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*International Journal of Fracture*, Vol. 209, No. 1/2 (January 2018): 231-234. [DOI](http://dx.doi.org/10.1007/s10704-017-0247-6). This article is © Springer and permission has been granted for this version to appear in [e-Publications@Marquette](http://epublications.marquette.edu/). Springer does not grant permission for this article to be further copied/distributed or hosted elsewhere without the express permission from Springer.

A Degradation Function Consistent with Cocks–Ashby Porosity Kinetics

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The load carrying capacity of ductile materials degrades as a function of porosity, stress state and strain-rate. The effect of these variables on porosity kinetics is captured by the Cocks–Ashby model; however, the Cocks–Ashby model does not account for material degradation directly. This work uses a yield criteria to form a degradation function that is consistent with Cocks–Ashby porosity kinetics and is a function of porosity, stress state and strain-rate dependence. Approximations of this degradation function for pure hydrostatic stress states are also explored.

Degradation function; Porosity; Damage; Ductile fracture; Strain-rate dependence

# [Introduction](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#toc)

The load carrying capacity of ductile materials degrades as a function of porosity (Vaz and Andrade Pires [14] ). However, this degradation is also a function of stress state (Benzerga et al. [3] ) and strain-rate (Rodriguez et al. [12] ). Gurson’s model (Gurson [7] ) for ductile materials accounts for this degradation through a yield criteria that is an explicit function of stress state and porosity. Gurson’s model, however, accounts for strain-rate dependence only implicitly, as Gurson’s yield criteria is function of flow stress, and flow stress may be strain-rate dependent.

For strain-rate dependent problems, the Cocks–Ashby model (Cocks and Ashby [5] ) provides a porosity evolution (kinetics) expression that is a function of porosity, stress state, and strain-rate exponent. However, their model does not give a corresponding yield criteria to evaluate material degradation. Inversely, since porosity evolution is related to a yield behavior, Cocks–Ashby’s porosity evolution expression does not necessary correspond with any general yield criteria. This work proposes a degradation function that is consistent with Cocks–Ashby’s porosity kinetics and gives thoughts on its implementation and uses.

# [Background](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#toc)

Gurson’s model gives a yield criteria where is hydrostatic stress, is flow stress, is porosity, and is strain-rate exponent. Strain-rate exponent characterizes the power-law relation between stress and strain-rate where is the rate of small strain evolution, is equivalent stress, and are material parameters.

Combining mass conservation with the flow rule for plastic strain gives a relation between porosity evolution and Gurson’s (or any) yield criteria,

(1)

where is a strain rate multiplier and σ is the Cauchy stress. Cocks–Ashby’s porosity evolution expression is given by:

(2)

where is the effective plastic strain rate and Cocks–Ashby’s original form has been modified to include two calibration parameters and . Equation (2) does not adhere to Eq. ( [1](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#bib1)) and (possibly as a result) is used in crystal scale calculations where a yield criteria is not generally known in a functional form. In these cases, phenomenological degradation is used. Specifically, Kweon ([8] ) used the degradation function and Barton et al. ([2] ) used the degradation function where was a degradation parameter independent of Eq. (2). In both cases in Eq. (2) was replaced with the sum of shearing rates in a crystal.

Several yield criteria for strain-rate dependent materials have been proposed. Marin and McDowell ([10] ) expressed these criteria in the form

(3)

where is the second invariant of deviatoric stress (related to equivalent stress by is a function to be discussed later, and and are functions of and corresponding to the various yield criteria of Cocks ([4] ), Michel and Suquet ([11] ) and Duva and Hutchinson ([6] ), or Sofronis and McMeeking ([13] ).

# [Theory](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#toc)

A general yield criteria is given as (Vaz and Andrade Pires [14] ):

(4)

Marin and McDowell ([10] ) express porosity evolution in terms of the yield criteria in Eq. (3) as

(5)

where is the first invariant of stress.

Marin and McDowell ([10] ) equate this to Cocks–Ashby’s porosity evolution expression by assuming that and integrating with respect to I1 to yield1 [[1](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#bib1)]:

(6)

The yield criteria in Eq. (3) can be written in terms of a degradation function (as in Eq. 4) if can be written as . Thus, Eq. (6) could be multiplied by to achieve this form. However, a more natural and general solution is to integrate Eq. (5) exactly without the small sinh assumption to give:

(7)

using and including the two calibration parameters gives:

(8)

With this modification, Eq. (3) can be written as:

(9)

which fits in the form and gives a degradation function consistent with Cocks–Ashby porosity kinetics:

(10)

Equation (10) is plotted for several strain-rate exponents in Fig. 1 highlighting the explicit dependence of degradation on m absent from Gurson’s model.

Figure 1. The degradation function’s porosity dependence for several strain-rate exponents and a triaxiality of 0.3

Figure 1. The degradation function’s porosity dependence for several strain-rate exponents and a triaxiality of 0.3

This is simply a reinterpretation of the general yield criteria in Marin and McDowell ([10] ) (neglecting only the small sinh assumption), but it gives physical insight into the degradation of the material, and gives a functional form for degradation that both captures strain-rate dependence explicitly and is consistent with Cocks–Ashby porosity kinetics.

# [Implementation](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#toc)

This section implements Eqs. (2) and (4) using the new degradation function in Eqs. (8) and (10), and gives some thoughts on a useful approximation for Eq. (8). These expressions are implemented in the ale3d finite element code developed at Lawrence Livermore National Laboratory (Anderson et al. [ [1](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#bib1)] ). Simulations are executed on a single 3D finite element loaded with a constant uniaxial velocity and no lateral constraints. Cocks ([4] ) yield criteria is used giving and . Kirchhoff’s stress is used in evaluation of the yield criteria, and is converted to Cauchy stress via , where , for calculation of internal forces. A Swift-type flow stress expression is used with parameters set to MPa, . The Young’s modulus and Poisson’s ratio are 73.3 GPa and 0.33 respectively; The calibration parameters are ; The strain-rate exponent is , where strain-rate is considered constant; and Initial porosity is 0.07 for the simulation without nucleation.

It is of consequence for implementation that Eq. (2) is singular for pure hydrostatic stress (i.e., ). The appendix of Cocks and Ashby ([5] ) accounts for this by providing a separate expression for pure hydrostatic tension conditions; however, this expression requires both an ad-hoc barrier between high triaxialities (that approach pure hydrostatic stress) and low triaxialities as well as implementation of a mechanism for switching between cases.

Figure 2. The degradation function with different approximation of triaxiality (T). When nucleation is present no initial porosity is prescribed

Figure 2. The degradation function with different approximation of triaxiality . When nucleation is present no initial porosity is prescribed

To protect against singularity for pure hydrostatic stress states, this work approximates the triaxiality in Eqs. (2) and (8) by replacing σeq with . The same approximation was made by Lee and Dawson ([9] ) and was motivated by their observation that is “very close” to for rate-independent inelastic materials. The effect of this approximation on degradation (under a uniaxial load) with and without porosity nucleation are shown in Fig. 2. The scenario with nucleation has no initial porosity and porosity nucleation is defined by:

(11)

In Eq. (11), the porosity available for nucleation is , the reference strain is , and the standard deviation of strain-based nucleation equivalent plastic strain is .

Figure 2 shows that, while Eq. (10) allows for increasing degradation (with finite initial porosity) before the onset of plastic strain, this elastic degradation has minimal effect (and is barely distinguishable in Fig. 2). Figure 2 also shows that approximating triaxiality using results in negligible error for the loading and parameters studied.

# [Conclusions](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#toc)

This work shows that the general yield criteria outlined by Marin and McDowell ([10] ) can be expressed as a degradation function. This degradation function has the following advantages over its predecessors: It.

* gives physical insight into the degradation of the material which may be opaque from a yield criteria;
* is both consistent with Cocks–Ashby porosity kinetics and an explicit function of strain-rate exponent and stress state;
* is useful for crystal scale (i.e., crystal plasticity) problems where a yield function is not used directly.

Substituting for in the degradation function protected against a singularity at pure hydrostatic stress and resulted in negligible error for the loading and parameters studied

# [Acknowledgements](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#toc)

The author would like to thank Nathan Barton for his input and discussions regarding this work. This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344 (LLNL-JRNL-730381).

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[*1*](https://0-web-a-ebscohost-com.libus.csd.mu.edu/ehost/detail/detail?vid=3&sid=fd3ef5c9-fb44-41a3-9ce5-054214fa47a5%40sdc-v-sessmgr03&bdata=JnNpdGU9ZWhvc3QtbGl2ZQ%3d%3d#bib1up)*a typographic error in Marin and McDowell (1996) gives the leading term as (2 / 8).*