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# Processing Induced Voxel Correlation in SENSE fMRI Via the AMMUST Framework

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# Processing Induced Voxel Correlation in SENSE



## FMRI Via the AMMUST Framework

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### Synopsis

Many preprocessing steps are applied prior to fMRI statistical analysis and their effects ignored. Nencka et al. (2009) presented the AMMUST- $k$  model to examine preprocessing and reconstruction induced correlation. We extend the AMMUST- $k$  model to include the SENSE multi coil image reconstruction method. We find induced correlations between a voxel and its unfolded counterparts. This body of work has null hypothesis baseline fMRI implications.

### Introduction

In fMRI, images of objects are Fourier encoded [1] similar to the Fourier transform in Figure 1,

$$(\bar{\Omega}_{yR} + i\bar{\Omega}_{yI}) * (V_R + iV_I) * (\bar{\Omega}_{xR} + i\bar{\Omega}_{xI})^T = (F_R + iF_I) \quad (1)$$

Figure 1. Fourier encoded image of an object.

the image of an object is reconstructed via the inverse Fourier transform as in Figure 2.

$$(\Omega_{yR} + i\Omega_{yI}) * (F_R + iF_I) * (\Omega_{xR} + i\Omega_{xI})^T = (V_R + iV_I) \quad (2)$$

Figure 2. Image reconstruction via inverse FT.

the inverse FT process in Figure 2 can be represented with a real-valued isomorphism [2]

$$v = O_I * \Omega_a * O_k * f \quad (3)$$

Figure 3. Alternative image reconstruction via inverse FT.

as in Figure 3 when  $O_k$  and  $O_I$  are identity matrices and  $\Omega_a = \Omega$  however preprocessing in  $k$ -space can be performed with  $O_k$  and in image space with  $O_I$  and adjusting for  $T_2^*$  and  $-\Delta B_0$  through  $\Omega_a$ . It was shown that the reconstructed voxels covariance matrix  $\Sigma$  can be represented as

$$\text{cov}(v) = (O_I \Omega_a O_k) \Gamma (O_k^T \Omega_a^T O_I^T) = \Sigma \quad (4)$$

where  $\Gamma$  is the covariance matrix for the spatial frequencies [3]. With  $\Gamma = I$ , it was shown that preprocessing and reconstruction operations can induce spatial correlation between voxels [3].

### Methods

Equation 3 is generalized to include SENSE multi coil image reconstruction

$$v_{SE} = (S^H \Psi_{SE}^{-1} S)^{-1} S^H \Psi_{SE}^{-1} a \quad (5)$$

where in a given voxel,  $S_C$  is the complex-valued coil sensitivity,  $\Psi_C$  is the complex-valued noise covariance matrix [5], and  $a_C$  is the vector of complex-valued aliased voxel measurements [4]. This generalization is

$$v = P_2 \begin{matrix} U \\ u_1 \\ \vdots \\ u_p \end{matrix} P_1 \begin{matrix} O_I \Omega_a O_k \\ O_{I1} & O_{k1} & & 0 \\ & O_{I2} & O_{k2} & \\ & & O_{I3} & O_{k3} \\ & & & O_{I4} & O_{k4} \end{matrix} f \quad (6)$$

Figure 4. SENSE Image reconstruction with processing.

where  $f_j$  is the aliased spatial frequency vector for coil  $j$ ,  $O_{kj}$  the operations on aliased spatial frequency vector for coil  $j$ ,  $\Omega_{aj}$  the adjusted reconstruction operator for aliased spatial frequency vector for coil  $j$ ,  $O_{Ij}$  the image space operations on the reconstructed image vector from the aliased spatial frequency vector for coil  $j$ ,  $P_1$  is a permutation that reorders values from by coil image to by voxel,  $u_q$  denotes an isomorphism matrix representation for a SENSE reconstruction of voxel  $q$ ,  $p$  is the total number of voxels,  $P_2$  is a permutation that reorders values from by unaliased voxel to unaliased image, and  $O_I$  are the image space operations on the combined reconstructed image vector. If Equation 6 is written as  $v = Of$ , then the covariance between voxels is  $\Sigma = OO^T$ , with an identity spatial frequency correlation. The correlation matrix  $R$  can be found.

### Results

A true noiseless vector of  $n_c=4$  aliased image spatial frequencies was generated for a  $96 \times 96$  Shepp-Logan phantom image scaled by 50 with an AP reduction of  $R=3$ . The coil covariance matrix is  $\text{real}(\Psi_C) = \text{imag}(\Psi_C) = [1, \rho, \rho^2, \rho; \rho, 1, \rho, \rho^2; \rho^2, \rho, 1, \rho; \rho, \rho^2, \rho, 1]$  where  $\rho=0.33$ . In reconstruction,  $O_{kj}$  included apodization of each image with a FWHM=2 voxels,  $\Omega_{aj} = \Omega$ ,  $u_j$  contains the true sensitivity and coil covariance matrix, while  $O_I = I$ . The induced correlation image for magnitude squared data for the center voxel is in Figure 5 (left) overlaid upon the reconstructed mean image. It is apparent that there is induced local correlation from apodization and induced correlation of the center voxel of interest with two other regions from the SENSE unfolding procedure. In Figure 5 (right) the voxel standard deviation is displayed with true value being one.

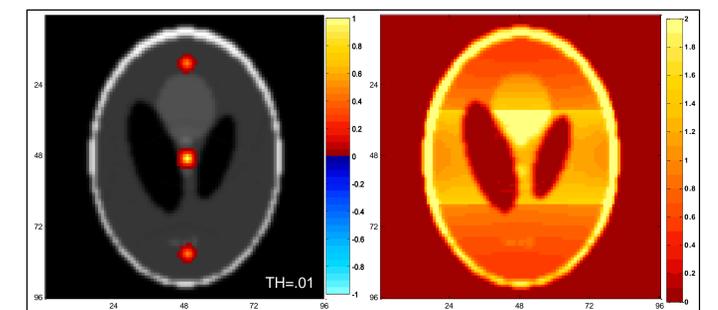


Figure 5. Induced correlation for the center voxel (left) and voxel standard deviations (right).

### Discussion

Previous work that theoretically describes induced correlation between image voxels from spatial preprocessing and reconstruction operations has been summarized [3]. This previous work has been extended to include the SENSE multi coil image reconstruction method [4]. This has null hypothesis fMRI connectivity implications as the no connectivity scenario is not for no spatial correlation but is rather for the spatial correlation induced by preprocessing.

### References

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