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Franck O. Hounkpevi
Marquette University

Edwin E. Yaz
Marquette University, edwin.yaz@marquette.edu

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Franck O. Hounkpevi
Department of Electrical and Computer Engineering, Marquette University, Milwaukee, WI
Edwin E. Yaz
Department of Electrical and Computer Engineering, Marquette University, Milwaukee, WI

Abstract

Linear minimum variance unbiased state estimation is considered for systems with uncertain parameters in their state space models and sensor failures. The existing results are generalized to the case where each sensor may fail at any sample time independently of the others. For robust performance, stochastic parameter perturbations are included in the system matrix. Also, stochastic perturbations are allowed in the estimator gain to guarantee resilient operation. An illustrative example is included to demonstrate performance improvement over the Kalman filter which does not include sensor failures in its measurement model.

Introduction

This work is on the robust optimal design of linear state estimators in case of sensor failures. In the past, models for sensor measurement have included the effect of possible uncertain observations in the measurement (Nahi, 1969, NaNacara and Yaz, 1997, Yaz et al., 1998). These uncertain observations in sensor measurement often result, in addition to noise from various sources such as sensor noise or environment noise, from sensor failures where the measurement may not contain the signal component with a nonzero probability. The problem of failing sensors has received particular attention in harsh industrial environments where sensors are liable to fail more frequently. The solutions that have been proposed (Matveev and Savkin, 2003, Nahi, 1969, NaNacara and Yaz, 1997, Savkin et al., 1999, Sinopoli et al., 2004, Smith and Seiler, 2003, Yaz et al., 1998) introduce the use of stochastic parameters into the measurement model to represent probable sensor failures. Some of the proposed solutions to the sensor failure problem add robustness to the estimator by assuming deterministic parameter perturbations in the system matrix.

Because of the importance of robust performance in practice where models are only approximate, much effort has been devoted to robust filter design (Petersen and Savkin, 1999, Savkin and Petersen, 1997, Savkin et al., 1999, Theodor and Shaked, 1996; Wang, Ho, & Liu, 2003; Wang, Yang, Ho, & Liu, 2005; Wang, Zhu, & Unbehauen, 1999). Another important issue in estimator design is resilience, which is robustness against perturbations in the estimator gain. This may be due to computational (round off) errors or changes (drifts) in the gains during operation for hardware realizations (Yaz, Jeong, Yaz, & Bahakeem, 2005). Resilience for deterministic and stochastic perturbations in estimator gain matrices is discussed in Yaz et al. (2005) and Yaz, Jeong, and Yaz (2006), respectively, among others.

In previous works on estimation with sensor failures, it has always been assumed that all sensors have identical failure characteristics. However, real systems usually involve multiple sensors with different characteristics. With today's rapid development of these sensor network systems, we cannot bypass these considerations anymore. One major contribution of the present research is to finally address this critical issue by including multiple sensors with different failure rates in the system model and the estimator design. In the present work, it is assumed that each sensor may fail at any sample time independently of the others and the probability of the occurrence of the failure is assumed to be known from the reliability data. So, the present work represents an improvement and generalization of Nahi (1969), Yaz et al. (1998), Theodor and Shaked (1996), Savkin and Petersen (1997), Petersen and Savkin (1999), Wang et al., 2003, Wang et al., 2005, Wang et al., 1999.

Another major contribution is stochastic robustness. As in Theodor and Shaked (1996), Savkin and Petersen (1997), Petersen and Savkin (1999), Wang et al., 2003, Wang et al., 2005, Wang et al., 1999, we also robustify the state estimator. However, in our case, stochastic parameter perturbations are included in the system matrix rather than deterministic parameter perturbations. This is more in line with the stochastic model used for missing measurements and is known to give less conservative results than the deterministic perturbation model (Yaz et al., 2006).

A further contribution is made in the present work by generalizing the state estimator to include a stochastic resilience feature (Yaz et al., 2006). This results in a much improved estimator that can account for computational errors and other perturbations in the estimator gain.

In addition, this novel estimator contributes to the general area of state estimator design for systems with stochastic parameters (DeKoning, 1984, Rajasekaran et al., 1971, Tugnait, 1981, Wu et al., 1997, Yaz, 1987, Yaz, 1988, Yaz, 1992) and may be useful in associated applications.

In Section 2, the problem statement is provided. In Section 3, our state space model is given which also includes a measurement equation for multiple sensors featuring probable failures. Then in Section 4, we present our main results on the derivation of the robust and resilient minimum variance unbiased state estimator. The details of the derivation are provided in the two appendices. In Section 5, a simulation example is presented. Section 6 is the conclusion section.

Section snippets

Problem statement

Existing models for sensor failure only model sensors with the same failure rate. Also, additional performance features such as robustness against system modeling errors and resilience against estimator gain perturbations are very desirable. Therefore the problem we consider in this paper is:

Problem

Find the linear minimum variance unbiased state estimator which is robust against stochastic perturbations in the system matrix and resilient against stochastic perturbations in the estimator gain for a

Modeling for stochastic robustness and multiple sensors featuring probable failure

This section is devoted to the modeling part of the solution to the problem posed in Section 2.

Consider the dynamical system and the measurement model

$$\begin{cases} x_{k+1} = \left(A_k^0 + \sum_{j=1}^{n_\alpha} \alpha_k^j A_k^j \right) x_k + F v_k, \\ y_k^i = \gamma_k^i C_k^i x_k + D^i w_k^i, i = 1, \dots, p, \end{cases}$$

where $x_k \in R^n$, with x_0 having the expected value $E\{x_0\} = \bar{x}_0$ and covariance P_0, v_k is a zero mean white noise vector uncorrelated with x_0 and with covariance V, $y_k^i i = 1, \dots, p$ are scalar sensor outputs with zero mean white scalar sensor noise wki with variance $\sigma_{w_i}^2$ that are uncorrelated with v_k and x_0

Main results

Theorem 1

The linear robust and resilient minimum variance unbiased state estimator of the form(3)*for the system and measurement model*(2)*is given as follows*: $\hat{x}_{k+1} = A_k^0 \hat{x}_k + K_k^0 (y_k - \bar{\Gamma}_k C_k \hat{x}_k), \hat{x}_0 = \bar{x}_0$ *with the gain* $K_k^0 = \Lambda_k \Omega_k^{-1}$, *where* $\Omega_k = \bar{\Gamma}_k C_k P_k C_k^{\mathrm{T}} \bar{\Gamma}_k^{\mathrm{T}} + \Xi \otimes C_k X_k^{\mathrm{T}} C_k^{\mathrm{T}} + \mathrm{DWD}^{\mathrm{T}}$, $\Lambda_k = A_k^0 P_k C_k^{\mathrm{T}} \bar{\Gamma}_k^{\mathrm{T}}$. *The error covariance evolution*: $P_{k+1} = L_k - \Lambda_k \Omega_k^{-1} \Lambda_k^{\mathrm{T}}, P_0 = \mathrm{Cov}(x_0)$, *where* $L_k =$

$$\begin{pmatrix} A_k^0 P_k (A_k^0)^{\mathrm{T}} + \sum_{j=1}^{n_\beta} \sigma_{\beta j}^2 K_k^j \bar{\Gamma}_k C_k P_k C_k^{\mathrm{T}} \bar{\Gamma}_k^{\mathrm{T}} K_k^{j\mathrm{T}} \\ + \sum_{i=1}^{n_\alpha} \sigma_{\alpha i}^2 A_k^i X_k^{\mathrm{T}} A_k^{i\mathrm{T}} \\ + \sum_{j=1}^{n_\beta} \sigma_{\beta j}^2 K_k^j (\Xi \otimes C_k X_k^{\mathrm{T}} C_k^{\mathrm{T}}) K_k^{j\mathrm{T}} \\ + \sum_{j=1}^{n_\beta} \sigma_{\beta j}^2 K_k^j \mathrm{DWD}^{\mathrm{T}} K_k^{j\mathrm{T}} + \mathrm{FVF}^{\mathrm{T}} \end{pmatrix}, X_k = E\{x_k x_k^{\mathrm{T}}\}, X_{k+1} = A_k^0 X_k (A_k^0)^{\mathrm{T}} + \sum_{i=1}^{n_\alpha} \sigma_{\alpha i}^2 A_k^i X_k^{\mathrm{T}} A_k$$

A simulation example

We have applied the new estimator to the following system and measurement model where the sensor has nonzero failure probability:

$$\begin{cases} x_{k+1} = \left(\begin{bmatrix} .3 & .7 \\ .2 & .6 \end{bmatrix} + \alpha_k \begin{bmatrix} .1 & .05 \\ .2 & .1 \end{bmatrix} \right) x_k + \begin{bmatrix} 1 \\ .5 \end{bmatrix} v_k, \\ y_k = \begin{bmatrix} \gamma_k^1 & 0 \\ 0 & \gamma_k^2 \end{bmatrix} \begin{bmatrix} .5 & 1 \\ 1 & 1 \end{bmatrix} x_k + \begin{bmatrix} 0.5 & 0 \\ 0 & .2 \end{bmatrix} \begin{bmatrix} w_k^1 \\ w_k^2 \end{bmatrix}, \end{cases}$$

, where $x_k \in R^2$, with x_0 having $E\{x_0\} = [21]^{\mathrm{T}}$ and covariance $P_0 = \begin{bmatrix} .1 & 0 \\ 0 & .1 \end{bmatrix}$, v_k is zero mean white Gaussian noise with variance $V = 0.3$, w_k^1 and w_k^2 are zero mean Gaussian noise with covariance $\sigma_{w_1}^2 = \sigma_{w_2}^2 = 0.3$. α_k is zero mean Gaussian noise with variance $\sigma_\alpha^2 = 0.1$. $\Sigma_k^\beta = \beta_k \begin{bmatrix} .1 & .1 \\ .1 & -.1 \end{bmatrix}$ where β_k is zero mean Gaussian noise

Conclusions

Models in estimation theory usually assume that measurement always contains the signal to be estimated. However, for failing sensors, this is no longer valid. Several solutions have been proposed to deal with this problem. However, none of the proposed solution takes into account the realistic nature of a true sensor network in which different sensors might have different failure rates. The current paper has developed a solution to that particular challenge by incorporating multiple sensors

Franck O. Hounkpevi received his M.S. degree in Electrical and Computer Engineering from Marquette University, Milwaukee, WI, in 2003. He is currently working toward his Ph.D. degree in the same department. He is a student member of the IEEE and a member of Eta Kappa Nu. His research interests include signal processing, communication systems and control systems.

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