1-1-2002

Extension Risk in Commercial Mortgages

Charles C. Tu
California State University - Fullerton

Mark Eppli
Marquette University, mark.eppli@marquette.edu

Accepted version. *Real Estate Finance*, Vol. 18, No. 4 (Winter 2002): 53-63. Publisher Link.
Mark Eppli was affiliated with George Washington University at the time of publication.

Extension risk in commercial mortgages

Charles C. Tu

Department of Finance, California State University Fullerton, CA

Mark J. Eppli

Department of Finance, George Washington University Washington, DC.

The risk in mortgages has been investigated by academicians and practitioners for decades. While the development of our understanding of the risk in commercial mortgages has paralleled that of the residential mortgage research, it lagged by 10-20 years and has some different emphases.[1] Unlike recent studies on residential mortgages that primarily focus on prepayment risk,[2] the research on commercial mortgages concentrates on default risk because commercial mortgages are generally nonrecourse and have prepayment protection in the form of lockouts, defeasance, or yield maintenance agreements.

Commercial mortgage default risk has been examined from a variety of perspectives. Some studies examine the performance of commercial mortgages with loan-level data (Snyderman [1991, 1994]; Ciochetti [1997]; Esaki, L'Heureux, and Snyderman [1999]);[3] others investigate the probability of default using statistical models (Vandell [1992]; Vandell et al. [1993]).[4] Researchers have also analyzed the pricing of commercial mortgages in a contingent-claims framework

Real Estate Finance, Vol. 18, No. 4 (Winter 2002): pg. 53-63. Permalink.

(Titman and Torons [1989]; Kau et al. [1990]; Childs, Ott, and Riddiough [1996]).[5] These studies focus on the risk that the borrower may default before the loan matures but generally pay little attention to the possibility that a borrower may have difficulty paying off the loan at maturity even though the loan is not in default.[6]

Unlike residential mortgages, most of which are fully amortizing loans, commercial mortgages often are interest-only or amortizing loans with a balloon payment at maturity. This balloon payment is usually made by refinancing the current mortgage and the extension risk (also referred to as balloon risk or refinance risk) arises from the borrower's inability to refinance at mortgage maturity. Borrowers with a mortgage at maturity that is not in default but does not meet contemporaneous underwriting standards will often request to extend the loan with the lender (see Harding and Sirmans [1997]; Jacob and Fastovsky [1999]). The borrower's ability to refinance a mortgage is largely dependent on changes in four factors between mortgage origination and loan maturity: mortgage interest rate, property net operating income (NOI), debt coverage ratio (DCR), and loan-to-value ratio (LTVR).

The inability of a borrower to refinance a mortgage can lead to extension and ultimately default. Lenders and mortgage service companies often agree to extend a mortgage only after establishing a new set of standards for the repayment of principal and interest; some of these include: assigning property income to a lock box, hyper-amortization, payment of mortgage extension points, higher interest rates, and floating interest rates, among others. While many extended loans are eventually worked out with no direct loss (and possibly a gain) to the lender, other extended loans fail.

In this article we use both historical data and Monte Carlo simulation to examine the likelihood of loan extension and potential losses associated with extension. We find that extension probability is highly sensitive to property NOI growth, to NOI volatility, to the amortization schedule, and to the loan term. We also find that extension risk is largely unaffected by changing credit spreads, changing yield curve assumptions, and changing term default assumptions.[7] We also find that changing the underwriting standards (i.e., tighter or looser DCR and LTVR ratios) affects the probability of loan extension; however, in a somewhat muted way. Finally, we estimate that the loss during extension is approximately 2%-3% of the outstanding loan amount at maturity.

The remainder of the article is segmented into four additional sections. In the next section we discuss extension risk using historical data. Using data from the American Council of Life Insurance (ACLl), we assess the likelihood of loan extension across a variety of property income growth rates and property types. In the section that follows we estimate the probability that a loan will be extended using Monte Carlo simulation. With Monte Carlo simulation we are able to control for a variety of factors that are not accounted for when using historical data including default, NOI volatility, and interest rate volatility. With reasonable estimates of mortgage extension rates, we measure extension loss in the next to last section and close the article with the conclusion.

Extension Risk Using Historical Information

We first examine a borrower's ability to refinance a mortgage at maturity using data on commercial mortgages compiled by the American Council of Life Insurance (ACLI).[8] There are two primary lender ratios that are used by permanent lenders to measure risk in commercial mortgages—debt coverage ratio and loan-to-value ratio. While lenders also assess borrower credit history and asset quality, the riskiness of commercial mortgages is primarily based on the ability of the asset to generate sufficient cash flow to make periodic mortgage payments (return on investment) and the expected asset value at loan maturity to repay principal (return of investment).

The debt coverage ratio measures how many times property income covers debt service. In other words, DCR is a cash flow adequacy test. To determine the justified loan amount based on property cash flows, we calculate the following:

$$Justified\ Loan\ Amount(JLA_{DCR}) = \frac{Net\ Operating\ Income/Debt\ Coverage\ Ratio}{Mortgage\ Constant} \qquad (1)$$

In this equation NOI divided by DCR reveals the justified debt service amount. Dividing the justified debt service amount by the mortgage constant (MC) returns the justified loan amount. The mortgage constant is the installment to amortize a dollar for amortizing loans and the mortgage interest rate for interest-only loans.

In addition to determining a justified loan amount based on property cash flows, lenders also determine a justified loan amount based on the collateral value of the asset:

$$Justified\ Loan\ Amount(JLA_{LTVR}) = \frac{Net\ Operating\ Income}{Capitalization\ Rate} \times Loan\ to\ Value\ Ratio \qquad (2)$$

Dividing the NOI by the capitalization rate (CR) returns an asset value, which is then multiplied by the loan-to-value ratio to arrive at the justified loan amount based on property value. If the requested loan amount is less than or equal to the lesser of the JLA_{DCR} and JLA_{LTVR} the borrower has a good credit history, and the asset is of sufficient quality, the loan is made.

Moving forward in time to loan maturity, if the property has been reasonably well maintained and the borrower remains in good standing, the loan is likely to be refinanced if both the contemporaneous JLA_{DCR} and JLA_{LTVR} are greater than or equal to the outstanding loan balance at maturity (OLB_M, which is also the balloon payment). Using the ACLI data on mortgage interest rates, capitalization rates, debt coverage ratios, and loan-to-value ratios, we measure the likelihood that the borrower would be unable to refinance the loan at maturity.

The ACLI data for the period 1966 to 1998 is summarized in Exhibit 1. The average interest rate across the three property types (office, retail, and industrial) on commercial loans was approximately 9.7%, with interest rates varying widely from just over 6% to just under 16%. Capitalization rates were approximately 10% with a standard deviation of about 1%, about one-half the variability of mortgage interest rates. The two primary loan-underwriting standards, DCR and LTVR, show little variance around their mean values of 1.3 times income and 71 % of value.

To estimate whether an average loan would be extended we use the loan underwriting standards and interest rates reported at mortgage origination and compare them to the standards and rates at mortgage maturity as follows:[9]

$$\begin{cases} \text{If } JLA_M > OLB_M, \\ \textit{the borrower refinances the loan and outstanding loan} \\ \textit{balance at maturity } (OLB_M) \textit{ is paid off.} \\ \\ \text{If } JLA_M < OLB_M, \\ \textit{the borrower cannot refinance the loan (extension risk).} \end{cases} \tag{3}$$

Extension risk is therefore a function of the justified loan amount when the loan is refinanced (*JLAM*) and the outstanding loan balance at maturity (i.e., balloon payment). It should be noted that this measure of extension risk does not imply that the value of the property is not adequate to pay off the loan at maturity. What is suggested is that using underwriting standards at loan maturity, the outstanding balance exceeds what can be justified to refinance the outstanding loan balance.

Panel A of Exhibit 2 presents the results of estimating extension risk of five-year mortgages using the underwriting standards set forth in Equations (1) and (2). The extension risk is based on rolling five-year periods beginning 1966:1 and running through 1998:3. For each of 111 five-year periods (1966:1 to 1970:4, 1966:2 to 1971:1, and so on) we determine whether the outstanding loan balance of a mortgage originated five years earlier can be underwritten based on contemporaneous interest rates, cap rates, LTVRs, and DCRs given a certain NOI growth rate. The exhibit presents the frequency of loan extension for both interest-only and 30-year amortizing loans with a balloon at the end of a five-year loan term.[10]

The results reveal that across all property types and income growth rates, the extension potential approximately doubles from a 30-year amortizing loan with a five-year balloon payment to an interest-only (i.e., five-year bullet) loan, all else equal. For example, assuming a 30-year amortizing mortgage on an office property with a 3% NOI growth rate, 9.0% of the time (10 out of 111) income growth is not adequate to offset an increase in interest rates or a change in underwriting standards five years later; 18.9% of the time (21 out of 111) an interest-only loan would be subject to extension risk. These loans may not be underwater (i.e., the outstanding loan amount is greater than the property value); however, either the property income or property value is not adequate to refinance the outstanding loan amount at maturity. As expected, when property income growth rates increase extension risk drops in a geometric pattern across both amortizing and interest-only loans.

Panel B of Exhibit 2 presents the extension risk for 10-year mortgages. The results are similar to Panel A; however, the extension risk differentials are even greater between amortizing and interest-only loans. These results indicate that amortizing loans are much less likely to run the risk of being extended. While amortizing loans are expected to have a lower extension risk, the magnitude of the difference was not expected. Additionally, longer-term loans have lower extension risk than shorter-term loans. Greater loan amortization and the upward drift of NOI may explain lower extension risk for longer-term loans.

While the results presented in Exhibit 2 are interesting and informative, there are some limitations to these findings. The results are based on aggregate data, not loan-specific data. Also, it is assumed that all originated loans survive to maturity, i.e., there is no default or prepayment before maturity. Additionally, the analysis does not consider NOI volatility. In the next section we use the Monte Carlo simulation to measure the extension risk of individual loans while explicitly accounting for interest rate volatility, NOI volatility, and term default.

Extension Risk Using the Monte Carlo Simulation

To more accurately assess extension risk, it is necessary to first estimate the probability that the mortgage has been terminated prior to maturity. In other words, loans that default during the term of the loan are no longer outstanding at maturity and therefore have no possibility of being extended. To measure extension risk while accounting for the possibility of term default, we employ the framework developed by Titman and Torous [1989] and Kau et al. [1990], whereby the borrower chooses to exercise the default option if the property value falls below the market value of the mortgage at any payment date.

Two stochastic state variable—property NOI and interest rates—are included in the model. We assume that NOI follows a standard lognormal diffusion process,[11] and that interest rates follow the mean-reverting, square root model of Cox, Ingersoll, and Ross [1985].[12] Property values are determined by direct capitalization where value is equal to NOI divided by capitalization rate (CR), $V = NOI/CR$. Historical data shows that capitalization rates are correlated with mortgage interest rates; we therefore estimate contemporaneous cap rates as a function of mortgage interest rates.[13]

Following other studies on commercial mortgages (e.g., Titman and Torous [1989]; Childs, Ott, and Riddiough [1996]), we assume that all mortgages are nonrecourse and that prepayment will not occur due to defeasance, lock-out, and yield maintenance provisions. The base-case simulation uses an upward-sloping Treasury yield curve,[14] and the credit risk spread on 10-year mortgages is assumed to be 180 basis points over similarly termed Treasuries.[15]

Since only mortgages that survive to maturity are subject to extension risk, an important step in the analysis is to establish the borrower's default decision criteria. Early commercial mortgage pricing research assumes that borrowers default ruthlessly (i.e., the borrower defaults when the property value falls below the mortgage value). Subsequent evidence shows that transaction costs for both borrowers and lenders are relevant to an appropriately specified model.[16] We therefore assume that if the property value is 5% less than the market value of mortgage the borrower will default. Panel A of Exhibit 3 presents the timing of term default for the base case analysis where we assume a 3% property NOI growth rate and a 12% NOI standard deviation. Both the timing and cumulative default rates (10.46%) are consistent with Esaki, L'Heureux, and Snyderman [1999]. Panel B of the exhibit shows the distribution of loan-to-value-ratio at the end of year 10 for a mortgage that does not default before maturity. The distribution reveals that a mortgage has a 76.66% chance of being refinanced, and a 12.88% chance of being extended.

Exhibit 4 reveals the cumulative default levels and extension risk levels for 10-year mortgages with 30-year amortizations for a range of NOI growth rates and NOI growth volatilities.[17] NOI growth rates range from 1% to 5%, which reflects investor and lender expectations over the past 15 years. The NOI standard deviation range is from 6% to 18%.[18] At origination the loan is expected to be underwritten using a 1.3 DCR and a 75% LTVR; the same standards are used to underwrite the loan at refinancing.

Cumulative default rates in Exhibit 4 range from a 0.0% for the 5% NOI growth and 6% NOI standard deviation case to 40.90% for the 1% NOI growth and 18% NOI standard deviation case. As expected default rates increase monotonically as the volatility of NOI increases and decrease monotonically as expected NOI growth increases. However, the pattern of extension risk is less clear.

Extension risk in Panel B of Exhibit 4 generally reveals a decreasing pattern for the 1% NOI growth rate case as the standard deviation increases, while the 3% and 5% growth rate cases reveal an increasing pattern of extension risk that is increasing at a decreasing rate. One possible explanation for this anomaly is the high probability of term default tor the 1% NOI growth rate case, which significantly reduces the chance of loans remaining till maturity to be extended. For the 3% and 5% NOI growth cases, the high probability of term default may also explain why extension risk largely stabilizes, or even declines, after the 12% NOI standard deviation cases.

Exhibit 5 shows the simulation results for 10-year, interest-only mortgages. The extension risk reveals similar patterns across NOI growth and volatility. Interestingly, extension risk is much higher for the non-amortizing loans than the 30-year amortizing loans when NOI standard deviation is relatively low, but becomes comparable to Exhibit 4 across amortization schedules when NOI is more volatile. Exhibit 6 presents the results for the base case analysis (3% NOI growth, 12% NOI standard deviation, and 30-year amortization) where underwriting standards at maturity are allowed to differ from those at loan origination. Across reasonable changes in underwriting standards the risk of extension remains relatively stable; however, as underwriting standards are taken to relative extremes, extension risk changes become more volatile.

Additional simulations were completed where the following variables were permitted to change: credit risk spreads, the term structure of interest rates, and the correlation between NOI and interest rates. For each of these simulations we find the extension risk to be largely unaffected over reasonable changes in these attributes.

Extension Loss

With an understanding of the effect of changing interest rates, underwriting standards, and property income growth rates and volatilities on the probability of mortgage extension, we now estimate lender's expected loss on an extended loan. Extension loss comes from two factors: 1) delays in receiving cash flows from extended mortgages, and 2) mortgage default during extension.

By definition, extended loans are riskier than loans that can be refinanced. As such, cash flows from extended loans need to be discounted at a rate that reflects the increase in yield necessary to offset the quality shrinkage. Therefore, the discount rate to take the present value of the mortgage cash flows (back to the mortgage maturity date) should exceed mortgage interest rate at maturity to account for the uncertainty and illiquidity that comes with investing in extended loans.

Default during extension is another risk that must be addressed. We assume that during the extension period 1) loans with a DCR of greater than 1.30 and a LTVR of less than 75% will be refinanced; 2) default will occur when property values fall below 95% of the mortgage value after year 10 (the same assumption used to model term default) and defaulted mortgages are assumed to incur a loss of 35% of the outstanding loan balance;[19] and 3) loans that are not refinanced and not in default will be extended for another one-year period.

Exhibit 7 presents default, extension, and refinance rates assuming a 3% NOI growth rate. Assuming each loan can be extended for as many as 10 consecutive years, we analyze mortgage extension through year 20.[20] The first column of the exhibit presents the probability that a loan has defaulted before maturity, that it can be refinanced, and that it needs to be extended. For example, using the 12% NOI standard deviation case in Exhibit 7, the simulation results reveal a 10.46% term default rate, 76.66% refinance rate, and 12.88% extension rate.

For the 12% standard deviation case (where 12.88% of the loans are extended) subsequent default, refinance, and extension are simulated for each year after the original maturity. For instance, in year 11, of the extended loans 0.66% default, 4.14% are refinanced, and 8.08% are extended again.[21] This process is continued each year till year 20 when all remaining loans are refinanced. Within five years of maturity, less than 1 % of all loans continue to be extended across all simulated levels of NOT variance. In the first year of mortgage extension (year 11), as NOI standard deviation increases from 6% to 15% default rates increase dramatically from 0.04% to 1.08%, but the rate of default levels off for the 18% NOI standard deviation case. The change in default rates across NOI standard deviations becomes much more muted in subsequent years, and after the fifth year of extension (year 15) default and extension rates become negligible across all levels of NOI variance.

The large increase in default levels across increasing NOI variance creates significantly higher loss rates for higher NOI standard deviations as can be seen in Exhibit 8. In the exhibit loan losses are stated as a percentage of the outstanding loan balance at maturity. Since little empirical evidence exists on the appropriate cash flow discount rate we present a range of risk premiums over contemporaneous mortgage interest rates. The results reveal that the 6% NOI standard deviation case has an expected loss that is half of the 12%-18% standard deviation cases. Interestingly, expected loss rates for the 12%-18% NOI standard deviation cases when using a 100-600 basis point risk premium over contemporaneous mortgage interest rates are remarkably stable at approximately 2%-3% of the outstanding loan balance at maturity.

Conclusion

Extension risk in commercial mortgages arises from the borrower's inability to refinance a property at maturity. The risk of a loan extending primarily comes from adverse changes in the loan-to-value ratio, the debt coverage ratio, the property's net operating income, and/ or interest rates. While a loan may run the risk of extension, extended loans may not create losses. Most loans that are extended have stepped-up amortization schedules, mortgage extension points, and interest rate adjustments.

In this article we use both historical data and Monte Carlo simulation to assess the probability that a borrower is unable to refinance the mortgage at maturity. As expected, we find that extension risk is sensitive to NOI drift and mortgage amortization: properties with lower NOI growth are more likely to experience difficulty refinancing; and interest-only loans are subject to higher extension risk than amortizing loans. While NOI volatility has a dramatic effect on term default risk, reasonable ranges of NOI volatility have a muted effect on extension risk. One potential reason for the minor impact of NOI volatility on extension risk may be attributable to the interaction of default and extension (i.e., a defaulted loan is no longer outstanding at maturity and therefore cannot be extended).

Other potential factors in predicting extension risk are the underwriting standards at maturity. As expected, tighter standards at maturity, as opposed to loan origination, increase extension risk. Ho\\'ever, the results are somewhat surprising when the same underwriting standards are used at loan maturity and loan origination. While the tighter underwriting requirements at origination substantially reduce the probability of default during the life of a loan, they increase the possibility of extension. Again, this result may be attributable to the interaction between default and extension.

For all loans that cannot be refinanced at maturity we continue the simulation for 10 more years. There the extended mortgages are refinanced as soon as underwriting standards are met. By year 15 we find that less than 1 % of all mortgages continue to be extended across all models of NOI variance. Using a range of discount rates we find that expected losses from extension are relatively stable at 2%-3% of the original loan amount at loan origination.

To date, extension risk has largely been overlooked in the literature, and possibly over (under) estimated by mortgage lenders due to the uncertainty (ignorance) of losses during extension. This article is a first cut at understanding extension risk and we find that this risk is not trivial, but may be less than some might expect.

Endnotes

1. For a comprehensive review of the early literature on mortgage credit risk, see Vandell [1993].
2. Residential mortgage-backed securities (MBS) are generally guaranteed by government-sponsored agencies, such as Fannie Mae and Freddie Mac, effectively eliminating the default risk tor MBS investors.
3. Snyderman [1991, 1994] and Esaki, L'Heureux, and Snyderman [1999] examine cumulative default risk and loss severity of commercial mortgages made by insurance companies. The results indicate that investor; in commercial mortgages generally cam more yield than Treasury securities but overall performance of these loans was extremely volatile. Ciochetti [1997] describes the loss characteristics associated with commercial mortgage foreclosure and finds that the average net loss recovery was approximately 69% and this amount is related to loan size, geographical location, and, most importantly, the jurisdictional foreclosure method.
4. Vandell [1992] and Vandell et al. [1993] use statistical models to evaluate the relationship between commercial mortgage default and loan,

borrower, property, and market characteristics. The results confirm that property value and market value of the mortgage are the dominant factors affecting default.

5. Titman and Torous [1989] are the first to use contingent claims modeling in commercial mortgage pricing. They find that the model can explain the observed default premiums for a sample of fixed-rate, bullet mortgages. Kau et al. [1990] model the pricing of commercial mortgages and their mortgage-backed securities. In their analysis, the valuation of an MBS is explicitly tied to that of the underlying mortgage. The authors conclude that option-pricing models provide an accurate and flexible approach to valuing MBS. Childs, Ott, and Riddiough [1996] apply the contingent-claim model to the pricing of multi-class commercial mortgage-backed securities.

6. For a discussion of reasons why defaults will tend to be delayed, and thus balloon risk will become more significant, see Corcoran [2000].

7. Term default is defined as borrower default during the term of the loan and is exclusive of default during the extension period.

8. The data reported by the ACLI are quarterly averages by property type. No loan-specific data is provided. ACLI reporting companies account for approximately two-thirds of non-farm mortgages held in the U.S. by life insurance companies. The data in this report is from the ACLI's *Investment Bulletin*.

9. In the analysis it is assumed that there are no defaults or prepayments before loan maturity.

10. Given a 1% NOI annual growth, for example, an office mortgage with 30-year amortization would not be refinanced at maturity in 27 of the 111 rolling five-year periods (24.3%) as either the LTVR is too high or the DCR is too low to underwrite based on the contemporaneous standards. Although this analysis approach is sometimes called "historic simulation," it involves only mechanical computations with historical data, but no random variables.

11. It is assumed that the net operating income of a property follows the lognormal process:

$$dNOI = \gamma\, NOI\, dt + \sigma_{NOI} NOI dz$$

$$\frac{dNOI}{NOI} = \gamma\, dt + \sigma_{NOI} dz$$

Where

γ ≡ instantaneous expected growth rate of NOI

δ*NOI* ≡ instantaneous standard deviastion of NOI growth.

z ≡ standardized Wiener process.

12. In the Cox-Ingersoll-Ross model, instantaneous risk-free rate is assumed to follow the stochastic process:

    $$dr = \kappa(\nu - r)dt + \sigma_r \sqrt{r} dz_r$$

    where

    $\kappa \equiv$ speed of reversion parameter,

    $\nu \equiv$ long-run mean of r,

    $\sigma_r \equiv$ instantaneous standard deviation of r (interest rate volatility parameter), and

    $Z_r =$ standard Wiener process.

    Additionally, unanticipated changes in NOI growth are assumed to be correlated with unanticipated changes in interest rates, $dz_{NOI}dz_r = pdt$, where p denotes the correlation coefficient. A 0.2 correlation is used in the analysis.

13. The relationship between capitalization rates and mortgage rates is estimated with data from the American Council of Life Insurance (ACLI) for the period 1996 to 1998, using the following regression model:

    $$Capitalization\ Rate = a + b \times Mortgage\ Rate + \delta_{CAP}dz$$

    where a and b are the intercept and slope of the regression line.

14. In the model, κ, υ, and σ_r, are 10%, 7.5%, and 8%, respectively. These parameters are consistent with studies by Riddiough and Thompson [1993] and Childs, Ott, and Riddiough [1996]. A 6% short-term risk-free rate is assumed. Other shapes of the yield curve and parameters used in Cox, Ingersoll, and Ross [1979]. Dunn and McConnell [1981], Kau, Keenan, and Kim [1994], and Hilliard, Kau, and Slawson [1998] are also considered, but do not significantly affect the results.
15. The average spread between commercial mortgages and U.S. Treasuries from 1966 to 1998 was 167 basis points; the average spread in the last 10 years was 187 basis points. We run the simulation with spreads of 160, 170, 180, 190, and 200 basis points, and find similar results.
16. Borrower default cost has been treated in a variety of ways. For example, Ciochetti and Vandell [1999] consider the borrower default cost as a constant percentage of property value. Riddiough and Thompson [1993] model the costs as a function of loan characteristics.

17. At the beginning of the simulation process, it is assumed that mortgage interest rate is 8.03% and the property NOI is $1,000 (which is arbitrarily selected and has no effects on the results). In each simulation path, the interest rate, NOI growth rate, and capitalization rate are generated by computer based on the specified stochastic processes. With the simulated parameters, mortgage value and property value are calculated for each of the following time periods. In each period, if the property value is less than 95% of the mortgage value, we assume that the borrower defaults so the simulation path is terminated. In a path where the borrower docs not default prior to maturity, a justified loan amount is calculated based on the LTVR and DCR at maturity, the contemporaneous mortgage rate, and the property NOI at the time of refinancing. If the justified loan amount is greater than the outstanding loan balance at maturity, the mortgage is refinanced; otherwise, it needs to be extended. Loan extension is represented by a binary variable (which has a value of 1 if the loan is extended. and 0 otherwise). Extension risk in the following exhibits is the mean value of this binary variable using 5,000 simulation iterations.

18. Jacob, Hong, and Lee [1999] estimate a 6% volatility of NOI with large and diversified pool of properties and expect the volatility of individual properties to range between 9% and 15%. Meanwhile, Ciochetti and Vandell [1999] and Geitner, Craff, and Young [1994] suggest an implied annual volatility of property value of 14%-18%. We therefore consider the range between 6% and 18% for the simulation analysis.

19. See Curry, Blalock, and Cole [1991], Snyderman [1994], and Ciochetti [1997] for discussion of loss severity associated with commercial mortgage foreclosure. It could be argued that a risk-adjusted discount rate already accounts for default losses. Here we model extension default separately and suggest that increases in the discount rate are attributable to the uncertainty of the timing of the cash flows and lack of investment liquidity.

20. All loans extended for each of the years 11-19 are assumed to be refinanced at the end of year 20.

21. The rate of default, refinance, and extension are calculated based on the original mortgage amount. The sum of default, refinance, and extension is equal to the extension rate in the previous year.

References

Childs, P.D., S.H. Ott, and T.J. Riddiough. "The Value of Recourse and Cross-Default Clauses in Commercial Mortgage Contracting." *Journal of Banking and Finance,* 20 (1996), pp. 511-536.

Ciochetti, B.A. "Loss Characteristics of Commercial Mortgage Foreclosures." *Real Estate Finance* Vol. 14, No.1 (1997). pp. 53-69.

Ciochetti. B.A. and K.D. Vandell. "The Performance of Commercial Mortgages." *Red Estate Economics,* Vol. 27, No.1 (1999), pp. 27- 61.

Corcoran. P. "Stress and Defaults in CMBS Deals: Theory and Evidence." *Real Estate Finance,* Vol. 17. No.3 (2000), pp. 63-71.

Cox, J.C., J.E. Ingersoll, and S.A. Ross. "A Theory of the Term Structure of Interest Rates." *Econometrica* , 53 (1985), pp. 385-407.

Curry, T.J. Blalock, and R. Cole. "Recoveries on Distressed Real Estate and the Relative Efficiency of Public versus Private Management." *Journal of the American Real Estate and Urban Economics Association,* 19 (1991), pp. 495-515.

Dunn, K.B., and J.J. McConnell. "Valuation of GNMA Mortgage Backed Securities," *Journal of Finance,* 36 (1981), pp. 599-616.

Esaki. H., S. L.'Heureux, and M. Snyderman. "Commercial Mortgage Defaults: An Update." *Real Estate Finance,* Vol. 16, No.1 (1999), pp.80-86.

Geltner, D., R. Graff and M. Young. "Appraisal Error and Commercial Property Heterogeneity: Evidence from Disaggregate Appraisal-Based Return." *Journal of Real Estate Research,* 4 (1994), pp. 403-420.

Harding, J.P., and C.F. Sinnans. "Commercial Mortgage-Backed Securities: An Introduction to Professional Investors." *Real Estate Finance,* Vol. 14, No. 1 (1997). pp. 43-51.

Hillard, J.E., J. B. Kau, and V. C. Slawson. "Valuing Prepayment and Default in a Fixed-Rate Mortgage: A Bivariate Binomial Option Pricing Technique." Real Estate Economics*,* Vol. 26. No. 3 (1998), pp. 431-468.

Jacob, D.P., and P. Fastovsky. "Understanding and Managing the Balloon Risk of Commercial Mortgages in CMBS." In F.J. Fabozzi and D.P. Jacob, eds., *The Handbook of Commercial Mortgage-Backed Securities.* New Hope. PA: Frank J. Fabozzi Associates. 1999. pp. 275-286.

Jacob, D.P., T. Hong, and L.H. Lee. "An Options Approach to Commercial Mortgages and CMBS Valuation and Risk Analysis." In F.J. Fabozzi and D.P. Jacob. eds .. *The Handbook of Commercial Mortgage-Backed Securities.* New Hope, PA: Frank J. Fabozzi Associates, 1999, pp. 317-346.

Kau, J.B., D.C. Keenan, W.S. Muller, and J.F. Epperson. "Pricing Commercial Mortgages and Their Mortgage-Backed Securities." *Journal of Real Estate Finance and Economics,* 3 (1990), pp. 333-356.

Kau, J.B., D.C. Keenan, and T. Kim. "Default Probabilities for Mortgages." *Journal of Urban Economics,* 35 (1994). pp. 278-296.

Riddiough, T.J., and H.E. Thompson. "Commercial Mortgage Pricing with Unobservable Borrower Default Costs." *Real Estate Economics* Vol. 21, No.3 (1993). pp. 265-291.

Snyderman. M. "Commercial Mortgages: Default Occurrence and Estimated Yield Impact." *Journal of Portfolio Management,* Vol. 18. No. 1 (1991). pp. 82-87.

——. "Update on Commercial Mortgage Defaults." *Real Estate Finance,* Vol. 10, No.1 (1994), p. 22.

Titman, S., and W. Torous. "Valuing Commercial Mortgages: An Empirical Investigation of the Contingent-Claims Approach to Pricing Risky Debt." *Journal of Finance*, Vol. 44, No.2 (1989), pp. 345-373.

Vandell, K.D. "Predicting Commercial Mortgage Foreclosure Experience." *Journal of the American Real Estate and Urban Economics Association,* Vol. 20, No.1 (1992), pp. 55-88.

——. "Handing Over the Keys: A Perspective on Mortgage Default Research." *Journal of the American Real Estate and Urban Economics Association*, Vol. 21. No.3 (1993), pp. 211-240.

Vandell, K.D ., W. Barnes, D. Hartzell, D. Kraft, and W. Wendt. "Commercial Mortgage Defaults: Proportional Hazards Estimation Using Individual Loan Histories." *Journal of the American Real Estate and Urban Economics Association,* Vol. 21. No.3 (1993). pp. 451-481.

Appendix

Exhibit 1: Summary Statistics of Quarterly Commercial Loan Characteristics (1966:1-1998:3)

	Mean	Std. Dev.	Minimum	Maximum
Office				
Mortgage Rate (%)	9.66	1.95	6.04	14.97
Cap Rate (%)	9.93	1.08	8.20	13.30
Debt Coverage Ratio	1.34	0.13	1.14	1.84
Loan-to-Value Ratio (%)	72.67	3.46	63.90	88.60
Retail				
Mortgage Rate (%)	9.65	1.98	6.12	15.46
Cap Rate (%)	9.87	1.09	8.30	13.70
Debt Coverage Ratio	1.34	0.11	1.15	1.67
Loan-to-Value Ratio (%)	71.13	3.04	61.70	76.10
Industrial				
Mortgage Rate (%)	9.72	2.08	6.06	15.95
Cap Rate (%)	10.01	1.07	8.40	13.70
Debt Coverage Ratio	1.31	0.10	1.14	1.68
Loan-to-Value Ratio (%)	71.36	3.01	62.70	79.60

Source: American Council of Life Insurance (ACLI).

Exhibit 2: Extension Risk Measured by Historical Data

Panel A: Extension Risk of 5-Year Commercial Loans

	Property Type					
	Office		Retail		Industrial	
NOI Growth	30-Year Amortization	No Amortization	30-Year Amortization	No Amortization	30-Year Amortization	No Amortization
0%	37.8	68.5	28.4	64.2	16.5	53.8
1%	24.3	53.2	17.4	40.4	8.8	44.6
3%	9.0	18.9	9.2	15.6	5.5	19.8
5%	4.5	9.0	4.6	7.3	0.0	7.7

Panel B: Extension Risk of 10-Year Commercial Loans

	Property Type					
	Office		Retail		Industrial	
NOI Growth	30-Year Amortization	No Amortization	30-Year Amortization	No Amortization	30-Year Amortization	No Amortization
0%	17.9	61.1	15.7	50.6	16.5	53.8
1%	11.6	35.8	7.9	30.3	6.6	34.1
3%	4.2	9.5	1.1	6.7	0.0	5.5
5%	2.1	4.2	1.1	1.1	0.0	0.0

Exhibit 3: Mortgage Default, Extension, and Refinancing

Term: 10-Year Commercial Mortgages
Average NOI Growth Rate: 3%
Standard Deviation of NOI Growth: 12%

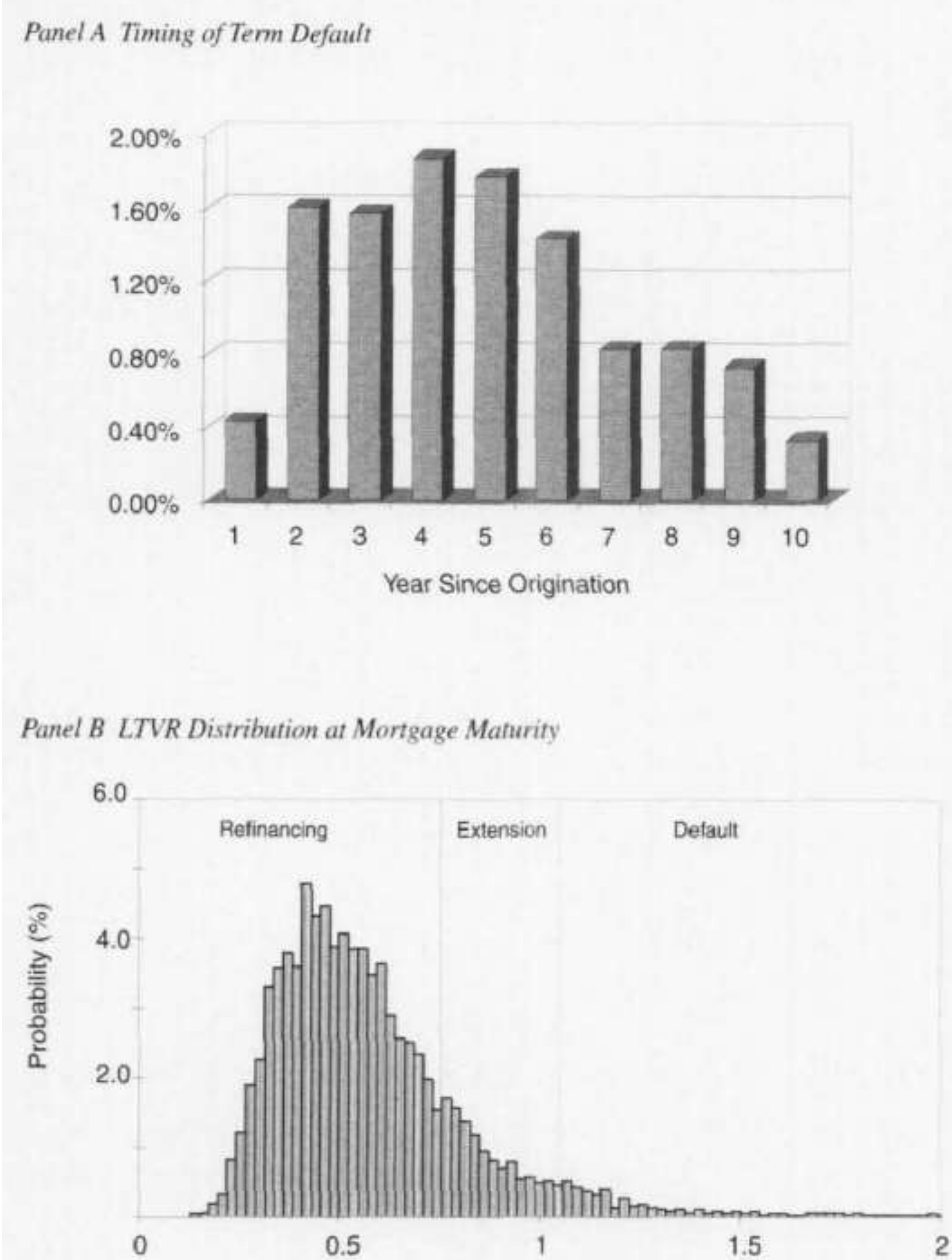

Exhibit 5: Cumulative Default Rates and Extension Probabilities (No Amortization)

Panel A: Cumulative Default Rate (in percent)

	Standard Deviation of NOI Growth				
NOI Growth	6%	9%	12%	15%	18%
1%	4.20	16.40	29.42	40.94	48.77
3%	0.54	5.58	16.24	26.82	37.47
5%	0.02	1.26	7.34	17.00	27.43

Panel B: Probability of Extension (in percent)

	Standard Deviation of NOI Growth				
NOI Growth	6%	9%	12%	15%	18%
1%	37.26	30.82	23.78	16.50	12.57
3%	14.74	17.82	17.60	13.70	11.43
5%	3.54	7.36	10.22	9.80	8.83

Exhibit 6: Extension Risk Under Various Underwriting Standards

NOI Growth: 3%
Standard Deviation of NOI Growth: 12%

	Underwriting Standards at Loan Origination				
Underwriting Standards at Refinance	LTVR=0.85 DSCR=1.20	LTVR=0.80 DSCR=1.25	LTVR=0.75 DSCR=1.30	LTVR=0.70 DSCR=1.35	LTVR=0.65 DSCR=1.40
LTVR=0.85 DSCR=1.20	9.06	8.22	7.38	6.62	5.68
LTVR=0.80 DSCR=1.25	11.48	10.70	10.20	8.60	6.90
LTVR=0.75 DSCR=1.30	14.76	14.12	12.88	11.00	9.40
LTVR=0.70 DSCR=1.35	18.86	18.72	17.92	15.24	11.56
LTVR=0.65 DSCR=1.40	24.50	23.06	22.10	18.78	16.54

Exhibit 7: Mortgage Default and Refinancing During Extension

Standard Deviation of NOI: 6%

	At Maturity	11	12	13	14	15	16	17	18	19	20
Default	0.14%	0.04%	0.04%	0.04%	0.14%	0.08%	0.00%	0.00%	0.00%	0.00%	0.00%
Refinance	93.38%	2.80%	1.44%	0.94%	0.48%	0.38%	0.06%	0.04%	0.00%	0.00%	0.00%
Extension	6.48%	3.64%	2.16%	1.18%	0.56%	0.10%	0.04%	0.00%	0.00%	0.00%	0.00%

Standard Deviation of NOI: 9%

	At Maturity	11	12	13	14	15	16	17	18	19	20
Default	2.40%	0.28%	0.34%	0.24%	0.26%	0.18%	0.06%	0.04%	0.02%	0.00%	0.00%
Refinance	85.54%	3.76%	2.72%	1.70%	1.30%	0.60%	0.34%	0.04%	0.06%	0.08%	0.02%
Extension	12.06%	8.02%	4.96%	3.02%	1.46%	0.68%	0.30%	0.22%	0.14%	0.06%	0.04%

Standard Deviation of NOI: 12%

	At Maturity	11	12	13	14	15	16	17	18	19	20
Default	10.46%	0.66%	0.40%	0.52%	0.40%	0.18%	0.06%	0.02%	0.06%	0.04%	0.02%
Refinance	76.66%	4.14%	2.40%	1.46%	1.18%	0.60%	0.34%	0.12%	0.10%	0.08%	0.06%
Extension	12.88%	8.08%	5.28%	3.30%	1.72%	0.94%	0.54%	0.40%	0.24%	0.12%	0.04%

Standard Deviation of NOI: 15%

	At Maturity	11	12	13	14	15	16	17	18	19	20
Default	21.28%	1.08%	0.82%	0.46%	0.34%	0.12%	0.10%	0.04%	0.00%	0.00%	0.02%
Refinance	67.14%	3.62%	1.74%	1.34%	0.68%	0.64%	0.22%	0.18%	0.08%	0.04%	0.02%
Extension	11.58%	6.88%	4.32%	2.52%	1.50%	0.74%	0.42%	0.20%	0.12%	0.08%	0.04%

Standard Deviation of NOI: 18%

	At Maturity	11	12	13	14	15	16	17	18	19	20
Default	30.56%	1.10%	0.78%	0.52%	0.26%	0.24%	0.14%	0.04%	0.00%	0.00%	0.04%
Refinance	59.12%	3.80%	1.48%	1.00%	0.56%	0.24%	0.02%	0.04%	0.02%	0.00%	0.02%
Extension	10.32%	5.42%	3.16%	1.64%	0.82%	0.34%	0.18%	0.10%	0.08%	0.08%	0.02%

Exhibit 8: Expected Loss from Mortgage Extension*

Discount Rate Risk Premium**	Standard Deviation of NOI 6%	9%	12%	15%	18%
100 bp	0.84	1.58	2.09	2.09	2.08
300	1.05	2.01	2.51	2.47	2.37
600	1.34	2.60	3.08	2.98	2.77
900	1.59	3.13	3.59	3.44	3.13
1200	1.83	3.61	4.06	3.86	3.47

* The expected losses are stated as a percentage of the outstanding loan balance at maturity.
** Discount rate risk premium over contemporaneous mortgage interest rate.

Real Estate Finance, Vol. 18, No. 4 (Winter 2002): pg. 53-63. Permalink.