MATHEMATICAL MODELING EXPERIENCES: NARRATIVES FROM A PRESERVICE TEACHER AND AN INSTRUCTOR

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Regardless of the benefits of engaging in mathematical modeling, few preservice teachers (PTs) have experienced mathematical modeling firsthand. This study offers an example of how to make sense of the interaction between the teaching and learning of mathematical modeling by examining a teacher educator's decision making, her analysis of 36 PTs' learning, and an in-depth narrative from a PT. Findings show the value of engaging with structurally relevant mathematical modeling tasks and considering social issues via mathematical modeling, resulting in task designs which aim to deepen students' understanding of society and mathematics.

Keywords: Mathematical Modeling; Preservice Teacher Education

The benefits of engaging with mathematical modeling (i.e., the process of using mathematics to provide insight into real-world situations) have been presented by numerous research studies (Consortium for Mathematics and Its Applications & Society for Industrial and Applied Mathematics [COMAP & SIAM], 2016). Students develop reasoning abilities, entrepreneurial thinking, conceptual understanding, and procedural fluency when they engage with mathematical modeling (e.g., Blum & Niss, 1991; Lesh, Hoover, Hole, Kelly, & Post, 2000; Zbiek & Conner, 2006). However, despite the advantages of learning mathematical modeling, many teachers have limited experience with learning and teaching the related content and processes (e.g., Burkhardt, 2006; COMAP & SIAM, 2016). The inclusion of mathematical modeling in preservice teachers' (PTs') learning has been recommended so that they may be prepared to engage their future students with authentic problem-solving experiences (Association of Mathematics Teacher Educators, 2017); still, few PTs have experienced mathematical modeling firsthand. Research remains to be done about mathematical modeling learning opportunities provided to PTs, especially on the purpose of such learning opportunities and how learners respond to them. This study offers an example of how to make sense of the interaction between the teaching and learning of mathematical modeling by examining a teacher educator's curricular intentions and her PTs' responses to tasks designed around those intentions. The focus here is on the teacher educator (second author)'s decision-making, her analysis of PTs' learning, and a more in-depth narrative from one of her PTs (first author). Specifically, the research questions are "How does a teacher educator make decisions about implementing mathematical modeling to preservice teachers?" "How do PTs respond to the implementations of mathematical modeling tasks?" and "How does a PT reflect on and narrate specific aspects of the implementation?"

Methods

36 PTs enrolled in three sections of a problem-solving mathematics course taught by the second author (Jung) at a Midwestern university in the U.S. Thirteen teams of PTs solved, designed, and revised mathematical modeling tasks in this course. Specifically, the PTs solved relevant mathematical modeling tasks, including Model-Eliciting Activities (MEAs), which are known to showcase the nature and usefulness of mathematics while developing valuable

everyday skills (Lesh et al., 2000). PTs also reflected on their own learning and relevant literature related to mathematical modeling (e.g., English, Fox, & Watters, 2005). They then collaboratively designed mathematical modeling tasks and revised them based on peer feedback. The course resulted in the following data sources: (a) audio-recordings of individual interviews focusing on PTs' learning of mathematical modeling; (b) 26 team solutions for two mathematical tasks (13 solutions for each task); (c) 25 mathematical modeling tasks designed by PT teams; (d) and PTs' individual reflections on their learning.

Data analysis involved two phases. During the first phase, Jung and her colleagues analyzed data sources collected from 36 PTs and found the overall learning opportunities related to mathematical modeling (Jung & Magiera, 2018). This analysis was used to describe Jung's analyses of the 36 PTs' learning opportunities. The second phase focused on the learning experiences of the first author (Brand). Among the 36 PTs, Brand was uniquely interested in multitiered teaching experiments (Lesh & Kelly, 2000) and the analysis of her learning of mathematical modeling. Brand's products included her team's written solutions to two MEAs, two written mathematical modeling problems designed by her team, and seven individual journal reflections that documented her learning of mathematical modeling. Upon dissecting her solutions to tasks, Brand noted themes embedded within her processes of developing mathematical constructs as she solved or designed mathematical modeling problems. Then, analyzing her reflections, Brand focused on what she learned from each activity and documented evidence of changes revealed from each journal reflection (Strauss & Corbin, 1998). The following section outlines the overall learning experiences of 36 PTs as intended and analyzed by Jung, followed by Brand's narratives of her own learning.

Results

PTs as Problem Solvers

Jung's intention and initial analysis of student work. To engage PTs with mathematical modeling as learners, Jung first selected the Fun on the Field MEA (Chamberlin, 2000), which required PTs to split 15 individuals into three equal teams for a school's field day. To sort the students, four categories of data were provided: each individual's 100-meter dash time, 800-meter run time, high jump height, and whether they passed a fitness test. When she analyzed PTs' initial work on this MEA, Jung found that only five out of 13 teams demonstrated a sophisticated sorting method that considered all four categories of data. Jung asked each team to review other teams' work and provide peer feedback (West, Williams, & Williams, 2013). She then provided the Volleyball MEA as a follow-up activity because it was mathematically connected to the Fun on the Field MEA but required more complex thinking about large datasets. By way of explication, the Volleyball MEA engaged PTs to split 18 individuals into three equal teams for a volleyball summer camp. Compared to the Fun on the Field MEA, the Volleyball MEA required a more involved usage of ranking systems and quantification processes, as it supplied additional data sets, including qualitative data (e.g., coach's comments). Jung's anlysis of PTs' work on the Volleyball MEA revealed that most teams (12 out of the 13 teams) considered all the data sets and used ranking methods to represent their sorting system.

Brand: My narrative. As my partner and I began solving the Fun on the Field MEA, I proposed using a point system which assigned points to individuals' results based on predetermined intervals for each category, excluding the fitness test results. My partner, however, expressed a concern that my procedure failed to differentiate between individuals whose results fell in the same point interval. Suggesting we rank individuals within each category, my partner encouraged me to consider both our avenues for solution. As we continued, my partner and I wanted to ensure that our solution strategy utilized all the data provided. Thus, we needed to determine how to include the participants' fitness test scores. Initially, we chose to assign 1 point to individuals who passed the test and 0 points to those who did not. Upon review, though, we recognized two flaws with this plan. For one, according to our ranking system, a lower score indicated a more athletically inclined individual. Receiving additional points for passing a fitness test would increase that individual's score, making them appear less physically adept than he or she was. To fix this, we quickly decided to add 0 points to an individual's score for passing the test. For our second concern, I was apprehensive about how uninfluential the fitness test results were on the subjects' final scores. I recommended that we add 5 points to the scores of those who failed, while still adding 0 points to the scores of those who passed. My partner agreed that this would generate an appropriate effect on the subjects' final scores.

Overall, my learning from this MEA included growth in my recognition and application of justifiable strategies, open-mindedness to others' skillsets, and receptivity to revisions. I also grasped that mathematical modeling allows for students to continuously revise their models until a desirable outcome is reached. Resultingly, when my partner and I completed the Volleyball MEA, we paid close attention to our peers' feedback and strategies from the Fun on the Field MEA. Having two opportunities to engage in similar MEAs encouraged my peers and me to develop more evolved mathematical reasoning skills and problem-solving strategies.

PTs as Designers: First Task

Jung's intention and initial analysis of student work. Jung later incorporated a problem-posing activity in her course. She also provided PTs opportunities to revise their tasks based on peer feedback, followed by instructor feedback. Her analysis of 36 PTs' invented tasks revealed improvements from the initial tasks to the revised tasks. Most PTs refined their tasks to make them more realistic and mathematically sound. Specifically, they developed diverse mathematical modeling tasks, including contexts such as Christmas tree decorating, a dream vacation plan, and designing a park.

Brand: My narrative. When designing my first modeling task, I frequently evaluated my group's problem to ensure it met the mathematical modeling problem criteria discussed in class and required students to (a) make assumptions about and predict a realistic context; (b) create and verify mathematical models; and (c) provide complex solutions beyond numerical results. Because my group was anxious about meeting these standards in our first original MEA, we chose to create a task which elicited strategies we had applied when solving previous MEAs. The result was an MEA, titled "Ms. Penny's Classroom," similar in mathematical structure to the Fun on the Field and Volleyball MEAs. Specifically, the task required students to split 15 students into three groups of equal academic caliber for a group project.

Despite the similarities between our problem and those we had solved in class, a new strategy could emerge from this problem: using qualitative data as a tool for revision. My team recognized that quantifying comments on students' behavior (e.g., "well-behaved and a great student") was not entirely feasible. We instead chose to use that dataset as a mechanism to adjust our initial groupings. Creating a new MEA thus broadened my supply of problem-solving strategies. Additionally, I developed an understanding of mathematical modeling as an enriching instructional experience because of its relevance to students' realities.

PTs as Designers: Second Task

Jung's intention and initial analysis of student work. Although the contexts of the first tasks that PTs developed were relevant to target students of their choices (e.g., Christmas tree, academic grouping), Jung wanted PTs to consider critical aspects of the real world. She believed it to be crucial that PTs encourage students to critically interpret the world using mathematics (Gutstein & Peterson, 2005). PTs were asked to share unfair or unjust experiences their future students may face in their lives or world. With these experiences in mind, PTs collaboratively developed a mathematical modeling task with a critical context.

Brand: My narrative. My partner and I wished to create a problem which did not ask students to group individuals or items, as we felt we had exhausted this type of context. After much consideration, we recalled the garbage pollution in our city and chose to focus our task on a solution for widespread littering. We also wanted to involve spacial concepts in our problem through the use of a map. Our problem “Preventing Litter in the City” emerged, involving data reasoning strategies we’d encountered in previous MEAs. Further, a new strategy, which we dubbed “grouping,” became necessary for my team to both teach and solve our problem. For example, I chose to group two trash cans with every recycling bin in my solution. I also wanted the task to demonstrate my own beliefs that since we should involve “students in social and political conflicts…we should ensure that the implications in our math problems allow this” [Reflection 6]. To do so, I incorporated my students’ city of residence in the problem, thereby encouraging them to take action within their own community. Ultimately, by creating this task, I learned the importance of choosing justice-oriented contexts for mathematical modeling tasks. I saw how teaching mathematics can be more engaging, interdisciplinary, and useful for developing students’ knowledge of social justice.

Conclusion

Through the reflective processes, the authors illustrated practical and research knowledge that influenced the decisions a teacher educator and a PT made throughout mathematical modeling problem-solving and problem-posing experiences. As Diefes-Dux and Capobianco (2008) proposed, reflection supports a better understanding of the complex, interrelated sets of situations that teaching and learning requires. When this line of thinking is organized and connected, it often leads to new knowledge or action (Diefes-Dux & Capobianco, 2008).

The reflective nature of this self-study contributes to mathematics education in three ways. First, this-self study describes both the instructor’s intentions for and analyses of 36 PTs’ mathematical modeling experiences, as well as one PT’s narrative of her learning. The perspectives from both the instructor and PT provide evidence of learning and changes in action, which would remain uncaptured were only one of these lenses considered. The results show that each participant was an active decision maker (Mundry, Britton, Raizen, & Loucks-Horsley, 2000), acting in different, evolving directions (Lesh & Kelly, 2000). Second, both the instructor’s and PT’s reflections reveal the value of engaging with structurally relevant mathematical modeling tasks (Ärlebäck, Doerr, & O’Neil, 2013; Doerr, 2016). The PTs used more sophisticated ways of data reasoning, including the analysis of qualitative data, when they engaged with two related mathematical modeling tasks. Brand’s narrative detailed decisions she made when solving and posing mathematical modeling problems, ultimately contributing to a better understanding of her process of engaging with complex mathematical modeling activities. Last, the study details the rationales of how a PT created mathematical modeling tasks regarding societal issues. PTs were encouraged to design tasks that aimed to deepen students’ understanding of society. Such results around mathematical modeling problem-posing activities extend findings from previous studies about PTs’ problem-posing activities focusing on word problems (Tichà & Hošpesovà, 2010) or other modeling problems (Paolucci & Wessels, 2007).

As Brand narrated, choosing socially-aware contexts for a mathematical modeling problem enabled her to see the value of mathematics as a tool for broadening problem solvers' knowledge of the world.

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