GROUP PRESENTATIONS AS A SITE FOR COLLECTIVE MODELING ACTIVITY

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We approach student presentations of solutions to modeling tasks as occasions for whole-class reflection on the rich conceptual work that small-group teams have done in parallel. Analyzing and interpreting these interactions can offer insights into how a classroom group negotiates a shared sense of what they have learned and what they collectively view as "newsworthy" across groups from their recent (and ongoing) model-building. We describe analytical tools to interpret a classroom's work during presentations, and we illustrate their use in a single case. This work offers a foothold for design-based research to harness presentations to improve learning, drive instructional decisions, and illuminate modeling processes at both individual and group levels.

Keywords: Modeling; Problem solving; Communication

Orchestrating presentations and the surrounding discussions can be critical in classroom practice (Ball, 1993; Lampert, 2001), where it can be challenging to avoid a simple "show and tell" pattern (Stein, Engle, Smith, & Hughes, 2008; 2015). In the context of rich modeling problems, we framed group presentations as a scene of second-order, shared mathematical modeling. That is, we viewed interactions around presentations as reflecting the class's shared effort to make sense of what they have learned and to identify continued uncertainty or disagreement. To proceed analytically, we developed a scheme for coding presentations along three *dimensions*, characterizing utterances in terms of (a) how they referred to phases of modeling, (b) how they enacted rhetorical stances, and (c) how they contributed to whole-group-level modeling (e.g., building toward consensus or strengthening a sense of multiplicity in the class's solutions). We present this scheme in Brady & Jung (2019). Here, we focus on a single case, analyzing a class discussion that provoked reflection and shared modeling. We present this case to demonstrate how our coding scheme functions to capture dynamics that reframe presentations as shared modeling opportunities. A single case cannot show changes over time in a class's practices, but it illustrates our method, the dimensions, and connections among them.

Literature Review: In Search of a Group-Level Model of Modeling

Mathematical modeling processes involve the development and refinement of purposeful *models* that describe or provide insight into real-world problem-solving situations (e.g., COMAP & SIAM, 2016; Kaiser & Stender, 2013; Lesh, English, Sevis, & Riggs, 2013; Lesh, Yoon, & Zawojewski, 2007). Lesh, Hoover, Hole, Kelly and Post (2000) defined a *model* as a system used to describe another system. Thus, a *mathematical model* is a system that consists of mathematical elements (e.g., numbers and variables); relationships among the elements (e.g., equivalence relationships); operations and representations that describe how the elements interact (e.g., graphs, symbols, equations); and patterns or rules that show how it can describe another system (Lesh et al., 2000). Developing a mathematical model in the face of a situation, or *mathematizing* reality, involves organizing, quantifying, and/or coordinatizing a real-world situation (Lesh et al., 2007). To characterize students' *presentations* of modeling we consider: (a) features of modeling they refer to; (b) discourse moves they enact; and (c) emergent structure across presentations.

A significant strand in the international research on mathematical modeling has articulated modeling as a cycle with phases (see, Blum, 2015). "Modeling competencies" (Kaiser, 2007; and cf Niss, 2003) are under debate, with a broadly shared caveat that such competencies are not isolated skills, but develop together (e.g., Blum, 2015; Zbiek & Conner, 2006). For example, Galbraith and Stillman (2006) endorse a generally regular image of the modeling cycle, but they show that students can experience barriers in moving from any phase to the next. On the other hand, Borromeo Ferri (2007) found that the work sequence for individual students could be quite idiosyncratic and non-linear, departing from any canonical cycle. In our study of *presentations*, we conjectured that struggles or insights characteristic of model cycle phases (and transitions) might be milestones for groups, but that these might occur in any order.

Our conjectures were also supported by research (a) at the small group level, and (b) on learners' meta-cognitive reflections on their modeling. Czocher (2016) visualized patterns exhibited by *groups* in modeling phases and the transitions between them. She found diversity and non-linearity similar to Borromeo Ferri (2007), but these patterns could be understood in terms of groups' *styles* of problem solving. Research on metacognition by Magiera and Zawojewski (2011) building on work by Wilson and Clarke (e.g., 2004) used students' talk in stimulated-recall interviews about Model-Eliciting Activities or MEAs (Lesh et al, 2000).

As we worked to understand modeling at the whole-class level during presentations we foregrounded *constructivist* and *situated* perspectives on knowing and learning (e.g., Beth & Piaget, 1966; Lesh & Lehrer, 2003) attending particularly to how the specific conditions or framing could affect students' behavior. This led us to begin inductively from cases and focus on characterizing emergent structures of agreed-upon and disputed features of models.

Methods

We present data collected during a two-week summer camp on mathematical modeling for Grade 6-8 students and led by the second author and a mathematics teacher. Twenty-one middle school students (ten females and nine racially diverse students) from five schools participated. On the first day of the camp, the group as a whole constructed norms on interacting with other students (e.g., listen to your classmates, respect others' ideas). Throughout the summer camp, students worked in teams and presented their ideas and solutions to modeling problems to the whole class. The case we present here involved the *Counting Caribou* problem (Lesh & English, n.d.). Students were given aerial photographs of caribou herds and asked to develop a procedure for estimating their numbers. Their "client" was the Alaska Department of Fish and Game. Features of the two sample photographs added to the problem's complexity: (a) there were too many caribou in each photo to count easily; (b) the density of populations differed within and across the photos; (c) some caribous' bodies extended beyond the edge of the photos.

Our analysis focuses on student discourse around six small groups' solutions to this problem, which occurred in a 24-minute exchange at the end of the sixth day of the camp. We selected this case as it was also fundamental in generating our larger codebook for describing references to *modeling phases* and students' *discourse moves* in presentations (see Brady & Jung, 2019).

Findings: Patterns in Discourse across Mathematical Modeling Presentations

We analyzed the class's presentations and discussions of their solutions focusing on understanding moments where questions from the audience provoked shared reflection. The six groups' presentations were very diverse, yet patterns emerged across all of our dimensions. In terms of *modeling phases*, we used the following descriptors:

- **Understand the problem**. Return to the problem statement (including text, tables, or figures) to clarify what constitutes a solution for the Client.
- **Construct / structure**. Frame the problem situation and solution criteria to address them with mathematical tools. Choose a way of looking at the problem.
- **Patch**. Adapt the model in process, responding to "unruly" features of the problem situation that emerge as the initial model is applied.
- **Work mathematically**. Do (or explain) arithmetic or algebraic manipulations.
- **Validate**. Consider the reasonableness of the answer or process. Check extreme or special cases; check assumptions/validity of the mathematical procedures used.
- **Interpret**. Explain the answer as referring back to the context of the problem.

We found that presenters emphasized the Construct/Structure and Work Mathematically phases. In contrast, questioners emphasized the Understand the Problem and Patch phases. This complementarity struck us. We then asked whether we could characterize *forms* of question that had high leverage for raising new ideas. We found questioner contributions of "Seeking Explanations" and "Inquiring about Omitted Features" forms often provoked presenters to introduce novel features of their models, which had been unstated. In particular, unique aspects of the presenters' work did not always show up until students in the audience asked questions of these kinds. We felt that this might be a feature of interactions around presentations that could be supported or learned. In our case, the first instance of this came in the questions for Group 1: Hope, Tim, and Kevin (all proper names are pseudonyms). Their presentation had introduced several key ideas, but many important themes that later arose in the discussion as a whole had not yet emerged. The audience's questions quickly led three of these to surface:

01 Uri: So, I really liked that yours was really exact, but it was also pretty easy and simple. I also have a question. What did you do with the overlapping caribou? Like the ones that are half on the page and...
02 Hope (presenter 1): Oh, those ones we connected.... If it was half a body, we found another half body we would smush it together to make one.
03 Teacher: Any other comments or questions?
04 Irene: Why did you decide to do it in the most crowded...count the ones in the most crowded area?
05 Hope (presenter 1): Well, we kind of like.... All three of us, we counted different areas, so one of them counted up to like 110 or so, some of them counted like to 95 or so. We kind of just rounded them to 100.
06 Tim (presenter 2): Yeah.
07 Uri: So how did you account for.... Like the second one, how did you account for...the instance where there was like none of them...Like the second picture... It's just like...
08 Kevin (presenter 3): We eliminated those squares so I could get an accurate estimate.

The class's questions prompted Group 1 to elaborate on aspects of their modeling that they had left out of their presentation. They also served to identify issues in the public forum that any future presenters should attend to. As such, they represented bids to contribute to an emergent, shared *specification for an adequate solution*. Uri's question surfaced a "patch" that Group 1 had devised as they applied the procedure they had described simply as, "We counted the caribou in one section." Irene called into question the logic of choosing a highly populated grid cell (rather than using a "medium" cell, which emerged with later groups and became a 'standard'). Here, Hope's answer was not fully responsive: instead, she used the question as an occasion to elaborate the counting strategy beyond what Group 1 had presented. (In their solutions, they sampled three *different* crowded grid cells and took a rough average of caribou counts across these.) Finally, Uri's question led Kevin to mention a feature that came to be shared across many groups' presentations: "patching" the grid approach for Herd 2 by removing the number of empty or very sparsely populated grid cells from the count used in the calculation.

These question contributions were coded in our *discourse-moves* dimension as "Inquiring about omitted features of the problem or solution process" (Uri's questions) and "Seeking explanations for aspects of the modeling process" (Irene's question). We find it interesting that the questioning session here operated in collaboration with the presentation to articulate complementary aspects of Group 1's solution and to raise features of the problem and solution that would highlight the diversity of later groups' modeling work and feed consensus and debate.

A more extensive account of our data analysis describes how the sequence of presentations led to the emergence of a shared *solution specification* and a shared *"skeleton" model* at the class level. That is, the class came to agree on what would count as a solution for the Client and also on some key components of good approaches. Without this class-level discussion—constituted across contributions by presenters and questioners—the solution specification and the structural similarities among groups' approaches might have remained implicit. In contrast, this class appeared aware that a shared model had indeed emerged by the end of the presentation sequence. Group 5 began by saying, "Okay. So our method was essentially the same as basically everybody else's method." And Group 6 repeated a similar pronouncement, twice: at the start ("Okay, so we pretty much did the same procedure as everyone else") and after describing a unique aspect of their work ("…and then we did the exact same thing like everybody else").

Discussion and Conclusion

We have explored presentations as occasions where a whole class can identify common ground across their work and identify "newsworthy" elements of each others' approaches. This lens allowed us to ask questions about how the class converted a small-group modeling activity into a second-order modeling experience at the whole-class level. We used a case from our larger study to illustrate the phenomena we are focusing on and to demonstrate our approach.

Reflecting on generalizability, we expect that similar conceptual issues and challenges would face any classroom engaged in processing its small-group work on a rich modeling activity, but that there would be variation in the specific results. This variation might be used to characterize aspects of "expert" group-level modeling practices and/or to guide instructional decision-making. With this in mind, we intend our approach to illuminate the texture of whole-class modeling work, rather than to categorize it neatly or uni-dimensionally. In particular, we expect different phases of modeling to be highlighted in different classrooms or for different activities. For instance, Counting Caribou is a Model Eliciting Activity, or MEA (Lesh et al, 2000), but it places a relatively low emphasis on the details of the client's situation, relative to other MEAs. This might in part account for a relatively low occurrence of Understand the Problem, Interpret, and Validate codes in our *model-phase* dimension.

Having groups present their solutions to the whole class can be critiqued on the basis of the value of the use of time. In fact, Lesh (2010) proposes an alternative approach, modeled on conference Poster Sessions, to allow groups to be exposed to and critically assess alternative solutions. However, our analysis suggests it is possible for a class to engage in collective modeling at the whole-class level, and this may be an instructionally useful experience. Our larger study and future work in applying and interpreting the coding approach presented here promise to offer insights into collective modeling during presentations, and to suggest instructional interventions that could support students in formulating and recognizing high-leverage discourse moves that drive collective modeling processes.

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