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A New Generalized Modified Weibull Distribution

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# Abstract

We introduce a new distribution, so called A new generalized modified Weibull (NGMW) distribution. Various structural properties of the distribution are obtained in terms of Meijer’s *G*–function, such as moments, moment generating function, conditional moments, mean deviations, order statistics and maximum likelihood estimators. The distribution exhibits a wide range of shapes with varying skewness and assumes all possible forms of hazard rate function. The NGMW distribution along with other distributions are fitted to two sets of data, arising in hydrology and in reliability. It is shown that the proposed distribution has a superior performance among the compared distributions as evidenced via goodness–of–fit tests.

# Keywords

Odd log-logistic distribution, modified Weibull distribution, moment, simulation study, maximum likelihood estimation

# 1. Introduction

The Weibull distribution is a popular distribution for modeling and analyzing lifetime data with monotonic hazard rates. On the other hand, for complex systems, the hazard rate function can often be of non–monotonic shape, which the Weibull distribution cannot accommodate. To overcome such shortcomings, various generalizations of the classical Weibull distribution have been investigated by several authors in the recent years; among them, the extended flexible Weibull distribution [2], the generalized modified Weibull distribution [3], the exponentiated Weibull distribution [17], the additive Weibull distribution [20] , Weibull Birnbaum-Saunders distribution [12] and the modified beta Weibull distribution [11] and new type 1 half-logistic weibull [1]. Also, Sarhan and Zaindin [28] introduced the modified–Weibull (MW) distribution having three parameters and , with the cumulative distribution function (cdf) and probability density function (pdf)

(1)

and

(2)

respectively. It is worth noting that the MW distribution has submodels the exponential and Weibull .

Consider starting form a parent continuous cdf , where and . The cdf of generalized odd log–logistic family of distributions (Gleaton and Lynch [10]) is defined by

(3)

Its corresponding pdf is

(4)

where is the density of the baseline distribution.

Here, we introduce a new distribution having four parameters, so called A new generalized modified Weibull (NGMW) distribution, which is a generalization for the MW distribution. Our motivation for introducing the NGMW distribution is due to the simple analytic expressions of and of the MW distribution and it can be represented as infinite linear combination of MW distributions. Furthermore, the distribution provides a wide range of shapes with varying skewness, varied tail weights and shifting modes based on its additional parameters. It also accommodates most forms of hazard rates that are encountered in a variety of real-life problems.

The NGMW distribution is obtained by taking in (3) to be the cdf (1) of the MW distribution. Accordingly,

(5)

The NGMW density function is

(6)

The survival function and the hazard rate function of the NGMW distribution are

(7)

(8)

An interpretation of the NGMW family (5) can be given as follows. Let be a random variable describing a stochastic system with cdf . If the random variable represents the odds ratio, the risk that the system following the lifetime will not working at time is given by . If we are interested in modeling the randomness of the odds ratio by the Marshall-Olkin log-logistic pdf (and cdf , for ), the cdf of is given by

which is exactly the cdf (5) of the new family.

The remainder of this article is organized as follows. Some statistical functions of the NGMW distribution are provided in Section 2, such as the quantile function, moments, moment generating function, conditional moments, mean deviations, reliability curves and the distribution of order statistics. Certain characterizations of the proposed distribution are presented in section 3. The estimation of the parameters by maximum likelihood (ML) method is investigated in Section 4. Simulation study is performed in Section 5. In Section 6, the NGMW distribution along with other distributions are fitted to two sets of data. It is shown that, the proposed distribution has a better fit as verified by several goodness–of–fit tests.

Table 1. Some special cases of the NGMW distribution

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | Reduced distribution |
| 1 | - | 0 | - | 1 | Exponential distribution |
| 1 | - | 0 | - | - | Marshall-Olkin Exponential distribution |
| 1 | 0 | - | - | 1 | Weibull distribution |
| 1 | 0 | - | - | - | Marshall-Olkin Weibull distribution |
| 1 | 0 | - | 2 | 1 | Rayleigh distribution |
| 1 | 0 | - | 2 | - | Marshall-Olkin Rayleigh distribution |
| 1 | - | - | 2 | 1 | Linear failure rate distribution |
| 1 | - | - | 2 | - | Marshall-Olkin Linear failure rate distribution |
| 1 | - | - | - | 1 | Exponential weibull distribution |
| 1 | - | - | - | - | Marshall-Olkin Exponential weibull distribution |
| - | - | 0 | - | 1 | OLL exponential distribution [9] |
| - | - | 0 | - | - | Marshall-Olkin OLL exponential distribution [10] |
| - | 0 | - | - | 1 | OLL weibull distribution [9] |
| - | 0 | - | - | - | Marshall-Olkin OLL weibull distribution [10] |
| - | 0 | - | 2 | 1 | OLL Rayleigh distribution [9] |
| - | 0 | - | 2 | - | Marshall-Olkin OLL Rayleigh distribution [10] |
| - | - | - | 2 | 1 | OLL linear failure rate [9] |
| - | - | - | 2 | - | Marshall-Olkin OLL linear failure rate [10] |
| 1 | - | - | - | - | Marshall-Olkin MW [25] |
| - | - | - | - | 1 | OLLMW [26] |

# 2. Structural properties of the NGMW distribution

Using generalized binomial expansion we can write

where and

where. Therefore, we can write

(9)

where and for ,

(10)

and



Figure 1. Plots of the NGMW pdf for some parameter values

The pdf specified by (6) can be written as

(11)

and

It is clear from (11) that can be expressed as infinite linear combinations of exponentiated MW distributions and hence many properties of the NGMW can be deduced from the corresponding ones of the MW distribution. In what follows, we discuss some properties of the NGMW distribution and consider several associated statistical functions.



Figure 2. Plots of the NGMW hrf for some parameter values

If *X~* NGMW, then one can easily show that the random variable has Marshall-Olkin log-logistic distribution with shape parameter *α* and scale parameter equal 1.

## 2.1. Asymptotic and Shapes

In this section, we discuss the asymptotic and possible shapes of the pdf (6) and the hazard rate function (8).

The asymptotics of equations (5), (6) and (8) as are given by

The asymptotics of equations (5), (6) and (8) as are given by

Observed that plots (a), (b), (c) and (d) indicate how the parameters and affect the NGMW density and show flexibility of density shapes where skewness, heavy tails and modality can be observed. Figures (e) and (f) represent increasing, decreasing, unimodal, v–shaped, upside–down bathtub and bathtub–shaped hazard rate function.

## 2.2. Quantile function

For simulation of NGMW, if *U* ∼ *u*(0*,* 1), then the solution of non-linear equation

(12)

has cdf (5).

## 2.3. Moments and moment generating function

We now obtain representations of the moments and moment generating function of the NGMW random variable on the basis of the result developed in [19]:

(13)

where and is a rational number with and integers.

Making use of (13), the *rth* order moment and moment generating function of the NGMW distribution can be expressed in terms of Meijer’s –functions as

(14)

The *hth* order moment can readily be determined by replacing with in (14). The moment generating function of the NGMW is

(15)

## 2.4. Order statistics

Suppose is a random sample from the NGMW family of distributions. Denote the ordered random variables in the ascending as ; the pdf of is (David and Nagarajah [6])

(16)

where .

Here denotes the exp-MW density function with power parameter ,

(17)

and is defined by Equation (10). Here, the quantities is obtained recursively by and (for )

Thus one can easily obtain moments, generating function and incomplete moment of order statistics of NGMW. In the remainder of this section, we shall make use of the next lemma.

### *Lemma 1*

Let

where . Then, we have

### *Proof*

The proof follows ny considering the arbitrary function *.* letting where are natural co-prime numbers and making use of the identity

which results from Equation (13) of [4].

## 2.5. Conditional moments and mean deviations

In connection with lifetime distributions, it is important to determine the conditional moments which are of interest in predictive inference. The *rth* conditional moment of the NGMW distribution can be obtained as

(18)

The mean deviations provide useful information about the characteristics of a population and it can be calculated from the first incomplete moment. Indeed, the amount of dispersion in a population may be measured to some extent by all the deviations from the mean and median. The mean deviations of about the mean and about the median can be expressed as and , where is calculated from (5) and

(19)

## 2.6. Reliability curves

The Bonferroni and Lorenz curves have various applications in economics, reliability, insurance and medicine. The Bonferroni curve for the NGMW distribution is given by

and the Lorenz curve of is

(20)

The scaled total time on test transform of a distribution function is defined by *,* and it is important for the ageing properties of the underlying distribution and can be applied to solve geometrically some stochastic maintenance problems.

# 3. Characterizations of NGMW Distribution

This section deals with various characterizations of NGMW distribution. These characterizations are based on: (*i*) a simple relationship between two truncated moments; (*ii*) the hazard function and (*iii*) the reverse (or reversed) hazard function. It should be mentioned that for characterization (*i*) the cdf may not have a closed form.

We present our characterizations (*i*) *−* (*iii*) in three subsections.

## 3.1. Characterizations based on two truncated moments

In this subsection we present characterizations of NGMW distribution in terms of the ratio of two truncated moments. This characterization result employs a theorem due to Glänzel [7], see Theorem 1 in Appendix B. Note that the result holds also when the interval *H* is not closed. As shown in [7], this characterization is stable in the sense of weak convergence.

### *proposition 1*

Let be a continuous random variable and let, and for *.* The random variable has pdf (6) if and only if the function defined in Theorem 1 has the form

### *Proof*

If *X* has pdf (6), then

and

and finally

Conversely, if is given as above, then

and hence

Now, in view of Theorem 1, *X* has density (6).

### *Corollary 1*

Let be a continuous random variable and let be as in Proposition 1. Then, has pdf 6 if and only if there exist functions and defined in Theorem 1 satisfying the differential equation

### *Proof*

The general solution of the differential equation in Corollary 1 is

(21)

where is a constant. Note that a set of functions satisfying the above differential equation is given in Proposition 1 with *.* However, it should be also noted that there are other triplets satisfying the conditions of Theorem 1.

## 3.2. Characterization based on hazard function

It is known that the hazard function, , of a twice differentiable distribution function, , satisfies the first order differential equation

For many univariate continuous distributions, this is the only characterization available in terms of the hazard function. The following characterization establish a non-trivial characterization of NGMW, in terms of the hazard function, which is not of the above trivial form.

### *proposition 2*

Let be a continuous random variable. The pdf of is (6) if and only if its hazard function satisfies the differential equation

(22)

with boundary condition *.*

### *Proof*

If has pdf (6), then clearly the above differential equation holds. Now, if the differential equation holds, then

(23)

or

which is the hazard function of the NGMW distribution.

## 3.3. Characterization in terms of the reverse hazard function

The reverse hazard function, , of a twice differentiable distribution function, , is defined as

### *proposition 3*

Let be a continuous random variable. The pdf of is (6) if and only if its reverse hazard function satisfies the differential equation

(24)

with boundary condition *.*

### *Proof*

If has pdf (6), then clearly the above differential equation holds. Now, if this differential equation holds, then

(25)

or

which is the reverse hazard function of the OLLMW distribution.

# 4. Estimation, inference and goodness-of-fit statistics

Estimating the unknown parameters of a distribution is an essential issue in applied statistics. In this section, we obtain the maximum likelihood (ML) estimation of the parameters of the NGMW distribution based on a random sample from this distribution. Further, five goodness-of-fit statistics are given to compare the density estimates and selection of the models.

## 4.1. Maximum Likelihood Estimation

By using the loglikelihood of the sample in conjunction with the NMaximize command in the symbolic computational package Mathematica, we can estimate the unknown parameters of a distribution. Given the observed values of the taken sample from the NGMW distribution, the maximum likelihood estimators (MLEs) of the parameters are obtained by maximization of the log-likelihood function given by

(26)

where and the associated nonlinear loglikehood system , where

(27)

where

(28)

By solving the above equations simultaneously, we obtain the MLEs of the parameters. The numerical iterative techniques may be used for estimating the parameters and the global maxima of the log-likelihood is possible to investigate by putting different starting values for the parameters. The information matrix will be required for interval estimation. The elements of the total observed information matrix for can be obtained from authors upon request. Under conditions that are fulfilled for parameters in the interior of the parameter space but not on the boundary, the asymptotic distribution of is , where is the expected information matrix. The approximate multivariate normal

distribution, where is the inverse observed information matrix evaluated at , can be used to set up the approximate confidence intervals of the distribution parameters.

# 5. Simulation study

The performance of the MLEs of the NGMW distribution with respect to sample size n is considered. Simulation study is done based on the following steps:

1. generate five thousand samples of size n from (6). This work is done by solving Equation (12) with numerical method based on R program (”uniroot” function).
2. compute the MLEs for the five thousand samples, say for .
3. compute the biases and mean squared errors given by

and

for .

We repeated these steps for with special case of parameters and . The shape of pdf for this case of parameters is bimodal and it is presented in Figure 1. So computing and for and .

Figure 3 shows how the five biases vary with respect to . The biases for each parameter either decrease or increase to zero as .

Figure 4 shows how the five mean squared errors vary with respect to *.* The mean squared errors for each parameter decrease to zero as .

The reported observations in both figures are for only one choice for , namely . The results, however were similar for a wide range of other choices for (. In particular, 1) the biases for each parameter either decreased or increased to zero and appeared reasonably small at ) the mean squared errors for each parameter decreased to zero and appeared reasonably small at .



Figure 3. (top left), (top right), (middle left), (middle right) and (bottom left) versus .

# 6. Applications

In this section, we compare the NGMW model with other related lifetime models, namely: generalized modified Weibull (GMW) [3], Mcdonald Lomax (McLomax) [14], beta modified Weibull (BMW) [18] and transmuted generalized modified Weibull (TGMW) [5] distributions. To do so, we make use of two real data sets: first, the carbon fibre data [4] and, secondly, the bladder cancer data [13]. More specifically, the fitted models are (for ):

1. The GMW density function [3]



Figure 4. (top left), (top right), (middle left), (middle right) and (bottom left) versus .

* The McLomax density function [14]
* The BMW density function [18]

Table 2. MLEs (standard errors in parentheses) for the carbon fibres

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Distributions | Parameter | estimates |  |  |  |
| GMW | 5.49894 | 0.43639 | 0.14811 | 0.51628 |  |
|  | (8.02208) | (0.64986) | (0.53839) | (0.16932) |  |
| McLomax | 4.01844 | 44.9998 | 3.37645 | 1499.98 | 5.43418 |
|  | (16.155) | (177.75) | (0.79071) | (7941.2) | (3.49922) |
| BMW | 0.44730 | 0.13899 | 0.49618 | 5.87258 | 1.12967 |
|  | (0.72868) | (0.54698) | (0.46150) | (12.2267) | (2.95269) |
| TGMW | 0.19212 | 3.31948 | 0.27486 | 0.58561 | 0.67440 |
|  | (0.40783) | (4.54722) | (0.82386) | (0.23501) | (0.37119) |
| NGMW | 1.312813 | 0.739076 | 0.088051 | 2.211504 | 43.07359 |
|  | (1.28462) | (2.74113) | (0.942847) | (4.20357) | (207.414) |

Table 3. Goodness-of-fit statistics for the carbon fibres

|  |  |  |
| --- | --- | --- |
| Distributions | *A*∗ | *W* ∗ |
| GMW | 0.38543 | 0.06279 |
| McLomax | 0.49648 | 0.08398 |
| BMW | 0.38423 | 0.06261 |
| TGMW | 0.33211 | 0.05279 |
| NGMW | **0.26541** | **0.03948** |

* The TGMW density function [5]

## 6.1. The Carbon fibre data

The first data set which is uncensored pertains to the breaking stress of carbon fibres (in Gba) as reported in [4].

## 6.2. The bladder cancer data

The second data set represents the remission times (in months) of a random sample of 128 bladder cancer patients as reported in *LeeW ang* : 2003.

The estimated pdf’s and cdf’s of the TGMW model are plotted in Figures 4 and 5 for the carbon fibres and cancer data, respectively. The estimates of the parameters as well as the values of the Anderson-Darling (*A*∗) and Cramér-von Mises statistics are listed in Tables 2 to 5. We note that the TGMW model provides the best fit for both data sets.

# 7. Conclusions

There has been a growing interest among statisticians and applied researchers in constructing flexible lifetime models in order to improve the modelling of survival data. As a result, significant progress has been made towards the generalization of the traditional Weibull model. In this paper, we propose a five–parameter model named the *odd log-logistic Marshal-Olkin modified Weibull* (NGMW), which is obtained by applying the odd log- logistic Marshal-Olkin technique to the modified Weibull model. The new model extends several important lifetime distributions. We studied some of its statistical properties and obtained representations of the positive, negative and factorial moments, as well as the quantile function and the density of the order statistics. The proposed distribution as applied to two actual data sets turned out to provide better fits than other competing lifetime models.

Table 4. MLEs (standard errors in parentheses) for the bladder cancer data

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Distributions | Parameter | estimates |  |  |  |
| GMW | 2.79601 | 0.45369 | 0.65441 |  |  |
|  | (1.85772) | (0.37182) | (0.24811) | (0.00628) |  |
| McLomax | 0.8085 | 11.2929 | 1.5060 | 4.1886 | 2.1046 |
|  | (3.364) | (15.818) | (0.243) | (25.029) | (3.079) |
| BMW | 0.46965 | 0.66613 |  | 2.73477 | 0.90825 |
|  | (0.47875) | (0.31225) | (0.00639) | (2.02018) | (1.52196) |
| TGMW | 0.25215 | 2.24129 | 0.72431 |  | 0.72252 |
|  | (0.31749) | (1.74023) | (0.38549) | (0.00795) | (0.35566) |
| NGMW | 1.994473 | 0.015967 | 0.022496 | 0.417419 | 0.026061 |
|  | (1.22619) | (0.033625) | (0.051937) | (0.338987) | (0.123865) |

Table 5. Goodness-of-fit statistics for the bladder cancer data

|  |  |  |
| --- | --- | --- |
| Distributions |  |  |
| GMW | 0.27198 | 0.04050 |
| McLomax | 1.81435 | 0.3550 |
| BMW | 0.27197 | 0.04051 |
| TGMW | 0.18733 | 0.02732 |
| NGMW | 0.08753 | 0.01364 |



Figure 5. The estimated densities superimposed on the histogram

# Appendix A. Meijer *G*–function

The symbol denotes Meijer’s *G*−function (MEIJ) which is defined in terms of a Mellin–Barnes integral as

(29)

where and the poles are such that no pole of *,* coincides with any pole of *,* ; i.e. , while being a suitable integration contour, see [p. 143]Ref8 and *MEIJformoredetails*. The *G*-function’s Mathematica code reads

MeijerG

# Appendix B.

*Theorem 1*

Let be a given probability space and let be an interval for some *.* Let be a continuous random variable with the distribution function and let and be two real functions defined on such that

is defined with some real function . Assume that and is twice continuously differentiable and strictly monotone function on the set . Finally, assume that the equation has no real solution in the interior of . Then is uniquely determined by the functions and , particularly

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