Dynamic Behavior of Granular Earth Materials Subjected to Pressure-Shear Loading

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ABSTRACT
DYNAMIC BEHAVIOR OF GRANULAR EARTH MATERIALS SUBJECTED TO PRESSURE-SHEAR LOADING

Jeff W. LaJeunesse, M.S.
Marquette University, 2018

The dynamic response of granular earth materials such as sand has been of interest for many years. Multiple previous works have explored the shock response of sand in various grain shapes, sizes, and moisture contents, but the response during rapid combined loading has been relatively unexplored. The current study contributes to that lack of data by performing pressure-shear experiments on Oklahoma #1 silica sand, with quasi-smooth grains of 63 – 120 µm diameter and 99.8 wt.% SiO₂ composition. In these experiments, an oblique flyer plate impacts an equally inclined target, imparting a longitudinal (pressure) and transverse (shear) wave into a material of interest. The final loading states within the sand were inferred by measuring the normal and transverse components of particle velocity from the rear surface of the target using Photon Doppler Velocimetry (PDV). Tests were performed over a range of impact velocities to vary the magnitude of combined loading on the sand. Uncertainty in the calculated transverse particle velocity was explored for a variety of normal and angled PDV collimator setups to minimize the measurement uncertainty in shear stress. Combined loading in the experiments reached 0.25 – 1 GPa and 0.02 – 0.10 GPa of normal and shear stress, respectively. Yield surface models originally derived for lower strain rate loading of granular materials were shown to fit the experimental data in normal-shear stress space. The failure surface had a slope, or shearing resistance, of $\mu = 0.130$ and potential failure caps were presented. Scanning electron microscope images were taken of the recovered samples for 9 of 12 shots. Three-dimensional mesoscale simulations using an Eulerian hydrocode, CTH, were performed to better understand the experimental results and explore the boundaries of mesoscale formulations in Eulerian frameworks. Two different grain surface treatments were utilized, stiction and slide, to determine the influence of mixed cell treatments within CTH. Simulated normal stress - shear stress responses resulted in a shearing resistance of $\mu = 0.172$ and $\mu = 0.176$, for the sliding and stiction case, respectively, but failure caps were not observed for either mixed cell treatment.
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DEDICATION

To everyone who has given me the chance to succeed, believed in my abilities, and supported me along the way.

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D.2 Raw data from shot 2. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.3 Raw data from shot 3. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.4 Raw data from shot 4. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
D.5 Raw data from shot 5. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.6 Raw data from shot 6. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.7 Raw data from shot 7. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
D.8 Raw data from shot 8. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.9 Raw data from shot 9. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.10 Raw data from shot 10. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
D.11 Raw data from shot 11. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.12 Raw data from shot 12. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.

D.13 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 50 m/s.

D.14 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 70 m/s.

D.15 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 90 m/s.

D.16 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 110 m/s.

D.17 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 130 m/s.

D.18 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 150 m/s.

D.19 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 170 m/s.

D.20 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 190 m/s.

D.21 Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 210 m/s.
CHAPTER 1
INTRODUCTION

The response of granular, earth materials to various types of loading is of interest to disciplines including planetary science, mining and civil engineering industries, and various defense applications. Fully characterizing these materials requires testing over a range of strain rates and loading configurations. The most important implication of these responses is the determination of failure strength. A wealth of data from soil mechanics studies exists for triaxial loading and direct shear loading [51] at strain rates ranging from $10^{-2}$ - $10^{2}$ s$^{-1}$, but data for various types of loading at higher strain rates ranging from $10^{2}$ - $10^{4}$ s$^{-1}$ and upwards is much sparser. At extreme strain rates, additional mechanisms begin to contribute to yielding, such as the failure of constituent materials that makes up individual grains in sands and soils.

Understanding these mechanisms is fundamental to creating physics-based models that are applicable in a wide range of conditions. Experiments at strain rates upwards of $10^{3}$ s$^{-1}$ generally require more advanced apparatuses such as a split-Hopkinson bar [50, 57], or light-gas guns [38]. Both experiments use planar impacts to load materials in uniaxial plane strain and, combined, cover a range of strain rates roughly $10^{2}$ - $10^{7}$ s$^{-1}$. However, the plane strain loading accomplished using these methods does not reveal information about deviatoric stresses experienced within the sample of interest. Deviatoric stresses are a direct measure of the strength of materials. For example, a commonly used constitutive model for homogenous materials, the elastic-perfectly-plastic Von Mises yield criterion [99], relies on the second invariant of the deviatoric stress tensor, $J_2$. Likewise, the Drucker-Prager failure model for non-cohesive granular materials [33], predicts that failure is a function of $J_2$ and pressure. This model is famous for capturing the pressure-dependent, work hardening behavior of granular materials. Therefore, there is a need to investigate the dynamic shear response of granular materials to accurately model their behavior over a wide range of strain rates and complicated loading conditions.

Quartz sand was chosen as a representative material given its widespread presence on Earth as well as other planetary bodies. Being comprised of 99.9% silicon dioxide (SiO$_2$), quartz sand represents a wide range of brittle, non-cohesive earth materials and is abundant in natural regions as well as manmade structures such as concrete. Traditional high strain rate testing of sands or other brittle granular materials has been limited to uniaxial plane strain loading experiments [7, 11, 19, 20, 22, 30, 65–67, 73, 74, 78, 84, 85, 90, 93–95]. This uniaxial loading results
in a quasi-one-dimensional, plane wave, which is ultimately interpreted using the one-dimensional Rankine-Hugoniot jump equations for shock waves. However, the one-dimensionality of these experiments does not provide any information for how the material responds when deviatoric components are present in the stress tensor. To probe these regions, pressure-shear experiments are necessary.

This study focuses on loading sand to combined normal-shear stress states using dynamic pressure-shear experiments, also referred to as oblique plate impact [2]. Pressure-shear experiments use a traditional flyer plate setup, but implement an inclined projectile and target face at a set skew angle to impart a longitudinal (normal) and transverse (shear) wave into the target material upon impact. Both waves can be observed from the rear surface as normal and transverse surface (or particle) velocities, using interferometry techniques. Once these wave profiles have been measured, impedance matching can be used to determine characteristics of the material such as flow stress and yield strength. The use of flyer-plate techniques for pressure-shear experiments is extremely useful because it can investigate the effect of much higher strain rates, both normal and shear.
CHAPTER 2

LITERATURE REVIEW

Various mechanisms for testing granular materials and soils have been developed within
the last century. Examples of applications for strain rate testing in the $10^{-2} - 10^3 \text{s}^{-1}$ range are:
determining the ability of military or civilian vehicles to travel on certain soils based on their
weight; determining seismic wave speed [9], and predicting landslide probability in high-risk
areas [21]. Common tests in this regime are the Jenike direct shear test [45, 51] and the triaxial
test [29]. The direct shear test is the most applicable to this study because it measures the shear
stress necessary to yield a granular sample based on an explicit amount of normal force.
Applications for high strain rate tests $10^3 \text{s}^{-1}$ and upwards have relevance for: planetary impact
and crater formation [26, 83], dart penetration into sand- and other soils [3, 4, 80, 96], and blast
wave propagation resulting from buried explosives [41, 58].

Most applicable to this work are the applications of higher strain rate testing. As
mentioned earlier, split-Hopkinson bars and light-gas guns are used to achieve strain rates of $10^2 -
10^4 \text{s}^{-1}$ and $10^4 - 10^7 \text{s}^{-1}$, respectively. First, split-Hopkinson bar apparatuses sandwich a sample
of interest between two large, elastic bars. One of the bars is used as a “striker” bar and the other
is used as a “backer” bar. The striker bar is subjected to a single impulse loading, which is then
transmitted into the sample and subsequently into the backer bar. The response of the material is
then inferred from strain gauges mounted in the striker and backer bar. This apparatus has
successfully been used to study granular materials by utilizing a compressive sleeve in the radial
direction to confine the material throughout the loading process [84].

Flyer-plate experiments performed with gas guns are used to load materials at strain rates
ranging from $10^4$ to $10^7 \text{s}^{-1}$. Samples are subjected to rapid uniaxial plane strain loading upon
impact from a flat-faced projectile traveling at velocities ranging from 100 - 6000 m/s. The name
“flyer plate” refers to the flat-face projectile and corresponding target. Projectiles are accelerated
down a barrel, generally 10 to 40 ft in length, using compressed light gases, explosives, or a
combination of both. The response of the material is measured using stress gauges, piezoelectric
timing pins, and velocimetry techniques such as VISAR (Velocity Interferometer System for Any
Reflector) [10] and PDV (Photon Doppler Velocimetry) [86]. Multiple works have used flyer plate
experiments to investigate the response of sand [7, 19, 20, 22, 30, 73, 74, 94]. Unfortunately, a
wealth of data does not equate to a wealth of agreement between studies. Slight variations in the
mesoscale composition of these granular materials such as grain size and shape [65–67], as well as moisture content [74], have been shown to affect the overall response of the system. Therefore, it is necessary to characterize the mesoscopic features of a granular sample to understand its macroscopic response.

In the same range of strain rates as flyer-plate experiments, penetration studies have been performed to study the response of sand to high velocity impact from a variety of objects such as spheres and long rod penetrators of different nose shapes [37, 55, 80, 96]. Penetration experiments result in quasi-planar compaction waves that appear as a bow shock in front of the projectile as well as rapid shearing near the nose of the projectile. Penetration is tracked using witness plates, high speed cameras [96], and x-ray imaging [55]. Stresses experienced in the compaction wave can be measured using buried quartz stress gauges. Compaction wave speed can be observed using high speed cameras and x-ray imaging and then tracked using digital image correlation. The goal of these studies is to determine terminal velocity and final penetration depth. To reproduce experimental results in a computational framework requires having accurate continuum models built upon a solid understanding of the longitudinal wave propagation and rapid shear loading of sand. Therefore, the flyer-plate pressure-shear experiments are an ideal way to impart easily characterized normal and shear waves into sand, or other materials, at extremely high strain rates.

Pressure-shear experiments were originally developed by Clifton et al [2] to study the response of materials subjected to rapid shear loading. These experiments have been used to measure the strength of multiple homogeneous materials such as aluminum [24, 42, 62], copper [39, 92], elastomers [52], soda lime glass [25, 88], brittle nano-composites [35], and ceramics [97]. Additionally, the rheology of lubricants [77] and transient friction at material interfaces [76] has been studied using this technique. However, only two studies used this technique to subject granular materials to dynamic pressure-shear loading. Sundaram et al [87, 89] conducted the first pressure-shear experiments on powdered aluminum to study the strength of ceramic rubble created during penetration events. Sundaram used shearing resistance to determine strength by observing the relationship between flow stress and pressure, i.e. the internal friction coefficient. Vogler et al [97] followed the original framework, with slight modifications, and performed pressure-shear experiments on granular tungsten carbide as well as sand. This study assumed the material exhibited a von Mises flow behavior to compare the measured shear stress to the flow stress under uniaxial stress. This study highlighted choice of anvil material as a main concern to ensure no-slip at the anvil-grain interface. Additionally, it
suggested that PDV systems could be used to measure transverse particle velocity, which will be included in the present work.

Recent works by Parab et al [70, 71] and Herbold et al [48] investigated the limit at which individual grains begin to break using soda lime glass microspheres compressed by Hopkinson bar or light gas gun impact. These experiments utilized a recent leap in technological capability within the Dynamic Compression Sector located at Argonne National Laboratory’s Advanced Photon Source (APS). The facility provides an X-ray source that can be used make phase contrast images throughout the loading process thereby enabling the visualization of mesoscale processes at time scales on the order of 150 ns. Combining images of the grain fracture with corresponding loading curves, conclusions can be made about fracture mechanisms.

Characterizing propagation of waves, both longitudinal and shear, in granular, earth materials is fundamental to building physics-based models that can accurately portray a variety of applications. Events involving the loading of granular, brittle materials are extremely complicated because of the phenomena occurring at each of the length scales: macro, meso, and micro. Observation of the macroscale reveals that non-cohesive granular materials such as dry sand have zero strength in tension, but also exhibit work-hardening under compressive loading [33]. This is useful because it allows one to make continuum based conclusions about the response of the overall system. On the meso-scale, a plethora of energy dissipating mechanisms reveal themselves such as plastic deformation of grains, microkinetic energy of grains, friction and melting at grain contact sites, grain fracture and pulverization, and gas compression in pore regions [64, 69]. Lastly, the microscale behaves according to the constituent material of individual grains where energy absorption is now associated with lattice mechanisms such as the production of point, line, and interfacial defects, dislocation motion, and twinning [103]. Understanding how the mesoscale mechanisms influence the macroscopic response of granular, earth materials is the goal this study. Modeling larger scale events such as the ones described above with reasonable computational expense rely on continuum models that have been constructed based upon the smaller scale processes. Therefore, a firm understanding of meso- and micro-scale influence on the macro-scale is necessary.

There are a variety of approaches commonly used for modeling porous and/or granular systems, but a few general approaches are: discrete element method simulations, peridynamic formulations, Eulerian based continuum simulations, and Eulerian and Lagrangian based mesoscale methods. Discrete element method (DEM) approaches treat elements as individual
grains and use Hertzian based contact models as well as friction models to transfer energy between grains. This approach is best suited for low stress and strain rate testing due to the non-linearity of the contacts between grains. It has been used to simulate direct shear testing [45] and reproduce pressure-dependent yield models [36]. However, this approach cannot capture fracture or plastic yielding of grains.

Peridynamic simulations use inter-particle potential functions to propagate forces through a network of nodes. Mesoscale simulations for sand have been performed using Peridigm [72], a peridynamics code developed by Sandia National Laboratories, in which grains were modeled as a clustering of nodes. A major advantage of peridynamics simulations is that they can resolve grain contact as well as capture fracture [61]. However, the potential functions that are used to communicate between nodes are often difficult to stabilize and lack physical meaning compared to the constituent material they represent.

Continuum models using Eulerian hydrocodes such as CTH [63], also developed by Sandia National Laboratories, involve empirically fitting porosity models such as the P-α [49] and P-λ [40] to modify equations of state for porosity changes during compaction. Since they are empirical, they require a considerable about of experimental data before they can be fit. Additionally, they do not provide insight into the mesoscopic phenomena dictating the macroscopic response. However, continuum models are best suited for large scale simulations where it is too computationally expensive to explicitly resolve the mesoscale features of the system. Mesoscale simulations using Eulerian hydrocodes such as CTH resolve grain-level features of the system by explicitly including grains and pores into the computational domain. This allows mesoscopic features of the system to control the observed macroscopic response. Individual grains are created by designating a cluster of neighboring computational cells to have the same material properties. Grains are distinguished from one another by assigning different material numbers to neighboring grains, but simultaneously assigning them the same material properties. This approach, along with all hydrocode formulations, have the major benefit of using equations of state as well as constitutive models to incorporate strength. Therefore, mesoscale simulations can be constructed from a knowledge of the physical geometry, equation of state, and constitutive law for the fundamental constituent material. For mesoscale simulations on sand, this means that individual grains in the computational domain behave based on the thermo-mechanical properties of quartz, but the macroscopic behavior of the entire domain is indicative of the network of grains. This approached was developed to model the compaction of
copper powders [13, 14], and since then has been adapted to model the response of heterogeneous geologic and planetary materials [28], granular tungsten carbide [16, 98], composites of polytetrafluoroethylene (PTFE), tungsten (W) and aluminum (Al) powders [47], quartz sandstone in dry and saturated conditions [34], and quartz sand in dry [15, 81], and saturated conditions [17, 60, 82]. An interesting upside to the mesoscale approach is that the macroscopic response can be constructed from a sum of thermo-mechanical states associated with individual grains [13]. Statistical methods can then be utilized to probe the internal mechanisms not readily available in experiments. Relationships between thermodynamic and mechanical state space can then be established, which provides a better understanding of experimentally observed physical phenomena.
CHAPTER 3

THEORY

3.1 Pressure-Shear Theory

A schematic of the pressure-shear setup is shown in figure 3.1. The projectile body is guided down the barrel by the projectile keys. Attached to the face of the projectile body is a nose piece machined to a specific angle of obliquity defined by \( \theta \). The target face is aligned to match the projectile nose piece ensuring planar impact across the surface of the driver plate (front anvil). Planar impact is of utmost importance because any misalignment can cause the slippage at the interface between the nose piece and the front anvil. To ensure full transmission of a shear wave into the target capsule, no-slip at the projectile-anvil interface must be achieved. No-slip is achieved by limiting the angle of impact to satisfy a no-slip condition explained later, as well as placing emphasis on minimizing impact tilt, i.e. misalignment of projectile and target face. Four piezoelectric pins (PZT pins) are placed around the circumference of the target to quantify impact tilt for each shot. Upon impact, components of particle velocity are transmitted into the sample as:

\[
\begin{align*}
    u_0 &= V \cos(\theta) \\
    v_0 &= V \sin(\theta)
\end{align*}
\]

where \( u_0 \) is the longitudinal (normal) velocity, \( v_0 \) is the transverse (shear) velocity, and \( V \) is the initial projectile velocity. Figure 3.2 presents an ideal position-time diagram for the pressure-shear schematic shown in figure 3.1. The longitudinal wave, travelling at a velocity of \( c_1 \), transmitted into the driver plate reaches the sample first and reverberates to a final state, \( \sigma_{max} \) in figure 3.3.
before the shear wave, traveling at a velocity of $c_2$, arrives. Achievement of a final normal stress state is important to ensure no-slip between the anvil and sample as well as to simplify the conditions in which the sample is loaded. When the shear wave reaches the sample, it also reverberates within the sample due to the sample being confined between two high-impedance anvils. The projectile, front anvil, and rear anvil must remain elastic to enable the use of linear stress-particle velocity relationships for impedance matching. This requirement allows both stress and deformation states within the thin sample to be easily inferred from particle velocity measurements off the free surface of the rear anvil.

The longitudinal and transverse waves reverberating within the sample transmit normal and shear stress waves into the rear anvil, which can be observed at the free surface of the rear anvil. The length of a test is designated by the shear window, which corresponds to the time between arrival of transverse waves at the rear surface of the anvil and release of the normal stress compressing the sample longitudinally. Once the initial longitudinal wave reaches the free surface of the rear anvil, the stress wave releases back toward the rear surface of the thin sample, and partially releases the normal stress on the sample. Without normal stress, shear stresses are less likely to be transmitted within the sample and the experiment is over. Figure 3.3 describes the impedance matching used to characterize the normal and shear stresses within the sample and both anvils. Using one-dimensional elastic wave propagation theory for both longitudinal and
transverse waves, all states at the front anvil - sample interface follow:

\[ \sigma = (\rho c_1)_A (u - u_0) \]  
\[ \tau = (\rho c_2)_A (v - v_0) \]

Likewise, states at the sample - rear anvil interface follow:

\[ \sigma = (\rho c_1)_A u \]  
\[ \tau = (\rho c_2)_A v. \]

where \( \rho \) is the mass density, \( c_1 \) and \( c_2 \) are the p- and s-wave speeds, \((\rho c_1)_A\) and \((\rho c_2)_A\) are the longitudinal and shear impedances of the anvil material, and \( u_0 \) and \( v_0 \) are the components of initial velocity along the longitudinal and transverse directions, respectively. Using equations 3.3 and 3.5, subsequent longitudinal particle velocity and normal stress states in the sample are defined as:

\[ u_{F,i} = \frac{Z_{S1}u_{B,i-1} + Z_{S1}u_0 - \sigma_{B,i-1}}{Z_{A1} + Z_{S1}} \]  
\[ \sigma_{F,i} = -Z_{A1}(u_{F,i} - u_0) \]  
\[ u_{B,i} = \frac{Z_{S1}u_{F,i} + \sigma_{F,i}}{Z_{A1} + Z_{S1}} \]  
\[ \sigma_{B,i} = Z_{A1}u_{B,i} \]
where $Z_{A1} = (\rho c_1)_A$ is the longitudinal anvil impedance, $Z_{S1} = (\rho c_1)_S$ is the longitudinal sample impedance, $u_{F,i}$ and $u_{B,i}$ are particle velocity states at the front and back of the sample, $\sigma_{F,i}$ and $\sigma_{B,i}$ are normal stress states at the front and back of the sample. Each state is denoted with subscript $i$ and the series of states is initialized with $\sigma_{F,0} = \sigma_{B,0} = 0$ and $u_{F,0} = u_{B,0} = 0$. Shear stress states at the front and rear sample interface can be calculated following equations 3.7 - 3.10 by replacing the normal velocity components, $u_i$, with transverse velocity components, $v_i$, and the longitudinal impedances, $Z_1$, with shear impedances, $Z_{A2} = (\rho c_2)_A$ and $Z_{S2} = (\rho c_2)_S$, which results in:

\[
v_{F,i} = \frac{Z_{S2}u_{B,i-1} + Z_{S2}u_0 - \sigma_{B,i-1}}{Z_{A2} + Z_{S2}} \tag{3.11}
\]

\[
\tau_{F,i} = -Z_{A2}(v_{F,i} - v_0) \tag{3.12}
\]

\[
v_{B,i} = \frac{Z_{S2}v_{F,i} + \tau_{F,i}}{Z_{A2} + Z_{S2}} \tag{3.13}
\]

\[
\tau_{B,i} = Z_{A2}v_{B,i}. \tag{3.14}
\]

Figure 3.3 depicts a series of normal and shear stress states at the front and rear sample interface, $(u, \sigma)_{F,i}$, $(u, \sigma)_{B,i}$, $(v, \tau)_{F,i}$, and $(v, \tau)_{B,i}$. In the case that the sample begins to yield and plastically flow, the shear stress is approximately equal at the front and rear sample surface, but a velocity difference exists between the front at rear sample surface. This velocity difference directly corresponds to the plastic flow, i.e. shear strain rate and shear strain, for a given shear stress.

It is important to note that equations 3.7 - 3.14 assume the anvils and sample remain elastic and the sample impedance does not change with increasing normal or shear stress. Depending on the type of sample material, these properties can vary greatly. Typically, the strength and impedance of homogeneous metals does not change significantly for loading below the elastic limit. However, for heterogeneous, granular materials, the elastic limit and impedance can vary directly with the slightest compressive loading. Appendix B.1 details work performed at Marquette University aimed at measuring the longitudinal impedance of sand for a variety of quasi-static compressive loadings. The work demonstrated a significant increase in longitudinal impedance for relatively small increases in normal stress. The dependence of shear impedance on compressive loading has yet to be characterized and could benefit future works similar to the present study greatly.

The next important thing to address is the assumption of 1D-planar wave propagation in equations 3.7 - 3.14. Wave fronts observed in dynamically loaded granular materials typically have a plethora of additional structure as compared to wave fronts observed in homogeneous
materials. This structure ranges from curvature of the shock front in the tangential direction to precursors in the axial direction. Ultimately, the validity of the 1D-planar wave assumption is assessed based on whether or not a steady wave has had time to form in the granular material. Since the samples in these experiments are thin and undergo reverberation loading, steady waves most likely do not have time (or space) to form between each reverberation. However, the upside to these experiments is that the sand sample is confined between two high-impedance anvils that buffer out most of the wave structure and provide an averaged, bulk response. Therefore, any waves that reach the free surface of the rear anvil are directly related to the average, instantaneous, stress state within the sand sample.

Normal (compressive) strain rate, $\dot{\epsilon}(t)$, is defined as the difference in normal velocity at the front, $u_F(t)$, and rear, $u_B(t)$, of the sample divided by the initial sample thickness, $h$:

$$\dot{\epsilon}(t) = \frac{u_F(t) - u_B(t)}{h}. \quad (3.15)$$

After a sufficient number of reverberations within a sample, a nominally homogeneous state of stress is achieved and the difference in stress between the front and rear anvil goes to zero, i.e. $\sigma_B \rightarrow \sigma_F$. This implies that the sample has run up to its final state, $\sigma_{\text{max}} = p$, where $p$ is the pressure in the sand. At this point, the difference in velocity between the front and rear anvil also goes to zero and

$$u_F(t) = u_0 - u_B(t) \quad (3.16)$$

which results in

$$\dot{\epsilon}(t) = \frac{u_0 - 2u_B(t)}{h} = \frac{u_0 - u_{fs}(t)}{h}. \quad (3.17)$$

Starting with an identical form of equation 3.15, shear strain rate is defined as

$$\dot{\gamma}(t) = \frac{v_F(t) - v_B(t)}{h}, \quad (3.18)$$

the difference in shear stress at the front and rear of the sample also goes to zero after a sufficient number of reverberations, i.e. $\tau_B \rightarrow \tau_F$. Likewise, the difference in velocity between the front and rear anvil goes to zero and

$$v_F(t) = v_0 - v_B(t) \quad (3.19)$$

which results in an expression for shear strain rate

$$\dot{\gamma}(t) = \frac{v_0 - 2v_B(t)}{h} = \frac{v_0 - v_{fs}(t)}{h}. \quad (3.20)$$
The nominal normal and shear strains are then obtained via time integration as

\[ \epsilon(t) = \int_0^t \dot{\epsilon}(t) dt \]  

(3.21)

and

\[ \gamma(t) = \int_0^t \dot{\gamma}(t) dt, \]  

(3.22)

respectively. The normal and shear stress, as a function of time, experienced by the sample

\[ \sigma(t) = \frac{1}{2} (\rho c_1 A) u_{fs}(t) \]  

(3.23)

and

\[ \tau(t) = \frac{1}{2} (\rho c_2 A) v_{fs}(t) \]  

(3.24)

is then measured using the longitudinal, \( u_{fs} \), and transverse, \( v_{fs} \), particle velocity components observed from the free surface of the rear anvil.

### 3.2 Strength Models for Granular Materials

When characterizing the response of granular materials, a convenient way to express strength is by comparing shear stress and normal stress. This expression states that the shear flow stress of a granular material is related to the normal stress compressing the material. Therefore, the internal friction coefficient can be defined as

\[ \mu = \tan \beta = \frac{\tau_{flow}}{\sigma} \]  

(3.25)

where \( \beta \) is the internal friction angle, \( \tau_{flow} \) is the shear flow stress, and \( \sigma \) is the normal stress. This relationship can then be related to yield surfaces for granular materials such as the Mohr-Coulomb failure model, Drucker-Prager Failure model and Drucker-Prager Cap model. The Drucker-Prager failure model was made famous for capturing the pressure-dependent yielding that granular materials exhibit. Figure 3.4 compares failure criterion for granular pressure-dependent materials, i.e. Drucker-Prager and Mohr-Coulomb, and homogeneous non-pressure-dependent materials, i.e. Von Mises and Tresca. A convenient way to express the three-dimensionality of principal stress space is in terms of effective Von Mises stress

\[ q = \sqrt{3J_2} = \sqrt{\frac{3}{2} s : s} = \sqrt{\frac{1}{2} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \]  

(3.26)
Figure 3.4: Yield surfaces for (a) Drucker-Prager and Von Mises and (b) Mohr-Coulomb and Tresca.

and mean stress, e.g. pressure,

$$
\sigma_m = p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)
$$

(3.27)

where \( s \) is the deviatoric stress tensor, \( J_2 \) is the second invariant of \( s \), and \( \sigma_1, \sigma_2, \sigma_3 \) are the principal stresses, \( \sigma_m \) is the mean stress, and \( p \) is the hydrostatic pressure.

A limitation of the Drucker-Prager and Mohr-Coulomb yield surface is the implication that strength, or straight line distance from the hydrostat to the yield surface, goes to infinity as the hydrostatic pressure is increased to infinity. However, as hydrostatic pressure increases, grains will begin to fracture and re-compact to high density states. Eventually the porosity is removed the strength of the originally porous, granular material will be based solely on the strength of the constituent material. In addition, the equation of state will begin to dominate the response due to higher pressures producing higher temperatures, which ultimately leads to melt. Multiple works [43, 44], have described pressure dependent yield surfaces that capture the plastic behavior after a certain threshold of hydrostatic compression. Figure 3.5, depicts a modified Drucker-Prager Cap model in \( q - p \) space. The surface \( F_s \) represents the pressure-dependent shear failure of the material and the cap surface, \( F_c \), represents the plastic failure due to hydrostatic compression. The transition surface \( F_t \) is a means of ensuring numerical differentiability between the two regions for computational implementation. The main motivation for presenting these cap models is to formulate hypotheses about the response observed when the sand is subjected to rapid compression and shear loading. They are promising for dynamic behavior of granular materials because of their inclusion of constituent material failure, which has been observed in
Figure 3.5: Drucker-Prager Cap Model with shear failure surface, $F_S$, transition surface, $F_T$, cap surface, $F_C$, cohesion, $c$, and shearing resistance, $\mu$.

pressure-shear experiments on granular tungsten carbide [98].
CHAPTER 4

EXPERIMENTAL METHODOLOGY

4.1 Design Considerations

4.1.1 Anvil Material Selection

One of the first things to consider when designing pressure-shear experiments is the choice of anvil material based on what types of loading conditions are desired for the sample. From the requirement that the anvils must remain elastic, a combined loading limit has been derived using a von Mises yield condition to determine possible combinations of skew angle and projectile velocity [92]

$$\left(\frac{1 - 2\nu_A}{1 - \nu_A}\right)^2 \left(\frac{\rho c_1 A}{2\sqrt{3}}\right)^2 \cos^2\theta + \left(\frac{\rho c_2 A}{2}\right)^2 \sin^2\theta\right] V^2 < \kappa_A^2$$ \hspace{1cm} (4.1)

where \(\nu_A\) is the Poisson ratio of the anvil, \(\theta\) is the skew angle, \(V\) is the initial flyer velocity, and \(\kappa_A^2\) is the yield strength in shear of the anvil. Along with yield criterion for the anvil material, a no-slip condition must be enforced to ensure the transmission of shear waves into the anvils and subsequently the target [92]. This no-slip condition is based on the skew angle, longitudinal and transverse sound speeds of the anvil, and the coefficient of friction, \(\eta\),

$$\tan\theta < \frac{c_1 A}{c_2 A} \eta$$ \hspace{1cm} (4.2)

An ideal anvil material is one that has a high yield strength, but also a reasonable ratio of longitudinal to transverse wave speed. If the transverse sound speed is too close to the longitudinal sound speed, the longitudinal wave will not have enough time to reach a nominal compressive stress state in the sample before the shear wave arrives. Conversely, if the transverse sound speed is much smaller than the longitudinal wave speed, the transverse waves will not have enough time to reach the free surface of the rear anvil before the longitudinal waves eventually release the compressive stress. Table 4.1, lists typical materials used in previous pressure-shear studies. Using the values listed in table 4.1, the yield criterion and no-slip condition can be combined to visualize potential combinations of initial velocity and skew angle for the various anvil materials, figure 4.1 shows the resulting plot.

\(\text{ZrO}_2\) is not pictured because its friction coefficient was unknown. However, its drastically higher yield strength requires velocities above 1000 m/s to yield the material. Therefore, for most
Table 4.1: Properties of Various Anvil Materials

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm$^3$)</th>
<th>P-wave Speed (km/s)</th>
<th>S-wave Speed (km/s)</th>
<th>Poisson Ratio</th>
<th>Yield Strength (GPa)</th>
<th>Friction Coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti-6Al-4V [98]</td>
<td>4.415</td>
<td>6.120</td>
<td>3.170</td>
<td>0.317</td>
<td>1.280</td>
<td>0.36</td>
</tr>
<tr>
<td>ZrO$_2$ [98]</td>
<td>6.082</td>
<td>7.100</td>
<td>3.670</td>
<td>0.318</td>
<td>9.400</td>
<td>N/A</td>
</tr>
<tr>
<td>Hampton Tool Steel [89]</td>
<td>7.612</td>
<td>5.893</td>
<td>3.624</td>
<td>0.270</td>
<td>1.600</td>
<td>0.74</td>
</tr>
<tr>
<td>Tungsten Carbide [89]</td>
<td>14.600</td>
<td>6.630</td>
<td>4.030</td>
<td>0.220</td>
<td>2.063</td>
<td>0.25</td>
</tr>
<tr>
<td>7075-T6 Aluminum [89]</td>
<td>2.800</td>
<td>6.230</td>
<td>3.100</td>
<td>0.320</td>
<td>0.275</td>
<td>0.40</td>
</tr>
<tr>
<td>1045 Carbon Steel</td>
<td>7.750</td>
<td>5.730</td>
<td>2.700</td>
<td>0.290</td>
<td>0.531</td>
<td>0.75</td>
</tr>
</tbody>
</table>

Figure 4.1: Combined criteria for no-slip and yielding of the anvil materials

Applications, the main concern when using ZrO$_2$ is slip. Above the horizontal line, the theoretical no-slip condition breaks down and to the right of the curved line, the anvil material begins to fail the yield criterion. Experimentally, ensuring planar impact, i.e. minimal tilt, for skew angles above 30 degrees becomes difficult. However, one could theoretically use aluminum for its high coefficient of friction to exploit the regions of low velocity and high skew angle to impart large amounts of shear (relative to normal) stress into a sample. The downside to this is a decreased shear strain rate due to the decreased flyer velocity and sound speed of aluminum.

Another important factor to consider when choosing an anvil material is the theoretical maximum combined loading state for a given material. Impedance matching equations 3.23 and 3.24 and assuming a symmetric impact and no slip, the maximum normal and shear stress for
a given anvil material is

\[ \sigma_{\text{max}} = \frac{1}{2} (\rho c_1) A V_0 \cos \theta \]  \hspace{1cm} (4.3)

and

\[ \tau_{\text{max}} = \frac{1}{2} (\rho c_2) A V_0 \sin \theta . \] \hspace{1cm} (4.4)

From this, the maximum state achievable in the sample after it has reach both pressure and shear equilibrium can be compared for different materials. Figure 4.2 compares the results for 1045 steel, 7075-T6 aluminum, tungsten carbide, and Ti 6Al 4V. Each normal-shear stress combination is computed with a range of initial velocities from 40 m/s to their maximum initial impact velocity from the Von Mises yield criterion, equation 4.1. Ti 6Al 4V appears to cover a wide range of normal-shear stress, but was not feasible with in-house machining capabilities. A combination of 7075 T6 aluminum and 1045 steel covered a considerable range of normal-shear stress based on their initial velocities of roughly 40 - 140 m/s and 40 - 90 m/s for 7075 T6 aluminum and 1045 steel, respectively. Both of these materials were cost-effective and machinable in-house. Therefore, they were selected as anvil materials for the pressure-shear experiments.

Figure 4.2: Maximum normal and shear stress for a given anvil material and impact angle. The lower limit for each was chosen to be 40 m/s based on feasibility when using gas guns and the upper limit was calculated from the Von Mises yield criterion, equation 4.1.
4.1.2 Timing of Longitudinal and Transverse Waves

As discussed in section 3.1, the timing of longitudinal and shear waves in pressure-shear experiments is crucial to achieving a final combined loading state that is characterizable. This timing is controlled by the thickness and corresponding p- and s-wave speeds of the anvils and sample material. The front anvil (driver) thickness should be thick enough such that the longitudinal wave has a chance to reach the sample and reverberate between the front and rear anvil before the transverse wave arrives. In homogeneous materials, the point at which this equilibrium is achieved is less ambiguous than in heterogeneous, granular materials. Granular materials have a tendency to compact and greatly change their density under the slightest load. As the density of the samples increases, so does the sound speed, which makes predicting the time to achieve longitudinal stress equilibrium difficult. Therefore, the current work sought to make reasonable assumptions about the time required to achieve normal stress equilibrium in the sand sample based on elastic p- and s- wave speed measurements described in section B.1.

As mentioned in the introduction section, the stress limit at which grains begins to break represents a significant increase in complexity when characterizing the response of sand, or any brittle granular material. The sound speed (p- or s-wave) in this intermediate range is still a major question due to the fact that the material is no longer elastic nor completely compacted, as observed in strong shock loading from uniaxial flyer plate experiments. In reality, there exists an elastic portion until grains begin to break and then an either partial- or full-compaction regime as the grain fragments begin to re-compact. The desired normal stress regime for the current experiments was less than the point of full compaction, approximately 4 GPa, observed in strong shock loading experiments [60]. Therefore, an understanding of the elastic and “plastic” p-wave speed was important for target design.

The only current pressure-shear experiments on sand by Vogler et al [97] presented four experimental particle velocity profiles for 250 µm thick sand samples with grains diameters on the order of 50 – 90 µm. The rise time for normal velocity was anywhere from 1.5 to 3.0 µs. Based on the p-wave speed of the 1045 steel and aluminum, the desired front anvil thickness can be calculated such that the shear wave arrives just after normal equilibrium is achieved in the sample.
using

\[ x = \frac{t_{eq}C_S C_L}{C_L - C_S}. \]  

(4.5)

(4.6)

Using an equilibrium time of \( t_{eq} = 3.0 \mu s \) the p- and s-wave speeds for 1045 steel and aluminum, the front anvil thickness needs to be \( x = 17.4 \) mm for aluminum and \( x = 16.8 \) mm for steel. However, in any flyer-plate impact experiment, uniaxial or pressure-shear, radial release waves are a major concern. The thickness of the target ensemble needs to be such that radial release waves do not have a chance to relieve the axial stress imparted on the target before the shear wave has a chance to reach the rear anvil free surface. Considering the rear anvil should be thicker than the driver to allow the longitudinal and shear waves to reach the sample, the target capsule would need to be roughly 40 mm thick. Since the Marquette University Shock Physics Laboratory (MUSPL) slotted barrel is 50 mm in diameter, the target diameter needs to be slightly less than that; ensuring a plane wave is imparted into the sample. Therefore, the impact domain on the target would be approximately 40x40 mm, almost guaranteeing radial release would occur before even the first longitudinal wave reached the rear surface of the target.

Using the same Oklahoma #1 sand test in the dry and water-saturated shock experiments by LaJeunesse et al [60] was a pillar of the current work. As discussed in section 4.2.1, the grain diameter distribution was 63 - 120 \( \mu m \). The minimum number of grain diameters across the axial thickness was approximately 5, which resulted in an axial thickness of roughly 0.4 - 0.5 mm. As section B.1 explains, the p-wave speed within the sand was roughly 0.580 mm/\( \mu s \). Since the anvil thickness required to achieve full longitudinal equilibrium before arrival of the shear wave did not fit within the design constraints, a front anvil thickness was chosen such that the sand was able to fully load to at least the first stress state predicted from impedance matching. With a sample thickness of 0.5 mm, it would take approximately 0.86 \( \mu s \) for the sand to reach the first loading state. Therefore, using equation 4.5, a driver (front anvil) thickness of 4 mm was chosen.

The rear anvil thickness was selected with consideration to reduce the effect of radial release as well as size restrictions for the target apparatus. A rear anvil thickness of 4 mm, 6 mm, and 8 mm was selected for experiments 1 - 6, 7 - 10, and 11 - 12, respectively. The rear anvil is the main limiting factor (aside from radial release) for the maximum normal stress achieved in the sample. Once the longitudinal wave has released off the free surface of the rear anvil and returned to the sample, the normal stress will decrease, i.e. a drop in confinement pressure. Fortunately,
both anvils remain elastic, which makes tracking wave propagation within them a reliable feature for which to make conclusions about the sample loading state. Since the longitudinal release states can be tracked at the free surface of the anvil, multiple shear measurements can be made within a single experiment. This represents a potentially exciting result. Many complicated phenomena involve the creation and then reloading of rubble. Therefore, modifications could be made to these experiments to explore this phenomena.

Figure 4.3 shows a position-time (XT) plot for each of the combinations of front and rear anvil thicknesses used. Each plot is time-shifted such that time zero is the point at which the first longitudinal wave reaches the free surface of the target. The flyer-driver impact site is designated x = 0 mm and the flyer is not included in the plots to enable wave interaction in the sand to be more easily visualized. Longitudinal (normal) waves are shown in blue and transverse (shear) waves are shown in red. The longitudinal wave can be seen reaching the sand first and reaching the sand-rear anvil interface slightly before the arrival of the shear wave at the sand. Once the longitudinal wave has begun to propagate into the rear anvil, the first normal stress state, $\sigma_1$, is achieved. It then propagates through the rear anvil and reflects off the free surface, $F_{N1}$, back towards the rear surface of the sand. During that time, the longitudinal wave reverberates within the sand until the first longitudinal wave comes from the free surface of the rear anvil, blue dashed line, and changes the normal stress state to $\sigma_2$. This longitudinal wave propagates back to the free surface, then back to the sand interface and the normal stress is again changed to $\sigma_3$.
In reality, a steady wave will not have enough time or distance to form within the sand before it reaches the sand-anvil interface due to there only being 5 - 8 grains across the axial thickness of the sample. The idealized reverberation depicted with impedance matching, figure 3.3, will most-likely not have distinct loading states, but a gradual ramp-like loading profile. Therefore, the instantaneous velocity measured from the back surface of the anvil, is directly related to the (average) stress in the sample. The same applies for the ramp-loading of the shear wave within the sand. Any transverse velocity that reaches the free surface of the anvil will be directly related to the shear loading on the sand. The different states recorded throughout the process will be based upon the timing of the longitudinal release waves that reflect in the anvil, \( R_{N1}, R_{N2}, R_{T1}, \) and \( R_{T2} \), where

\[
\begin{align*}
  t_{R_{N1}} - t_{F_{N1}} &= t_{R_{N1}} - 0 = \frac{x_{\text{arr}}}{C_L} \quad (4.7) \\
  t_{R_{N2}} &= 2t_{R_{N1}} \quad (4.8) \\
  t_{R_{T1}} &= x_{\text{arr}} \left( \frac{1}{C_L} + \frac{1}{C_S} \right) \quad (4.9) \\
  t_{R_{T2}} &= t_{R_{T1}} + x_{\text{arr}} \left( \frac{1}{C_L} + \frac{1}{C_S} \right) \quad (4.10)
\end{align*}
\]

It should be noted that the sand p- and s-wave speeds used to calculate the position time plots were the zero-stress elastic wave speeds described in section B.1. In actuality, the longitudinal and shear waves will propagate faster relative to the zero-stress during loading. Therefore, figure 4.3 merely serves as an estimate of arrival times for the waves at different locations and helps guide the determination of the final combined loading state.

4.2 Experimental Setup

The target schematic in section 3.1 provides the basic layout of the projectile-target assembly. Two high-impedance, high-strength anvil materials confined the sand sample in the longitudinal direction. A thin, rubber gasket was laser-cut and placed between the anvils to provide a gap for the sand to occupy. The anvils were held together using either 6 or 8 bolts near the perimeter of anvil. Bolt holes on the up-range face of the front anvil (driver) were countersunk so the samples could be bolted together, filled with sand, and then laid onto a flat surface for further measurements. PMMA rings were laser cut from a 0.33” sheet and fixed to the free surface of the rear anvil to provide a mount point for the gimbal as well as standoff distance for the PDV collimators. The thickness of these rings were specifically chosen based on the geometry of the
PDV collimator arrangement. A 0.25” hole was drilled into each sidewall of the PMMA ring, which allowed the capsule to be connected to a larger steel ring by a 0.25” diameter nylon shoulder bolt. Lastly, a disk was laser cut from a PMMA sheet and fixed to the rear surface of the PMMA ring to hold the PDV collimators. The fabrication process for these disks, or PDV bridges, is further discussed in section 4.3.1. Figure 4.4 shows the front anvil, gasket, rear anvil, and PMMA gimbal ring / PDV bridge combination.

![Figure 4.4: Blown up view of target pieces](image)

Figure 4.4: Blown up view of target pieces

![Figure 4.5: (a) Pre- and (b) post-assembled target pieces for shot 2.](image)

Figure 4.5: (a) Pre- and (b) post-assembled target pieces for shot 2.
4.2.1 Target Preparation

U.S. Silica Oklahoma #1 sand was chosen for these experiments, the same used for uniaxial shock loading described in LaJeunesse et al [60]. A pillar of this work was to contribute to a better overall understanding of a particular type of sand. The previous work aimed to characterize the Hugoniot response of the sand from 1 - 11 GPa, while the current work set out to understand its strength properties using combined pressure-shear loading, below the point of full-compaction near 4 GPa. The sand consisted of 99.8% wt. SiO$_2$ and had smooth, quasi-spherical grain shapes. Figure 4.6 shows a scanning electron microscope image of the sand. The sand was washed, baked, and sieved to grain diameters of 63 - 120 µm, corresponding to the “fine” sand from LaJeunesse et al [60]. Final packed density of the sand was determined to be $\rho_0 = 1.758 \pm 0.013$ g/cm$^3$, approximately 64 - 69% theoretical max density (TMD), based on the amount of mass poured into the capsule and volume of the capsule. Shots 3 - 6 used a slot design in the rear anvil to enable normal and transverse velocity measurements off the rear-surface of the front anvil, which modified the sand volume slightly from a right-cylinder. Appendix B.3 outlines the density calculation for both circle and slot modified capsule areas.

The steel and aluminum anvils were prepared by cutting wafers from a cylindrical rod of stock material, milling to a specific thickness, drilling the necessary bolt and diagnostic holes, and then either grinding or wet sanding for a final finish. Steel anvils were ground using an in-house diamond grinding wheel. Aluminum samples were wet sanded with a final paper grit of 1200.

Figure 4.6: SEM image of silica sand used for the pressure-shear plate impact experiments and Hugoniot experiments performed by Georgia Tech and Harvard.
Aluminum samples were not ground as a result of their non-ferrous nature and the fact that aluminum has a tendency to melt and coat diamond grinding wheels. Surface roughness measurements were then made using a Pocket Surf III profilometer. Both the ground steel and wet sanded aluminum had a starting surface roughness of approximately $R_a = 0.10 - 0.20 \mu m$. The face of each projectile nose-piece, inner anvil surface, and outer anvil surface were then directionally sanded perpendicular to the direction of shear with 200 or 300 grit sand paper to increase the traction at interfaces as well as to provide diffuse light return for the transverse PDV measurements. Surface roughnesses for each anvil and projectile nose piece were measured parallel to the direction of shear to be $R_a = 0.40 - 0.70 \mu m$. A table of all measured surface roughnesses can be found in appendix B.2. Once the surface roughness was characterized for each anvil, p- and s-wave speeds (longitudinal and transverse, respectively) were measured using a traditional pulse receiver technique. Appendix B.1 describes these measurements in detail and provides the sound speeds for each anvil. The average values for longitudinal and transverse wave speeds were $C_L = 5.757 \pm 0.049 \text{ km/s}$ and $C_S = 2.922 \pm 0.130 \text{ km/s}$ for 1045 steel and $C_L = 6.165 \pm 0.106 \text{ km/s}$ and $C_S = 2.985 \pm 0.051 \text{ km/s}$ for aluminum 7075-T6.

The gasket material selected to confine the sand between the anvils was an Oil-Resistant, High-Strength Aramid Fiber/Buna-N Rubber Blend from McMaster-Carr, with a thickness of $\frac{1}{64}$”. This material was chosen for its rubber-like finish, combination of pliability/rigidity, and ability to be cut in-house with a laser cutter. Once laser cut, the gasket was super glued into place on the rear anvil. A 0.125” fill hole was drilled into the capsule edge by installing half of the bolts and fixing the capsule in a vice. The fill hole extended to the inner-edge of the gasket to reduce the effect on the sand area. Sand was then poured into the capsule through the fill hole using a small funnel. A small amount of sand was poured into the capsule and then the entire capsule was vibrated using a sieving platform. This process was repeated until the capsule was full and the amount of mass added was in agreement with the calculated sand mass desired. The final, axial thickness of the sand domain was on the order of 0.4 – 0.5 mm. A steel dowel pin was then press fit into the fill hole and the remaining material was ground. If the capsule was slightly overfilled, the sand would occupy the fill hole. The amount of overfill was calculated using the fill hole depth and dowel pin length. The mass of sand in the capsule area was then corrected. A practice target capsule was created with a steel front anvil and PMMA rear window to test the filling methodology. The PMMA window enabled the sand to be visualized inside the capsule during the packing process. Figure 4.7 shows the practice capsule with steel rear anvil, PMMA front
anvil, gasket, and fill hole. Once the sand was packed into the capsule and sealed a dowel pin, additional sound speed measurements were attempted to get the p- and s-wave speed of the sand. Table B.8 in appendix B presents an average longitudinal wave speed of 
\[ C_L = 0.555 \pm 0.018 \text{ km/s} \]
and a transverse wave speed of 
\[ C_S = 0.202 \pm 0.004 \text{ km/s}. \]

![Figure 4.7: (a) Pre- and (b) during and (c) post-filled target capsule with test PMMA anvil for practice packing.](image)

### 4.2.2 Projectile

The projectile used in each experiment consisted of a polycarbonate sabot and either a 1045 steel or aluminum 7075 T6 nose piece. Each sabot was machined to a length of 4.75” and outer diameter of 1.975”. A 0.4 x 1.76” diameter, flat cup was lathed into the face of each sabot so the projectile nose piece could be attached. The rear face of the sabot was bored to create a 1.5 x 1.5” cavity. This was particularly useful because steel slugs of varying sizes could be placed into the cavity to change the overall mass of the projectile and ultimately the projectile velocity. Two o-ring channels span the perimeter of the cylinder wall, which house Buna-N o-rings that provide contact points with the barrel. A 0.3 x 1.5” slot was milled into the top surface of the sabot and a brass key was press-fit into place. A tapered end mill was used to create a triangular fin in the brass key to match the angled slot in the barrel. The projectile nose pieces were machined from either 1045 steel or aluminum 7076 T6; identical to the corresponding anvil material. Each nose piece was cut from a cylindrical stock, lathed to a diameter of 1.970” and a smaller diameter of 1.760” for the plug section that mated with the sabot. Once the outer diameters were established,
each nose piece was placed into a sine vise at 17° and milled/faced. Steel nose pieces were then transferred, while still locked in the sine vise, to a diamond grinder and the face was ground to a finish consistent with that of the steel anvils. Aluminum nose pieces were wet/dry sanded using the same procedure as the corresponding aluminum anvils. A minimum of 0.4” of cylinder was left behind the inclined face of the nose piece to provide adequate time before release from the projectile rear surface.

4.2.3 Target Mount and Alignment

The target mounting mechanism consisted of a one-dimensional gimbal fixed inside the target tank and allowed the capsule to rotate within the mount ring and match the inclination angle of the projectile face. A steel outer and PMMA inner ring, contained a 0.25” hole that housed a partially threaded, nylon shoulder screw. Figure 4.9 shows a front view of the target impact face, steel mount ring, and nylon shoulder screw. Figure 4.10 shows a rear view of a target fixed to the steel mount ring. The nylon shoulder bolt can be seen protruding into the PMMA inner ring, where a nut was used to fasten the shoulder screw in place. A Buna-N o-ring with inner diameter of 0.25” was placed between the inner PMMA and outer steel ring to provide spacing and resistance when rotating the target in the steel mount. The nylon shoulder bolt served as primary break point during each shot. Therefore, after the projectile-target impact, the nylon bolts failed, which allowed the target and projectile to pass through steel mount plate and ring. This was convenient because the steel mount plate could be reused. Additionally, the PMMA ring
detached from the anvil-sand target capsule leaving the capsule intact.

Each target was first mounted into the steel ring and then fixed to the target plate, figure 4.11. A rubber-coated steel wire was fixed to the rear bored section of the sabot via a steel eye hook, figure 4.8. The projectile was then pushed to the end of the barrel where a mechanical mate was made between the projectile face and target face. A depth measurement was made from the rear target surface to the projectile face through the three or four alignment holes in the target. Once agreement between each of the depth measurements was found, the target was rotationally locked into place using four set screws mounted to the steel ring via PMMA arms, figure 4.11. The projectile was then retrieved from the barrel and the alignment was completed. Each of the PDV collimators and PZT pins were then connected to their corresponding patch cables, which exited the target tank through CONAX vacuum sealed feedthroughs. A cylindrical blast shield in the target tank was used to protect the fiber optic and BNC patch cables. Figure 4.12 shows the inside of the target tank with patch cables and blast shield in place prior to closing the target-catch tank.

Figure 4.9: Steel mount ring with target installed and rotated to different angles.
Figure 4.10: (a) Rear view of target mounted to steel ring with PDV collimators and PZT pins installed. (b) Target ready to be mounted into target tank with fiber optic and BNC cables.

Figure 4.11: Target assembly fixed to mount plate inside of target tank.
Figure 4.12: (a) Rear view of target mounted inside the target tank, (b) target tank with blast shield in place, diagnostic feed-throughs showing patch cable bulkheads, and (c) close-up view of blast shield protecting fiber optic and BNC patch cables.
4.3 Diagnostic Setup

Eight holes were placed around the edge of the impact surface so as to be contacted by the outer-most edge of the projectile face. Piezoelectric timing pins (PZT pins) were placed in four of the holes and the other four holes were left blank to enable depth measurements from the free surface of the rear anvil to the projectile face during target alignment. For shots 2 - 6, a slot was milled into each of the rear anvils to allow a particle velocity measurement to be made off the rear surface of the front anvil. This slot can also be seen laser cut into the gasket. The PDV bridge contained either three or five holes to hold PDV collimators, depending on whether there was a slot in the rear anvil. Every shot had three collimators pointed at the rear surface of the rear anvil, one oriented normal to the anvil surface and two oriented at angles relative to the surface normal. Shots 2 - 12 used an additional barrel probe embedded in the rear anvil that measured the normal component of initial velocity through a through hole in the front anvil. Shots that had a slotted rear anvil aligned the remaining two collimators onto the rear surface of the front anvil via the slot in the rear anvil and gasket.

4.3.1 Velocimetry Measurements

Section 3.1 explained how the normal and shear stress in the sand are directly related to the longitudinal and transverse velocity measured from the free surface of the rear anvil. Therefore, the most important aspect of these experiments was making believable velocity measurements with minimal uncertainty. Photon Doppler Velocimetry (PDV) was used to make these measurements based on its growing popularity in the shock physics community, its relatively inexpensive, fiber-coupled components, and its ability to measure a wide range of particle velocities [86] with minimal light return. PDV systems measure surface velocity using the basic principles of a Michelson displacement interferometer. A monochromatic laser, i.e. single wavelength, is transmitted through fiber optic cable and emitted onto a reflective surface using a variety of collimators. The laser light reflects off the surface and is recollected either back onto the original collimator or onto a different collimator. This reflected light is then recombined with the original light. If the reflective surface is moving, i.e. displacement as a function of time, the frequency of the reflected light is Doppler shifted, relative to the frequency of the incident light, and a differential beat frequency is formed. This beat frequency, $f_b \approx 10^6 - 10^9$ Hz, can then be digitized using photo receivers since it has a frequency on the order of the difference between the
unshifted and shifted frequencies, \( f \approx 10^{15} \text{Hz} \) [53]. The apparent velocity, \( v^* \), of the reflective surface is then proportional to the differential beat frequency, \( f_b \),

\[
v^* = \frac{1}{2} f_b \lambda_0
\]

(4.11)

where \( \lambda_0 \) is the wavelength of the unshifted, monochromatic light source. If the surface is stationary, the beat frequency will be zero, and no velocity will be observed. This approach is referred to as Homodyne velocimetry. More advanced techniques combine an additional monochromatic laser, set at a different initial wavelength, \( \lambda_{ref} \), with the original monochromatic laser to provide a differential beat frequency even with no surface motion. This is referred to as Heterodyne velocimetry. Similar to equation 4.11, the apparent velocity is proportional to the difference in initial and shifted frequencies:

\[
v^* = \frac{1}{2} (f_b - f_0) \lambda_0
\]

(4.12)

where \( f_0 \) is now the differential beat frequency between the two monochromatic lasers and \( f_b \) is the beat frequency formed from the Doppler shifted light. Heterodyne techniques have a major advantage of supplying a beat frequency well above the frequency noise floor, i.e. \( f_0 \gg 0 \), which provides better resolution at lower velocities, i.e. lower frequency shifts, as compared to Homodyne systems. This idea leads to the methods in which time-dependent velocity is extracted from the spectral content.

Once the time-varying beat frequency is digitized by the photo receiver, the signal is recorded using a high-bandwidth oscilloscope (upwards of 4 GHz bandwidth). A variety of spectral analysis techniques are then employed to determine the time histories of frequency. The main software packages used to perform this analysis are PlotData [100] and SIRHEN [6]. The fundamental approach to any technique is to use a short-time Fourier transform (STFT) to determine variations in frequency over time. This is accomplished by breaking the original signal into smaller portions and performing an FFT of each section. The changes in spectral content between sections reveal time-varying frequency content that is then related to time varying velocity content.

The most traditional use of PDV or other velocimetry techniques is for measuring the normal component of surface velocity, i.e. the incident and reflected light travels parallel to the surface normal. However, recent works have investigated methods for capturing the transverse component of surface velocity as well as the normal component [23, 27, 31, 68, 104]. This requires
apparent velocity to be recorded from a collimator that has a non-zero angle relative to the surface normal. Arranging collimators such that they observe a component of normal and transverse velocity can be accomplished using a number of collimator arrangements, but most arrangements simplify to one of two approaches. First, a single collimator can emit and collect light at a non-zero angle relative to the surface normal or, second, multiple collimators can be used such that one collimator emits and collects and the other collimator only collects. Any collimator that emits light will be referred to as an “active probe” and any collimator that does not emit light and only collects light will be referred to as a “passive probe.” Figure 4.13 shows a general case light being transmitted on a send probe and collected on a receive probe. Apparent velocity observed on the receiving probe is expressed as a function of surface velocity along the components of incident, \( \hat{s} \), and reflected light, \( \hat{r} \),

\[
V^* = \vec{V} \cdot \left( \frac{\hat{r} - \hat{s}}{2} \right) = \frac{V_N}{2} (\cos \alpha + \cos \beta) + \frac{V_T}{2} (\sin \alpha + \sin \beta) + \frac{V_E}{2} (\sin \gamma + \sin \phi)
\]  

(4.13)

where \( \vec{V} \) is the velocity vector of the surface, \( V_N \) is the normal component of velocity, \( V_T \) is the transverse component of velocity, \( V_E \) is the out-of-plane component of velocity, \( \alpha \) is the angle between observed light and the surface normal, \( \beta \) is the angle between the incident light and the normal vector, and \( \gamma \) and \( \phi \) are angles of the out-of-plane components of velocity. Typically, the out-of-plane component of velocity, \( V_E \), is assumed to be zero if the impact tilt is minimal.

Shot 1 used a single normal, active probe and two angled, active probes at \( \beta = \alpha = +15^\circ, -20^\circ \) relative to surface normal. Shots 2 - 12 used a single active normal probe, \( \beta = 0 \), and two passive angled probes at \( \alpha = 20^\circ \) and \( \alpha = -20^\circ \). Figure 4.18 shows a schematic of the probe arrangement for shots 2 - 12 where both passive probes collected diffuse light from a
normal probe at ± 20° from the surface normal. These angles were chosen based on two main factors. First, relative error in the transverse velocity and second, light return on the passive, angled probes. Appendix C details the error propagation performed for a variety of probe configurations. Additionally, Johnson et al [53] explored the idea of light return as a function of angle of incidence for active angled probes collecting their own light. A similar process was used to determine that 20° provided adequate light return from the active normal probe to the passive angled probes. Section B.4 provides diagrams of optical components for the PDV system used for each shot.

PDV probes used for both normal and transverse free surface measurements were AC Photonics collimated 70 mm working distance, single fiber collimators. PDV probes used to measure the normal component of the projectile velocity, i.e. the “barrel probe”, was a 7 mm working distance, single mode fiber collimator. The 7 mm collimators were chosen for the barrel probes because of their price relative to the larger working distance collimators. Each PDV bridge was laser cut from a PMMA sheet. Probe locations on the bridge were calculated based on the thickness of the inner gimbal ring, i.e. the standoff distance between the anvil surface and collimator face. These holes were spotted/outlined with the laser cutter and then drilled with high-precision using a mill. Each bridge was fixed to a custom holder with the same bolt hole
pattern as the bridge and then fixed in 4” sine vise. The sine vise was propped up with the appropriate gage blocks and then each angled hole was drilled. To ensure the drill bit did not “walk” on the angled surface, an end mill was used first to create a small, flat surface that could be drilled with a regular drill bit. Figure 4.15 shows an example of an angled probe hole being drilled into a bridge. The bridge was then super glued to the inner PMMA gimbal ring and fixed to the target and PDV probes were installed.

Figure 4.15: Sine vise fixed on milling platform with custom mount plate for drilling angled collimator holes in PDV bridges.

Since velocity measurements are only possible if there is light return on each of the PDV probes, the probes were mounted to the bridge and all active probes were aligned using the 1550 nm PDV laser. First, the normal probe was inserted into the central hole and super glued into place once adequate light return of greater than -25 dBm was achieved. Next, a probe was placed into the +20° hole and connected directly to an inline digital Eigenlight power monitor. The angled probe was then maneuvered until it collected reasonable light return from the normal probe, roughly greater than -45 dBm. This level of light return proved to be adequate for passive
channels even without amplification. The passive, $-20^\circ$ probe was placed in the same fashion as its symmetric counterpart. As a double check, the probes were hooked up to the PDV system and reference light was added to the collected target light. A real-time FFT function was employed on the oscilloscope to ensure that a beat frequency had formed at the same spectral location for all active and passive probes. Once the light intensity and formation of a beat frequency were checked, the angles of the passive probes were measured.

Accurately measuring the angle of incidence for the fiber optic collimators (angled probes) was an important step in the target construction process. A table top apparatus was constructed using three translation stages fixed to an optical breadboard, figure 4.16. First, the probe bridge was attached to the PMMA gimbal ring and mounted to one of the translation stages such that the probes were aiming parallel to the table surface. The other two translation stages were fixed together to provide two-degrees of motion for a grid printed onto a sheet of PMMA with millimeter grid spacing. A red, test laser was connected to a 1x4 splitter to provide visible light for both the normal and angles probes. Initially, the grid was placed flush with the bottom side of the gimbal ring so the three, visible laser spots could be focused onto the same point on the grid. The grid was then translated away from the gimbal ring and the divergence of the angled probes was tracked every millimeter. Additionally, alignment of the normal probe was tracked by making sure its laser spot did not move as the grid was moved. The angle of incidence for the angled probes, and the associated uncertainty, was measured using a weighted linear fit, appendix A, of the x-y position data. Table B.4 in appendix B provides the collimator incidence angles measured for each shot.

![Figure 4.16: Translation stage used to measure the incidence angle of the angled probes for each PDV bridge.](image)
4.3.2 Other Diagnostics

A series of four ThorLabs laser diodes and optical collimators were installed in spring loaded mounts and aimed across the gap between the barrel and target face to act as a light gate for initial projectile velocity measurements. Each beam was approximately 0.094” in diameter and the beams were spaced approximately 0.75” apart for one another. Helminiak describes in more detail the specifics and construction of the light gate [46]. As the projectile exits the barrel, it begins to block the beams sequentially, which provides a series of voltage drops that are used to calculate initial velocity. Figure 4.17 provides an example of output signals from each diode/detector pair.

Time-of-Arrival transducers or piezoelectric timing pins (PZT pins) from Dynasen were used designate time or arrival, calculate impact tilt, as well as trigger the PDV system. Each pin is 0.064” in diameter and varies in length so they can be place in a range of location with minimal intrusion on the experiment. A 0.00005” layer of copper is vapor deposited on the impact face of each pin to cover a piezoelectric crystal either 0.040”, 0.010”, or 0.020” in diameter [1]. The mechanical strain on the face of the pin generated from impact causes the piezoelectric crystal to generate an output voltage of up to 100 Volts. The signal from multiple pins placed around the target face are sent to a pin mixer (Model CS2-50-300), which acts as a summing circuit for the multiple pin signals. The summing circuit combines the voltage from each input channel (pin) and provides a singular output voltage. This feature is useful for triggering subsequent oscilloscopes such as the PDV scope. Without the summing circuit, the oscilloscope would need to be triggered directly from a single pin. If that particular pin does not create a signal upon impact,
even if the rest of the pins do, the oscilloscope will not trigger and no data will be collected. With the summing circuit, any and/or all of the pins can be used as a trigger for an oscilloscope.

Figure 4.18: (a) Front face of target showing four PZT pins proud at the impact surface and (b) rear target surface showing epoxied PZT pins and connection points for pin cables.
Figure 4.19: Diagnostic tower showing (from top to bottom) 6 km delay leg, Dynasen pin mixer, digital display power monitors, fiber optic component drawer, Keopsys amplifiers, NKT Photonics target and reference laser, and power booster.
CHAPTER 5

SIMULATION METHODOLOGY

5.1 Mesoscale Simulations

Mesoscale simulations originated in the late 1990’s with works by Baer [11] and others. This approach incorporates heterogenous features of materials into the computational domain and aims to resolve their effect on the macroscopic response. Mesoscale is defined as the length scale between microscale and macroscale. Microscale directly resolves the crystalline or molecular structure of materials, while macroscale typically is reserved for large systems where homogenization is used to capture the behavior of collections of heterogeneous, or mesoscale, features. When performing mesoscale simulations, features such as sand grains, interface cracks, fiber ligaments, etc. are placed into the domain with considerations to the physically observed meso-structure. Each feature is given properties such as equations of state and constitutive models representative of the constituent material. Various types of loading can then be imparted onto the sample and the influence of heterogeneous features on wave propagation can then be observed at the mesoscale and macroscale in a non-intrusive fashion.

5.2 Hydrocode Background

The use of hydrocodes for simulating dynamic loading conditions and shock wave propagation has been prevalent since their creation. Their ability to capture massive deformations, phase transitions, elastic-plastic breakdowns, explosions, etc. make them a versatile tool. The name “hydrocode” implies that materials are treated “hydrodynamically,” or above the point at which they can support shear stresses. In most situations when a shock wave is formed, this treatment is sufficient. However, extensive effort has been placed into capturing material behavior below this limit using various constitutive, or strength, models. These models range from a linear elastic, perfectly plastic Von-Mises yield criterion, all the way to rate-dependent Johnson-Cook and viscoelastic models. The ability of these codes to capture material behavior in both elastic and plastic regimes make them a powerful tool.

CTH, a hydrocode developed and maintained by Sandia National Labs [63], was chosen for its robustness as well as being a natural extension of previous work on shock propagation in dry and water-saturated silica sand [59]. Groundwork by Borg and Vogler [16, 18] explored best
practices for performing mesoscale simulations on granular materials using hydrocodes such as CTH. Therefore, a solid framework existed for which to setup the pressure-shear simulations on sand. CTH is an arbitrary Lagrangian-Eulerian code that uses a finite volume mesh to solve the conservation equations for mass, momentum and energy:

\[
\begin{align*}
\text{Mass:} \quad \frac{\partial \rho}{\partial t} &= -\rho \nabla \cdot \mathbf{V} \\
\text{Momentum:} \quad \rho \frac{\partial \mathbf{V}}{\partial t} &= -\nabla P - \nabla \cdot \left[ \mathbf{\tilde{s}} + \mathbf{\tilde{Q}}(\mathbf{V}, c_s) \right] \\
\text{Energy:} \quad \rho \frac{\partial E}{\partial t} &= -P \nabla \cdot \mathbf{V} - \mathbf{\tilde{s}} \cdot \nabla \mathbf{V}
\end{align*}
\]

where \( \rho \) is density, \( t \) is time, \( \mathbf{V} \) is velocity, \( P \) is pressure, \( \mathbf{\tilde{s}} \) is the deviatoric stress tensor, \( E \) is energy, and \( \mathbf{\tilde{Q}} \) is an artificial viscosity term used as a numeric smoothing function. This artificial viscosity term is used to modify the inviscid forms of the conservation equations to allow for the treatment of near-discontinuous jumps in state variables observed across shock fronts [8]. Discretized versions of equations 5.1 - 5.3 are available in [101].

CTH operates with a continuum mechanics formulation that solves the conservation equations using a two-step Lagrangian-Eulerian process where the fixed, finite-volume mesh distorts with material motion and remaps back onto itself. Lagrangian formulations use a mesh that deforms with material motion whereas Eulerian formulations have a fixed mesh that advects mass through finites volumes, typically referred to as computational “cells.” Eulerian meshes are advantageous when simulating events with materials undergoing large deformations where as Lagrangian meshes tend to run into “mesh entanglement” problems. However, a major advantage of Lagrangian codes is the conservation of material shapes and interfaces. Finite element codes use a Lagrangian approach centered around computational nodes that form small “elements” on the surface, and within, materials of interest. These elements can be placed on the surface of objects, which helps track interfaces and surface deformation.

Eulerian codes cannot exactly resolve the surfaces of objects due to geometric overlap between objects and square cells. The ability of these codes to resolve features such as spheres is determined by the size of each computational cell and the resulting number of cells across the diameter of said sphere. Borg et al [16] found that 11 cells across the diameter of a sphere was adequate to resolve each grain in a given sand realization. Each grain then appears as a cluster of pure quartz cells surrounded by a spherical shell of partially filled quartz cells where the round surface of the sphere intersects the square mesh. The cells that intersect the surfaces of grains are
said to be partially filled and are treated differently compared to completely filled cells. First, the state variables in these partially filled cells are computed using volume weighted averages of each material present. Second, the strength within these cells can be designated using a few different options. The most common of which is volume weighted average calculated using the strength of each material present and the volume fraction of each material present. This allows grains to maintain their strength on their surface as well as at contact points with neighboring grains. The next common option is to set mixed cells to zero strength. This artificially weakens the grains by essentially coating the surface with a thin liquid layer that has no strength. Setting mixed cell strength to zero is convenient for porous bed compaction when large amounts of grain motion are expected. Maintaining strength at the surface and contact point between grains is in most cases advantageous, however, it can sometimes result in a response that is more stiff as compared to the experimental data. This phenomena was observed in the present work as well as in previous work on the shock compaction of dry and water-saturated sand [60].

The last obstacle is creating a way to ensure grains are not “welded” together when they come into contact with other grains. This is circumvented by labelling neighboring grains with different material numbers. CTH is able to distinguish to materials based on their designated material numbers, but not necessarily their material properties. If two grains come into contact within a single computational cell, having two different material numbers allows them to not “weld” together and enables the use of a variety of mixed cell strength treatments. This leads to the last useful CTH command for these mesoscale simulations, slide. Slide is a feature that allows two different materials in the same cell to move by one another by setting the shearing velocity components equal to zero, all while maintaining the strength of each material, if desired. Borg et al [16] found that mesoscale simulations with slide turned off, i.e. stiction, yielded a slighter stiffer response as compared to the experimental data. Conversely, slide on, or sliding, resulted in a response that was slightly less stiff as compared to the experimental data. Therefore, the two types of mixed cell, or contact, treatments tend to create an upper and lower bound on experimental data.

CTH uses a variable time step to control how the code increments forward in time, and integrates in space. The maximum possible time step is calculated using the Courant Stability
Criterion:
\[
\Delta t = f_{safety} \cdot \min \left( \frac{\Delta x}{|v_x| + c_s}, \frac{\Delta y}{|v_y| + c_s}, \frac{\Delta z}{|v_z| + c_s} \right)
\] (5.4)
\[
(5.5)
\]

where \( f_{safety} \) is a safety factor typically set to 0.6, \( c_s \) is the sound speed of the materials in the current cell, and \( \Delta x, \Delta y, \Delta z \) and \( |v_x|, |v_y|, |v_z| \) are the grid sizes and particle velocity in the x-, y-, and z-direction respectively. \( \Delta t \) is calculated in each cell and the minimum value is taken as the current time step. Without the time step criterion in place, waves might begin to propagate a distance larger than the smallest cell size, which causes a multitude of errors.

5.3 Simulation Setup

The process of creating a grain packing, or grain realization, within CTH was performed using an in-house code created by John Borg at Marquette University called Mesogrow. The code consists of a collection of Fortran subroutines that grow point sources into circles (2D) or spheres (3D) all while randomly packing into a specified domain. Each grain starts out as a point source with a spatially random location. The number of grains (point sources) inserted into the domain during this first time step is calculated using the volume (area) of the computational domain, volume (surface area) of an individual grain, and the desired packed density. The code then incrementally grows, i.e. increases the diameter, of each grain until they begin to overlap. Once the grains begin to overlap, they begin to move away from each other by the necessary amount to ensure no overlap. The code continues to iterate through this process until the desired grain diameter and sand bed density are achieved. As grains begin to have multiple contact points with multiple overlap sections, grain networks begin to form and the code iterates through the bed until a maximum threshold of overlap is achieved. Typically, the threshold of overlap between grains is set to be less than the size of the smallest computation cell. The final density of the grain bed is calculated using the number of grains and their corresponding diameters,
\[
\rho_0 = \frac{\sum_{i=1}^{N} m_i}{V_{box}}
\] (5.6)

where \( \rho_0 \) is the initial packed density of the sand, \( N \) is the total number of grains, \( m_i \) is the mass of grain \( i \), and \( V_{box} \) is the volume of the box, i.e. computational domain. The final packed density of the sand was 65% TMD or 1.723 g/cm\(^3\). The last operation Mesogrow performs is to go through the entire grain realization and give a material number to each grain, ensuring that no two
neighboring grains have the same material number. Each material number is then assigned the same material properties of quartz and the entire sand bed then has the same properties. Additional effort can be placed into created poly-dispersed grain size distributions and a distribution of quartz materials properties if so desired.

The ability to include periodic boundary conditions has been implemented into Mesogrow, which is useful when simulating grain beds that are semi-infinite in the lateral domain. During the grow and move process, if a grain leaves through the box wall, it enters back into the box from the opposite wall, with the same amount of overlap between the grain and the new wall. When these geometries are uploaded into CTH, any material that extends out of the computational domain is removed, which results in grains with flat surfaces where the plane of the wall intersects the spherical grain. It is desirable that grains near the walls in the axial direction do not get “cut off” when other material is inserted at the grain bed-anvil interface. Therefore, the axial thickness is adjusted at the beginning of the Mesogrow process to allow the shape of periodic grains at the x-walls to be conserved. The adjusted thickness, $x'$, is calculated as

$$x' = x_0 - 2(1 - f)D$$

(5.7)

where $x_0$ is the final desired thickness, $D$ is the grain diameter, $f$ is the percent overlap between the grain and wall. Values for $f$ range from 0, implying no overlap at the grain-wall interface, to 1, implying the entire grain would be removed outside of the boundary. Typical values of overlap are between 0.8 - 0.9. For uniaxial loading, $f$ was set to 0.9 since less contact area is needed to transmit a longitudinal wave and for pressure-shear loading, $f$ was set to 0.8 to obtain slightly more contact area between the grain bed and anvils for shear wave transmission.

The axial thickness of the sand bed was matched to that of the experiments, i.e. 0.5 mm, but the lateral domain was selected to be 2 mm in the y-direction and 1 mm in the z-direction. The sand domain size in y and z was chosen to provide a reasonable amount of grains such that a macroscopic response could be observed. It should be noted that x was chosen as the axial direction, i.e. longitudinal direction, and y was chosen to be the direction of shear velocity. A mono-dispersed grain diameter of 90 µm was chosen as the median size within the experimental distribution of 63 – 120 µm. This diameter ensured the number of grains across each thickness
was:

\begin{align}
N_x &> 5 \quad (5.8) \\
N_y &> 20 \quad (5.9) \\
N_z &> 10. \quad (5.10)
\end{align}

The entire axial thickness was chosen to match that of the experimental apparatus, i.e. the flyer, driver, sample, and rear anvil. Void space was inserted at the rear of the projectile and free surface of the rear anvil. This was desirable so symmetric boundary conditions could be used on both the up-range and down-range x boundaries. Periodic boundary conditions were used on the y- and z-boundaries to allow the shear wave to propagate into and out of the domain. 10 x 10 planes of Lagrangian tracer particles, or tracers, were placed in the lateral direction at the front and rear surface of the sand as well as at the free surface of the rear anvil to enable the tracking of both longitudinal and shear waves. These planes spanned 1.64 mm in the y-direction and 0.82 mm in the z-direction, yielding a surface area of 1.345 mm². An additional line of 40 tracer particles were placed from the flyer-driver to driver-sand interface along the axial direction to track the longitudinal and shear wave speeds in the anvil materials.

The resolution of mesoscale simulations is dictated by the smallest feature or characteristic length in the computational domain. Therefore, the diameter of an individual sand grain was chosen to be the smallest characteristic length and the mesh was sized such that 12 cells spanned the diameter of an individual sand grain. Therefore, the resolution, or minimum cell size, was calculated as \( dx = D/12 = 7.5 \mu m = 7.5 \times 10^{-4} \text{ cm} \) for a grain diameter of 90 \( \mu \text{m} \). Adaptive Mesh Refinement was used to cut down the computational expense of resolving the entire projectile and anvil package. For a square mesh with a cell size of 7.5 \( \mu \text{m} \), the total number of cells in the computational domain, \( N_{tot} \), becomes immense,

\begin{align}
N_x &= \frac{L_x}{dx} = \frac{2.5 \text{ cm}}{7.5 \times 10^{-4} \text{ cm}} \approx 3333 \\
N_y &= \frac{L_y}{dy} = \frac{0.2 \text{ cm}}{7.5 \times 10^{-4} \text{ cm}} \approx 267 \\
N_z &= \frac{L_z}{dz} = \frac{0.1 \text{ cm}}{7.5 \times 10^{-4} \text{ cm}} \approx 133 \\
N_{tot} &= N_x N_y N_z \approx 118 \text{ Million cells}
\end{align}

where \( N_x, N_y, N_z \) and \( L_x, L_y, L_z \) are the number of cells and domain length in the x-, y-, and z-direction, respectively. It is suggested somewhere between 30,000 - 40,000 cells per processor are
Figure 5.1: (a) Full simulation domain in CTH showing projectile nose, driver, sand bed, and anvil (b) sand bed generated using mesogrow and imported into CTH

used for most simulations. Therefore, a square mesh with this resolution will require between 2959 - 3945 processors, not to mention the memory requirements for storing the information from each of the computational cells. This is a considerable load to ask even the largest clusters to run. However, with proper implementation of adaptive mesh refinement (AMR), the computational expense can be dramatically decreased. AMR allows regions of interest to be resolved greater than other regions. Considering the sand bed makes up approximately 2% of the computational domain, implementing an AMR scheme is extremely beneficial in cutting down the computational cost. Mesh refinement schemes are defined using a set of “indicators.” Indicator sets provide a set of logical checks that guide the code when refining the mesh. For the present simulations, the sand sample was forced to be at maximum resolution throughout the duration of the simulation,
While the resolution in the front and rear anvil was half that of the sand, $dx_{anvils} = 2dx_{sand\ bed}$, and the projectile resolution was $dx_{proj} = 4dx_{sand\ bed}$. An additional indicator was established to resolve pressure gradients to the maximum resolution, which ensured that wave fronts propagating in the anvils would be resolved. The resulting AMR scheme allowed these simulations to run using 32 processors over the course of approximately 72 hours.

5.4 Desired Outcomes

The underlying motivation for pursuing mesoscale simulations revolves around two main ideas. First, the formulation is built on previously established materials properties and geometric representations of the heterogeneous material. Therefore, it represents a predictive model in contrast to homogenized material models empirically-fit from experimental data. Simulations such as these expose the strengths and weaknesses of current computational abilities and preexisting equations of state and constitutive models. Second, inherently resolving mesoscale features within the bulk wave propagation allows the effects of heterogeneity to be observed at the macroscale. This information provides insight into features of the bulk response that may not have previously been understood and allows for quantitative, statistical data to be extracted from within the sand sample. Chapter 6 displays histogram data of state variables such as normal and transverse velocity, stress components, and Von Mises stress, $J_2$, leading up to as well as at the final combined loading state. The information gathered from the mesoscale simulations was used to explore the effect of

1. Time of arrival of the longitudinal and shear wave on the loading state achieved in the sand
   - By varying the front anvil thickness, the difference in time of arrival between the p- and s-wave were varied
   - Varying the time of arrival allows for the effect of confining stress/pressure to be explored
   - Front anvil thickness of 4 and 16 mm were chosen to provide two distinct cases where the magnitude of normal stress is increased in unison with the shear stress as well as a case were normal equilibrium is achieved before the arrival of the shear wave

2. Effect of material strength at contact points between grains within the computational domain
• The treatment of mixed cell strength in mesoscale simulations that use an Eulerian mesh has been shown to effect the macroscale response, especially in loading regimes below full-compaction of the granular material
• Two different mixed cell strength options were tested within CTH, slide on (sliding) and slide off (stiction)
• Slide off maintains shear stresses within computational cells that have multiple materials, i.e. a material interface or grain contact point
• Slide on zeros the shear components of velocity within mixed cells
6.1 Data Analysis

Frequency data recorded on the oscilloscope after each shot was converted to velocity profiles using an analysis software, PlotData, developed by Sandia National Laboratories. The theory behind this is described in section 4.3.1. The time-varying spectrogram calculated by PlotData is directly related to the time-dependent velocity profile. Peak finding algorithms built into PlotData extract the frequency bin with maximum amplitude at each time step, which is then taken to be a singular point along the velocity profile. The peak finding algorithm is initialized by dragging the cursor over the spectral profile, which provides an initial guess for the maximum peak location within the spectral band and helps the code discern important portions of the spectral content. Figure 6.1 shows a typical PlotData user interface.

Figure 6.1: Example PlotData user interface for the rear surface normal collimator of shot 7. The upper left box shows the raw signal collected by the oscilloscope. The lower left corner shows the analysis control settings such as window length, window shift, window type, and contrast. The upper right shows the time-dependent spectrogram and the lower right shows the corresponding velocity profile selected from the maximum amplitude frequency band at each time step.
Once the particle velocity profiles from each collimator were extracted from the spectral data using PlotData, the transverse component of particle velocity was calculated based on the probe configuration as discussed in appendix C. The apparent velocity profiles were time synced with the signal measured on the normal probe by ensuring identical STFT (Short-Time Fourier Transform) settings were chosen in PlotData. This is crucial because each apparent velocity profile is compared to the normal profile at every time step throughout the duration of motion. If the profiles do not have identical time vectors, their corresponding velocity magnitudes will not be correct. A loop was constructed to calculate an average transverse velocity from the normal, $V_N$, and apparent, $V^-$ and $V^+$, equations C.4 and C.5, velocities at each time step. The uncertainty in transverse velocity was also calculated at each time step using the taylor method described in section C.3.

6.2 Experimental Results

Determining the final combined loading state or states in pressure-shear experiments builds upon analysis from traditional one-dimensional shock loading experiments. In analyzing one-dimensional experiments, the transit time through the sample is tracked along with the velocity magnitude achieved at the free-surface of the target. Similar phenomena are tracked in pressure-shear experiments. Previous pressure-shear experiments on homogeneous and heterogeneous materials by Clifton [24], Sundaram [89], Prakash [76], Grunschel [42], Ramesh [77], have outlined methods to determine the final combined loading state or states. With well-timed wave arrivals and minimal tilt, these experiments provide a standard with which to proceed. As discussed in chapter 3, the sample rings up in longitudinal stress and achieves equilibrium before the arrival of the shear wave. Then, the longitudinal wave propagates into the rear anvil, reaches the free-surface, and reflects back toward the sample such that the shear wave has a chance to fully load the sample and subsequently reach equilibrium or yield the sample. This orchestration of waves comes from a precise understanding of the p- and s-wave speeds of both the anvils and sample material. If the wave speeds of the sample significantly depend on changes in pressure/stress and density, the analysis becomes challenging to predict. However, the wave speed measured with pulse-receivers at the initial pack density and negligible stress can be used to place bounds on arrival times at different locations within the target capsule. Therefore, estimated p- and s-wave speeds were used for each experiment to create a position-time and stress-particle velocity plot, which, in conjunction with particle velocity time histories, guided the
determination of the final loading states.

Figure 6.2 shows the normal, $V_N$, and transverse, $V_T$, particle velocity profiles measured from each of the experiments. Appendix D provides the particle velocity - time, position - time, and stress - particle velocity plots for each experiment.

Figure 6.2: Measured normal and calculated transverse particle velocity measured from the free surface of the rear anvil for shots (a) 7 - 10 (b) 3 - 6 (c) 1 and 2 and (d) 11 and 12. Each profile is labeled with its initial velocity as $v_{0,i}$ where $i$ is the shot number. Black vertical dashed lines denoting $R_{N1}$, $R_{N2}$, and $R_{N3}$ are subsequent reverberations of the elastic wave in the rear anvil that change the normal stress state on the sample. Red dashed lines denoting $R_{T1}$, $R_{T2}$, and $R_{T3}$ are the point at which a given shear window has ended based on the current normal stress state.
The profiles in figure 6.2 are separated by the type of anvil material selected and rear anvil thickness. Shot 1 and 2 were grouped together because there is question about whether or not the anvils remained elastic during the loading. Significant deformation was observed in the anvils collected post shot 1 and 2, while the remaining 10 shots show mostly surface scratches with no major deformation. Considering the initial velocities of shot 1 and 2, $V_0 = 130$ m/s and $V_0 = 112$ m/s respectively, were above the calculated initial velocity limit based on the steel material properties and impact angle of $17^\circ$, it is believed that the loading from impact caused the deformation as opposed to late-time impact with the stopper plate or tank wall. However, it is difficult to quantify the effect of the elastic-plastic transition in the anvils to the loading conditions imparted onto the sand. This avenue idea is well-suited for future studies.

Each particle velocity profile was time-shifted such that the arrival of the longitudinal wave at the free surface of the rear anvil, $F_{N1}$, was time zero. This provided a consistent starting point to analyze each experiment. The transverse particle velocity history for each shot was truncated to remove the effect of tilt prior to the arrival of the shear wave, which helped for identifying properties of the shear wave. Each experimental particle velocity profile in appendix D shows a non-zero transverse velocities observed on both the positive and negative angled collimators prior to the arrival of the shear wave, which is the manifestation of tilt at the free surface of the target. Tilt, in this context, refers to the tilted p-wave that is generated from a non-planar impact. The arrival of the tilted p-wave at the target free surface creates an apparent transverse velocity as the longitudinal wave sweeps across the free surface of the target. Once the shear wave arrives at the free surface, the surface begins to move according to the shear loading and both angled collimators converge, roughly, onto the same transverse velocity. This phenomena can be observed in the raw, experimental velocity profiles in appendix D. Each shot shows some amount of tilt prior to arrival of the shear wave and the magnitude of the tilt wave can be calculated using analysis performed by Clifton [56].

The first event that was tracked for each experiment was the arrival of the longitudinal release wave at the free-surface of the anvil, $R_{N1}$ in Figure 6.3. At this point, the original longitudinal wave has propagated through the entire target package, reflected back to the target, and then back to the free surface. This represents the first change in confining stress on the sample. Based on design constraints discussed earlier, normal stress equilibrium was not achieved in the sample before longitudinal release occurred. This is evident in the normal velocity profiles from each shot by noting that release from the anvil free surface occurs, $R_{N1}$, is before the normal
velocity reaches a plateau near the expected value of \( u_0 = V_0 \cos(\theta) \). Some shots did, however, almost reach normal equilibrium prior to arrival of the first release wave. However, since stress waves propagate slower in the sample than in the anvils, the sample is still being loaded from the driver-sand interface. Therefore, the sample maintains a confining stress while the shear wave loads the sample. Determining the normal stress on the sample is accomplished by tracking the time of arrival and magnitude of normal velocity at each subsequent arrival of release waves. Since radial release needs to be taken into account, a limited number of stress states were considered based on the elapsed time after impact.

Time of arrival for the release waves were calculated from the p-wave and s-wave speed of the rear anvil and the corresponding anvil thickness. These release states are shown in each of the velocity history plots as \( R_{N1} \) and \( R_{N2} \). The states \( R_{T1} \) and \( R_{T2} \) signify the end of the shearing window for each normal stress state. Any transverse velocity that arrives at the free surface of the rear anvil before this point is associated with the corresponding normal stress. Tracking the normal and shear stress windows enables the determination of multiple states throughout a single experiment. Once the windows in which stress states are expected to occur are determined, the particle velocity inside those windows is used to calculate the normal and shear stress via impedance matching. Figure 6.3 provides an example of the impedance matching used in shot 3.

As noted in chapter 3, material impedance is defined as \( Z = \rho C \), where \( \rho \) is density and \( C \) is sound speed. Since the rear anvil remains elastic during the experiment, the impedance is constant, unlike that of the sand, which drastically changes. Therefore, the normal stress states within the sand, \( \sigma_1, \sigma_2, \) and \( \sigma_3 \), are calculated as:

\[
\sigma_1 = \frac{1}{2} \rho_{\text{anvil}} C_L u_{F_{N1}} \\
\sigma_2 = \frac{1}{2} \rho_{\text{anvil}} C_L (u_{R_{N1}} - u_{F_{N1}}) \\
\sigma_3 = \frac{1}{2} \rho_{\text{anvil}} C_L (u_{R_{N2}} - u_{F_{N2}}). 
\]

The shear stress associated with each of these normal stress states is calculated as:

\[
\tau_1 = \frac{1}{2} \rho_{\text{anvil}} C_S v_{F_{T1}} \\
\tau_2 = \frac{1}{2} \rho_{\text{anvil}} C_S v_{F_{T2}} \\
\tau_3 = \frac{1}{2} \rho_{\text{anvil}} C_S v_{F_{T3}}. 
\]

The impedance matching for normal and shear stress differs because the shear wave does not have time to reach the free surface of the rear anvil and reflect back to the sample. Instead, the
shear stress, i.e. transverse velocity, is “sampled” each time a normal release wave reflects from the sand-rear anvil interface. This is based on the assumption that normal and shear waves propagate independently of each other in these experiments.

Figure 6.3 provides an example of the impedance matching defined in equations 6.1 - 6.6 that was performed for shot 3. The first velocity state observed in each particle velocity profile, $F_{\text{N1}}$, and subsequent release states, $R_{\text{N1}}$, and $R_{\text{N2}}$ are shown at the $\sigma = 0$ axis because their arrivals correspond to a release in stress at the free surface of the rear anvil each time the longitudinal wave reflects from the free-surface to the sample interface. Each of these states, $u_{F_{\text{N1}}}$, $u_{R_{\text{N1}}}$, and $u_{R_{\text{N2}}}$ are connected by red, anvil impedance lines to signify the stress in the sample, $\sigma_1$, $\sigma_2$, $\sigma_3$ after each reflection. Blue lines represent estimated, elastic impedance lines of sand. In reality, the slope will vary with the slightest change in stress and might even be better represented by a series of linear impedance lines, i.e. a “fan”, or a parabolic fit such as that used in shock-jump conditions. To show what effect this has on the stress determination, a fan of sand impedance lines are plotted with impedance ranging from $Z = Z_0 = \rho_0 c_1(\sigma = 0)$ to $Z = 2Z_0$. The fan can be shown to quickly spread out and create a range of potential states within the sand. Regardless of the functionality of the sand impedance lines, the anvil impedance line will remain linear after reflecting from the
sand interface. The intercept of these two anvil impedance lines therefore signifies the stress at the sand-anvil interface. The number of states for each experiment was determined on a shot-by-shot basis from the timing of normal and radial release. The time of arrival of the shear wave dictated whether a state only had a normal component or had both a normal and shear state.

Shots 1 - 6, figure 6.2b, used 4 mm front and 4 mm rear anvils, which did not provide enough time for the shear wave to reach the free surface of the rear anvil before the normal stress changed from $\sigma_1$ to $\sigma_2$. Figure 6.2b shows the shear wave arriving after the first shear window ($t < R_{T1}$) and part way into the second ($R_{T1} < t < R_{T2}$). Therefore, the sand was initially loaded to a normal stress of $\sigma_1$ and transitioned to $\sigma_2$. After the sand was loaded to $\sigma_1$, the shear wave began to propagate through the sand and subsequently into the rear anvil. The magnitude of shear stress in the sample continued to rise until the sample fully unloaded in compression or radial release occurred. Since the transition between normal states does not fully relieve the stress on the sample to zero, the shear wave will continue to load the sand. Shot 3 and 6 were the only to have a third stress state calculated due to their gradual increase in normal velocity at the free surface of the anvil.

Shots 7 - 10, figure 6.2a, had 4 mm front and 6 mm rear aluminum anvils, which allowed the shear wave to traverse the entire target package and reach the free surface during the first shear window, i.e. $t < R_{T1}$. The increased rear anvil thickness also allowed the normal wave to ring up closer to equilibrium for the two higher velocity shots, $v_{0,7} = 83$ m/s and $v_{0,9} = 109$ m/s. This is observed as the point at which $R_{N1}$ intersects the normal velocity. The blue and grey velocity histories show that approximately 70 and 90%, respectively, normal stress equilibrium was achieved for these shots. Since the initial normal loading state on these shots was close to the maximum expected, the subsequent normal states are close to zero, figure D.6c - D.10c. It is interesting to note that shear waves for both the 4-6 mm aluminum anvil and 4-4 mm steel anvil shots converge near each other around the second shear window. This could be an manifestation of grain fragments re-compacting and locking up to again support shear. This idea is explored further in the conclusions and discussion section.

Calculated normal-shear stress for each shot is shown in figure 6.4. The maximum combination of shear and normal stress calculated from elastic impedance matching is depicted with green and blue lines for aluminum 7075 T6 and 1045 steel, respectively. This limit is based on anvil material impedance, $Z$, initial velocity, $v_0$, and angle of impact, $\theta$. It represents the sample reaching full normal and shear equilibrium. Blue triangles signify shots 1 - 6, 11, and 12 that used
steel anvils. Red triangles signify shots 7 - 10 that used steel anvils. Solid red or blue marker faces imply the first combined loading state, \((\sigma_1, \tau_1)\). Open face (white) red or blue triangle markers imply the second combined loading state, \((\sigma_2, \tau_2)\). Open face (white) red or blue square markers imply the third combined loading state, \((\sigma_3, \tau_3)\). Data points connected by a dashed line indicate a single experiment. Aluminum anvil shots showed the presence of a shear wave at the first loading state so there is a non-zero shear stress. Steel anvil shots did not have a shear wave in the first shear window. Therefore, the first stress state can be seen along the \(\tau = 0\) axis. Most of the steel anvil experiments show a drop in normal stress from state 1 to 2 along with an increase in shear stress. The slight drop in normal stress is expected based on impedance matching and the shear stress lands near the expected, maximum normal-shear stress line. Overall, the data shows a section of increasing shear stress with normal stress up to about 0.5 - 0.6 GPa and another portion that drops off after that. The drop off is associated with a failure cap when increasing levels of normal stress subsequently decreases the ability of the sand to support shear loading.

Figure 6.4: Normal stress - shear stress plot for current experimental data set indicating the multiple states calculated from each shot. Sandia data by Vogler et al [97].
The main application of study was to explore the pressure-dependent strength of sand under rapid strain rates. A variety of analytical yield surfaces have been derived to capture pressure-dependent strength properties of granular materials. Many of these models can be fit to the current experimental data so as to parametrize constitutive models. Models, that when implemented into continuum codes, represent a homogenized version of sand. Most models refer to the linear relationship between normal and shear stress,

$$\tau = \mu \sigma + c$$  \quad (6.7)$$
as the failure surface, where $\mu$ is the shearing resistance and $c$ is the cohesion. To calculate these parameters, combined loading states with both a normal and shear component were isolated. A weighted linear fit [102] was applied to data points spanning 0 - 0.5 GPa normal stress. The calculated shearing resistance and cohesion was $\mu = 0.130 \pm 0.019$ and $c = 0.011 \pm 0.007$ GPa, respectively. Figure 6.5 shows only loading states for which combined loading was expected, i.e. the shear wave had a chance to propagate through the sample, as well as the linear fit. Additionally, experimental data from Volger et al. [97], shots Ti 78 and Ti 118, were shown for comparison to the extrapolated linear fit.

Higher-order, exponent shaped models relating normal stress to shear stress are also common, which relate normal stress to shear stress as

$$\tau(\sigma) = \frac{b}{a} \sqrt{\frac{1}{a} \left( \sigma + \sigma_t - \frac{\sigma_c}{2} \right)}$$  \quad (6.8)$$

where $a$ and $b$ are fit parameters and $\sigma_t$ is the distance from the onset of the failure cap to the $\sigma$-intercept and $\sigma_c$ is related to the cohesion [43]. An exponent shaped failure surface was approximately fit over the same range as the linear fit range with parameters, $a = 9$, $b = 1.13$, $\sigma_t = 0.30$ GPa, and $\sigma_c = 0.55$ GPa. Further numerical methods should be used to more accurately parameterize complex surfaces such as these. The next portion of the yield surface to address is the cap surface. This portion describes the decrease in shear strength with an increasing confining stress.

A modified Cam Clay model [79], describing both the linear failure surface and cap surface, is the simplest method to describe both regions of the yield surface and is defined as

$$\tau(\sigma) = \mu \sigma + c \quad \sigma < \sigma_a$$  \quad (6.9)$$

$$\tau(\sigma) = \mu \sqrt{\sigma \left( 2\sigma_a - \sigma \right)} \quad \sigma \geq \sigma_a$$  \quad (6.10)$$
where $\sigma_a$ is the point at which the yield surface begins to transition to the failure surface. It should be noted that the intersection of the regions is piece-wise continuous, but not differentiable. Numerically, this creates issues when the model is implemented into most plasticity frameworks. Figure 6.5 shows multiple cap surfaces corresponding to the Modified Cam Clay model, equation 6.10, with transition limits of $\sigma_a = 0.25, 0.55, 0.80, 1.10, \text{and } 1.45 \text{ GPa}$. These limits were chosen to fit experimental data points where a decrease in shear stress was believed to occur.

Multiple works have developed analytic functions for differentiable yield surfaces with both a failure and cap region. The most common is the modified Drucker-Prager [5, 44]. Other works such as Hammi et al [43] described internal state variable models built in a similar fashion to that of the modified Drucker-Prager, but modify the yield surface based on density. All of these models incorporate three portions: the shear failure surface, $F_s$, transition surface, $F_t$, and cap surface, $F_c$. The shear failure surface is typically represented with something like equation 6.9 or 6.8. The cap surface is typically some type of downward facing parabolic function of normal stress. The transition surface is another analytic function of normal stress and shear stress that ties the shear failure surface to the cap surface. A simplified cap surface adapted from Hammi [43] can be tied to a variety of failure surfaces such as the Drucker-Prager as follows

$$\tau(\sigma) = f_t(\sigma)f_c(\sigma)$$

(6.11)

where

$$f_t(\sigma) = \mu \sigma + c$$

(6.12)

and

$$f_c(\sigma) = \begin{cases} 1 & \sigma < \sigma_a \\ 1 - \frac{(\sigma - \sigma_a)^2}{2(\sigma_b - \sigma_a)^2} & \sigma \geq \sigma_a \end{cases}$$

(6.13)

(6.14)

where $\sigma_b$ is the distance from the onset of the cap to the cap surface intersection with the x-axis. This approach uses a piecewise cap surface, $f_c(\sigma)$, that, when multiplied by the failure surface, $f_t(\sigma)$, becomes continuous and smooth. The same cap surface, equation 6.14, can also be applied to the exponent shaped failure surface. This particular method is simple and convenient to demonstrate functionality, while not requiring density-dependence and/or transition surfaces to be included. Both of these yield surfaces are shown with the combined normal-shear stress data from the current experiments in figure 6.5. Transition limits of $\sigma_a = 0.01, 0.35, 0.63, 0.95, 1.10 \text{ GPa}$ were selected as potential fits to the current data.
Complete yield surfaces with failure, transition, and cap regions are more comprehensive and require an accurate measure of density before and during loading \([5, 43, 44]\). As the density of the sample is increased, either pre-shot or during uniaxial loading, the onset of the failure surface occurs at higher normal stresses. Therefore, multiple cap surfaces are typically shown to represent increasing failure strength at higher relative densities. Figure 6.5 shows the approximate fit lines corresponding to potential failure caps for the modified Cam Clay model, Drucker-Prager, and Exponent Shaped failure surface, but density was not used to locate the caps.

Stress-density loading curves were calculated for each of the experiments in an attempt to relate the potential failure surfaces with relative density. Normal stress was calculated as a function of instantaneous normal velocity starting from the arrival of the longitudinal wave, \(F_{NL}\), up to the first change in normal stress, \(R_{NL}\), as \(\sigma_N(t) = \frac{1}{2} \rho_{ane} C_L u(t)\). Density of the sample was calculated by assuming plane strain on the sample in the axial direction, equation 3.21. Therefore,
axial strain was used to first calculate a new thickness, $h'$ at every time step as

$$h'(t) = h_0(1 - \epsilon(t)). \quad (6.15)$$

The instantaneous density, $\rho(t)$, was then calculated as

$$\rho(t) = \rho_0 \frac{h_0}{h'(t)} \quad (6.16)$$

where $\rho_0$ and $h_0$ are the initial density and thickness of the sample, respectively. This method is most applicable when the loading is rapid and the sample takes a small amount of time to ring to stress equilibrium [42, 62]. Since the sand samples have a much lower impedance relative to the anvils and require an extended amount of time compact and reach longitudinal stress equilibrium, the axial strain is over-predicted. This is a result of the approximation used to calculate strain rate, i.e.

$$\dot{\epsilon}(t) = \frac{u_F - u_B(t)}{h} \approx \frac{u_0 - 2u_B(t)}{h} = \frac{u_0 - u_{fs}(t)}{h}, \quad (6.17)$$

which assumes $u_F(t) - u_B(t) \approx u_0 - u_{fs}(t)$. This is most applicable as the sample approaches equilibrium and $u_F \rightarrow u_B$. This is not an issue for simulations because the anvil interfaces can be tracked, but future experimental work could greatly improve upon this calculation to obtain better final density values.

Figure 6.6 shows the normal stress - density plot for the current experimental data as well as the solid quartz curve, single shock compression curve from LaJeunesse et al [60], and quasi-static compression curve from Perry et al [75]. The type of loading imparted on the sand while the normal stress waves ring up between the high-impedance anvils could be considered quasi-isentropic. It lies somewhere between the single shock Hugoniot and quasi-static range. The solid quartz Hugoniot, originating from $\rho_0 = 2.65 \text{ g/cm}^3$, represents the limit at which all the porosity has been crushed out of the sample. The response of any porous silica sand sample, whether subjected to shock or quasi-static loading, is expected to approach, but not exceed, that of the solid quartz Hugoniot. Figure 6.6 shows that the final density state for shot 1 exceeded the fully-dense quartz Hugoniot. This is where the validity of the axial strain rate approximation, equation 6.17, comes into question. For most of the remaining shots, the loading curve did not reach full longitudinal stress equilibrium before release waves in the rear anvil changed the normal stress state. Therefore, the end point of each loading curve does not represent the stress-density expected based on the anvil impedance, impact velocity, and impact angle. Future
work desiring to correlate the final density into the normal stress - shear stress comparison to populate density dependent failure cap models should strategically populate this normal stress - density space, while observing the shear stress. The combination of these three factors should provide a complete picture of failure cap density-dependence observed in quasi-static testing.

![Figure 6.6: Calculated normal stress - density for the current pressure-shear experiments with quasi-static compression [75] and single shock compression [60] curves for reference.](image)

An interesting feature of the loading curves measured in the current study is the presence of a precursor near 0.2 GPa. Initially, the response starts above both the single shock and quasi-static curves and then converges onto the single shock curve. This is a useful result because precursors have been shown to exist in other brittle granular materials such as soda lime glass microspheres [67]. Additionally, the single shock loading data [60] was fit over a range of 1 - 9 GPa, leaving a gap from the zero-stress initial density state to the bottom end of the single shock data set near 1 GPa. The magnitude of this precursor is believed to signify the onset of compaction phenomena such as micro-kinetic energy (grain network dissociation) and grain fracture. Both of these mechanisms result in a drastic increase in density for similar amounts of
confining stress. Eventually as the intact and fractured grains rearrange and re-compact, the response stiffens again and more closely represent the single shock Hugoniot. The end point of the loading curves lands somewhere between the quasi-static compression and single shock compression, which is to be expected based on the strain rate of ramp loading being somewhere between that of quasi-static and strong shock. However, a clear relationship is not obvious between the final density of the sample (while at a state with both normal and shear stress present) and the potential location of each cap. The last convenient space to explore is stress-relative density. Figure 6.7 shows the same experimental data with the single shock and solid quartz Hugoniot as well as the quasi-static loading curve. Again, the precursors are visible up to roughly 0.2 GPa and about 5% relative density change. Above this limit, significantly more compression is observed until about 20% compaction when the response begins to again stiffen. It is interesting to note that shot 1 did not seem to have a precursor, which suggests the first compressive loading state was enough to cause compaction to occur.

Figure 6.7: Calculated normal stress-relative density for the current pressure-shear experiments with quasi-static compression [75], single shock compression [60]
6.3 Post-Shot Qualitative Analysis

Qualitative analysis of the sand was performed before and after the pressure-shear experiments to investigate the damage resulting from various loading conditions. A single SEM image was taken of the pristine sand and is shown in figure 6.8. Post-shot SEM images were captured for all shots except for shots 1, 2, and 6. As mentioned earlier, the steel anvils for shots 1 and 2 yielded at some point during the loading process and the capsule broke open. The anvils from shot 6 appeared to remain elastic throughout the loading process, but the some of six bolts that held the capsule together broke when the capsule was thrown into the catch tank. The sand sample was therefore lost. Shots 7 - 12 used additional rubber matting and aluminum foam in an attempted to soften the impact within the catch tank, well after initial impact. Soft recovery for pressure-shear is promising because of the slower impact velocities relative to traditional flyer plate studies. However, the inclined projectile nose tends to act as a wedge and drive the target capsule vertically once the experiment is finished. Therefore, a special catch mechanism would need to be implemented at a non-zero angle relative to the shot direction. Additionally, qualitative characteristics of soft-recovered samples should be approached with caution due to obscure loading states potentially imparted on the sample as it is stopped post impact. Regardless, the underlying assumption for the present post-shot analysis is that the damage incurred by the sand is primarily a result of the stress loading imparted by the projectile. Figures 6.9 - 6.17 provide SEM images of each recovered sample at four different magnification levels.

Shots 7, 8, and 10 appeared to have many large grains still intact, while shot 9 seemed to show more smaller grain fragments. The loading states for shots 7, 8, and 10 all lie close to the linear portion of the yield surface whereas shot 9 falls more on the failure cap portion of the yield surface. Shot 11 and 12 showed a large amount of grain fracture, which is supported by the larger normal stress during state 1 relative to other shots. Shot 4 also shows a significantly greater amount of smaller fragments relative to shot 5, which is supported by shot 4 having high normal-shear stress states for both state 1 and state 2.
Figure 6.8: Pre-shot SEM image of the Oklahoma #1 sand at x60 magnification.

Figure 6.9: SEM images of the sand sample post-shot 3. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading state(s) \((\sigma, \tau)_i = (0.412, 0.000)_1, (0.455, 0.096)_2, (0.599, 0.059)_3\) GPa.
Figure 6.10: SEM images of the sand sample post-shot 4. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading state(s) $(\sigma, \tau)_i = (0.746, 0.000)_1, (0.447, 0.051)_2$ GPa.
Figure 6.11: SEM images of the sand sample post-shot 5. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading state(s) \((\sigma, \tau)_i = (0.512, 0.000)_1, (0.325, 0.048)_2\) GPa.
Figure 6.12: SEM images of the sand sample post-shot 7. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading state(s) $(\sigma, \tau)_i = (0.568, 0.068)_1$ GPa.
Figure 6.13: SEM images of the sand sample post-shot 8. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading states $(\sigma, \tau)_i = (0.206, 0.010)_1, (0.184, 0.044)_2$ GPa.
Figure 6.14: SEM images of the sand sample post-shot 9. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading states $(\sigma, \tau) = (0.785, 0.058)$ GPa.
Figure 6.15: SEM images of the sand sample post-shot 10. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading states $(\sigma, \tau)_i = (0.279, 0.027)_1, (0.166, 0.033)_2$ GPa.
Figure 6.16: SEM images of the sand sample post-shot 11. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading states $(\sigma, \tau)_i = (1.140, 0.000)_1, (0.279, 0.052)_2$ GPa.
Figure 6.17: SEM images of the sand sample post-shot 12. Magnifications of (a) 60x, (b) 100x, (c) 250x, and (d) 600x. Loading states $(\sigma, \tau)_i = (0.252, 0.000)_1, (0.288, 0.052)_2$ GPa.
6.4 Mescoscale Simulation Results

Two types of data extraction methods were used to obtain information from the mesoscale simulations. First, the planes of 100 Lagrangian tracer particles at the driver-sand interface, \(x_f(t)\), and sand-rear anvil interface, \(x_r(t)\), were used to calculate average x- and y-position as well as x- and y-velocity at the front and rear surface of the sand, figure 6.18. The average x- and y-position of the front and rear plane, \(x_f(t)\), \(x_r(t)\), \(y_f(t)\), \(y_r(t)\) respectively, were calculated as follows:

\[
x_f(t) = \frac{1}{100} \sum_{i=1}^{100} x_{f,i}(t)
\]

\[
x_r(t) = \frac{1}{100} \sum_{i=1}^{100} x_{r,i}(t)
\]

\[
y_f(t) = \frac{1}{100} \sum_{i=1}^{100} y_{f,i}(t)
\]

\[
y_r(t) = \frac{1}{100} \sum_{i=1}^{100} y_{r,i}(t).
\]

The strain was then calculated from the average position as

\[
e(t) = 1 - \frac{(x_{r,i}(t) - x_{f,i}(t))}{h_{0,x}}
\]

\[
\gamma(t) = \frac{(y_{r,i}(t) - y_{f,i}(t))}{h_{0,y}}
\]

where

\[
h_{0,x} = x_r(0) - x_f(0)
\]

\[
h_{0,y} = \max(y_r(0)) - \min(y_r(0)).
\]

Note that the computational sand domain spanned \(y = [-1, 1]\) mm, which made the average y-position at \(t = 0\) sec, \(y = 0\) mm. To correct for this, the initial y-thickness was taken to be the width of the plane of tracers in the y-direction, \(h_{0,y} = 1.64\) mm. Average longitudinal and transverse velocity, \(v_N(t) = v_x(t)\) and \(v_T(t) = v_y(t)\), were calculated similar to equation 6.18. To calculate normal and shear stress imparted on the sand, the x- and y-velocity at the sand-anvil interface was impedance matched using the anvil density and sounds speeds. Based on impedance matching, the velocity on at the interface of two materials is directly related to the
stress as

\[ \sigma = v_{x,r}(t) \rho C_L \]  \hspace{1cm} (6.26)

\[ \tau = v_{y,r}(t) \rho C_S. \]  \hspace{1cm} (6.27)

This method was chosen to mimic the stress calculation used in the experiments. The next data extraction method created histograms from the thermo-mechanical states experienced by the constituent material. This was accomplished by selecting state variables from computational cells (nodes) within the sand domain that contained quartz. Histograms of pressure, x-velocity, y-velocity, and von Mises stress \( J_2 \) were calculated periodically throughout each of the individual simulations.

Figure 6.18: Mesoscale sand realization uploaded into CTH showing the location of the front and rear plane of Lagrangian tracer particles.

A single impact angle of \( \theta = 17^\circ \) was chosen to correspond to the experiments and initial velocities ranging from \( v_0 = 50 - 210 \) m/s were selected based on an upper maximum normal stress of \( \sigma_{\text{max,expected}} = \rho_{\text{steel}} C_{L,\text{steel}} \left( 1/2 v_0 \cos \theta \right) \approx 4.3 \) GPa. Therefore, the upper range of initial velocities would be expected to create combined loading conditions above that of the specified quartz yield strength, \( Y = 4.1 \) GPa. Figure 6.19 shows the axial strain on the sand for the simulations with 4 mm front and 12 mm rear anvil as well as the 16 mm front and 16 mm rear anvil simulations. Additionally, the effect of mixed cells or partially filled cells at the interface of
multiple grains is shown with a slide off (stiction) case and slide on (sliding) case. First, axial strain for both cases using stiction was decreased relative to that of the sliding case. This is to be expected considering the strength between cells at grain interfaces is higher in the stiction case, which resists compacts and slows axial strain. Comparing the 4 - 12 mm anvil case to the 16 - 16 mm anvil case, the axial strain does not noticeably differ, suggesting that the arrival of the shear wave during normal compression in the 4 - 12 mm simulation does not significantly effect compaction.

![Graphs showing axial strain](image)

Figure 6.19: Time histories of axial strain calculated from mesoscale simulations with initial velocities ranging from 50 - 210 m/s and an impact angle of 17°. (a) 16 - 16 mm anvils with slide condition off, (b) 4 - 12 mm anvils with slide off, (c) 16 - 16 mm anvils with slide on, and (d) 4 - 12 mm anvils with slide on. Note: slide off refers to the stiction interface between grains.

However, the treatment of mixed cells at grain contact points and the front anvil thickness significantly effects the shear strain. The most noticeable effect is that of the front anvil thickness. This thickness determines how much time the p-wave has to rush ahead of the s-wave and
confine the sample. If the s-wave arrives before the p-wave and begins to shear the sample, an increased amount of shear strain is observed resulting from the lack of confining stress, which is to be expected. Next, both cases with slide on, i.e. sliding between grains, exhibited a greater cumulative shear strain relative to the stiction cases. The last interesting thing to notice is that when the sample is loaded to the expected normal stress prior to the arrival of the shear wave, the shear strain is essentially the same regardless of initial velocity. In contrast, if the shear wave arrives prior to normal stress equilibrium, the transverse velocity imparted on the sand has a significant effect on the overall shear strain. CTH has the ability to render images of the

three-dimensional computational domain as well as place state variable contours onto material surfaces. This helps greatly to grain a qualitative understanding of mesoscale processes inherent

![Figure 6.20: Time histories of shear strain calculated from mesoscale simulations with initial velocities ranging from 50 - 210 m/s and an impact angle of 17°. (a) 16 - 16 mm anvils with slide condition off, (b) 4 - 12 mm anvils with slide off, (c) 16 - 16 mm anvils with slide on, and (d) 4 - 12 mm anvils with slide on. Note: slide off refers to the stiction interface between grains.]
in these experiments. Figures 6.21 - 6.25 provide three-dimensional state variable plots of pressure, longitudinal velocity, transverse velocity, and von Mises stress ($J_2$) for the 16 - 16 mm anvil case with slide ON and simulated initial velocities of 50, 90, 130, 170, and 210 m/s. Each figure reads left to right in time and displays pressure (dyne/cm$^2$), x-velocity (cm/s), y-velocity (cm/s), and von Mises stress denoted in CTH as "J2P" (dyne/cm$^2$) from top to bottom. Note the unit conversion for pressure/stress is 1 GPa = 10 dyne/cm$^2$. Three different locations in time were chosen to signify, $(t_1 = 3.1 \mu s)$, arrival of the longitudinal wave in the sand, $(t_2 = 4.4 \mu s)$ the point at which normal stress equilibrium is achieved in the sand, and, $(t_3 = 5.2 \mu s)$, the point at which the shear wave has begun to load the sample.

Figure 6.21: Surface plot of (a) pressure (b) longitudinal velocity, $v_x = v_N$, (c) transverse velocity, $v_x = v_N$, and (d) von Mises stress, $J_2$, for an impact velocity $v_0 = 50$ m/s, impact angle $\theta = 17^\circ$, and stiction between grains.
In each simulation, the longitudinal wave can be seen loading the sand starting from the negative x-direction and propagating forward to the positive x-direction. During this period stress bridges begin to form between the grain networks and the sample begins to densify as grains move and compact. Since velocity is driven by differences in pressure, normal stress equilibrium, i.e. the maximum confining pressure, is reached when the velocity at the front surface of the sand matches that of the rear surface. Also interesting to note is the presence of non-zero y-velocity components during longitudinal loading. This supports the idea that there exists a zero-centered, distribution of velocity components perpendicular to the axial direction during compaction. Once the shear wave arrives at the front surface of the sand, $t_3$, the grains can be seen collectively moving in the direction of shear loading.

Figure 6.22: Surface plot of (a) pressure (b) longitudinal velocity, $v_x = v_N$, (c) transverse velocity, $v_y = v_N$, and (d) von Mises stress, $\sigma_2$, for an impact velocity $v_0 = 90$ m/s, impact angle $\theta = 17^\circ$, and stiction between grains.
Comparing the shear wave front thicknesses in the sand sample for different confinement pressures, i.e. different initial velocities, it is clear that increasing the confinement pressure, shortens the rise time of the shear wave and make the shear wave front much thinner. This is rather intuitive since increased confinement pressures will result in more dense samples, which provide more paths for the shear wave to propagate and form a more homogenized planar wave front. This also helps to qualitatively support the idea that the shear wave speed within the sand is expected to increase with confinement pressure.

![Surface plots](image)

Figure 6.23: Surface plot of (a) pressure (b) longitudinal velocity, $v_x = v_N$, (c) transverse velocity, $v_x = v_N$, and (d) von Mises stress, $J_2$, for an impact velocity $v_0 = 130$ m/s, impact angle $\theta = 17^\circ$, and stiction between grains.

The last interesting observation to be made revolves around the $J_2$ stress plotted in the bottom row for each initial velocity. Again, this stress signifies the point at which the constituent material, quartz, begins to reach the elastic, perfectly-plastic limit. The colormap range is set such
that red indicates stresses (and pressures) that have reached the specified yield strength of 4.1 GPa. Shortly after the arrival of the longitudinal wave, $t_1$, it is apparent that grains locked in force chains experience much higher stresses. This is visualized as paths of increased $J_2$ stress that follow force chains arranged in the x-direction. Additionally, the contact points between grains are shown to have stresses much closer to, or at, the quartz yield strength. From $t_2$, it is clear that the ring-up of longitudinal stress causes a significant amount of the computational domain to experience stress magnitudes close to the yield limit. Once the shear wave has fully loaded the sand, $t_3$, more of the quartz is loaded to the plastic limit. This can be seen by comparing the $J_2$ stress between $t_2 = 4.4 \mu s$ and $t_3 = 6.75 \mu s$ for each of the initial velocities.

Figure 6.24: Surface plot of (a) pressure (b) longitudinal velocity, $v_x = v_N$, (c) transverse velocity, $v_x = v_N$, and (d) von Mises stress, $J_2$, for an impact velocity $v_0 = 170 \text{ m/s}$, impact angle $\theta = 17^\circ$, and stiction between grains.
Figure 6.25: Surface plot of (a) pressure, (b) longitudinal velocity, $v_x = v_N$, (c) transverse velocity, $v_x = v_N$, and (d) von Mises stress, $J_2$, for an impact velocity $v_0 = 210$ m/s, impact angle $\theta = 17^\circ$, and stiction between grains.

A more quantitative measure of constituent material failure, i.e. grain failure, is to look at a histogram of the normalized von Mises stress, $J_2/Y$, in each computational cell that contains quartz, where $Y$ is the yield strength of quartz. The resulting distribution provides an idea of loading states within the quartz grains where $J_2/Y = 0$ signifies no deviatoric loading and $J_2/Y = 1$ signifies material that is on the elastic-plastic yield surface. Figure 6.26 shows the normalized $J_2$ stress distributions for two sets of simulations: (1) sliding off and (2) sliding on for the 16 mm front and 16 mm rear anvil simulations. The occurrences in each bin, $N_x$, are normalized by the total number of occurrences, $\Sigma N_x$. Four points in time were chosen (1) just after the arrival of the longitudinal wave - dashed blue line, (2) after the longitudinal wave had rung-up (normal stress equilibrium) - solid blue line (3) just after the arrival of the shear wave - dashed red line, and (4) the final combined loading state once the shear wave had a chance to
propagate through the sand - solid red line.

After the longitudinal wave arrives, the normal stress begins to increase within the sand, which corresponds to the first dashed blue peak around \( J_2/Y < 0.1 \). This peak takes on a similar shape for both the stiction and sliding case. Next, the distribution at the final compression state indicated by the solid blue line begins to spread out as grains trapped in force chains approach the yield surface, \( J_2/Y = 1 \). As the shear wave arrives and loads the sand to a final shear state, the distributions can be seen shifting further towards the yield surface. The changes do not appear to be drastic in this space, but when integrated reveal significant shift in stress towards the yield surface. Figure 6.27 compares the normalized distribution of \( J_2 \) stress for stiction and sliding cases at the final combined loading state for each initial velocity. It is immediately apparent that less of the quartz is near the yield surface, \( J_2/Y = 1 \), for each initial velocity when slide between grains is used. This is an intuitive result considering the slide command sets the shearing velocity gradients to zero at each time step. Therefore, less stress is maintained and transmitted through the central portion of each grain. Considering stiction maintains material strength at the surface of grains, there should be an increased amount of deviatoric stress within the quartz. This is reflected in the amount of quartz present at the elastic-plastic yield surface after normal and shear equilibrium is achieved for each simulation. Integrating each histogram from 0.95 to 1.0 provides a quantitative measure of the percent of the sand domain with fundamental constituent (quartz) at the yield surface, table 6.1 provides these results. For each initial velocity, the amount of quartz at the yield surface in stiction simulations is almost double that of the simulations that used sliding between grains.

Table 6.1: Percent of quartz with a normalized \( J_2 \) stress greater than 95% at the final combined loading state for both the stiction and sliding mesoscale simulations.

<table>
<thead>
<tr>
<th>Initial Velocity (m/s)</th>
<th>Stiction</th>
<th>Sliding</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>9.3%</td>
<td>2.0%</td>
</tr>
<tr>
<td>90</td>
<td>25.5%</td>
<td>11.6%</td>
</tr>
<tr>
<td>130</td>
<td>40.7%</td>
<td>22.3%</td>
</tr>
<tr>
<td>170</td>
<td>48.4%</td>
<td>28.8%</td>
</tr>
<tr>
<td>210</td>
<td>52.2%</td>
<td>30.3%</td>
</tr>
</tbody>
</table>

The normal stress - shear stress relationship calculated from the 16 mm front and 16 mm rear anvil simulations is shown in 6.28. There was a slight difference in impedance between the steel used for the experiments and the steel within the CTH database. Therefore, the maximum
expected normal-shear stress line for the simulations is slightly steeper than that of the experiments. The initial velocities of \( v_0 = 50, 70, 90, 110, 130, 150, 170, 190, \) and \( 210 \) m/s were tested for the stiction and sliding case. It is immediately clear that the simulations populate a higher stress regime compared to the experiments. This is a result of the sand samples not reaching the full normal stress in the experiments based on the extended ring up times. Additionally, the initial projectile velocity was limited in the experiments based on the yield criterion of the anvil and projectile nose piece. Since yield strength in CTH is merely a stress limit and does not effect the wave speed, the projectile and anvil strength were set arbitrarily high to enable larger initial velocities. This enabled the anvils to create stresses within the sand domain that were above the yield strength of the fundamental constituent, quartz.

Comparing the sliding and stiction case, white and black markers, respectively, it can be observed that stiction between grains allowed the sand to support an increased amount of shear stress. A least-squares estimation [102] was used to calculate a linear fit of

\[
(sliding) \quad \tau = (0.172 \pm 0.012) \sigma - (0.059 \pm 0.039) \text{ GPa} \\
(stiction) \quad \tau = (0.176 \pm 0.013) \sigma - (0.090 \pm 0.041) \text{ GPa}
\]

for the sliding and stiction case, respectively. Interestingly, the shearing resistance, \( \mu \), of the stiction and sliding cases are nearly identical, but the cohesion, \( c \), is roughly 30 MPa lower in the sliding case. The error bars for each simulation data point are representative of the standard deviation of particle velocities across the final loading state. Slight fluctuations in average velocity throughout the loading process are believed to be the result of small numerical fluctuations, but also a result of wave interactions present in any heterogeneous material. Additionally, the planes of tracer particles were located at the interface of spherical quartz grains, high-density steel, and void, which resulted in many interfaces with high impedance mismatches. These fluctuations can be seen in the simulated particle velocity profiles, figures D.13 - D.21 in appendix D.2.
Figure 6.26: Distributions of normalized von Mises stress, $J_2/Y$, for impact velocities of $v_0 = 50, 90, 130, 170,$ and $210$ m/s, impact angle $\theta = 17^\circ$, and (a) stiction between grains and (b) sliding between grains. Simulations times arrival of longitudinal wave $t_1 = 3.00 \mu$s, normal stress equilibrium $t_2 = 4.50 \mu$s, arrival of shear wave $t_3 = 5.50 \mu$s, final shear stress $t_4 = 6.75 \mu$s.
Figure 6.27: Distributions of normalized von Mises stress, $J_2/Y$, at the final combined loading state (normal and shear equilibrium in the sample) for impact velocities of $v_0 = 50, 90, 130, 170$, and $210$ m/s, impact angle $\theta = 17^\circ$, and stiction between grains (dashed lines) and sliding between grains (solid lines).
Figure 6.28: Normal stress - shear stress for the mesoscale simulations with impact velocities of $v_0 = 50, 70, 90, 110, 130, 150, 170, 190,$ and $210$ m/s, impact angle $\theta = 17^\circ$, with (a) stiction between grains (black circles) (b) sliding between grains (white circles). Red solid lines indicate the maximum expected normal-shear stress combination based on the initial velocity, $V_0$, anvil impedance, $\rho c_L$ and $\rho c_S$, and impact angle, $\theta$. 

\[
\begin{align*}
\sigma_{\text{max}} &= 0.5\rho c_L V_0 \cos \theta \\
\tau_{\text{max}} &= 0.5\rho c_S V_0 \sin \theta
\end{align*}
\]
CHAPTER 7

CONCLUSIONS

The response of Oklahoma #1 silica sand to rapid combined loading was experimentally and computationally investigated using oblique flyer-plate impacts and three-dimensional mesoscale simulations. Sand was chosen due to its abundance in nature as well as in man-made structures. This particular type of sand was selected as an extension of previous work that characterized its shock response in dry and fully water-saturated conditions [60]. To recreate the macroscopic response of brittle granular materials subjected to a variety of loading conditions, an equation of state and constitutive model must be formulated/parameterized. Equations of state describe the thermodynamics at play well above the elastic limit while constitutive models control strength properties below the elastic limit. A number of previous studies have characterized the shock response, or equation of state properties, of various types of sand, but only one has attempted to perform pressure-shear experiments on sand. Pressure-shear experiments are somewhat rare in the dynamic materials community and most works have focused on homogeneous metals. A framework was developed by Vogler et al [97] to perform pressure-shear experiments on brittle granular materials, but the amount of data produced was limited. As discovered by the vast amount of data pertaining to the shock response of dry sand, multiple works are necessary to gain a true understanding of the dynamic response of such complicated materials. The current study has significantly contributed to this data set as well as proposed new ideas on the mechanisms that dictate the observed response.

The current simulation methodology was chosen based on a desire to explore the ability of CTH, an Eulerian hydrocode, to capture mesoscale phenomena in a new loading regime, i.e. below the point of full compaction (zero porosity). Multiple previous studies have used CTH to capture the shock response of granular materials with two- and three-dimensional mesoscale simulations and found that the mesoscale method performs well above the point of full compaction, but tends to struggle below this limit. It has performed well because Eulerian codes typically handle large deformation much better than finite element codes where mesh entanglement becomes an issue. Additionally, CTH hosts a suite of well-established equations of state that are able to accurately depict material response in shock loading conditions. However, Eulerian codes tend to struggle when it comes to loading regimes near the elastic limit of the constituent material when material interface tracking, strength properties, and fracture dictate
material response. Previous mesoscale simulations by LaJeunesse et al. [60] accurately depicted
the shock loading of dry and water-saturated silica sand, but the applicability of this framework
was a major question at the beginning of this work.

Nevertheless, the pressure-shear response of sand was investigated using CTH by
creating mono-dispersed grain packings of quartz spheres and observing the transmission of
normal and shear stress waves through the computational domain. Two different grain contact
treatments (mixed cell strength): 1) sliding and 2) stiction were utilized to observe their influence
on the macroscopic response. The shearing resistance was calculated to be \( \mu = 0.172 \) and
\( \mu = 0.176 \) for sliding and stiction simulations respectively. This is within the expected regime and
is approximately near the experimentally measured shearing resistance of \( \mu = 0.130 \). The
cohesion calculated for the stiction simulations were slightly higher than that of the sliding case,
\( c = -0.059 \) GPa versus \( c = -0.090 \) GPa, which supports the concept that treating cells at grain
contact points with a sliding condition decreases the shear transmission through the grain bed.
Tracking shear strain between the front and rear face of the sand suggested that the sand was able
to support shear wave propagation, but macroscopic failure did not occur and a failure cap was
not observed. For this to happen, large amounts of plastic shear strain, i.e. flow, would be
expected beyond a certain crush pressure. Distributions of normalized von Mises stress, \( J_2 \),
extracted from individual quartz grains showed that the portions of the sand domain had begun
to reach the yield surface. This suggests that the constituent material had failed, even though the
collection of grains supported shear stresses. It is hypothesized that the mix of yielded and
un-yielded computational cells allowed the sand to continue to support shear stress even after
some grains had failed. Also, the representative computational domain, approximately 5 - 8
grains across the axial thickness, 20 - 25 grains across in the shear direction, and 11 - 15 grains in
the third dimension might be small enough such that significant granular flow would not be
observed. Lastly, the fracture of individual grains becomes a limiting factor when simulating the
response of sand in this regime. Since the quartz grains were treated as ductile, using an elastic
perfectly plastic strength model, failed sand grains flow rather than fracturing as would be
expected based on the brittle nature of quartz. In this flow, grains may support stresses, but will
plastically deform without any additional stress. If the grains had fractured, no stress would be
supported until the fragments re-compact. Future work aiming to resolve fracture in a grain bed
should pursue other strength models for the quartz and pay particular attention to the resolution
of grain fragments. The current simulations were resolved based on an individual grain being the
smallest artifact in the domain. Once fracture initiates, smaller objects will be produce, which requires increased resolution. Regardless, fracture continues to be a challenging problem for many computational frameworks and the present computational results merely serve as further motivation for better computational tools.

Oblique flyer-plate experiments were performed using the Marquette University Shock Physics Laboratory single stage, slotted barrel, gas gun. Sand samples roughly 0.5 mm thick were confined between two high-impedance anvils, either 1045 steel or aluminum 7075 T6, to create a target capsule. This target capsule was mounted at the end of the gun barrel and oriented such that its angle relative to the shot direction matched that of the projectile nose piece, approximately 17°. Projectiles were accelerated from 40 − 140 m/s prior to impact resulting in stresses up to $\sigma < 1.4$ GPa and $\tau < 0.150$ GPa. Upon impact at the projectile-target interface, longitudinal and transverse waves were imparted forward into the target and backward into the projectile. The waves propagate through the front anvil, into the sand layer, and subsequently into the rear anvil where the response was measured from the free surface using photon doppler velocimetry (PDV).

To enable measurements of shear stress, a reliable method to measure transverse surface velocity was established based on works that performed foundational measurements and calculations [23, 31, 104]. The current diagnostic approach utilized a traditional PDV system to make measurements of transverse surface velocity. Previous studies accomplished these measurements by tracking the transverse motion of lithography deposited diffraction gratings. This study concluded that with proper surface preparation, selection of fiber optic collimators, and probe arrangement, transverse surface velocity measurements can be made with reasonable uncertainty. Starting from the generalized equation used to calculate transverse components of surface velocity, this study laid out the possible collimator arrangements that could be used to observe the surface motion. Uncertainty for each collimator configuration was assessed using a general multi-component surface velocity case, which enabled comparison between each of the collimator configurations. Each of the methodologies had strengths and weaknesses based on criteria such as measurement uncertainty, ease of implementation, and durability. Ultimately, it was concluded measurement uncertainty was lowest for arrangements in which the angled collimator emits and collects its own light. The results of appendix C are useful to a multitude future studies that wish to make transverse surface velocity measurements and inform their system design with measurement uncertainty calculations and practicality discussions.
Methods to controllably limit the initial project velocity were accomplished using steel slugs added to a bored section of the polycarbonate sabot. This ensured that both the front anvil and projectile remained elastic, which allowed the stress states to be determined from elastic impedance matching. Shots 1 and 2 performed for the current study were the first attempts at MUSPL to limit final velocity in a controllable way. The anvils recovered post-test for shots 1 and 2 showed considerable levels of plastic deformation, which introduces uncertainty into whether they were able to transmit shear waves in an easily predictable fashion. With a few tweaks to the projectile and pressurizing methodology, the initial velocity was varied in a controllable manor under 100 m/s for the remaining 10 shots. This velocity limit was established using the von mises yield criterion described in section 4.1.1.

The measurement of normal and transverse surface velocity from the back of the target capsule was used to calculate the normal and shear stress imparted on the sand. Tracking the propagation of elastic waves within the rear anvil allowed for multiple combined loading states to be inferred for a single shot. What started out to be a design constraint, turned into a useful tool for probing the shear strength at multiple points in stress-density space. A relationship between normal stress and shear stress was measured from the experimental data and parameters for a modified Cam Clay failure model and exponent-shape shear failure yield surface were calculated. Many different pressure-dependent constitutive models exist for capturing the strength of granular materials. The current data set provides experimental measurements for which to parameterize these models.

Hammi [43], Almonstötter [5], and Han [44] presented multiple analytic formulations for density dependent failure cap models, which presents an interesting avenue for future work to explore the applicability of these models for dynamic loading. However, a better method to calculate density is needed for these experiments if density-dependent yield surfaces such as those proposed by Hammi [43] are to be parameterized. The density calculation stems from an approximation used to calculate axial strain rate that is mostly applicable as the velocity of the front anvil approaches that of the rear anvil, equation 6.17. Since granular materials take an extended amount of time to load, i.e. for the normal velocity to reach the expected equilibrium plateau value, the accumulated strain is too large and the resulting density in some cases results in densities much higher than that of the constituent material. Therefore, the applicability of this approximation should be further explored in future works.
Each of the shots revealed a significant amount of damage to the individual sand grains. This is not an unexpected result given that SiO$_2$ is brittle and has many different modes in which individual grains can fail. The magnitude of stress states achieved in this study were up to roughly 0.10 - 1.25 GPa in normal stress. Therefore, the sand samples were loaded up to roughly 25% of the limit at which full compaction was observed to occur from single shock loading, approximately 4 GPa [60]. As normal stress is increased from zero to the precursor limit, approximately 0.2 GPa, it is believed that compaction is occurring and stress bridges begin to form. As the stress is increased above this limit, the locked grain networks begin to fail as individual grains fracture. From this point up until full-compaction, i.e. zero porosity, many mesoscale mechanisms contribute to the complexity observed in the macroscopic response. Shots with multiple stress states, 3, 4, 5, 6, 8, 10, 11, and 12 all seem to end up near the linear portion of the failure surface. This supports the idea that grain fragments can re-compact and then again support shear stress at a new densified state. Future studies could aim to simultaneously populate the stress-density space as well as the normal stress - shear stress space, thereby investigating the evolution of potential yield cap surfaces. This would enable the population of density-dependent yield cap models such as that proposed by Hammi [43], Almonstötter [5], and Han [44].

In conclusion, the current work provided novel insights into the dynamic pressure-shear behavior of sand by obtaining pressure-dependent shear stress measurements as well as observing the emergence of potential failure caps. Advances to diagnostic capabilities were made through the design, characterization, and implementation of a PDV system used to make measurements of transverse surface velocities. Additionally, the measurement uncertainty resulting from a variety of collimator arrangements was quantified, which is useful for future works. Lastly, three-dimensional mesoscale simulations mimicking the experimental pressure-shear loading conditions were able to capture the shearing resistance of sand, but did not result in a failure cap, which simultaneously highlighted strengths and weaknesses of a well-established hydrocode.


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APPENDIX A

ERROR ANALYSIS

In this study, error (uncertainty) is defined in two forms: random and systematic [91]. Random uncertainty is a result of deviation between multiple measurements of the same value. If the quantity $x$ is measured $N$ times, the final value will be taken as the average, $\bar{x}$, and the uncertainty is taken as the standard deviation of the mean (SDOM):

$$\delta x_{\text{ran}} = \sigma_{\bar{x}} = \frac{\sigma_x}{\sqrt{N}}$$  \hspace{1cm} (A.1)

where $\sigma_x$ is the standard deviation across all measurements of quantity $x$. Therefore, it is useful to take a sizable number of measurements to minimize $\sigma_{\bar{x}}$. Systematic uncertainty is a result of measurement uncertainty from the observer or the device making the measurement. For example, an optical translation stage lists its uncertainty as $\pm 0.01$ mm. This uncertainty should be noted as the systematic error, $\delta x_{\text{sys}}$, inherent in the position of the translation stage. If both forms or uncertainty are present in a measurement, or series of measurements, of $x$, the random and systematic uncertainties can be listed in one of two ways. Either by explicitly listing both,

$$(\text{measured value of } x) = x_{\text{best}} \pm \delta x_{\text{ran}} \pm \delta x_{\text{sys}}$$  \hspace{1cm} (A.2)

or providing a single measure of uncertainty via addition in quadrature

$$\delta x = \sqrt{(\delta x_{\text{ran}})^2 + (\delta x_{\text{sys}})^2}.$$  \hspace{1cm} (A.3)

It should be noted that $x_{\text{best}}$ in equation A.2 is typically taken as the average value, $\bar{x}$. If a quantity is dependent on multiple variables that have uncertainty, uncertainty propagation must be used to quantify the contribution of each independent variable and its uncertainty to the dependent variable uncertainty. If multiple independent variables, $(x_1, ..., x_M)$, are needed to calculate a certain quantity, $q$, such that $q = q(x_1, ..., x_M)$, where $M$ is the number of independent variables, then uncertainty propagation must be used to calculate $\delta q$. If all independent variable are independent and random, the uncertainty has the form

$$\delta q = \sqrt{\left(\frac{\partial q}{\partial x_1}\delta x_1\right)^2 + \cdots + \left(\frac{\partial q}{\partial x_M}\delta x_M\right)^2}.$$  \hspace{1cm} (A.4)

If there is reason to believe the independent variables, $(x_1, ..., x_M)$, are not independent and random from each other, the worst case uncertainty in $q$ is

$$\delta q \leq \left|\frac{\partial q}{\partial x_1}\right| \delta x_1 + \cdots + \left|\frac{\partial q}{\partial x_M}\right| \delta x_M.$$  \hspace{1cm} (A.5)
A weighted average was used to calculate many of the velocity states, most importantly the transverse velocity state associated with experimental shear measurements. The average value, \( \bar{x} \) was calculated using

\[
\bar{x} = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}.
\]  

(A.6)

where \( w_i \) is the weight for each \( x_i \) value and is calculated from the uncertainty as

\[
w_i = \left( \frac{1}{\sigma_i} \right)^2.
\]  

(A.7)

The uncertainty in the weighted average is then

\[
\bar{\sigma} = \left( \sum_{i=1}^{N} w_i \right)^{-1/2}.
\]  

(A.8)
APPENDIX B

EXPERIMENTAL SETUP

B.1 Sound Speed Measurements

Obtaining p- and s-wave measurements for sand has historically been non-trivial and the data can vary greatly. Previous unpublished work by Beavers et al [12] at Marquette University has sought to obtain a relationship between stress, density, and p-wave speed in Ottawa sand samples. The work built a sand capsule that could be mounted into a material testing machine (MTS) with places for pulse-receivers to make sound speed measurements. P-wave measurements were made for stresses up to 4 MPa and densities around 65% TMD. Figure B.1 shows the sound speed data as a function of density as well as stress as a function of density. The data shows that for a sample loaded and unloaded for three cycles, p-wave speed increases roughly 50% for about a 2% increase in density. Also, for a stress increase of 3 - 5 MPa, the density changes from roughly 1.635 to 1.660 g/cm$^3$. Throughout the entire loading process, grains primarily remained intact and did not fracture such as they are known to do in dynamic experiments.

Each sand sample prepared for the present study attempted to make p- and s-wave measurements using the pulse-receiver technique, but values were only obtained for shots 7 - 12.

![Graph A](image1)

![Graph B](image2)

(a) (b)
Both longitudinal and shear sound speed measurements were made for each type of material using an Olympus High Voltage Pulse/Receiver, Model 5058PR, figure B.1. Each anvil for a given pressure-shear test was evaluated prior to assembly and then a measurement of the entire package was made after the sand was packed. For projectile nose pieces, a right cylindrical section of the stock material was tested prior to milling the inclined impactor face. The pulse/receive method was used for each of the sound speed measurements. Inherent with this technique is a delay between the pulse transmitter and the receiver. Therefore, this delay was measured first by placing a small amount of couplant, either ultrasound gel for p-wave and Olympus Shear Wave Couplant (Product Number SWC-2) for s-wave, on the receiver and then pressing the transmitter directly onto the receiver. A clear difference between the transmission and arrival of the pulse was apparent for each test, which raises an important point for these measurements. The size of the delay is directly related to the couplant thickness, therefore, the thinnest possible layer that still enables clean detection of the pulse on the receiver is desired. Once the delay was quantified, a sample, less than 1” thick, was placed between the transmitter and receiver and the time of transmission was observed on the oscilloscope. The final transit time through the material was calculated by subtracting the delay time from the total time of transmission. An example image showing the pulse, delay, and received signal is shown in figure B.2. Table B.3 provides the measured sound speed for each of the anvils used in the experiments.

B.2 Anvil and Projectile Surface Preparation Techniques

The faces of anvils and projectile nose pieces were prepped first by grinding and then directionally sanding using sandpaper. Grinding consistently produced surface roughness on the order of 0.5 µm and was directionally invariant. The projectile face and front and rear of both anvils were then sanded using sand paper, 100 grit for 1045 steel and 200 grit for aluminum 7075
Figure B.2: Example output of the pulse receiver used for sound speed measurements. The difference in time of arrival between the signals is taken as the transit time across the sample.

Figure B.3: Steel anvil post sanding with visible surface roughness

T6. This roughness was added to aid in the transmission of shear between the projectile-driver, driver-sand, and sand-anvil interfaces. Additionally, directional sanding was shown to adequately diffuse incident laser light for transverse particle velocity measurements, see appendix C. Sanding was performed perpendicular to the direction of shear for each surface by placing the sand paper on a steel flat and sliding the part back and forth roughly 40 times. The metal surface had visible directional surface roughness after sanding was completed, figure B.3.
B.3 Density Calculation for Sand Samples

As described in the target preparation section, 4.2.1, the sand targets were packed using a 0.125” fill hole and the density was calculated in the typical mass to volume ratio. Measuring the amount of mass inserted into the sample domain and calculating the volume of sample domain were, however, non-trivial. Since the combined mass of the targets that had steel anvils was too much for the range of the fine scale, the mass of sand could not be directly measured by comparing the mass of the filled and empty sample. Therefore, the mass of sand inserted into the target was measured by placing a small container of sand into the scale, zeroing the scale, and then recording the amount of sand removed from the container as it was poured into the capsule. This was not an ideal method, however, one born from desperation. Once the capsule was filled, a rough mass measurement of the entire capsule was made using the rough scale, which read to the 0.1 g. Next, the volume of the sand domain was calculated using the sample thickness and area of the sand-anvil interface. Sample thickness was measured as:

\[ h_{sand} = h_{total} - h_{front\ anvil} - h_{rear\ anvil} \] (B.1)
Figure B.5: Schematic describing proportions of sand domain area

where $h_{\text{total}}$ is the thickness of the assemble driver-sand-anvil capsule and $h_{\text{front anvil}}$ and $h_{\text{rear anvil}}$ are the thicknesses of the front and rear anvil, respectively. Shots 3 - 6 incorporated a rear anvil slot and, therefore, had a reduced volume. The surface area of the sand domain was calculated by knowing the sample radius, $R$, and the amount that was cut off from the sample for the front surface measurement, $r$. The area was thereby calculated by integrating the equation for a circle over the bounds of the sand domain:

$$A_{\text{sand}} = \int_{-R}^{r} \sqrt{R^2 - x^2} \, dx = \int_{-R}^{0} \sqrt{R^2 - x^2} \, dx + 2 \int_{0}^{r} \sqrt{R^2 - x^2} \, dx = \frac{1}{2} \pi R^2 + 2 \int_{0}^{r} \sqrt{R^2 - x^2} \, dx = a_1 + 2a_2$$

where

$$a_1 = \frac{1}{2} \pi R^2$$

and

$$a_2 = \frac{1}{2} \left( (R - r) \sqrt{R^2 - (R - r)^2} + R^2 \tan^{-1} \left( \frac{R - r}{\sqrt{R^2 - (R - r)^2}} \right) \right).$$

The surface area for shots 1, 2, and 7 - 10 was calculated in the typical form $A = \pi r^2$. Volume of the sand capsule was then calculated from the surface area and thickness, $V = Ah$. 
B.4 Photon Doppler Velocimeter Setup

Each experiment in the current test series used a target and reference laser to provide a non-zero beat frequency and better contrast in the spectral analysis. This setup, as explained in section 4.3.1, is considered a Heterodyne velocimeter. Figures B.6, B.7, and B.8 show the arrangement of optical components for each range of experiments. A target laser with output power of approximately 10 mW, and wavelength $\lambda_{\text{target}}$, is sent to a booster, which amplifies the power of the signal to around 500 mW. The boosted signal is then split into four individual channels with a 1x4 splitter. From this point, individual setups begin to distinguish themselves. For the case of an active Heterodyne channel, light of wavelength $\lambda_{\text{target}}$ is sent to a circulator where it enters connection one, exits connection two, and is delivered to the target using an optical collimator or "PDV probe." The reflected light, $\lambda'_{\text{target}}$, from the target is recollected and a beat frequency of wavelength $\lambda_0$ is formed. This combined signal then exits connection three of the circulator. Reference light with a power of 20 mW and wavelength $\lambda_{\text{ref}}$ is then added using a 2x1 splitter to form a final beat frequency of $\lambda_b$. The signal is then digitized using a photo receiver and sent to the oscilloscope for recording.

For the case of a passive Heterodyne channel, light from the 1x4 splitter is capped so the light from the target laser does not continue to propagate. However, an optical collimator is aligned to the target surface where it collects diffuse reflection of light emitted from an active channel. The collected light has the same initial wavelength as the active channel, but the apparent frequency shift is dependent on the angle the collimator makes relative to the angle of incidence of the emitted light, as detailed in equation 4.13. Light collected on the passive collimator is then combined with reference in the same fashion as an active channel and then digitized using a photo receiver. Each of the shots in the experimental test series used a combination of active and passive channels. The benefits of using active versus passive probes are discussed in appendix C.

Shot 3 - 6 utilized six PDV probes, which required additional optical components to make the, originally four channel, system work. Figure B.6 provides a schematic of the setup. Three probes were aligned to the rear surface of the target; one active, normal and two passive, angled. Two probes were aligned to the rear surface of the driver through the anvil slot; one active, normal and one passive, angled. The last probe was mounted in a through hole in the target to reflect off the projectile face just before impact, also known as a barrel probe. The face of the barrel probe was proud from the target face such that impact would cause the signal to drop out.
Therefore, the barrel probe was multiplexed, using a 1x2 splitter, with the rear target surface normal probe considering the normal probe signal would not pickup until the wave arrived at the rear target surface; multiple microseconds after impact. This combined channel was then digitized using the first photo receiver. Both passive probes on the rear target surface operated as passive _Heterodyne channels_ and were digitized separately on individual photo receivers. The active and passive probes focused to the rear driver surface were electronically multiplexed, i.e. using a BNC splitter, post photo receiver. Since both signals recorded on these probes were expected to rise at the same time, a time delay leg was added to the normal probe channel, which provided a 30 µs delay. The normal channel was delayed because the light return on the angled, passive probe was expected to drop out after roughly 10 - 20 µs as the surface moved. This phenomena is discussed further in appendix C. Using these additional components, 6 signals were condensed onto 4 oscilloscope channels. The signals were then decoupled during analysis in PlotData.

Shots 7 - 10 did not have a slotted anvil. Therefore, they utilized four PDV probes, instead of the six used in shots 3 - 6. Figure B.7 provides a schematic of the setup. The rear surface normal probe and barrel probe were again multiplexed and the passive probes on the rear surface were digitized on their own photo receivers; identical to shots 3 - 6. However, in an attempt to recreate a diagnostic method first described by Chen et al [23] and then later by Zuanetti et al [104], each passive channel was split using a 1x2 splitter post amplification. A signal from each passive channel was then combined with a 2x1 splitter and digitized, without the addition off reference light, on a single photo receiver. The combination of signals from these probes, angled symmetrically away from the surface normal, creates a single Homodyne PDV channel with a beat frequency that corresponds directly to the magnitude of transverse surface velocity, with zero contribution from the normal component of surface velocity. This direct measure of transverse surface velocity has a number of advantages compared to other methods that require the transverse signal to be decoupled. However, the small magnitude of transverse velocity, \( v_t < 20 \text{ m/s} \), results in a small beat frequency shift that is difficult to capture at the time scales necessary to resolve the wave arrival. This dilemma is explained further in section C. A direct measure of transverse surface velocity was not achieved for shots 7 - 10, so the additional combination of passive probes was abandoned for shots 11 and 12. Figure B.8 provides a schematic of the setup. The barrel probe, rear target surface probes, and angle passive probes were all digitized on their photo receivers and recorded on individual oscilloscope channels. This setup was the easiest to implement because it converted a traditional four active channel PDV
system to a two-active, two-passive system by capping two channels of the 1x4 input splitter. If additional power is needed for the active channels, a 1x2 splitter could be used to split the power 50:50 instead of a 25:25:25:25 split resulting from a 1x4.

Figure B.6: Schematic of PDV setup used for shots 3 - 6. Note: red arrows represent an active/emitting collimator and blue arrows represent passive/collecting collimators.
Figure B.7: Schematic of PDV setup used for shots 7 - 10. Note: red arrows represent an active/emitting collimator and blue arrows represent passive/collecting collimators.

Figure B.8: Schematic of PDV setup used for shots 11 and 12. Note: red arrows represent an active/emitting collimator and blue arrows represent passive/collecting collimators.
B.5 Data Tables

The following tables are for measured and calculated properties of the anvils, projectiles, and sand samples.

Table B.1: Anvil thickness and surface roughness for each shot. Inner surface implies metal-sand interface. Outer implies free-surface. Each surface roughness was calculated from an average of 11 individual measurements around the surface of the anvil. Uncertainty was calculated from the standard deviation across the 11 measurements.

<table>
<thead>
<tr>
<th>Shot #</th>
<th>Material</th>
<th>Front/Rear Anvil</th>
<th>Thickness (mm)</th>
<th>Surface Roughness Inner - $R_a$ (µm)</th>
<th>Surface Roughness Outer - $R_a$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1045 Steel</td>
<td>Front</td>
<td>4.003 ± 0.005</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>4.003 ± 0.008</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>2</td>
<td>1045 Steel</td>
<td>Front</td>
<td>4.003 ± 0.005</td>
<td>0.456 ± 0.039</td>
<td>0.685 ± 0.120</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>4.007 ± 0.008</td>
<td>0.515 ± 0.070</td>
<td>0.481 ± 0.050</td>
</tr>
<tr>
<td>3</td>
<td>1045 Steel</td>
<td>Front</td>
<td>4.037 ± 0.001</td>
<td>0.497 ± 0.038</td>
<td>0.488 ± 0.089</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>4.019 ± 0.001</td>
<td>0.818 ± 0.083</td>
<td>0.600 ± 0.061</td>
</tr>
<tr>
<td>4</td>
<td>1045 Steel</td>
<td>Front</td>
<td>4.036 ± 0.001</td>
<td>0.586 ± 0.149</td>
<td>0.554 ± 0.084</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>4.037 ± 0.003</td>
<td>0.580 ± 0.084</td>
<td>0.509 ± 0.093</td>
</tr>
<tr>
<td>5</td>
<td>1045 Steel</td>
<td>Front</td>
<td>4.034 ± 0.001</td>
<td>0.551 ± 0.087</td>
<td>0.514 ± 0.070</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>4.035 ± 0.002</td>
<td>0.489 ± 0.060</td>
<td>0.557 ± 0.091</td>
</tr>
<tr>
<td>6</td>
<td>1045 Steel</td>
<td>Front</td>
<td>4.064 ± 0.000</td>
<td>0.605 ± 0.052</td>
<td>0.667 ± 0.053</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>4.036 ± 0.002</td>
<td>0.368 ± 0.065</td>
<td>0.495 ± 0.068</td>
</tr>
<tr>
<td>7</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>4.254 ± 0.003</td>
<td>0.539 ± 0.042</td>
<td>0.628 ± 0.188</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>6.067 ± 0.006</td>
<td>0.723 ± 0.233</td>
<td>0.563 ± 0.153</td>
</tr>
<tr>
<td>8</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>4.225 ± 0.007</td>
<td>0.434 ± 0.046</td>
<td>0.517 ± 0.071</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>6.213 ± 0.009</td>
<td>0.781 ± 0.171</td>
<td>0.441 ± 0.162</td>
</tr>
<tr>
<td>9</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>4.236 ± 0.016</td>
<td>0.473 ± 0.153</td>
<td>0.566 ± 0.158</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>6.250 ± 0.009</td>
<td>0.467 ± 0.108</td>
<td>0.428 ± 0.118</td>
</tr>
<tr>
<td>10</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>4.229 ± 0.011</td>
<td>0.431 ± 0.077</td>
<td>0.479 ± 0.102</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>6.243 ± 0.021</td>
<td>0.372 ± 0.038</td>
<td>0.423 ± 0.054</td>
</tr>
<tr>
<td>11</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>4.006 ± 0.002</td>
<td>0.386 ± 0.086</td>
<td>0.365 ± 0.038</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>8.005 ± 0.003</td>
<td>0.359 ± 0.077</td>
<td>0.300 ± 0.042</td>
</tr>
<tr>
<td>12</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>4.003 ± 0.003</td>
<td>0.277 ± 0.035</td>
<td>0.267 ± 0.103</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Rear</td>
<td>8.003 ± 0.001</td>
<td>0.332 ± 0.070</td>
<td>0.283 ± 0.031</td>
</tr>
</tbody>
</table>
Table B.2: Measured inclination angle and surface roughness for each projectile nose piece. Shots 3 - 6 calculated the angled using a height gage and caliper. Shots 7 - 10 used a coordinate mapping machine (CMM) to map the plane of the projectile face and then determine the angle of inclination. Figure B.4 shows a surface roughness measurement being made on a nose piece.

<table>
<thead>
<tr>
<th>Shot #</th>
<th>Material</th>
<th>Inclination Angle (deg)</th>
<th>Surface Roughness $R_a$ (µm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1045 Steel</td>
<td>n/a ± 0.05</td>
<td>n/a</td>
</tr>
<tr>
<td>2</td>
<td>1045 Steel</td>
<td>n/a ± 0.05</td>
<td>0.446 ± 0.070</td>
</tr>
<tr>
<td>3</td>
<td>1045 Steel</td>
<td>17.06 ± 0.04</td>
<td>0.540 ± 0.167</td>
</tr>
<tr>
<td>4</td>
<td>1045 Steel</td>
<td>17.04 ± 0.04</td>
<td>0.472 ± 0.060</td>
</tr>
<tr>
<td>5</td>
<td>1045 Steel</td>
<td>16.98 ± 0.04</td>
<td>0.402 ± 0.071</td>
</tr>
<tr>
<td>6</td>
<td>1045 Steel</td>
<td>17.03 ± 0.04</td>
<td>0.504 ± 0.090</td>
</tr>
<tr>
<td>7</td>
<td>Al 7076 T6</td>
<td>17.05 ± 0.04</td>
<td>0.451 ± 0.043</td>
</tr>
<tr>
<td>8</td>
<td>Al 7076 T6</td>
<td>16.80 ± 0.01</td>
<td>0.615 ± 0.050</td>
</tr>
<tr>
<td>9</td>
<td>Al 7076 T6</td>
<td>16.99 ± 0.01</td>
<td>0.567 ± 0.046</td>
</tr>
<tr>
<td>10</td>
<td>Al 7076 T6</td>
<td>17.04 ± 0.01</td>
<td>0.518 ± 0.082</td>
</tr>
<tr>
<td>11</td>
<td>1045 Steel</td>
<td>17.59 ± 0.04</td>
<td>0.332 ± 0.086</td>
</tr>
<tr>
<td>12</td>
<td>1045 Steel</td>
<td>17.35 ± 0.04</td>
<td>0.290 ± 0.078</td>
</tr>
</tbody>
</table>

Table B.3: Anvil p- and s-wave speeds measured using pulse-receiver technique.

<table>
<thead>
<tr>
<th>Shot #</th>
<th>Material</th>
<th>Front/Rear Anvil</th>
<th>P-Wave (km/s)</th>
<th>S-Wave (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1045 Steel</td>
<td>Front</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>2</td>
<td>1045 Steel</td>
<td>Rear</td>
<td>n/a</td>
<td>n/a</td>
</tr>
<tr>
<td>3</td>
<td>1045 Steel</td>
<td>Front</td>
<td>5.747 ± 0.018</td>
<td>2.601 ± 0.11</td>
</tr>
<tr>
<td>4</td>
<td>1045 Steel</td>
<td>Rear</td>
<td>5.722 ± 0.020</td>
<td>2.840 ± 0.13</td>
</tr>
<tr>
<td>5</td>
<td>1045 Steel</td>
<td>Front</td>
<td>5.735 ± 0.016</td>
<td>2.859 ± 0.12</td>
</tr>
<tr>
<td>6</td>
<td>1045 Steel</td>
<td>Rear</td>
<td>5.741 ± 0.016</td>
<td>2.842 ± 0.12</td>
</tr>
<tr>
<td>11</td>
<td>1045 Steel</td>
<td>Front</td>
<td>5.611 ± 0.016</td>
<td>2.901 ± 0.13</td>
</tr>
<tr>
<td>12</td>
<td>1045 Steel</td>
<td>Rear</td>
<td>5.809 ± 0.009</td>
<td>3.155 ± 0.08</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>5.757 ± 0.049</td>
<td>2.922 ± 0.130</td>
</tr>
<tr>
<td>7</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>6.403 ± 0.018</td>
<td>2.894 ± 0.12</td>
</tr>
<tr>
<td>8</td>
<td>Al 7076 T6</td>
<td>Rear</td>
<td>6.166 ± 0.014</td>
<td>3.009 ± 0.09</td>
</tr>
<tr>
<td>9</td>
<td>Al 7076 T6</td>
<td>Front</td>
<td>6.070 ± 0.020</td>
<td>2.946 ± 0.13</td>
</tr>
<tr>
<td>10</td>
<td>Al 7076 T6</td>
<td>Rear</td>
<td>6.091 ± 0.014</td>
<td>3.016 ± 0.10</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td></td>
<td>6.165 ± 0.106</td>
<td>2.985 ± 0.051</td>
</tr>
</tbody>
</table>
Table B.4: Collimator angle relative to the surface normal measured with translation stage and calculated from linear fit from York [102].

<table>
<thead>
<tr>
<th>Shot #</th>
<th>Positive Probe (deg)</th>
<th>Negative Probe (deg)</th>
<th>Channel Probe (deg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>19.92 ± 0.26</td>
<td>20.21 ± 0.26</td>
<td>20.24 ± 0.26</td>
</tr>
<tr>
<td>2</td>
<td>20.98 ± 0.32</td>
<td>23.56 ± 0.26</td>
<td>19.93 ± 0.38</td>
</tr>
<tr>
<td>3</td>
<td>21.88 ± 0.34</td>
<td>20.41 ± 0.26</td>
<td>21.70 ± 0.34</td>
</tr>
<tr>
<td>4</td>
<td>23.32 ± 0.30</td>
<td>22.73 ± 0.22</td>
<td>21.31 ± 0.24</td>
</tr>
<tr>
<td>5</td>
<td>21.96 ± 0.24</td>
<td>21.88 ± 0.24</td>
<td>21.93 ± 0.28</td>
</tr>
<tr>
<td>6</td>
<td>20.89 ± 0.70</td>
<td>20.09 ± 0.31</td>
<td>n/a</td>
</tr>
<tr>
<td>7</td>
<td>20.14 ± 0.26</td>
<td>21.32 ± 0.28</td>
<td>n/a</td>
</tr>
<tr>
<td>8</td>
<td>20.20 ± 0.32</td>
<td>19.97 ± 0.26</td>
<td>n/a</td>
</tr>
<tr>
<td>9</td>
<td>20.23 ± 0.26</td>
<td>21.83 ± 0.28</td>
<td>n/a</td>
</tr>
<tr>
<td>10</td>
<td>19.53 ± 0.30</td>
<td>20.13 ± 0.48</td>
<td>n/a</td>
</tr>
<tr>
<td>11</td>
<td>19.53 ± 0.30</td>
<td>20.50 ± 0.30</td>
<td>n/a</td>
</tr>
</tbody>
</table>

Table B.5: Bulk properties of each material used in the mesoscale simulations

<table>
<thead>
<tr>
<th>Material</th>
<th>Density $\rho$ (g/cm$^3$)</th>
<th>P-Wave Speed, $C_L$ (km/s)</th>
<th>S-Wave Speed, $C_S$ (km/s)</th>
<th>Yield Strength, $Y$ (GPa)</th>
<th>Poisson Ratio $\nu$</th>
<th>Fracture Strength (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al 7075 T6</td>
<td>2.813</td>
<td>6.667</td>
<td>3.577</td>
<td>20.00*</td>
<td>0.32</td>
<td>5.00</td>
</tr>
<tr>
<td>Steel 4340</td>
<td>7.872</td>
<td>6.033</td>
<td>3.272</td>
<td>20.00*</td>
<td>0.29</td>
<td>5.00</td>
</tr>
<tr>
<td>Quartz</td>
<td>2.650</td>
<td>n/a</td>
<td>n/a</td>
<td>5.00</td>
<td>0.15</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table B.6: Calculated normal and shear stress states for each shot. * signifies that the shear wave had not arrived at free surface within the first shear window.

<table>
<thead>
<tr>
<th>Shot #</th>
<th>$\sigma_1$ (GPa)</th>
<th>$\tau_1$ (GPa)</th>
<th>$\sigma_2$ (GPa)</th>
<th>$\tau_2$ (GPa)</th>
<th>$\sigma_3$ (GPa)</th>
<th>$\tau_3$ (GPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.986 ± 0.014</td>
<td>*</td>
<td>1.236 ± 0.020</td>
<td>0.015 ± 0.008</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>0.980 ± 0.014</td>
<td>*</td>
<td>1.111 ± 0.017</td>
<td>0.010 ± 0.015</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>0.412 ± 0.006</td>
<td>*</td>
<td>0.455 ± 0.008</td>
<td>0.010 ± 0.008</td>
<td>0.599 ± 0.008</td>
<td>0.059 ± 0.008</td>
</tr>
<tr>
<td>4</td>
<td>0.746 ± 0.010</td>
<td>*</td>
<td>0.447 ± 0.013</td>
<td>0.051 ± 0.011</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>0.512 ± 0.007</td>
<td>*</td>
<td>0.325 ± 0.008</td>
<td>0.048 ± 0.007</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>0.458 ± 0.006</td>
<td>*</td>
<td>0.405 ± 0.008</td>
<td>0.000 ± 0.007</td>
<td>0.434 ± 0.008</td>
<td>0.085 ± 0.006</td>
</tr>
<tr>
<td>7</td>
<td>0.569 ± 0.021</td>
<td>0.068 ± 0.016</td>
<td>0.042 ± 0.021</td>
<td>0.000 ± 0.015</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>0.206 ± 0.007</td>
<td>0.010 ± 0.009</td>
<td>0.184 ± 0.093</td>
<td>0.044 ± 0.010</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>0.785 ± 0.028</td>
<td>0.006 ± 0.013</td>
<td>0.075 ± 0.076</td>
<td>0.000 ± 0.012</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>10</td>
<td>0.279 ± 0.010</td>
<td>0.027 ± 0.009</td>
<td>0.166 ± 0.011</td>
<td>0.033 ± 0.012</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>11</td>
<td>1.139 ± 0.015</td>
<td>0.000 ± 0.004</td>
<td>0.279 ± 0.016</td>
<td>0.052 ± 0.006</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>12</td>
<td>0.252 ± 0.003</td>
<td>0.000 ± 0.003</td>
<td>0.288 ± 0.004</td>
<td>0.052 ± 0.004</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
Table B.7: Initial thickness, volume, mass, and packed densities for shots 2 - 10.

<table>
<thead>
<tr>
<th>Shot #</th>
<th>Thickness (mm)</th>
<th>Volume (cm³)</th>
<th>Mass (g)</th>
<th>Density (g/cm³)</th>
<th>Packed Density (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.472 ± 0.002</td>
<td>0.650 ± 0.025</td>
<td>0.721 ± 0.001</td>
<td>1.108 ± 0.042</td>
<td>41.8</td>
</tr>
<tr>
<td>3</td>
<td>0.518 ± 0.005</td>
<td>0.715 ± 0.011</td>
<td>1.305 ± 0.001</td>
<td>1.810 ± 0.027</td>
<td>68.3</td>
</tr>
<tr>
<td>4</td>
<td>0.544 ± 0.009</td>
<td>0.750 ± 0.015</td>
<td>1.350 ± 0.001</td>
<td>1.753 ± 0.037</td>
<td>66.2</td>
</tr>
<tr>
<td>5</td>
<td>0.518 ± 0.010</td>
<td>0.715 ± 0.017</td>
<td>1.320 ± 0.001</td>
<td>1.786 ± 0.044</td>
<td>67.4</td>
</tr>
<tr>
<td>6</td>
<td>0.494 ± 0.009</td>
<td>0.707 ± 0.016</td>
<td>1.290 ± 0.001</td>
<td>1.825 ± 0.045</td>
<td>69.0</td>
</tr>
<tr>
<td>7</td>
<td>0.504 ± 0.012</td>
<td>0.784 ± 0.020</td>
<td>1.363 ± 0.001</td>
<td>1.739 ± 0.044</td>
<td>65.6</td>
</tr>
<tr>
<td>8</td>
<td>0.538 ± 0.013</td>
<td>0.835 ± 0.021</td>
<td>1.460 ± 0.001</td>
<td>1.748 ± 0.044</td>
<td>66.0</td>
</tr>
<tr>
<td>9</td>
<td>0.492 ± 0.020</td>
<td>0.756 ± 0.032</td>
<td>1.304 ± 0.001</td>
<td>1.703 ± 0.072</td>
<td>64.2</td>
</tr>
<tr>
<td>10</td>
<td>0.470 ± 0.024</td>
<td>0.731 ± 0.038</td>
<td>1.265 ± 0.001</td>
<td>1.703 ± 0.091</td>
<td>65.3</td>
</tr>
<tr>
<td>11</td>
<td>0.424 ± 0.005</td>
<td>0.672 ± 0.010</td>
<td>1.131 ± 0.001</td>
<td>1.683 ± 0.025</td>
<td>63.5</td>
</tr>
<tr>
<td>12</td>
<td>0.442 ± 0.010</td>
<td>0.702 ± 0.011</td>
<td>1.156 ± 0.001</td>
<td>1.687 ± 0.028</td>
<td>63.6</td>
</tr>
</tbody>
</table>

Table B.8: Zero-stress p- and s-wave speeds of SiO₂ sand for shots 7 - 12 measured using the pulse receiver technique.

<table>
<thead>
<tr>
<th>Shot #</th>
<th>P-wave, ( C_L ) (km/s)</th>
<th>S-wave, ( C_S ) (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>4</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>5</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>6</td>
<td>--</td>
<td>--</td>
</tr>
<tr>
<td>7</td>
<td>0.517 ± 0.016</td>
<td>0.210 ± 0.005</td>
</tr>
<tr>
<td>8</td>
<td>0.596 ± 0.017</td>
<td>0.214 ± 0.004</td>
</tr>
<tr>
<td>9</td>
<td>0.580 ± 0.018</td>
<td>0.201 ± 0.004</td>
</tr>
<tr>
<td>10</td>
<td>0.554 ± 0.018</td>
<td>0.214 ± 0.004</td>
</tr>
<tr>
<td>11</td>
<td>0.568 ± 0.020</td>
<td>0.183 ± 0.005</td>
</tr>
<tr>
<td>12</td>
<td>0.528 ± 0.017</td>
<td>0.186 ± 0.004</td>
</tr>
<tr>
<td>Average</td>
<td>0.555 ± 0.018</td>
<td>0.202 ± 0.004</td>
</tr>
</tbody>
</table>
APPENDIX C

TRANSVERSE VELOCITY MEASUREMENTS USING PHOTON DOPPLER VELOCIMETRY

Dolan [31] described what PDV measures, i.e. what a collimator observes and what we then interpret as velocity by describing the moving surface as a series of micro reflectors at quasi-random orientations relative to the surface normal. The apparent velocity observed on a collimator is not only a function of the angle it makes relative to the surface normal, but also a function of the angle the incident, un-shifted, light made relative to the surface normal. Therefore, apparent velocity, $V^*$, is defined as

\[
V^* = \vec{V} \cdot \left( \frac{\hat{r} - \hat{s}}{2} \right) = \frac{V_N}{2} (\cos \alpha + \cos \beta) + \frac{V_T}{2} (\sin \alpha + \sin \beta) + \frac{V_E}{2} (\sin \gamma + \sin \phi)
\]  

(C.1)

where $\hat{r}$ is the reflected light vector, $\hat{s}$ is the incident light vector, $V_N$ is the normal component of velocity, $V_T$ is the transverse component of velocity, $V_E$ is the out-of-plane component of velocity, $\alpha$ is the angle between observed light and the surface normal, $\beta$ is the angle between the incident light and the normal vector, and $\gamma$ and $\phi$ are angles of the out-of-plane components of velocity. In most cases where minimal tilt is present, the out of plane velocity is assumed to be negligible. Therefore, it is not shown in future definitions of apparent and transverse velocity. It should be stated that the following methods are based on the fact that all collimators are aligned onto the same spot. This is important for experiments involving heterogeneous materials that result in velocity profiles that vary spatially. Experiments on homogeneous materials have much less spatial variation, therefore, aligning collimators onto the same spot is not as relevant of an issue. “Normal” refers to a collimator that is aligned normal to the surface and “angled” refers to collimator that has a non-zero angle relative to the surface normal. “Active” refers to a collimator
that is emitting light and “passive” refers to a collimator that is not emitting light. Typically, active collimators are used to both emit and collect light and passive collimators are used to only collect light emitted from a different, active collimator.

C.1 Active - Passive Configurations

C.1.1 Method 1A - Active Normal, Single Passive Angled

The first method involves an active collimator aligned parallel to the surface normal (referred to as active, normal) and an additional passive collimator at some non-zero angle relative to the surface normal (referred to as passive, angled). The active, normal collimator emits and collects its own light, making a direct measurement of the normal component of surface velocity. The passive, angled collimator collects diffusely reflected light emitted from the active, normal collimator and observes a component of both the normal and transverse velocity. Since the incident light emitted from the active probe is parallel to the surface normal, $\beta = 0$, and the diffuse reflected light is collected at some angle, $\alpha$, relative to the surface normal, the apparent velocity is defined as

$$V^* = \frac{V_N}{2} (\cos \alpha + 1) + \frac{V_T}{2} (\sin \alpha) \quad \text{(C.2)}$$

Rearranging the apparent velocity, equation C.2, the transverse velocity is defined as

$$V_T = \frac{1}{\sin \alpha} (2V^* - V_N (\cos \alpha + 1)) \quad \text{(C.3)}$$

C.1.2 Method 1B - Active Normal, Double Passive Angled, Multiplexing

The next method is to emit light from an active, normal collimator and collect it on two passive, angled probes at symmetric angles relative to the surface normal. Signals from the angled collimators are combined optically (before the photo receiver) and sent to the oscilloscope. This results in a cancellation of the normal velocity term observed on the positive and negative angled collimators and enables a direct measurement of transverse velocity from two apparent velocity signals. Setting $\beta = 0$ and $\alpha_2 = -\alpha_1 \neq 0$ results in apparent velocities of

$$V^+_1 = \frac{V_N}{2} (\cos \alpha_1 + 1) + \frac{V_T}{2} \sin \alpha_1 \quad \text{(C.4)}$$

$$V^-_1 = \frac{V_N}{2} (\cos \alpha_1 + 1) - \frac{V_T}{2} \sin \alpha_1 \quad \text{(C.5)}$$
where $V_+^*$ and $V_-^*$ are the apparent velocities observed along collimator angles $\alpha_1$ and $\alpha_2$, respectively. Subtracting $V_+^*$ from $V_-^*$ and solving for $V_T$, yields a transverse velocity of

$$V_T = \frac{V_+^* - V_-^*}{\sin \alpha_1} = \frac{V_+^* - V_-^*}{\sin \alpha_2}. \quad (C.6)$$

It is important to note that there is no dependence on normal velocity, which results in a direct measurement of the transverse component of surface velocity. This is beneficial because it makes the data processing more straight-forward.

### C.1.3 Method 1C - Active Normal, Double Angled Passive, Averaging

The collimator setup of Method 1C is identical to 1B with the exception that the signals from each passive collimator are digitized individually by separate photo receivers, captured on the oscilloscope, processed to find transverse velocity, and then averaged together. Therefore, the equations for apparent velocity on the angled probes do not change from method 1B, i.e. equations C.4 and C.5, but the signals are processed individually following method 1A, and then averaged together using

$$V_T = \frac{V_{T+}^* + V_{T-}^*}{2}. \quad (C.7)$$

where $V_{T+}^*$ and $V_{T-}^*$ are the transverse velocities measured from the positive and negative angled collimators, respectively.

![Figure C.2: Active-Passive configurations for (a) method 1A, (b) method 1B, and (c) method 1C. A red arrow signifies an active collimator and blue arrow signifies a passive collimator. “S” is the send probe, i.e. the origin of the un-shifted light and “R” is the receiving probe that observes the apparent velocity.](image-url)
C.2 Active - Active Configurations

C.2.1 Method 2A - Active Normal, Single Active Angled

Method 2 implies that angled collimators are now active, i.e. they collect the light they transmit. For this method to work, light emitted at a non-zero angle relative to the surface normal needs to be reflected back at the same angle, which requires particular attention to surface preparation. Previous work by Johnson et al [54] has explored various surface treatment techniques to achieve surfaces with retro-reflective abilities. This is discussed further in the material preparation section. Since the angled collimator collects (a portion) of the light it emits, \( \beta = \alpha \), and the apparent and transverse velocity are given by

\[
V^* = V_N \cos \alpha + V_T \sin \alpha \\
V_T = \frac{1}{\sin \alpha} (V^* - V_N (\cos \alpha + 1)).
\]

An immediate difference between this method and method 1A, is the factor of two that multiplies the apparent velocity in equation C.3. This factor of two becomes important in the following error analysis calculations.

C.2.2 Method 2B - Active Normal, Double Active Angled, Multiplexing

Method 2B is similar to 1B, with the exception that the angled collimators are now active and collect the light they emit. Therefore, \( \beta = 0 \) and \( \alpha_2 = \alpha_1 \neq 0 \) which yields apparent velocities of

\[
V^*_+ = V_N \cos \alpha_1 + V_T \sin \alpha_1 \\
V^*_- = V_N \cos \alpha_2 + V_T \sin \alpha_2
\]

and a transverse velocity of

\[
V_T = \frac{V^*_+ - V^*_-}{2 \sin \alpha_1} = \frac{V^*_+ - V^*_-}{2 \sin \alpha_2}.
\]

Again, the factor of two in the denominator is the only difference between method 1B, equation C.6, and 1C, equation C.10.
C.2.3 Method 2C - Active Normal, Double Active Angled, Averaging

Method 2C is the same as 1C with the exception that the apparent velocities observed on the active angled collimators are given by method 2B, equations C.9 and C.9.

\[
V_T = \frac{V_{T+}^* + V_{T-}^*}{2}
\]  

(C.11)

Figure C.3: Active-active configurations for (a) method 2A, (b) method 2B, and (c) method 2C. A red arrow signifies an active collimator and blue arrow signifies a passive collimator. "S" is the send probe, i.e. the origin of the un-shifted light and "R" is the receiving probe that observes the apparent velocity. For an active-active scheme, it is assumed that the light from each send collimator is reflected back onto itself, i.e. \( S_1 = R_1 \) and \( S_2 = R_2 \).

C.3 Uncertainty Analysis

Starting with the general definition of apparent velocity, equation C.1, uncertainty propagation was performed to calculate the relative error in transverse velocity,

\[
\delta V_{T, rel} = \frac{\delta V_T}{V_T}
\]  

(C.12)

The uncertainty in transverse velocity, \( \delta V_T \), was calculated using the square root of a sum of squares where each term was the individual contribution from each independent variable, equation A.4. The uncertainty for case 1A and 2A had the form,

\[
(\delta V_T)^2 = \left( \frac{\partial V_T}{\partial V^*} \delta V^* \right)^2 + \left( \frac{\partial V_T}{\partial V_N} \delta V_N \right)^2 + \left( \frac{\partial V_T}{\partial V_E} \delta V_E \right)^2
\]

\[+ \left( \frac{\partial V_T}{\partial \alpha} \delta \alpha \right)^2 + \left( \frac{\partial V_T}{\partial \beta} \delta \beta \right)^2 + \left( \frac{\partial V_T}{\partial \gamma} \delta \gamma \right)^2 + \left( \frac{\partial V_T}{\partial \phi} \delta \phi \right)^2 \]

(C.13)

where \( \delta V^*, \delta V_N, \delta V_E, \delta \alpha, \delta \beta, \delta \gamma, \) and \( \delta \phi \) are the uncertainties in apparent velocity on the angled probe, normal velocity, out-of-plane velocity, light return angle, light incidence angle, out-of-plane
angle $\alpha$, and out-of-plane angle $\phi$. For cases 1C and 2C, the an uncertainty was calculated for each signal and then added in quadrature

$$\delta V_T = \sqrt{\left(\frac{\partial V_T}{\partial V_{T,1}} \delta V_{T,1}\right)^2 + \left(\frac{\partial V_T}{\partial V_{T,2}} \delta V_{T,2}\right)^2}$$

$$(C.14)$$

where $\delta V_1$ and $\delta V_2$ are the uncertainty in transverse velocity measured by each angled collimator.

The uncertainty for method 1B and 2B have a slightly different form since the calculation of $V_T$ involves the difference between $V_1$ and $V_2$ and a $\frac{1}{\sin \alpha}$ term. Even though the normal components of velocity cancel during the subtraction, they were included in the uncertainty calculations to enable the investigation of misalignment of the angled collimators. Therefore, the uncertainty for methods 1B and 2B has the form

$$(\delta V_T)^2 = \left(\frac{\partial V_T}{\partial V_{1}} \delta V_1\right)^2 + \left(\frac{\partial V_T}{\partial V_{2}} \delta V_2\right)^2 + \left(\frac{\partial V_T}{\partial V_{N}} \delta V_N\right)^2 + \left(\frac{\partial V_T}{\partial V_{E}} \delta V_E\right)^2 + \left(\frac{\partial V_T}{\partial \alpha_1} \delta \alpha_1\right)^2 + \left(\frac{\partial V_T}{\partial \alpha_2} \delta \alpha_2\right)^2 + \left(\frac{\partial V_T}{\partial \beta_1} \delta \beta_1\right)^2 + \left(\frac{\partial V_T}{\partial \beta_2} \delta \beta_2\right)^2$$

$$(C.15)$$

where the independent variables are the same as equation C.14, except the subscript 1 and 2 correspond to angled probe 1 and 2, respectively.

To compare the relative uncertainty in transverse velocity for each method, a symmetric flyer-target impact case was created to provide apparent and normal velocity components indicative of typical experimental data sets. A general, symmetric impact such as this allows the expected components of normal and transverse free surface particle velocity to be calculated, without any dependence on material density or sound speed, from elastic impedance matching as

$$u_{fs} = V_0 \cos \theta$$

$$(C.16)$$

$$v_{fs} = V_0 \sin \theta.$$  

$$(C.17)$$

This also allows each method to be directly compared in a fair and consistent manor. An initial velocity of $V_0 = 100$ m/s was chosen based on typical expected velocities for pressure-shear experiments. The impact angle, $\theta$, was varied between $15 - 25^\circ$, collimator angle, $\alpha$, between $0 - 90^\circ$, and three relative particle velocity uncertainties, $\delta u_{p,rel}$, of 1, 2.5 and 5 % were tested. The choice of relative particle velocity uncertainties come from Dolan [32]. This work concluded that based on the sampling rate, signal to noise ratio, and analysis time duration, the uncertainty of
PDV measurements is typically on the order of 1% of the particle velocity magnitude. To provide a conservative estimate of uncertainty, this relative uncertainty was varied in the present calculations between 1 − 5%. The three parameters, $\theta$, $\alpha$, and $\delta u_{p,rel}$, were chosen based on their significant influence on relative uncertainty. The solution sequence started by calculating $u_{fs}$ and $v_{fs}$ from equation C.16, calculating apparent velocity from equation C.1, then relative uncertainty in transverse velocity for each combination of $\theta$, $\alpha$, and $\delta u_{p,rel}$. Figure C.4 shows the results of this parametric study.

The first takeaway from figure C.4 is the sharp increase in relative error near values of $10 − 20^\circ$ collimator angle for all configurations. This is caused by the component of transverse velocity observed on the angled collimator going to zero as the collimator angle goes to zero, which causes the relative error to asymptotically approach positive infinity. The two,
black-dashed, vertical lines represent the ideal collimator angle range based on relative uncertainty and light return. Each red-to-purple band represents an assumed value for relative uncertainty in observed particle velocity. For each configuration, increasing the relative uncertainty from 1% to 2.5% and 2.5% to 5% increases the relative uncertainty by roughly a factor of two, with a slightly more dramatic increase for active-passive methods 1A, 1B, and 1C. Increasing the angle of obliquity or impact angle lowers the relative uncertainty. This is a result of larger magnitude transverse velocities that occur for increasing shear components of loading. The most significant takeaway is that active-active configurations, 2A - 2C, appear to have much lower overall relative uncertainty as compared to active-passive configurations, 1A - 1C. Simply put, it appears to be better if a collimator collects the same light that it emits. However, logistical hurdles exist for each of the configurations depicted here and the optimal method may change based on a variety of factors.

C.3.1 Pros and Cons of Active-Passive and Active-Active Methods

Both active-passive and active-active methods require surface treatment. Active-passive methods use a diffusely reflective surface that has the ability to reflect the normal-incident light at non-zero angles relative to the surface normal. After experimenting with sand blasting and special diffuse surface coating, a simple, directional sanding provided the best diffuse light reflection for the angled, passive collimators. Each anvil free surface was directionally sanded using approximately 50 paths across a piece of 200-grit sand paper. A square holder was fabricated to hold each of the metal anvils and prevent them from rotating. The directionality of the sanding was perpendicular to the transverse velocity direction to encourage light to be diffusely reflected onto the angled collimators. This is believed to be a significant improvement compared to sand blasting, which has no directionality and reflects the light in all directions, thereby wasting reflected light. Since light return is imperative with any velocimetry system, any opportunity to increase light return is worth pursuing.

Active-active schemes require light sent from an angled collimator be reflected back to onto the collimator face. To accomplish this, a retroreflective finish mush be placed onto the free surface of the anvil. Johnson et al [53] explored various ways to accomplish retro-reflectivity such as sanding, applying glass micro-spheres, and milling angled v-channels. Light return as a function of collimator angle was recorded for each surface treatment. The micro-spheres and angled v-channels provided the best light return for a considerable range of collimator angles, but
also required significant effort to apply to the anvil surface. Fixing the micro-spheres to the surface of the sample required pressed the beads into a thin layer of epoxy. SEM images revealed that this layer is rarely uniform and that the epoxy has a tendency to coat some of the spheres, which diminishes their ability to reflect light. Additionally, the ability of the beads to remain fixed to a target surface after it begins moving is still an open question. The study by Johnson et al revealed that beads were ejected from a target surface that was subjected to a release wave after normal loading. If beads begin moving (relative to the rear anvil free surface) before the shear wave has arrived, the beads will not move with any shear motion. Lastly, each bead has a considerable size on the order of 35 - 45 µm and an impedance close to that of glass. A collection of these beads, along with a thin epoxy layer, will have an impedance of its own that could alter the free surface release that is used to infer stress states within the sample.

In many applications of combined normal-transverse surface velocity measurements, it is desirable to observe the normal velocity and (component of) transverse velocity at the same spatial location. However, when using any configuration that has multiple emitting (active) collimators, cross-contamination of light can occur between probes. Figure C.5 depicts this phenomena by showing three collimators, each emitting light, depicted with solid lines. The reflected light from each collimator, dashed lines, then scatters back towards, as well as away from, the original collimator face. Light that is reflected elsewhere can end up reflecting onto a different collimator face. The light collected by the other collimator will then contain additional velocity spectra. Experiments that produce spatially varying velocity profiles, such as wave propagation in heterogeneous materials, require the normal and transverse components of velocity to be observed at the same spatial location to obtain a valid transverse velocity. The equation for transverse velocity is sensitive to slight fluctuations in particle velocity between the normal and angled probes. Therefore, if slight spatial variations occur in the wave front, a clean transverse velocity will not emerge. Experiments with minimal spatial variation in velocity, such as wave propagation in quasi-homogeneous materials, do not require that the normal active probe be aligned to the same spatial location as the active angled probe. However, appropriate time shifts must be applied to the signals, based on calculated tilt corrections, to ensure the angled and normal probes see the longitudinal wave arrive at the same time. Otherwise, small differences in velocity between the normal and apparent signals, arising from not having the appropriate time shift, can cause artificially large components of transverse velocity, which are not physical.
Figure C.5: Schematic of active-active configuration paths of incident (solid lines) and reflected (dashed lines) light from a single normal collimator (red) and two angled collimators (green and blue).

Shot 1 used the active-active scheme with $\alpha_1 = -20^\circ$ and $\alpha_2 = 15^\circ$. A spectrogram from the normal, positive, and negative angled probes can be seen in Figure C.6. Spectra collected by the normal and negative angled collimator revealed evidence of cross-contamination. This becomes an issue near the top of the rise in velocity when additional spectra emerges near the primary signal, which is thought to be the largest amplitude frequency. The spectrogram for the normal probe shows additional velocity spectra above and below the primary signal, suggesting that light from both the negative and positive probe was collected on the normal probe. The spectrogram for the negative angled probe shows additional spectra above the primary signal, suggesting light from the normal probe was collected. Ultimately, cross-contamination of light can be circumvented with proper data editing and peak finding in PlotData, but small differences in apparent velocity between the normal and angled collimators can be difficult to resolve if the signals lie too close to one another. An additional method to avoid cross-contamination, in heterodyne systems, is to use a different beat frequency for each probe. That can be accomplished by designating a different initial wavelength laser to each probe, i.e. PDV channel, and then adding a reference wavelength in the typical fashion. This shifts the spectra observed by each probe and allows the profiles to be distinguished in the spectrogram.
Figure C.6: Spectrogram data from shot 1 showing cross-contamination of light between collimators. (a) normal (b) $\alpha_1 = -20^\circ$ and (c) $\alpha_1 = 15^\circ$ probes.
D.1 Experimental Velocity Profiles

The following section provides the measured normal velocity (red line), apparent velocity for the positive (green) and negative (blue) angles probes, the calculated transverse velocity from the positive (green) and negative (blue) angled probes, as well as the average transverse velocity (black). Additionally, position-time and stress-particle velocity plots are provided to show how the stress state from each experiment was determined.
Figure D.1: Raw data from shot 1. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 2C - active normal probe, double active angled probes with averaging, appendix C.2.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.2: Raw data from shot 2. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.3: Raw data from shot 3. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.4: Raw data from shot 4. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.5: Raw data from shot 5. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.6: Raw data from shot 6. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.7: Raw data from shot 7. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.8: Raw data from shot 8. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.9: Raw data from shot 9. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0 \cos \theta$ and $v_0 = V_0 \sin \theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.10: Raw data from shot 10. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.11: Raw data from shot 11. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, \( u_0 = V_0 \cos \theta \) and \( v_0 = V_0 \sin \theta \), respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, \( \sigma_1 \), \( \sigma_2 \), and \( \sigma_3 \). (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, \( \tau_1 \), \( \tau_2 \), and \( \tau_3 \). Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
Figure D.12: Raw data from shot 12. (a) Normal and apparent velocity measured from the PDV collimators and the corresponding calculated transverse surface velocity. Transverse velocity was calculated using method 1C - active normal probe, double passive angled probes with averaging, appendix C.1.3. The horizontal dashed lines represent the expected normal and transverse particle velocity, $u_0 = V_0\cos\theta$ and $v_0 = V_0\sin\theta$, respectively, using elastic impedance matching. (b) Position-time plot showing longitudinal (blue) and transverse (red) waves propagating in the front anvil, sand, and rear anvil. (c) Normal stress - normal velocity plot depicting the impedance matching used to determine normal stress states, $\sigma_1$, $\sigma_2$, and $\sigma_3$. (d) Shear stress - transverse velocity plot depicting the impedance matching used to determine shear stress states, $\tau_1$, $\tau_2$, and $\tau_3$. Note: multiple lines are drawn for post the impedance matching and position-time plots to represent the range of impedances, both normal and shear, potentially present within the sand during loading.
D.2 Simulated Velocity Profiles

The following section provides the average velocity profiles (calculated from the planes of 100 Lagrangian tracer points) extracted from the three-dimensional mesoscale simulations at the front (red) and rear (blue) surface of the sand. The expected normal and transverse velocity is shown as a horizontal dashed line in each plot. The point at which the front and rear surface velocities converge onto each other represents stress equilibrium.

Figure D.13: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 50 m/s.
Figure D.14: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 70 m/s.
Figure D.15: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 90 m/s.
Figure D.16: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 110 m/s.
Figure D.17: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 130 m/s.
Figure D.18: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 150 m/s.
Figure D.19: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 170 m/s.
Figure D.20: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 190 m/s.
Figure D.21: Simulated profiles for 4-12 mm and 16-16 mm anvils and impact velocity of 210 m/s.