatrix}(-A_c{}^T P_c - P_c A_c)\sin\varphi_c & (P_c A_c - A_c{}^T P_c)\cos\varphi_c \\ (A_c{}^T P_c - P_c A_c)\cos\varphi_c & (-A_c{}^T P_c - P_c A_c)\sin\varphi_c\end{bmatrix} \\ &> \begin{bmatrix}(\Delta_K{}^T B^T P_c + P_c B\Delta_K)\sin\varphi_c & (\Delta_K{}^T B^T P_c - P_c B\Delta_K)\cos\varphi_c \\ (P_c B\Delta_K - \Delta_K{}^T B^T P_c)\cos\varphi_c & (\Delta_K{}^T B^T P_c + P_c B\Delta_K)\sin\varphi_c\end{bmatrix}\end{aligned} \quad (30)$$

The second line of (30) can be rewritten and bounded by applying Lemma 2,

$$\begin{aligned}&\begin{bmatrix}P_c B\sin\varphi_c & -P_c B\cos\varphi_c \\ P_c B\cos\varphi_c & P_c B\sin\varphi_c\end{bmatrix}\begin{bmatrix}\Delta_K & 0 \\ 0 & \Delta_K\end{bmatrix} \\ &+\begin{bmatrix}\Delta_K{}^T & 0 \\ 0 & \Delta_K{}^T\end{bmatrix}\begin{bmatrix}B^T P_c\sin\varphi_c & B^T P_c\cos\varphi_c \\ -B^T P_c\cos\varphi_c & B^T P_c\sin\varphi_c\end{bmatrix} \\ &\le \begin{bmatrix}P_c B\sin\varphi_c & -P_c B\cos\varphi_c \\ P_c B\cos\varphi_c & P_c B\sin\varphi_c\end{bmatrix}\begin{bmatrix}M_0 & 0 \\ 0 & M_0\end{bmatrix}^{-1}\begin{bmatrix}B^T P_c\sin\varphi_c & B^T P_c\cos\varphi_c \\ -B^T P_c\cos\varphi_c & B^T P_c\sin\varphi_c\end{bmatrix} \\ &+\begin{bmatrix}\Delta_K{}^T M_0 \Delta_K & 0 \\ 0 & \Delta_K{}^T M_0 \Delta_K\end{bmatrix}\end{aligned} \quad (31)$$

Using (31) and applying (7), we obtain the following sufficient condition for (30),

$$\begin{bmatrix} (-A_c^T P_c - P_c A_c)\sin\varphi_c - M_1 & (P_c A_c - A_c^T P_c)\cos\varphi_c \\ (A_c^T P_c - P_c A_c)\cos\varphi_c & (-A_c^T P_c - P_c A_c)\sin\varphi_c - M_1 \end{bmatrix} - \begin{bmatrix} P_c B\sin\varphi_c & -P_c B\cos\varphi_c \\ P_c B\cos\varphi_c & P_c B\sin\varphi_c \end{bmatrix} \begin{bmatrix} M_0 & 0 \\ 0 & M_0 \end{bmatrix}^{-1} \begin{bmatrix} B^T P_c\sin\varphi_c & B^T P_c\cos\varphi_c \\ -B^T P_c\cos\varphi_c & B^T P_c\sin\varphi_c \end{bmatrix} > 0 \quad (32)$$

By applying Lemma 1 to (32) we have (18).

3.4 Observer eigenvalue conditions

Using the inequalities (13)-(15) and following the similar derivation from (22) to (32), we will have observer vertical strip condition (19) and (20), and sector condition (21). This completes the proof.

4. FURTHER ANALYSIS

In this section, we focus on the controller and analyse how Δ_a and Δ_b are related to the maximum allowed perturbation obtained from the LMIs. For simplifying the exposition, we study a single input case and we let $M_0=I$, $M_1=\delta_K I$, $N_0=I$, $N_1=\delta_L I$, where δ_K and δ_L are positive scalars. The perturbed gain can be written as

$$K\Delta_b + \Delta_a - K = \begin{bmatrix} k_1 & \cdots & k_n \end{bmatrix}\left(\begin{bmatrix} \Delta_{b1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \Delta_{bn} \end{bmatrix} - I_n\right) + \begin{bmatrix} \Delta_{a1} & \cdots & \Delta_{an} \end{bmatrix} \quad (33)$$

Substituting (33) into (7) and using lemma 3, we have

$$\begin{bmatrix} (\Delta_{b1}-1)k_1 + \Delta_{a1} & \cdots & (\Delta_{bn}-1)k_n + \Delta_{an} \end{bmatrix} \begin{bmatrix} k_1(\Delta_{b1}-1) + \Delta_{a1} \\ \vdots \\ k_n(\Delta_{bn}-1) + \Delta_{an} \end{bmatrix} \le \delta_K \quad (34)$$

which is in the form of

$$\sum_{i=1}^{n} [(\Delta_{bi}-1)k_i + \Delta_{ai}]^2 \le \delta_K \quad (35)$$

For a second order system, (35) becomes

$$[(\Delta_{b1}-1)k_1 + \Delta_{a1}]^2 + [(\Delta_{b2}-1)k_2 + \Delta_{a2}]^2 \le \delta_K \quad (36)$$

which can be rewritten as

$$\left(\frac{\Delta_{b1} - (1 - \Delta_{a1}/k_1)}{\sqrt{\delta_K}/k_1}\right)^2 + \left(\frac{\Delta_{b2} - (1 - \Delta_{a2}/k_2)}{\sqrt{\delta_K}/k_2}\right)^2 \le 1 \quad (37)$$

If Δ_a=0, which means there are only multiplicative perturbations and no additive perturbations, (37) becomes

$$\left(\frac{\Delta_{b1} - 1}{\sqrt{\delta_K}/k_1}\right)^2 + \left(\frac{\Delta_{b2} - 1}{\sqrt{\delta_K}/k_2}\right)^2 \le 1 \quad (38)$$

Inequality (38) is in the form of an ellipse whose semi-major and semi-minor axes are $\sqrt{\delta_K}/k_1$ and $\sqrt{\delta_K}/k_2$, with center at (1,1). Similarly, (37) is a function of a set of ellipses with the same length of axes, but with different centers. Values of Δ_{a1} and Δ_{a2} shift the center positions of the ellipses.

If Δ_b=0, which means there are only additive perturbations and no multiplicative perturbations and the ellipses become circles.

So based on (37), it is possible to determine whether the perturbation is within the allowable range.

5. SIMULATION STUDIES AND ANALYSIS

This section contains simulation results of the controller performance analysis proposed in this work. An unstable second order system is chosen to demonstrate possible application of the proposed methodology. For illustrative purposes, in this example, we study a case with only multiplicative perturbations.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -0.2 & 2 \\ -1 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \quad (39)$$

The eigenvalues of the open loop system are $0.15 \pm 1.37i$ and the state variables of the open loop system diverge quickly.

Nominal controller and observer gains $K = \begin{bmatrix} -8.29 & -0.304 \end{bmatrix}$ and $L = \begin{bmatrix} -64.34 & 108.94 \end{bmatrix}^T$ are used to place the controller eigenvalues at $-2 \pm 2i$ and observer eigenvalues at $-11 \pm 7i$.

The desired controller eigenvalue region is chosen to be a strip between σ_b = 3 and σ_a = 1 and the sector region angle φ_c = 55º and observer eigenvalue region to be a strip between σ_d = 12 and σ_c = 10 and the sector region angle φ_o = 35º. By replacing LMI unknown variables M_1 and N_1 with $\delta_K I$ and $\delta_L I$, and from the feasible solution of LMIs (16)-(18), the maximum δ_K is found to be 1.08; similarly from LMIs (19)-(21), maximum δ_L is found as 0.73.

The accuracy of this analysis will now be discussed in what follows.

The multiplicative perturbation $\tilde{K} = K\Delta_b$ can be written as

$$\tilde{K} = \begin{bmatrix} K_1 & K_2 \end{bmatrix} \begin{bmatrix} \Delta_{b1} & 0 \\ 0 & \Delta_{b2} \end{bmatrix} = \begin{bmatrix} K_1\Delta_{b1} & K_2\Delta_{b2} \end{bmatrix} \quad (40)$$

Using the following inequality and (7),

$$(K\Delta_b - K)^T (K\Delta_b - K) \le (K\Delta_b - K)(K\Delta_b - K)^T I \quad (41)$$

we obtain

$$\begin{bmatrix} K_1(\Delta_{b1}-1) & K_2(\Delta_{b2}-1) \end{bmatrix} \begin{bmatrix} K_1(\Delta_{b1}-1) \\ K_2(\Delta_{b2}-1) \end{bmatrix} \le \delta_K I \quad (42)$$

The largest actual controller gain perturbations can be determined by finding the maximum δ_K for which the eigenvalues of $A + B\tilde{K}$ remain within the specified region as compared with the theoretical value. To systematically study this, we define the perturbation as

$$\begin{bmatrix} K_1(\Delta_{b1}-1) & K_2(\Delta_{b2}-1) \end{bmatrix} = \begin{bmatrix} \delta_c\cos\theta & \delta_c\sin\theta \end{bmatrix} \quad (43)$$

where $\delta_c = \sqrt{\Delta_K \Delta_K^T} = \sqrt{\delta_K}$.

By incrementing θ through 360 degrees with a step of 1 degree, and finding the maximum value for δ_c by incrementing δ_c until the eigenvalues of $A + B\tilde{K}$ exit the specified region for each degree increment, we obtain δ_c, which is the maximum perturbation tolerance for that particular angle. And the similar process from (40) to (43) can be done for the observer case to obtain the maximum value of δ_o for each angle.

The results for δ_c^2 and δ_o^2 values for each angle are shown in blue in Fig. 2 for the controller and observer gain tolerances. This figure shows the allowed perturbation in each direction is highly direction dependent.

As shown in Fig. 2, for the controller, the minimum among these maximum allowable values is 1.25 at 59º, determined from closed loop system simulation. And for the observer, the minimum among these maximum allowable values is 0.84 at 218º. The bound of δ_K=1.08 and δ_L =0.73 obtained from the LMIs are approximately 12% less than the error from the 360 degree validation. And these bounds are plotted in red in Fig. 2 for comparison with the 360 degree validation.

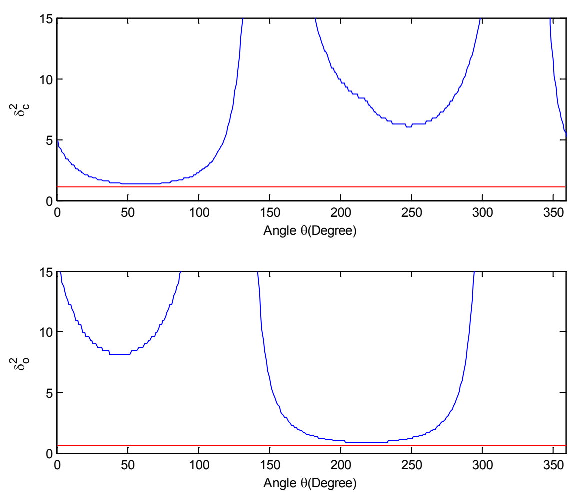

Fig. 2. Perturbation allowed in each direction for K and L comparing with bounds from LMIs

An alternative way to study perturbations in different directions is to transfer all these δ_c values to Δ_{b1} and Δ_{b2} coordinates by rearranging (43) in the following manner,

$$\begin{bmatrix}\Delta_{b1} & \Delta_{b2}\end{bmatrix} = \begin{bmatrix}\delta_c \cos\theta / K_1 + 1 & \delta_c \sin\theta / K_2 + 1\end{bmatrix} \quad (44)$$

After substituting all θ and δ_c values, we obtain 360 pairs of Δ_{b1} and Δ_{b2} values. In Fig. 3, a section of these values is shown in blue in comparison with the LMI result in red, which is calculated using (38).

We can see from the blue colored bounds in Fig. 3 that the minimum deviation distance from (1,1) for all 360 degrees is very close to the red ellipse, which is from the LMI result. This shows that the proposed bound is accurate for this particular system. While large perturbations may be allowed in certain directions, the LMI result provides a safe and guaranteed bound for all directions.

The minimum value of δ_c^2 from Fig. 2 is located at point C in Fig. 3. The center of the ellipse (38) is point A. Point B is located on the ellipse at the point closest to C. Point D represents perturbations which will exceed the bound on δ_c established by the simulation analysis. The Δ_{b1} and Δ_{b2} values for each point are shown in Table 1.

Table 1. Coordinates of the four points

	Δ_{b1}	Δ_{b2}
A	1	1
B	0.94	-1.90
C	0.92	-2.52
D	0.875	-4.64

A similar analysis for observers can be done to get a dynamic feedback controller co-plot showing in Fig. 4 for four different cases: unperturbed, bounds from LMI, minimum bounds from 360 degree validation and exceeding limits.

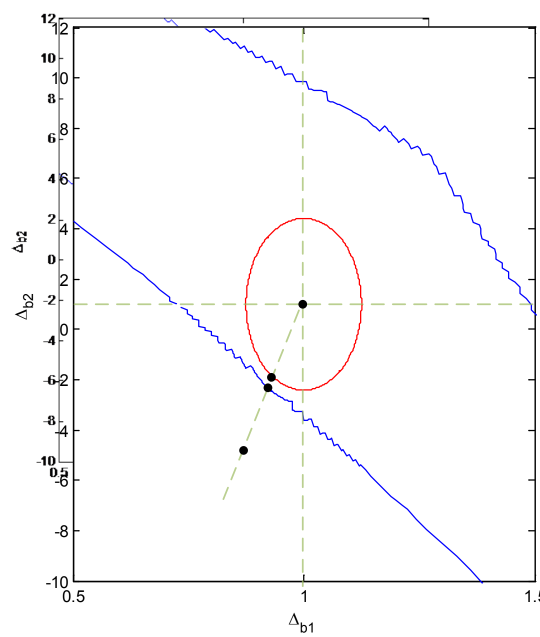

Fig. 3. Comparison of Δ_{b1} and Δ_{b2} behavior of LMI result and validation result

The co-plot of the state variables with x_0=[1 ; -1] for all cases is given in Fig. 4 and closed loop controller and observer eigenvalue positions for all cases are plotted in Fig. 5. We can tell that the responses are different in terms of settling time and overshoot. And we can also see in Fig. 4 and 5, although the system is still stabilized for all cases, the settling time and overshot exceed the bounds for case (d) because the eigenvalues are outside the desired regions.

As we stated in the introduction, the LMI result gives a safe and guaranteed bound for all directions, which provides us a quick and convenient way without conducting extensive simulations.

6. CONCLUSIONS

In this work, performance resilience analysis procedure for a dynamic feedback controller has been presented for continuous-time systems. By defining controller and observer eigenvalue regions, multiplicative and additive perturbation bounds allowed in controller and observer gains are found. With this method, designers can evaluate the resilience degree of their controllers based on the design parameters chosen and adjust design parameters to fit appropriate desired system performance and design requirements. In this work, results are given for performance involving response speed and magnitude bound on response. LMI techniques are used throughout the process. For future work, this method can be extended to stochastic perturbations on the gains, certain classes of nonlinear systems and systems with L_2 disturbances for general performance criteria and other eigenvalue regions in the complex plane.

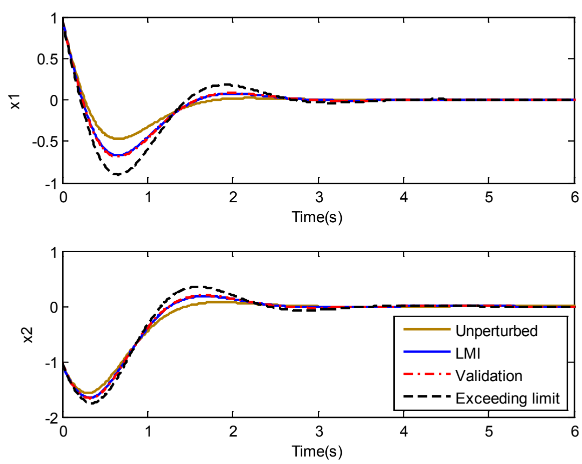

Fig. 4. Co-plots of the state variables in four cases, (a) No perturbation, (b) LMI result, (c) Validation result (d) Exceeding limit

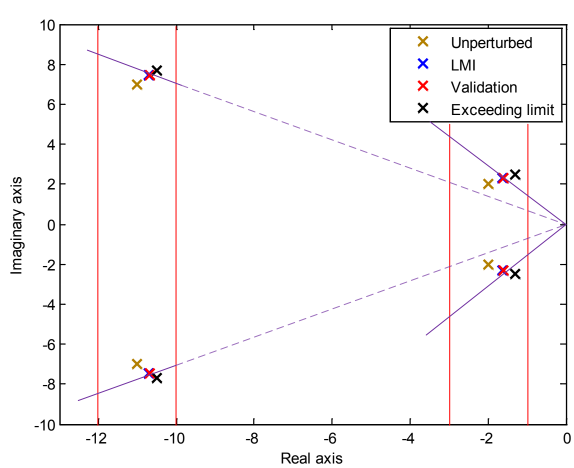

Fig. 5. Eigenvalue positions of all four cases

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