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# Using Historical Simulation to Compare the Accuracy of Nine Alternative Methods of Estimating the Present Value of Future Lost Earnings

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# Using Historical Simulation to Compare the Accuracy of Nine Alternative Methods of Estimating the Present Value of Future Lost Earnings

Brian C. Brush*

## Abstract

To estimate the present value of future lost earnings, forensic economists must employ some method to determine the interest rate and the earnings growth rate, or the net discount rate derived from them, to use in that estimation. Historical simulation can be used to determine how accurate any such method would have been had it been used in the past. In this paper, historical simulation is used to compare the accuracy of nine different methods of choosing the net discount rate to estimate present value for numerous 30-, 20- and 10-year loss periods. These methods include historical averages, current rates, recent rates, total offset, and a number of methods that combine historical averages with current or recent rates. While no one method is obviously superior in all cases, the results do provide some support for blending historical averages with current or recent rates.

## I. Introduction

In tort cases involving lost future earnings, the lump sum of money necessary to appropriately compensate the injured party depends in part on the future behavior of two key variables, the interest rate and the wage growth rate. These future values must be estimated, either separately or jointly as the *net* discount rate, and the present value of the future losses (the lump sum) can then be calculated using these estimates.

According to a 2009 survey of forensic economists, two methods of estimating the net discount rate continue to be most widely used for estimating the present value of long-term (30-year) future losses. The historical averages method, in which the net discount rate is derived from the average interest rate and average wage growth rate that prevailed over some lengthy past period, was being used by 43% of respondents (down from 58% in a similar 1990 survey), while 32% of respondents (up from 25% in 1990) were using the current rates method, in which the current or most recent interest rate and wage growth rate were used (Brookshire et al., 2009).

With the passage of time, the accuracy of any method of estimating the present value of future lost earnings can be examined by observing the actual

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experience subsequent to the estimate being made. A number of studies have used historical simulation to examine the historical accuracy of the two main methods just described as well as other methods (Schilling, 1985; Dulaney, 1987; Pelaez, 1989, 1991 and 1995; Brush, 2003 and 2004). However, given that these various studies have differed with respect to the time periods covered, the estimation methods examined, and the details of their approaches, the answer to the question of which method is most accurate remains elusive.

A recent paper by Brush (2003) used the historical simulation approach to analyze the accuracy of the historical averages method, using data from the period 1926-2001. In the present paper, we update that work using data through 2008 and then extend the analysis to eight other methods of estimating the net discount rate. As a result, we can compare the historical accuracy of all nine methods, which should be of considerable interest to forensic economists. These methods include two versions of the historical averages method as well as the use of current rates, recent rates, and the total offset method, all of which have been used in practice and discussed in the literature. In addition, we test several blends of historical averages and current or recent rates, following the results of Cushing and Rosenbaum (2006) indicating that a "compromise estimator," the *average* of the net discount rates obtained with the historical averages and current rates approaches, may work well.

In the rest of this paper, we will proceed as follows: Section II will briefly describe the implications of time series analysis for determining the optimal estimation method. Section III will then briefly review the literature on historical simulation. This will be followed in Section IV by a description of the nine methods to be compared in this study, while Section V will describe our data and computation methods. Our results are presented in Section VI, followed by further discussion of these results in Section VII and our concluding comments in Section VIII.

## II. Foundations in Time Series Analysis

Parallel to the flow of historical simulation studies previously cited, another stream of research has sought to determine the best method of estimating the net discount rate by examining the time series properties of interest rates and earnings growth rates, or the net discount rates derived from them, using time series analysis. While a review of this sizable literature is outside the scope of the present study, it may be useful briefly to describe the current state of knowledge as well as the relationship between the time series studies and the historical simulation studies of the kind contemplated in the present paper.

The basic ideas lying behind much of the time series research can be easily summarized. The time series of the net discount rate may be relatively stable and predictable provided that the interest rate and wage growth rate share a common stochastic trend. The net discount rate will be a stationary series and be mean-reverting if and only if the interest rate and wage growth rate are cointegrated. If the time series of the net discount rate is stationary about its mean, use of the historical averages method to determine the net discount rate for discounting future lost earnings to present value will be appropriate. If the

value of the net discount rate is stationary about its mean *and* that mean is approximately zero, the use of the total offset method (which assumes equality between the interest rate and the wage growth rate) will be appropriate. If the net discount rate exhibits drift or a deterministic trend, historical data may be useful in choosing a net discount rate, but the simple historical averages method will not be appropriate. Finally, if the net discount rate series has a unit root and is not a stationary series but a pure random walk without drift or a deterministic trend, the use of historical averages will again be inappropriate, and the use of current rates is recommended (Brush, 2004, p. 10).

Based on the foregoing, much of the research has involved the search for a unit root in the time series of the net discount rate. The results of the various studies have been decidedly mixed, varying with the time periods observed and the statistical methods utilized, with the abrupt upward shift in net discount rates in the late 1970s and early 1980s providing a significant challenge to arriving at firm conclusions. (A comprehensive summary of this literature can be found in Clark et al., 2008, Table 1, p. 234.) Perhaps the one firm conclusion arising from these various time series studies is "a near-consensus against use of the total offset method." (Brush, 2004, p. 11)

The investigation of the time-series properties of the net discount rate continues to progress with ever-increasing sophistication. Among the more recent contributions to the literature is that of Braun et al., (2005), who state that "most previous empirical findings in the literature support the view that net discount rates are mean-reverting." (p. 470) However, they go on to employ a two-break unit root test to reach the conclusion that net discount rates are mean *averting* and *non*-stationary, thus challenging the use of the historical averages method and supporting the use of current rates.

Another recent contribution comes from Cushing and Rosenbaum (2006), whose own review of the extensive literature on the time series analysis of the net discount rate led them to conclude that: "Based on the compendium of this research, it is unclear whether a net discount rate estimator that is derived from a stationary process is any better than one that is not." (p. 142) They then derive an optimal estimator based on the assumptions that the net discount rate follows a stationary first order autoregressive process with known parameters. This optimal estimator "depends on both the length of the forecast horizon and the rate at which a time series converges to its equilibrium level in response to a shock (or its degree of persistence)." (p. 139) Of particular interest for the present study, they find that in predicting the five-period-ahead average net discount rate, a "compromise estimator" that is an equally weighted average of the net discount rates based on current rates and historical averages worked about as well as their theoretical optimal estimator.[1]

We also must note the recent study of Clark et al. (2008), who conclude their own review of the literature with the following: "Clearly the findings to date on the time series properties of the net discount rate are mixed. Specifi-

[1]In a follow-up paper, Cushing and Rosenbaum (2007) provided 50% confidence intervals for the estimated future net discount rates based on historical averages, current rates, their optimal estimator, and their compromise estimator. Unfortunately, these confidence intervals appeared to be "surprisingly wide." (p. 10)

cally, there seems to be some disagreement as to whether it is stationary and mean-reverting, or non-stationary and mean-averting." (p. 235) Modeling the net discount rate as a fractionally integrated ($I(d)$) process, Clark et al. find that the series is non-stationary but also mean-reverting. "If shocked away from its historical average, a mean-reverting but nonstationary [net discount rate] will eventually revisit this level. However, there is a strong possibility that the time path away from this level will be much longer compared to the case of a stationary [net discount rate]." (Clark et al., 2008, p. 242) They interpret their findings as generally supportive of current practice (the use of historical averages), but also assert that "When working with long memory processes, forecasts that essentially rely on point estimates of the mean will generally be inferior to forecasts that place more emphasis on recent observations." (p. 242) They conclude that "more accurate estimates of lost earnings can be obtained by simulations that take long memory into account and accordingly incorporate the actual historical time path of the net discount ratio into the estimates." (p. 246)

These three recent studies suggest that, in deriving the net discount rate for the purpose of discounting lost future earnings to present value, forensic economists should use either (a) current rates (Braun et al., 2005), or (b) a blend of historical averages and current rates (Cushing and Rosenbaum, 2006), or (c) historical averages, while somehow also taking into account the actual time path of the net discount rate (Clark et al., 2008). Perhaps it is fair to state that the matter has not yet been settled.

Additional support for blending historical averages and current rates comes from the more recent work of Cushing and Rosenbaum (2010) which compared the performance of the traditional methods that rely entirely on historical and/or current interest rate and wage data (historical averages, current rates, and their blended compromise estimator) with two other approaches to estimating the five-year-ahead average net discount rate. These two other approaches were (1) the use of publicly-available professional forecasts of interest rates and wage rates, and (2) the use of time series forecasting techniques to estimate future interest rates and wage rates. As previously noted, 75% of the respondents to a recent survey of forensic economists indicated use of either the historical averages or current rate methods to estimate the net discount rate for long-term (30-year) losses. In the same survey, 9% of the respondents used professional forecasts, while the use of time series techniques was not a separately identified option (Brookshire et al., 2009). Cushing and Rosenbaum (2010) found that use of the professional forecasts outperformed the historical averages and current rates methods but performed worse than their compromise estimator. Also, they found no clear advantage for several time series techniques over the compromise estimator, suggesting that the latter remains the preferred method, at least for five-year forecasts.

In sum, time series studies such as those described in this section are potentially useful in examining the appropriateness of various methods of estimating the net discount rate for the purpose of estimating the present value of future lost earnings. Typically, however, they do not generally provide information on the degree of accuracy or inaccuracy that may arise from the use of the

various methods in practice. The comparative accuracy of the different methods is certainly of interest to forensic economists, but so too is the actual degree of accuracy/inaccuracy of each method. Historical simulation studies deal more explicitly with these measurement issues and deal more directly with the methods most forensic economists use in practice, so to these we now turn.

## III. Historical Simulation Research

In historical simulation studies, some method (historical averages, current rates, etc.) is used to determine the net discount rate for the purpose of estimating the present value of the future lost earnings. This *estimated* present value (also called the *ex ante* or *foresight* present value) is then compared to the *actual* present value (also called the *ex post* or *hindsight* present value), the latter being the amount of money that actually would have been needed to compensate the victim given the behavior of wages and interest rates during the future period for which the estimate was made. The net discount rate (*NDR*) used for estimating present value can be calculated as

$$NDR = \frac{1+R}{1+W} - 1 \tag{1}$$

where $R$ is the interest rate and $W$ is the wage growth rate. The issue at hand is to determine, through an examination of the historical record, which method of estimating $R$ and $W$ would have resulted in estimated present values that come closest to the actual present values.

Only a few historical simulation studies have been done. One of the most extensive was that of Schilling (1985), who used economy-wide data on wages along with interest rates on high-grade corporate bonds to test two versions of the historical averages method and also the total offset method, in which the interest rate $R$ is assumed to be equal to the wage growth rate $W$. Covering numerous rolling loss periods of 30-, 12- and 5-years in length extending over the period 1900-1982, he found a "clear if modest superiority" (p. 114) for the total offset method, although none of the methods proved to be particularly accurate. For the total offset method, the average forecast error was 27%, 16% and 8% for the 30-, 12- and 5-year loss periods, respectively.

Dulaney (1987) compared results with four alternative methods including historical averages, current rates, total offset, and a "base period" method which based the net discount rate on the interest rates and wage growth rates for the three year period ending in the current year.[2] Looking at 15 rolling 20-year periods from 1953-72 through 1967-86, and using interest rates on three-year Treasury notes and the rate of growth in compensation per hour in the U.S. business sector, he found average forecast errors for the various methods falling in the relatively narrow range from 8.5% to 11.9 %, with the base period method having the smallest average error and the current rates method having

[2]Dulaney (1987) called these four methods the historical period projection approach, the base year projection approach, the total offset approach, and the base period projection approach, respectively.

the highest. Unfortunately, Dulaney's test of the historical averages method was flawed in that he used average interest and wage growth rates for the entire 1953-86 period, so that his "historical" period completely overlapped each of the 20-year forecast periods in his study. This can result in bias towards finding the method to be more accurate than it actually is since the actual behavior of interest rates and wages during the future period will significantly influence the determination of the "historical" net discount rate which is used to estimate for the future period.

While correcting for this overlap problem, Brush (2004) updated Dulaney's study to cover the next 15 rolling 20-year periods from 1968-87 through 1982-2001, and all four methods performed much worse in the later 15 periods. Average forecast errors ranged from 18.3% to 27.4% with the total offset method having the smallest average error and historical averages having the highest. This worsening performance was the consequence of the movement towards higher net discount rates that occurred over time.

In a series of papers, Pelaez (1989, 1991, 1995) provided further evidence on both the total offset method and the historical averages method, with the innovation of using wage data at the individual sector or industry level, along with interest rates on one-year Treasury bills. With his data covering the period 1955 through 1986, he found forecast errors were typically less than 10% for both methods. However, he covered relatively few time periods and, like Dulaney, his method of assessing the historical averages method involved using a "historical" period that substantially overlapped the future periods for which the forecasts were being made.

Another study dealing with the accuracy of the historical averages method was that of Haydon and Webb (1992). While avoiding the overlap problem, they sought to determine the best duration of the past period on which to base the net discount rate for future loss periods of 10, 15 and 20 years, respectively, using economy-wide wage data and interest rates on three-month Treasury bills for the 40 years from 1948 through 1987. With a very limited set of time periods with which to work, it is not surprising that their results were inconclusive. However, considering the upward trend in the net discount rate over the time period covered in their study, they suggested that the net discount rate should be based on the average of rates based on longer and shorter historical periods. This is similar to the suggestion of Cushing and Rosenbaum (2006) to combine historical averages with current rates. Using a very different approach from Haydon and Webb, Rosenbaum and Guthmann (2007) also sought to find the best duration of the past period on which to base the net discount rate. They carefully examined how net discount rates vary with the duration of the period used to calculate them, but their results were also inconclusive.

Brush (2003) studied the accuracy of the historical averages method using 1926-2001 data on investment returns on U.S. Treasury bills and alternatively, on intermediate-term government bonds, along with data on wages in the U.S. manufacturing sector. For future forecast periods of 30, 20 and 10 years, the net discount rate was derived from data for the immediately preceding historical period of the same length (e.g., a 30-year historical period was used for each

30-year future period). The accuracy of the historical averages method was assessed for 17 rolling 30-year future periods, 37 rolling 20-year future periods, and 57 rolling 10-year future periods. The results were very similar using both Treasury bills and the intermediate-term government bonds, so here we describe only the results with Treasury bills. The 30-year rolling future periods had beginning years from 1956 through 1972, and the average forecast error was 94%, and always in the direction of over-compensation. The 20-year rolling future periods began in 1946 through 1982, and the average forecast error was 41%, and almost always in the direction of over-compensation. The 10-year rolling future periods began in 1936 through 1992, but here the average forecast error was just 16% and errors in both directions were often found, with 21 of 57 periods resulting in under-compensation. Clearly, the accuracy of the historical averages method improved as the length of the future loss period was shortened.

The very large forecast errors, and the direction of those errors, that would have resulted from use of the historical averages method for the 20-year and 30-year future loss periods covered in the Brush study (2003) are the consequence of determining the net discount rate by reaching back in time to periods during which, as it happened, net discount rates were much lower than they turned out to be during the future periods for which the forecasts were being made. Of course, such errors can go in the other direction, depending on shifts or trends in the net discount rate over time. And since large forecast errors may be expected to arise with *any* method, it will be useful to extend the analysis to a comparison of a variety of estimation methods that are either in common practice or that have been suggested in the literature.

## IV. Nine Estimation Methods to be Compared

In this paper, the forecast performance of nine alternative estimation methods will be examined and compared. The methods include:

(1) **HA1,** a historical averages method in which $W$ and $R$ are assigned the values of the average compound wage growth rate and the average compound interest return, respectively, for the historical period immediately preceding the future loss period for which the forecast is being made that is equal in length to the forecast period (i.e., a 30-year past period is used for a 30-year future period, a 20-year past period for a 20-year future period, and a 10-year past period for a 10-year future period). This approach was taken by both Schilling (1985) and Brush (2003, 2004). According to a recent survey of forensic economists, 27% of the respondents used a past period that matches the length of the future period, although 53% indicated use of a fixed period that is independent of the worklife. (Brookshire et al., 2009, p. 19) It is interesting, however, that among those using a fixed period independent of the worklife, the average length of the fixed period was 27 years for a 30-year forecast. As noted earlier, the historical averages method, in some form or another, is the most widely used method in practice for long-term future losses.

(2) **HA2,** a second historical averages method in which $W$ and $R$ are assigned the values of the average compound wage growth rate and the average compound interest return for the entire period 1926-2008. While this will involve overlaps between the historical period and the forecast periods, the potential bias will be much smaller than in Dulaney (1987) and Pelaez (1991), since the 83-year historical period is nearly three times as long as the 30-year future periods for which estimations will be made, with even larger multiples for the 20-year and 10-year future loss periods. Therefore, the actual behavior of wages and interest rates during the future loss periods will not dominate the value of the net discount rate to be used for estimation.

(3) **CR**, the current rates method, in which $W$ and $R$ are assigned the values of the wage growth rate and the interest return in the first year of the loss period. This method is the second most widely used in practice for long-term future losses.

(4) **BP,** the base period method, in which $W$ and $R$ are assigned the values of the simple average of the annual wage growth rates and interest returns for the three-year period ending in the first year of the loss period. This method was found to be slightly more accurate than three other methods by Dulaney (1987).

(5) **TO**, the total offset method in which it is assumed that the future wage growth rate will equal the future interest return ($W = R$). We noted earlier that the more recent time-series studies have led to a near-consensus view against use of this method, and it does not appear to be in widespread use. In the most recent survey of forensic economists, the median net discount rate used for a 30-year loss was 1.75%, far above zero, with an interquartile range of 1% to 2.19%. Yet there remain some adherents to the method, with 8% of the survey respondents indicating use of a net discount rate of 0% or lower (Brookshire et al., 2009).

The final four methods blend historical averages with either current or recent rates, as suggested by the work of Cushing and Rosenbaum (2006, 2010) and Haydon and Webb (1992). They are the following:

(6) **HA1-CR**, in which the net discount rate is the average of the net discount rates based on the **HA1** and **CR** methods. This is a blend of the first historical averages method and the current rates method.

(7) **HA2-CR**, in which the net discount rate is the average of the net discount rates based on the **HA2** and **CR** methods. This is a blend of the second historical averages method and the current rates method.

(8) **HA1-BP**, in which the net discount rate is the average of the net discount rates based on the **HA1** and **BP** methods. This is a blend of the first historical averages method and the base period method.

(9) **HA2-BP**, in which the net discount rate is the average of the net discount rates based on the **HA2** and **BP** methods. This is a blend of the second historical averages method and the base period method.

## V. Data, Method, and Computations

In this paper, data on interest returns and wages covering the period 1926-2008 are used to assess the historical accuracy of the nine methods of estimating lump sum awards for alternative future periods of 30, 20 and 10 years, respectively. Following Brush (2003), we use data on the annual returns on Treasury bills from Ibbotson (2009) along with data on the U.S. manufacturing wage (Council of Economic Advisors, 2002 and 2009, Table B-47, and U.S. Department of Commerce 1975, Series D 802-810.). While certainly not perfect, these two data series have the great advantage of being available on a consistent basis over a very long historical period.[3]

In all cases, the net discount rate calculated using the various methods described above is used to determine the *estimated* present value of the future earnings loss. These *estimated* present values are then compared to the *actual* present values that represent the actual lump sums that would have been required to replace the future lost wages, given the actual year-to-year interest returns and wage growth that prevailed during the periods for which the forecasts are made. For purposes of calculating both estimated and actual present values, it is assumed that investment returns are received and wages paid out at the end of each year, and that the injured party's wages would have increased at the same rate each year as the wages of the average worker. All calculated awards are based on a base annual loss of $1,000, as measured in the year just prior to the first year of the future loss period.

The estimated present value for a period of $n$ future years of wage loss can be calculated with the following formula,

$$PV_{EST.} = \sum_{t=1}^{n} \left[ \frac{1+W}{1+R} \right]^{t} * 1{,}000 \tag{2}$$

where $W$ and $R$ are the wage growth rate and interest rate chosen by the forensic economist for estimation.

The actual present value for a period of $n$ future years can be calculated with the following formula,

$$PV_{ACT.} = \sum_{t=1}^{n} \left[ \prod_{i=1}^{t} \frac{1+W_i}{1+R_i} \right] * 1{,}000 \tag{3}$$

[3]An alternative investment return series that is available for the same lengthy period would be the returns on intermediate-term government bonds from Ibbotson (2009). However, this series reports *total* returns, including capital gains and losses, on an annual basis. These annual total returns are much more volatile than the underlying interest rates and, as such, they are not suitable for use with either the current rate or base period methods. As for wage rates, there does not appear to be any good alternative to the manufacturing wage series for a long-period study. However, an index of total compensation per hour in the non-farm business sector has been available since 1947, so we describe some comparative results with the manufacturing wage and this broader compensation index for the 1947-2008 periods at the end of section VI.

where $W_i$ and $R_i$ are the actual wage growth rate and interest return in each year of the future loss period. It should be noted that the actual present value calculated in this way may be quite different from, and also much more accurate than, an alternative calculation based on the *average* net discount rate that prevailed during the forecast period.

Consider the case in which the first historical averages method, **HA1**, is used to estimate the present value of a 10-year wage loss running from1999 through 2008. Over the past period from 1989 through 1998, the average compound interest return was 5.29% and the average compound wage growth rate was 2.85%, so the net discount rate was 2.37%, and the estimated present value of the base amount of $1,000 for 10 future years is $8,811. However, given the actual interest returns and wage growth rates in each year of the future period, the actual present value is $9,761. (A demonstration that this is the correct amount can be found in Table A in the Appendix.) In this particular example, the estimation method resulted in under-compensation, and the *relative error* (the absolute value of the estimation error, in percentage terms) is 9.7%. The ratio of the estimated present value to the actual present value (the *award ratio*) is 0.90. An award ratio exceeding 1.0 indicates over-compensation, whereas an award ratio less than 1.0 indicates under-compensation. The relative error can be interpreted as a measure of *accuracy*, while the award ratio can be interpreted as a measure of *bias* towards either the plaintiff or defendant.

## VI. Results

Table 1 shows our comparative results for all nine estimation methods. The number of cases that can be considered is limited by the data requirements for the **HA1** method. For 30-year losses, this method requires 60 years of data for each case of comparing estimated to actual present values. Since our data go back to 1926 and end in 2008, this means that we have enough data to consider 24 rolling 30-year future periods ranging from 1956-85 through 1979-2008. For the 20-year forecasts using **HA1**, 40 years of data are needed for each case, and our data set therefore allows us to compare estimated to actual present values with all nine methods for 44 rolling future periods from 1946-65 through 1989-2008. For the 10-year forecasts using **HA1**, 20 years of data are needed for each case, and we can compare estimated to actual present values with all nine methods for 64 rolling 10-year future periods from 1936-45 through 1999-2008.

The comparative results for the 24 cases of 30-year future losses are shown in Table 1.A. For each method, we show, under "direction of error," the number of cases of over-compensation (+) and under-compensation (–), as well as the maximum percentage over-compensation (a *windfall* for the plaintiff), the maximum percentage under-compensation (a *shortfall* for the plaintiff), the mean relative error (in terms of absolute values), and the mean award ratio. Finally, since the mean relative error can be influenced by extreme values, we show in the final two columns the number of cases in which the relative error was <=20% and <=10%, respectively, as further indicators of accuracy.

Table 1
Estimated vs. Actual Present Values

| A. 30 Years Forward, First Years 1956 Through 1979 (24 Cases) | | | | | | | |
|---|---|---|---|---|---|---|---|
| Estimation Method | Direction Of Error | Maximum Windfall | Maximum Shortfall | Mean % Error | Mean Ratio | Error <=20% | Error <=10% |
| HA1 | 24+, 0- | 132% | ---- | 91% | 1.91 | 0 | 0 |
| HA2 | 24+, 0- | 67% | ---- | 24% | 1.24 | 13 | 9 |
| CR | 20+, 4- | 148% | 9% | 37% | 1.35 | 12 | 9 |
| BP | 21+, 3- | 136% | 3% | 33% | 1.35 | 12 | 6 |
| TO | 20+, 4- | 53% | 8% | 15% | 1.13 | 17 | 13 |
| HA1-CR | 24+, 0- | 120% | ---- | 63% | 1.63 | 0 | 0 |
| HA1-BP | 24+, 0- | 114% | ---- | 63% | 1.63 | 0 | 0 |
| HA2-CR | 24+, 0- | 100% | ---- | 30% | 1.30 | 14 | 6 |
| HA2-BP | 24+, 0- | 94% | ---- | 29% | 1.29 | 15 | 4 |
| **B. 20 Years Forward, First Years 1946 Through 1989 (44 Cases)** | | | | | | | |
| Estimation Method | Direction Of Error | Maximum Windfall | Maximum Shortfall | Mean % Error | Mean Ratio | Error <=20% | Error <=10% |
| HA1 | 42+, 2- | 88% | 13% | 38% | 1.37 | 14 | 2 |
| HA2 | 33+, 11- | 52% | 32% | 19% | 1.12 | 27 | 20 |
| CR | 27+, 17- | 181% | 26% | 25% | 1.17 | 31 | 13 |
| BP | 28+, 16- | 96% | 18% | 23% | 1.17 | 26 | 16 |
| TO | 20+, 24- | 43% | 36% | 17% | 1.05 | 26 | 20 |
| HA1-CR | 42+, 2- | 84% | 9% | 27% | 1.27 | 21 | 10 |
| HA1-BP | 42+, 2- | 64% | 12% | 28% | 1.27 | 18 | 11 |
| HA2-CR | 34+, 10- | 75% | 7% | 16% | 1.15 | 32 | 20 |
| HA2-BP | 39+, 5- | 59% | 18% | 16% | 1.15 | 30 | 21 |
| **C. 10 Years Forward, First Years 1936 Through 1999 (64 Cases)** | | | | | | | |
| Estimation Method | Direction Of Error | Maximum Windfall | Maximum Shortfall | Mean % Error | Mean Ratio | Error <=20% | Error <=10% |
| HA1 | 36+,28- | 34% | 37% | 15% | 1.04 | 43 | 27 |
| HA2 | 34+, 30- | 29% | 37% | 14% | 1.00 | 42 | 28 |
| CR | 30+, 34- | 58% | 29% | 11% | 1.03 | 52 | 44 |
| BP | 35+, 29- | 45% | 30% | 10% | 1.03 | 56 | 45 |
| TO | 28+,36- | 25% | 39% | 14% | 0.97 | 44 | 33 |
| HA1-CR | 37+, 27- | 26% | 31% | 11% | 1.03 | 53 | 34 |
| HA1-BP | 35+, 29- | 29% | 32% | 12% | 1.04 | 49 | 33 |
| HA2-CR | 42+, 22- | 20% | 30% | 8% | 1.01 | 57 | 43 |
| HA2-BP | 44+, 20- | 26% | 33% | 8% | 1.02 | 58 | 48 |

For the 30-year forecasts, there was a pronounced tendency towards over-compensation with all nine estimation methods. The total offset method **(TO)** was clearly superior to all others. It had a mean relative error of 15%, and although 20 of the 24 cases resulted in over-compensation, the mean award ratio was just 1.13, indicating moderate average bias towards plaintiffs. It also dis-

played the most cases of relative errors <=20% (17 out of 24) and 10% (13 out of 24), respectively. Next came **HA2**, the historical averages method that uses the average net discount rate for the entire 1926-2008 period, with a mean relative error of 24%, a mean award ratio of 1.24, and 13 and nine cases of relative errors that are <=20% and <=10%, respectively. The similarity of the performance of **HA2** to that of the total offset method is due to the fact that the average net discount rate for the entire 1926-2008 period is -0.57%, reasonably close to the value of zero assumed with **TO**. Bunched in the middle is a group with fairly close results, including current rates (**CR**), the base period method (**BP**), and the two methods that combine the longer term historical averages with current rates or the base period, **HA2-CR** and **HA2-BP**. These four methods had mean relative errors ranging tightly from 29% to 37% and were very similar in other respects as well. The worst methods were **HA1** and the two methods that combine this first historical averages method with current rates or the base period, **HA1-CR** and **HA1-BP**. **HA1** resulted in a mean relative error of 91% and a mean award ratio of 1.91, with zero cases of a relative error <=20%. The results with **HA1** are not surprising, as they simply update the results previously reported in Brush (2003) with seven more years of data. It is interesting that, while averaging **HA1** with either **CR** or **BP** does provide an improvement in results over **HA1** alone, the same is not true of **HA2**, which performs slightly better alone than when blended with **CR** or **BP**. However, **HA2-CR** and **HA2-BP** work better than **CR** and **BP** alone.

The results for the 44 cases of 20-year forecasts are displayed in Table 1.B. The tendency towards over-compensation is significantly reduced for most methods compared to the 30-year forecasts, and the mean relative errors are lower for all methods except **TO**. The performance of the various methods is now much closer, and there is no clear-cut "best" method. **TO** is again one of the best, as it has the best balance between over-compensation (20 cases) and under-compensation (24 cases) with a mean award ratio of 1.05, and it has one of the lowest mean relative errors (17%). However, **HA2-CR** and **HA2-BP** have slightly lower mean relative errors (16%) and have more cases in which the relative error is <=20%. **CR** and **BP** perform slightly worse than the above three methods, with higher mean relative errors because they are more sensitive to extreme values (note the maximum windfall value of 181% for **CR**). Again, the worst performing methods are **HA1** and the blended **HA1-CR** and **HA1-BP.** The performance of *both* historical averages methods is improved by averaging these methods with either **CR** or **BP**.

Table 1.C shows the results for the 64 cases of 10-year future losses. Now there is a reasonable balance between over-compensation and under-compensation across the board, with the mean award ratios close to unity for all methods. The mean relative errors are lower, and usually much lower, for all nine methods when compared to the 20-year forecasts. They range from 8% to 15%, compared to the range of 16% to 38% for the 20-year results. The relative performance of most of the various methods tends to be very close, but the two "best" methods, **HA2-CR** and **HA2-BP**, have a slight edge over **BP**, **HA1-CR**, and **CR**. These two best methods have a mean relative error of 8%, and the relative error is <=20% for approximately 90% of all cases and <=10% for 67-

75% of all cases. The "worst" performers for the 10-year periods are **HA1**, **HA2**, and **TO**. Once again we find that the performance of both historical averages methods is improved by averaging these methods with either **CR** or **BP**.

As previously mentioned, Cushing and Rosenbaum's (2006) recommendation for averaging the historical averages and current rates methods came from their results of estimating the average net discount rate for future periods of just five years. Forensic economists, of course, often must forecast for periods much longer than five years. However, our results show that the Cushing-Rosenbaum method may work relatively well also for 10 and 20-year future periods (but not necessarily for 30-year periods), even when allowing for the actual year-to-year movements in interest returns and wages during the future period, rather than just their averages.

In the foregoing results, the accuracy of all of the methods improved as the forecast periods were shortened from 30 to 20 to 10 years. It is to be expected that, in general, it would be easier to accurately forecast for shorter future periods. However, it is worth noting that, as the forecast periods were shortened, the number of cases that could be considered increased (from 24 to 44 to 64) and the forecast periods started earlier in time (from 1956 to 1946 to 1936). Thus, the results for the 30-, 20- and 10-year forecast periods are not completely comparable. This is due partly to the inclusion of **HA1** and its two blends among the estimation methods. Since **HA1**, **HA1-CR**, and **HA1-BP** were the worst overall performers, this comparability problem can be reduced and the number of cases can be significantly expanded by eliminating these three methods from consideration. This allows for the addition of 30 more cases of 30-year forecasts, 20 more cases of 20-year forecasts, and 10 more cases of 10-year forecasts. The results are shown in Table 2 for the remaining six estimation methods for all possible rolling 30-, 20-, and 10-year future loss periods starting with 1926 and moving forward.[4]

Table 2.A displays the outcomes for the expanded sample of 54 rolling 30-year future periods from 1926-1955 through 1979-2008. With the sample size now more than doubled compared to the 30-year results in Table 1.A, the **TO** method continues to be the most accurate, both in terms of the mean relative error (29%) and the number of cases with relative errors <=20% and <=10%. However, all of the six methods perform worse with this greatly expanded sample. Again, the performance of **HA2** is close to that of **TO** due to the fact that the net discount rate for the entire 1926-2008 period was close to zero. The sensitivity of the mean relative errors and mean award ratios to the presence of extreme values for **CR**, **BP**, and the blended measures that contain them is now clearly evident in Table 2.A. The mean relative error with **CR** is by far the highest at 118% with a maximum windfall of 1,713%. This would have occurred for the 30-year future period beginning in 1934, a year in which the wage growth rate was 20.45% and the interest return on Treasury bills was 0.16%, so the estimated present value was $1,496,857 while the actual present value was a mere $82,571. Interestingly, the relative errors for the years immediately preceding and following 1934 were only 64% and 44%, respectively.

[4]There are always two fewer cases for **BP** and **HA2-BP**, since they require two years of data preceding the first year of the loss period, so the first cases begin in 1928 instead of 1926.

Another whopping relative error with **CR** would have occurred for 1942 at 909%. The use of three years of data instead of a single year (the **BP** method) tends to lessen these errors somewhat, but the maximum windfall is still 511% for the future period beginning in 1943 with **BP**. Clearly the 1930s and 1940s were extraordinary times, but who can say when the next single year or three-year period will occur that may prove to be extraordinary relative to some future forecast period?[5]

Table 2
Estimated vs. Actual Present Values – Expanded Sample

| **A. 30 Years Forward, First Years 1926* Through 1979 (54 Cases)** | | | | | | | |
|---|---|---|---|---|---|---|---|
| Estimation Method | Direction Of Error | Maximum Windfall | Maximum Shortfall | Mean % Error | Mean Ratio | Error <=20% | Error <=10% |
| HA2 | 24+, 30- | 67% | 60% | 30% | 0.91 | 20 | 9 |
| CR | 36+,18- | 1,713% | 91% | 118% | 1.90 | 15 | 10 |
| BP | 39+, 13- | 511% | 86% | 68% | 1.48 | 17 | 9 |
| TO | 20+, 34- | 53% | 64% | 29% | 0.84 | 21 | 13 |
| HA2-CR | 39+, 15- | 826% | 70% | 65% | 1.41 | 19 | 10 |
| HA2-BP | 38+, 14- | 236% | 72% | 41% | 1.20 | 20 | 7 |
| **B. 20 Years Forward, First Years 1926* Through 1989 (64 Cases)** | | | | | | | |
| Estimation Method | Direction Of Error | Maximum Windfall | Maximum Shortfall | Mean % Error | Mean Ratio | Error <=20% | Error <=10% |
| HA2 | 35+, 29- | 52% | 52% | 24% | 0.98 | 32 | 23 |
| CR | 33+, 31- | 428% | 83% | 42% | 1.18 | 34 | 14 |
| BP | 34+, 28- | 168% | 77% | 31% | 1.12 | 31 | 17 |
| TO | 21+, 43- | 43% | 54% | 23% | 0.92 | 30 | 22 |
| HA2-CR | 39+, 25- | 188% | 60% | 25% | 1.08 | 37 | 22 |
| HA2-BP | 42+, 20- | 65% | 63% | 21% | 1.05 | 35 | 22 |
| **C. 10 Years Forward, First Years 1926* Through 1999 (74 Cases)** | | | | | | | |
| Estimation Method | Direction Of Error | Maximum Windfall | Maximum Shortfall | Mean % Error | Mean Ratio | Error <=20% | Error <=10% |
| HA2 | 40+, 34- | 29% | 37% | 15% | 1.00 | 47 | 30 |
| CR | 33+, 41- | 110% | 58% | 13% | 1.02 | 58 | 46 |
| BP | 37+, 35- | 45% | 52% | 11% | 1.01 | 60 | 48 |
| TO | 34+, 40- | 25% | 39% | 14% | 0.97 | 51 | 36 |
| HA2-CR | 47+, 27- | 39% | 32% | 9% | 1.01 | 63 | 45 |
| HA2-BP | 47+, 25- | 26% | 39% | 9% | 1.00 | 63 | 53 |

*First Years Beginning in 1928 for **BP** and **HA2-BP**, with two fewer cases.

[5]Recent interest rate changes may be illustrative. The three-month Treasury bill rate averaged 5.85% in 2000, then averaged just 2.85% over the 2001-2008 period, and has been near zero ever since. See *Economic Report of the President*, 2010, Table B-73.

Table 2.B provides the results for 64 rolling 20-year forecast periods from 1926-45 through 1989-2008. Here, four of the six methods are fairly closely bunched in terms of overall performance, including **HA2**, **HA2-CR**, **HA2-BP** and **TO**. All have similar mean relative errors in the 21-25% range, and result in relative errors of <=10% in approximately one-third of the cases. The other two methods, **CR** and **BP**, while suffering again from the presence of extreme values, appear to be less accurate also because of having fewer errors <=10%.

Finally, Table 2.C displays the outcomes for 74 rolling 10-year forecast periods from 1926-35 through 1999-2008. As was the case in Table 1, the 10-year forecasts are the most accurate, in this case with mean relative errors for the six methods ranging from 9% to 15%, and also the most free of bias, with the average award ratios ranging from 0.97 to 1.02. When considering the mean relative errors, the mean award ratios, and the number of cases with relative errors <=20% and <=10%, the "best" methods again appear to be **HA2-CR** and **HA2-BP**, followed closely by **BP** and **CR**, with **TO** and **HA2** performing least well.[6]

All of the results described up to this point were obtained using the manufacturing wage as the earnings series, since it is the only series that dates back over the entire period covered in this study. However, an ideal earnings series would provide much broader coverage of the economy, since manufacturing employment has declined as a share of total employment over time. The ideal earnings series would also include fringe benefits since they have become increasingly important as a share of total compensation over time. An index of compensation per hour in the non-farm business sector, which includes fringe benefits, has been available since 1947 (Council of Economic Advisors, 1994, Table B-47 and 2009, Table B-49). As a check on the sensitivity of our results to the choice of earnings series, we compared the results using the manufacturing wage with the results using the compensation index using the **HA1** method for 20-year future losses. With the restricted 1947-2008 data set, there are 22 future loss periods to consider, and the results for the two different earnings series were virtually identical. Using the manufacturing wage, all 22 cases resulted in over-compensation, with a mean error of 28%, a mean award ratio of 1.28, 10 cases with errors <=20%, and one case with an error <=10%. Using the compensation index, all 22 cases again resulted in over-compensation, with a mean error of 27%, a mean award ratio of 1.27, 11 cases with errors <=20%, and two cases with errors <=10%.

## VII. Further Discussion

In both Table 1 and Table 2, we see that estimating present values generally becomes more accurate the shorter is the future loss period. For 30-year future loss periods, the **TO** method turned out to be the most accurate based on our historical simulations, with **HA2** close behind. For the 20-year forecasts, **TO** is still one of the better methods, but with fewer "small" errors, it would

[6]We would expect the results in Table 2.C to be very similar to those in Table 1.C, since only 10 more forecast periods have been added, increasing the total from 64 to 74.

rank slightly behind **HA2-CR** and **HA2-BP.** For the 10-year forecasts, **TO** is one of the worst performers, with **HA2-CR** and **HA2-BP** again being the best.

As we expanded the sample to include earlier forecast periods that included data from the 1920s, 1930s, and 1940s, the forecasting performance of all six methods included in Table 2 deteriorated compared to the results for those same methods in Table 1 (although minimally for the 10-year forecasts). This suggests that economic conditions in those earlier periods made forecasting future net discount rates especially difficult. Looking at the results in Table 1, it may be that the very poor performance of **HA1** and the two blends that included **HA1** is due to the fact that it was the only method that, for many forecast periods, based the net discount rate largely on data from these unusual past economic times.

An even better explanation for the poor performance of **HA1** for the longer 30-year and 20-year forecasts can be found in the long-term behavior of net discount rates. Annual net discount rates based on Treasury bills and the manufacturing wage, while showing considerable fluctuations over the lengthy time period covered in this study, have nonetheless displayed a general upward trend since the early 1930s. This tends to create an overall bias towards over-compensation for many of the methods applied in this study, as the net discount rates used for estimation are lower than those that actually prevailed in the forecast periods. But this problem is most severe with **HA1** for the longer forecast periods.

If we examine the 30-year historical net discount rates beginning with the 1926-55 period and then moving forward one year at a time, we find that this series declines from the ending years 1955 through 1962, and then rises strongly, with only a few brief downward blips, all the way through our final ending year of 2008. This is why the **HA1** method resulted in substantial over-compensation for every single 30-year forecast period we considered. Similarly, the 20-year historical net discount rate series also displays a strong upward trend from the end year 1953 through the end year 1999 before the series turns down. But even with a similarly strong upward trend, the shorter 20-year periods reduce the magnitude of the forecast errors relative to those obtained for 30-year periods.

Total offset and **HA2** are the two methods that tend not to be biased towards either over- or under-compensation as the result of the long-term upward trend in net discount rates. For these methods, under-compensation for earlier forecast periods tends to balance out over-compensation for later forecast periods, resulting in average award ratios relatively closer to unity. But any advantage for using total offset seems to derive from the fact that, over the historical period covered it is statistically close to **HA2.** And while our results provide some support for the use of the total offset method for the longer term losses, this must be tempered by consideration of the nature of the data and methods employed in this study.

The interest returns used in this study are for Treasury bills, considered the ultimate "risk-free" securities in that they are free of both default risk and inflation risk. Many forensic economists base their discount rates on longer term Treasuries, which are free of default risk but typically have higher yields

to compensate for the risk of loss due to unanticipated inflation. Using intermediate-term government bonds as the investment vehicle, the 1926-2008 net discount rate is +1.07%, whereas using Treasury bills as the investment vehicle, the 1926-2008 net discount rate is -0.57%, and the absolute difference between these two numbers is 1.64 percentage points.[7]

For the forensic economist who believes that the net discount rate should be based on investment returns from instruments other than Treasury bills, possibly including intermediate- or long-term Treasuries, corporate bonds, or even balanced portfolios, the use of total offset would seem to be inappropriate. And total offset is the one method in this study for which the performance would clearly change if a riskier alternative to Treasury bills were chosen. Using total offset with an alternative, higher-yielding, investment instrument would mean that the expected present value would remain unchanged while the actual present value would decrease. All other methods we have considered would likely perform in a similar manner when the choice of investment instrument is changed since the change would affect the calculation of *both* the expected and actual present values in a similar way. For example, the performance of the historical averages method (**HA1**) has been found to be very similar with either Treasury bills or intermediate-term government bonds (Brush 2003).

The results in this paper also provide some support for the use of an estimation method that combines historical averages with current or recent rates as suggested by the work of Cushing and Rosenbaum (2006, 2010) and Haydon and Webb (1992). This often produced better results than using historical averages alone, or using either current rates or a base period alone. **HA2-CR** and **HA2-BP** were the best methods for 10-year future losses and among the best methods for 20-year losses. Certainly one advantage of this approach is to reduce the potential impact of extreme values that may arise when using either the current rate or base period methods.

## VIII. Concluding Comments

Estimating the present value of future lost earnings requires the forensic economist to make a projection of future interest rates and wage growth rates, or the future relationship between the two in the form of the net discount rate. Unfortunately, such estimation is a very inexact science and forecast errors are inevitable. Nonetheless, forensic economists should seek out and use estimation methods that reasonably may be expected to keep the forecast errors to a minimum.

In this paper we have used historical simulation to compare the forecast accuracy of nine different estimation methods that either appear to be in widespread use, or have been suggested in the literature, for numerous 30-, 20- and 10-year loss periods covering a total of 2,322 forecasts. We find that the best estimation method may depend partly on the length of the forecast period. Regardless of the method used, average forecast errors tend to decline sharply as

[7]Calculated using data from Ibbotson (2009), Council of Economic Advisors (2002, 2009), and U.S. Dept. of Commerce (1975).

the length of the forecast period is reduced. For longer (30-year) forecast periods, total offset tends to work relatively well, although this would likely change if the net discount rate were based on something other than risk-free Treasury bills. Our results also provide support for recent suggestions in the literature that the net discount rate should be based on a combination of historical averages and current or recent rates, especially for 20- and 10-year future loss periods.

Most of the estimation methods currently in widespread use rely on historical and/or current wage and interest rate data to estimate the net discount rate to be used to discount future economic losses to present value. Whether any of these "traditional" estimation methods are "accurate enough" is an open question, given the inherent difficulties of forecasting net discount rates 10, 20, 30, or even 40 years into the future.

With the advent of the "Great Recession" beginning in late 2007, interest rates have declined and we have entered a period of historically low interest rates, with Treasury bill rates close to zero. If this low interest rate regime were to continue for a significant period, the **HA1** method would be slow to adjust and would likely result in substantial under-compensation going forward for 30-year and 20-year forecast periods, while the use of current rates might provide much more accurate forecasts. On the other hand, if interest rates were to trend upward from their current levels over a long future period (it seems unlikely that rates could trend *downward* from their current levels), **HA1** might prove for a time to be superior to the use of current rates, which would tend to result in substantial over-compensation. Uncertainty regarding the future course of interest rates, and the future course of wages as well,[8] may suggest basing the net discount rate used to estimate the present value of future lost earnings on a blend of historical averages and current/recent rates, which is consistent with some of our empirical results.

Finally, it is encouraging that some important recent efforts have been made towards finding better methods of estimating present value, including the possible use of time series forecasting techniques (Cushing and Rosenbaum, 2010) and the use of the zero coupon Treasury yield curve to discount future economic damages (Rosenberg, 2010). But for traditional and "cutting-edge" methods alike, we can learn something about how well they might work in the present by looking at how well they would have worked had they been employed in the past. Thus, historical simulation studies should continue to be part of the research agenda in forensic economics.

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[8]The macroeconomic relationship between interest rates and wage growth is theoretically ambiguous, as causality may flow in either direction. A central bank might respond to perceived inflationary pressures (say, a rise in commodity prices) by raising interest rates, which would tend to reduce spending, output, and employment, causing downward pressure on wage growth. But if the perceived inflationary threat begins with rising wages, the central bank's actions to raise interest rates can be seen as the result of the rising wages. Finally, both interest rates and wage growth may be directly related to a third variable, the rate of inflation. A rising rate of inflation may result in a higher anticipated rate of inflation, leading workers to demand higher wages and lenders to demand higher nominal interest rates.

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## Appendix

In Table A, the actual interest returns and wage growth rates for the years 1999 through 2008 are shown in the third and fourth columns. Suppose the plaintiff is awarded $9,761.19 at the beginning of the first year of the loss period. To this is added interest (.0468*9761.19) and then the first year's payout of wages ($1,000*1.0304) is subtracted, leaving an ending balance at the end of the first year of $9,187.61. This is the beginning balance in the second year, to which the second year's interest (.0589*9187.61) is added and the second year's wages (1.0345*1,030.40) are subtracted, leaving a second year ending balance of $8,662.81. This is the beginning balance in the third year, and so on. At the end of the 10th year, the ending balance is zero, demonstrating that $9,761.19 is the actual lump sum of money required to exactly compensate the plaintiff for the lost future wages given the actual behavior of interest returns and wages during the loss period.

Table A
Demonstration that $9,761.19 is the Actual Present Value for a Base Loss of $1,000 for the 10-Year Period 1999-2008

| Year Number | Calendar Year | Interest Return % | Wage Growth % | Beginning Balance | Plus Interest | Minus Wages | Ending Balance |
|---|---|---|---|---|---|---|---|
| 1 | 1999 | 4.68 | 3.04 | 9761.19 | 456.82 | 1030.40 | 9187.61 |
| 2 | 2000 | 5.89 | 3.45 | 9187.61 | 541.15 | 1065.95 | 8662.81 |
| 3 | 2001 | 3.83 | 3.20 | 8662.81 | 331.79 | 1100.06 | 7894.54 |
| 4 | 2002 | 1.65 | 3.59 | 7894.54 | 130.26 | 1139.55 | 6885.25 |
| 5 | 2003 | 1.02 | 2.94 | 6885.25 | 70.23 | 1173.05 | 5782.42 |
| 6 | 2004 | 1.20 | 2.54 | 5782.42 | 69.39 | 1202.85 | 4648.96 |
| 7 | 2005 | 2.98 | 2.60 | 4648.96 | 138.54 | 1234.12 | 3553.38 |
| 8 | 2006 | 4.80 | 1.51 | 3553.38 | 170.56 | 1252.76 | 2471.18 |
| 9 | 2007 | 4.66 | 2.68 | 2471.18 | 115.16 | 1286.33 | 1300.01 |
| 10 | 2008 | 1.60 | 2.68 | 1300.01 | 20.80 | 1320.81 | 0.00 |