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Finite Element-based Multi-objective Design Optimization of IPM Considering Saturation Effects for Constant Power Region of Operation

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# Abstract:

This paper proposes a Finite Element (FE)-based multi-objective optimization procedure, under a set of constraints, for interior permanent-magnet synchronous machines (IPMSMs) to achieve a wide constant-power region of operation taking saturation effects and multiple operating points into account. Nonlinear models of the d-q flux linkages are derived based on magnetostatic FEA, which can significantly enhance computational efficiency with good accuracy (only 6 seconds per tentative design using a common PC). A new numerical method conforming to MTPA and flux weakening is presented based on the nonlinear models of the d-q flux linkage. Furthermore, this paper proposes to combine the new numerical method with the magnetostatic FEA using the differential evolution (DE) optimization algorithm to optimize the machine performance to achieve a wide constant power speed range (CPSR). Any operating points (different current/torque and speed) within the torque and speed boundary requirements can be also incorporated in the optimization procedure. Furthermore, saturation effects and multiple operating points are considered in the proposed optimization procedure. The design optimization procedure has been employed to recently developed low-cost IPM with a blend of magnet types. The proposed optimization procedure can be extended to other types of electrical machines.

# SECTION I. Introduction

Interior permanent magnet (IPM) synchronous machines are attractive candidates for industrial and traction applications among others due to their higher power density and efficiency. One of the important factors contributing to the high-power density is the robustness of the rotor structure without any winding and the presence of magnetic saliency that provides additional reluctance torque. These characteristics enable IPM machines to have good flux-weakening capability and can achieve a wide constant power speed range (CPSR).

Flux weakening control techniques have been extensively investigated over the past few decades. Maximum output power is obtained at each speed/operating point following the maximum torque per ampere trajectory under DC bus voltage limitation [1]–[2].

While designing high-performance electrical machines, it is important to evaluate the performance over the entire speed range (constant torque as well as constant power regions) and under different loading conditions. FEA is the most powerful and accurate tool for nonlinear analysis of electrical machines. Some researches in [3][4][5][6]–[7] have attempted to investigate the effects of motor parameters on the CPSR, maximum output torque and torque ripple by means of FEA. However, only three magnet geometries under the rated current operating point are optimized to maximize CPSR and output torque [5]. Additionally, the response-surface method (RSM) was used to derive an empirical relationship between input variables and response, which cannot ensure an accurate flux and torque evaluation. Therefore, a high accuracy FE-based design optimization taking multiple operating points and saturation into effects needs to be pursued. In [8], an FE-based multi-objective design optimization procedure of IPM machine was used to predict the machine performance over a wide constant power region. However, the maximum torque was optimized at only two operating points, namely rated and maximum speed. Authors in [9] introduced an FE-based design optimization algorithm for improving the drive-cycle efficiency of PMSM and took saturation effects into account. However, both torque and flux look-up tables have to be created beforehand to search for minimal current magnitude in the optimal control algorithm, which is not time-efficient. Maximizing CPSR and output torque have not been investigated in the presented optimal algorithm.

This paper proposes an FE-based multi-objective optimization procedure to achieve a wide constant-power region of operation taking saturation effects and multiple operating points into account. The main contribution of the paper is that a new numerical method conforming to MTPA and flux weakening is combined with the magnetostatic FEA with good accuracy and high computational efficiency to optimize the electrical machine performance to achieve a wide constant power region. The presented FE-based multi-objective optimization procedure has been applied to a recently introduced low-cost IPM with a blend of magnet types [10]. Fig. 1 shows the cross section of the developed low-cost IPM, which significantly reduces the rare-earth content while maintaining the high performance of a rare-earth design. It is considered as a baseline design in the paper.

[[Fig. 1 - 
Blended magnets, spoke IPM (Nd in grey, ferrite in purple)
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma1-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma1-p7-ma-large.gif)

**Fig. 1** Blended magnets, spoke IPM (Nd in grey, ferrite in purple)

# SECTION II. Model of Electrical machine

## A. Linear Model of IPM machine

Stator voltage equations can be expressed in the traditional d-q reference frame. The stator voltage and torque of IPM machine in steady-state can be written as:

(1) (2) (3)

Where, *Vd*, *Vq* are the d-axis and q-axis voltages; *Id*, *Iq* are the d-axis and q-axis currents; *λd*, *λq* are the d-axis and q-axis flux linkages; p is number of pairs of poles; *ω* is the electric speed in rad/s.

At a constant speed, and ignoring saturation and winding resistance, d-q axis flux linkage and torque equations can be simplified as:

(4) (5) (6)

Where, *λpm* is PM flux linkage; *Ld* and *Lq* are d axis and q axis inductances, respectively.

Fig. 2 shows optimal current trajectory and voltage ellipses in the rotating rotor reference frame. The angle of the current vectors with respect to the q-axis is defined as γ.

[[Fig. 2 - 
Current and voltage trajectory in the d-q reference plane [8]
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma2-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma2-p7-ma-large.gif)

**Fig. 2** Current and voltage trajectory in the d-q reference plane [8]

[[Fig. 3 - 
Ideal torque and voltage vs. speed characteristics
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma3-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma3-p7-ma-large.gif)

**Fig. 3** Ideal torque and voltage vs. speed characteristics

[[Fig. 4 - 
Optimal current angle and power output vs. speed characteristics
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma4-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma4-p7-ma-large.gif)

**Fig. 4** Optimal current angle and power output vs. speed characteristics

The base speed (or corner speed) *ωb*, which is the lower limit of the CPSR shown in Fig. 4, is defined as the maximum speed at which output torque can be maintained at the rated torque within the current and voltage limits. The upper limit of the CPSR is defined as the maximum speed at which the output power can be maintained at the rated power within the current and voltage limits. When speed is below the base speed and output torque is maintained at rated torque, the terminal voltage increases linearly with speed then reaches the rated voltage at base speed, which correspond to point B in Fig. 2. As the speed exceeds the base speed (Point B in Fig. 2 and Point *ωb* in Fig. 4), the electric machine transitions into flux-weakening region, where the maximum voltage is reached. In order to keep increasing the speed, the current angle γ has to be advanced/increased to weaken the air-gap flux. Consequently, the output torque tends to decrease and the current vector moves from point B to point P. When the current angle continues to increase to 90 deg, the flux cannot be weakened any more and the output torque and power are zero. Point C represents the speed that theoretically is infinite. However, if , point P, which represents the obtainable maximum speed, lies on the negative d-axis and outside of current limit circle [11].

## B. Nonlinear Model of IPM Machine

The variation of inductance and d-q flux linkage due to saturation effects is an important characteristic of the IPM machine. The predicted torque and flux error can increase as saturation level rises [9]. In [12], authors also pointed out that increasing saturation level in the stator teeth can result in larger cogging torque and therefore impact output torque ripple.

Using analytical models and response-surface methods in the electrical machine design optimization process is not very accurate and can be complicated, especially taking saturation effects into account. In general, FEA, is considered the most accurate analysis method of electrical machine. However, FEA is time-consuming and sometimes cannot provide in-depth insights like analytical models.

Accurate prediction of the flux linkage is critical because it provides the basis for accurate prediction of an IPM machine performance. Also, relying on flux linkage eliminates the typical errors involved in calculating inductance as well as separating the d-axis inductance and the magnet flux linkage. In order to evaluate d-q flux linkages with accuracy, a small number of FEA simulations are performed at different γ from 0 to 90 deg. Typically, the FEA simulations points are extracted every 10 deg in electrical degree but in order to obtain higher accuracy results, higher resolution has to be considered. A cubic spline interpolation is performed to approximate the flux linkages between these points. The overall d-axis and q-axis flux linkages functions can be expressed as following:

(7) (8)

where, *λdi* (*I*) and *λqi*(*I*) , which can be obtained by two dimensional cubic spline interpolation in Matlab, are d-axis and q-axis flux linkages with respect to different γ*i* current angle. Assuming a=0 deg, b=10 deg, c=30 deg, d=50deg, e=70deg, f=90deg, respectively, *λdi*(*I*) and *λqi*(*I*) can be plotted as shown in Fig. 5(a) and (b).

[[Fig. 5 - 
d-q axis flux linkage
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma5-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma5-p7-ma-large.gif)

**Fig. 5** d-q axis flux linkage

*λd*\_*inter*(*I*) and *λq*\_*inter* (*I*) are the approximations of d-axis and q-axis flux linkages between two current angles using cubic-spline interpolation.

According to [(7)](https://ieeexplore.ieee.org/document/#deqn7-deqn8) and [(8)](https://ieeexplore.ieee.org/document/#deqn7-deqn8), the d-axis and q-axis flux linkages of the recently introduced low-cost IPM with a blend of magnet types can be obtained. In order to investigate the electrical performance under different loading conditions, a flux-linkage map can be obtained using FEA by interpolation method. The flux-maps obtained using [(7)](https://ieeexplore.ieee.org/document/#deqn7-deqn8) and [(8)](https://ieeexplore.ieee.org/document/#deqn7-deqn8) will be generated. Using the flux linkage maps, the torque, voltage, speed and output power over the optimum current trajectory and different loading conditions can be calculated.

According to [(1)](https://ieeexplore.ieee.org/document/#deqn1-deqn3) and [(2)](https://ieeexplore.ieee.org/document/#deqn1-deqn3), mechanical speed can be derived as:

(9)

Torque can be calculated based on [(7)](https://ieeexplore.ieee.org/document/#deqn7-deqn8) and [(8)](https://ieeexplore.ieee.org/document/#deqn7-deqn8):

(10)

Output power can be given by:

(11)

## C. Optimal Current Angle for Maximum Torque Per Ampere (MTPA)

The current angle γ*MTPA* is defined as the maximum torque per Ampere (MTPA) angle. Below the base speed, the terminal voltage is less restrictive so the maximum nominal torque can be obtained for a constant current angle of γ*MTPA.* However, the MTPA current angle for a different given current (different load operation when motor operating under the point located within torque-speed envelop) can increase with increasing current since the saliency torque is proportional to the current squared while the magnet power is only proportional to current [13]. Therefore, optimal current angle should be obtained in the optimization process.

[[Fig. 6 - 
Maximum torque vs speed
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma6-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma6-p7-ma-large.gif)

**Fig. 6** Maximum torque vs speed

Fig. 7 - 
Efficiency contours


**Fig. 7** Efficiency contours

Using [(7)](https://ieeexplore.ieee.org/document/#deqn7-deqn8), [(8)](https://ieeexplore.ieee.org/document/#deqn7-deqn8) and [(10)](https://ieeexplore.ieee.org/document/#deqn10), torque curves can be plotted. Based on the MTPA search algorithm for maximum torque and the flux weakening strategy, the torque-speed curve with DC bus voltage limit of 350 V and peak current of 250Arms is plotted in Fig. 6 for the baseline design [10]. Efficiency map, as illustrated in Fig. 7, has also been generated using MotorCAD software [10].

# SECTION III. Optimization Process

The magnetostatic FE-based multi-objective optimization procedure taking saturation effects and multiple operating points into account is presented in this section.

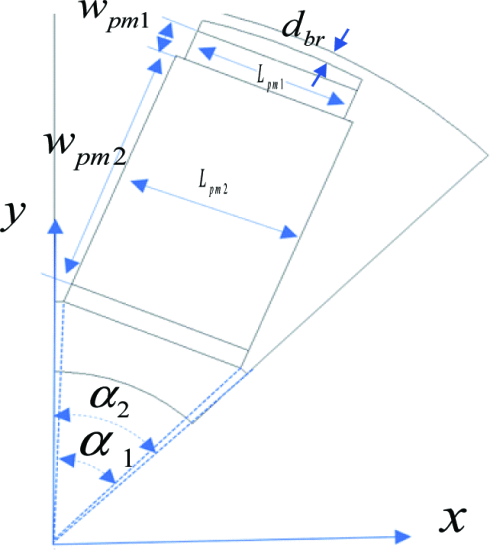
## A. Parametrized FE Model

The low-cost IPM with a blend of magnet types is optimized using parametric modelling so that the geometric dimensions of the machine are allowed to vary. In the parametric model, the outer diameter, inner diameter, number of poles, active stack length and air gap length are assumed to be constant and are summarized in Table 1.

**TABLE I**Constant Design Parameters

|  |  |
| --- | --- |
| Dimensions | constant Value [mm] |
| Stator outer diameter | 264 |
| Stator inner diameter | 161.9 |
| Rotor outer diameter | 160.4 |
| Rotor inner diameter) | 51 |
| Active stack length | 50 |
| Air gap length | 0.73 |
| Number of slots | 48 |
| Number of poles | 8 |

The parameterized finite element model was developed, with the volume of the NdFeB magnet fixed at 10% of the ferrite magnet volume. The key rotor design parameters are shown in Fig. 8. The model consists of four independent geometric variables in the rotor, with upper and lower boundaries to avoid geometric conflicts. The rotor geometric design parameters and corresponding ranges used in the design process are summarized in Table II.

[](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma8-p7-ma-large.gif)

**Fig. 8** Key rotor parameters for the proposed design

**TABLE II**Definition of Design Parameters

|  |  |  |  |
| --- | --- | --- | --- |
| Dimensions(mm) | DEFINIATION | MIN | MAX |
|  |  | 12 | 25 |
|  |  | 1 | 2.5 |
| Total PM width |  | 20 | 40 |
| Nd and ferrite length ratio |  | 0.4 | 0.9 |

## B. Optimization algorithm

With the use of magnetostatic FEA, a reduction of one to two orders of magnitude was achieved in terms of computational time in comparison to time-stepping FEA [14]. Nonlinear models of the d-q flux linkage can be derived based on the magnetostatics FEA. The proposed numerical algorithm is shown in Fig. 9. For each candidate design, the optimal angle and optimal torque for each operating point are obtained through the introduced algorithm shown in Fig. 9 The optimal current angle and torque can be calculated through the method introduced earlier if the speed is less than the base speed. Above the base speed, the introduced algorithm shown in Fig. 9 is applied to calculate the optimal current angle and torque. Accordingly, the constraint functions, such as maximum speed calculation, can be evaluated using the introduced algorithm. Through the developed algorithm in Fig. 9, MTPA and flux weakening strategies are successfully incorporated to ensure optimal current angle and torque while voltage and current limits are satisfied.

The developed numerical algorithm is combined with the magnetostatics FEA using the differential evolution (DE) optimization algorithm to optimize the CPSR performance. The DE search algorithm was selected to further reduce computational time [13]. The flowchart of the overall optimization process is shown in Fig. 10.

## C. Performance Metrics

[(1)](https://ieeexplore.ieee.org/document/#deqn1-deqn3) Evaluation of CPSR: the lower limit of CPSR can be calculated as:

(12)

where, *Vr* is the nominal voltage, γ*MTPA* is the optimal current angle, *Ir* is the nominal current.

The upper limit of CPSR can be calculated using [(9)](https://ieeexplore.ieee.org/document/#deqn9) when speed is Ω2.

[(2)](https://ieeexplore.ieee.org/document/#deqn1-deqn3) Active material cost: The active material cost can be calculated based on the masses of active material. The cost is evaluated using a weighted function as follows:

(13)

where, *mPM*, *mCu* and *mFe* are the masses of PM, copper and laminated steel, respectively. *cPM*, *cCu*and *cFe* are the material cost coefficients per unit of mass.

[[Fig. 9 - 
Developed numerical algorithm conforming to MTPA and flux weakening
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma9-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma9-p7-ma-large.gif)

**Fig. 9** Developed numerical algorithm conforming to MTPA and flux weakening

[[Fig. 10 - 
Flowchart of the overall optimization procedure
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma10-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma10-p7-ma-large.gif)

**Fig. 10** Flowchart of the overall optimization procedure

## D. Objectives and constraints

In order to maintain a wide constant power region and maximize the power output, the multiple objectives of the optimization process are selected as follows: (1) maximize output torque at base speed; (2) maximize output torque at maximum speed. The electrical machine design also needs to satisfy other constraints as follows: maximum torque below the base speed>120Nm, maximum torque at high speed > 20Nm maximum. All of these objectives have to be achieved within the voltage and current limits.

# SECTION IV. Optimization Results

In order to verify the proposed magnetostatic FE-based multi-objective optimization procedure, the developed low-cost spoke type IPM with a blend of magnet types was studied. Table III summarizes the key operating points. FEA is implemented using ANSYS/Maxwell to calculate the electric motor performance and is called by optimization algorithm, which is implemented in Matlab. The overall design optimization procedure is carried out on a common PC with Intel E3-1230 CPU and 16 GB RAM. The optimization process requires 180 generations, each with 10 members, making a total population of 1800 candidate designs. MTPA control, optimum current angle in flux weakening and torque calculations are evaluated in each iteration. Fig. 11 shows the torque performance of the 1800 candidate designs under both base speed and maximum speed. As illustrated in Fig. 11, the optimum candidate designs with maximum torque under base speed and maximum speed locate at right upper corner. The best candidate design specified as D1 with black dot in Fig. 11 can be selected for further investigation. Note that torque under the base speed and maximum speed are close to 130Nm and 30Nm respectively, which is comparable to the torque as the baseline (blended magnet IPM in [10]) can generate under the operating condition, as shown in table III.

**TABLE III**Operating points

|  |  |
| --- | --- |
| Main specifications | SPOKE DESIGN |
| Maximum DC bus voltage limit (V) | 350 |
| Rated current (A) | 106 |
| Current angle at base speed | Introduced MTPA in section C of II |
| Based speed | Calculated using Eq. (9) |
| Optimum current angle flux weakening | Proposed numerical algorithm in Fig.9 |
| Maximum speed | Depending on the optimization |

[[Fig. 11 - 
Optimization results through the developed algorithm
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma11-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma11-p7-ma-large.gif)

**Fig. 11** Optimization results through the developed algorithm

Fig. 12 shows the corresponding geometry of the selected candidate design. The optimal main design parameters corresponding to candidate design in Fig. 12 is summarized in Table IV. Fig. 13 shows a comparison of the torque-speed curves of the selected candidate design and initial design. As illustrated in Fig. 13, the output torque output is higher than the initial design over the entire speed range. This validates the effectiveness of the proposed design optimization procedure with considering both based speed and maximum speed. Torque waveform at corner speed is shown in Fig. 14. As illustrated in the figure, peak-to-peak torque ripple is 16.1%. The efficiency at the corner speed is around 96.3%. The efficiency calculation includes the copper and iron losses but not the mechanical losses.

**TABLE IV**Optimal Design Parameters

|  |  |
| --- | --- |
| Dimensions(mm) | VALUE |
|  | 11.9 |
|  | 1.6 |
| Total PM width | 38.5 |
| Nd and ferrite length ratio | 0.87 |

[[Fig. 12 - 
The selected candidate design
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma12-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma12-p7-ma-large.gif)

**Fig. 12** The selected candidate design

[[Fig. 13 - 
The comparison of torque-speed curve
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma13-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma13-p7-ma-large.gif)

**Fig. 13** The comparison of torque-speed curve

[[Fig. 14 - 
Torque waveform at corner speed
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma14-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma14-p7-ma-large.gif)

**Fig. 14** Torque waveform at corner speed

Fig. 15 shows the demagnetization analysis at low temperature (-20°C) and high temperature (120 °C) in the selected candidate design. The magnet materials selected for the study were TDK FB13B for the ferrite magnet and N45UH [15] for the rare-earth magnet. The flux density value at the knee points is 0.02 T for ferrite at -20°C and -0.1T for rare-earth magnet at 120 °C. As illustrated in Fig. 15, there is no demagnetization risk when the rated current is applied along the negative d-axis (current angle= 90 deg).

[[Fig. 15 - 
Demagnetization analysis
](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma15-p7-ma-large.gif)](https://ieeexplore.ieee.org/mediastore_new/IEEE/content/media/9235288/9235325/9236049/ma15-p7-ma-large.gif)

**Fig. 15** Demagnetization analysis

# SECTION V. Conclusions and future work

The paper presented an FE-based multi-objective design optimization procedure for an IPM to achieve a wide constant-power region of operation taking saturation effects and multiple operating points into account. Nonlinear flux linkage maps of the IPM machine have been derived using FEA, based on which a new numerical method conforming to MTPA and flux weakening has been developed. In order to save computation time, magnetostatic FEA is combined with the introduced numerical method through differential evolution (DE) search algorithm to optimize the electrical machine performance to maximize torque output and CPSR performance. The recently developed low-cost IPM with a blend of magnet types was studied to validate the effectiveness of the proposed design optimization procedure.

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