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Noise Assumptions in Complex-Valued SENSE MR Image Reconstruction

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Motivation
In MRI k-space for images is not measured instantaneously.

In parallel imaging, sub-sampled k-space points are measured in parallel and combined to form a single image.

Image and volume measurement time is decreased at the expense of increased image reconstruction difficulty and time.

The SENSE parallel imaging reconstruction technique utilizes a complex-valued least squares estimation process.

However, in SENSE the covariance is not properly modeled.

Background
In parallel imaging there is more than one receive coil.

Each coil measures a $k$-space array that is reconstructed into an aliased image then combined to form a single unaliased image.
Background

Image inverse Fourier Reconstruction for single coil.

\[(\Omega_{yR} + i\Omega_{yI}) \ast (F_R + iF_I) \ast (\Omega_{xR} + i\Omega_{xI})^T = (V_R + iV_I)\]
Background
Each coil measures a $k$-space array that is reconstructed into an aliased image then combined to form a single image.
**Background**

Each coil measures a $k$-space array that is reconstructed into an aliased image then combined to form a single image.

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Bruce and Rowe: In progress.
Methods

The SENSE model for aliased voxel values from \( n \) coils is

\[
a_C = S_C v_C + \varepsilon_C, \quad \varepsilon_C \sim CN(0, \Psi_C)
\]

where for each voxel

\( a_C \) is a vector of the \( n \) complex-valued aliased voxel values

\( v_C \) is a vector of the \( A \) unaliased voxel values

\( S_C \) is an \( n \times A \) matrix of complex-valued coil sensitivities

\( \varepsilon_C \) is a vector of the \( n \) complex-valued error values

\[
\]
Methods

The SENSE process

\[ a_C = S_C v_C + \varepsilon_C \], \quad \varepsilon_C \sim CN(0, \Psi_C) \]

\[ \Psi_C = \Psi_R + i\Psi_I \]

uses the complex normal distribution

\[ f(\varepsilon_C) = (2\pi)^{-n} |\Psi_C|^{-1} e^{-1/2\varepsilon_C^H \Psi_C^{-1} \varepsilon_C}, \quad H \text{ is the conjugate transpose (Hermetian)} \]

and for \( N_C \) coil measurements

\[ f(a_C) = (2\pi)^{-n} |\Psi_C|^{-1} e^{-1/2(a_C - S_C v_C)^H \Psi_C^{-1}(a_C - S_C v_C)} \]
Methods

From the distribution for the $n$ coil measurements

$$f(a_C) = (2\pi)^{-n} \left| \Psi_C \right|^{-1} e^{-1/2(a_C - S_C \nu_C)^H \Psi_C^{-1}(a_C - S_C \nu_C)}$$

the voxel values can be estimated as

$$\nu_C = (S_C^H \Psi_C^{-1} S_C)^{-1} S_C^H \Psi_C^{-1} a_C$$

with knowledge of $S_C$ and $\Psi_C$.

Bruce and Rowe: In progress.
Methods

Instead of writing the model with complex numbers as

\[ a_C \cdot n \times 1 = S_C \cdot n \times A \cdot A \times 1 + \varepsilon_C \cdot n \times 1, \]

\[ a_C = a_R + ia_I, \quad S_C = S_R + iS_I, \quad v_C = v_R + iv_I, \quad \varepsilon_C = \varepsilon_R + i\varepsilon_I \]

we can write the model using an isomorphism as

\[ a \cdot 2n \times 1 = S \cdot 2n \times 2A \cdot 2A \times 1 + \varepsilon \cdot 2n \times 1 \]

\[ a = \begin{pmatrix} a_R \\ a_I \end{pmatrix}, \quad S = \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix}, \quad v = \begin{pmatrix} v_R \\ v_I \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix} \]
Methods

Then the distribution for \( n \) coil measurements is

\[
 f(a) = \left(2\pi\right)^{-n} \left|\Psi_{SE}\right|^{-1/2} e^{-1/2(a-Sv)\Psi^{-1}_{SE}(a-Sv)}
\]

with

\[
 a = \begin{pmatrix} a_R \\ a_I \end{pmatrix}, \quad S = \begin{pmatrix} S_R & -S_I \\ S_I & S_R \end{pmatrix}, \quad v = \begin{pmatrix} v_R \\ v_I \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} \varepsilon_R \\ \varepsilon_I \end{pmatrix}
\]

and the complex normal distribution imposes skew-symmetric

\[
 \Psi_{SE} = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix}
\]

Bruce and Rowe: In progress.
**Methods**

The skew-symmetric covariance structure

\[ \Psi_{SE} = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix} \]

is incorrect.

What this says is that \( \text{cov}(I, I) = \text{cov}(R, R) \)

and that \( \text{cov}(I, R) = -\text{cov}(R, I) \).

The proper covariance structure should be

\[ \Psi_{SI} = \begin{pmatrix} \Psi_R & \Psi_{RI} \\ \Psi_{IR}' & \Psi_I \end{pmatrix} \]

(SE for SENSE and SI for new covariance model SENSE-ITIVE)

Bruce and Rowe: In progress.


Methods

Examine the difference between the two covariance structures

\[
\Psi_{SE} = \begin{pmatrix} \Psi_R & -\Psi_I \\ \Psi_I & \Psi_R \end{pmatrix} \quad \Psi_{SI} = \begin{pmatrix} \Psi_R & \Psi_{RI} \\ \Psi_{RI} & \Psi_I \end{pmatrix}
\]

in the distribution

\[
f(a) = (2\pi)^{-n} \left| \Psi_{SE/Sl} \right|^{-1/2} e^{-1/2(a-Sv)' \Psi_{SE/Sl}^{-1} (a-Sv)}
\]

through estimates

\[
\nu_{SE} = (S' \Psi_{SE}^{-1} S)^{-1} S' \Psi_{SE}^{-1} a
\]

\[
\nu_{SI} = (S' \Psi_{SI}^{-1} S)^{-1} S' \Psi_{SI}^{-1} a
\]

Bruce and Rowe: In progress.
Results

Noiseless multi-coil spatial frequency arrays are with

\[
\Psi_{SI} = \begin{pmatrix}
\Psi_R & \Psi_{RI} \\
\Psi'_{RI} & \Psi_I
\end{pmatrix},
\Psi_{R} = \begin{pmatrix}
1 & .33 & .11 & .33 \\
.33 & 1 & .33 & .11 \\
.11 & .33 & 1 & .33 \\
.33 & .11 & .33 & 1
\end{pmatrix},
\Psi_{RI} = \begin{pmatrix}
0 & -.11 & -.07 & -.11 \\
.33 & 0 & -.11 & -.07 \\
.42 & .26 & 0 & -.11 \\
.26 & .42 & .26 & 0
\end{pmatrix}
\]

\[
\Psi_{I} = \Psi_{R}
\]

Bruce and Rowe: In progress.
**Results**

**Magnitude**

Bruce and Rowe: In progress.
Results
Phase

SENSE

SENSE-ITIVE
Discussion

The SENSE image reconstruction method was described.

The SENSE reconstruction written with an isomorphism.

The covariance structure of complex SENSE described.

New SENSE-ITIVE method described with proper covariance.

Results of SENSE & SENSE-ITIVE reconstruction presented.

Ghosting present in SENSE magnitude and phase images.

Better reconstruction in SENSE-ITIVE reconstruction especially phase used for complex-valued time series activation.
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