Dynamic Modeling of a Belt Driven Electromechanical XY Plotter Cutter

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DYNAMIC MODELING OF A BELT DRIVEN ELECTROMECHANICAL XY PLOTTER CUTTER

by

Joseph V. Prisco, B.S.

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ABSTRACT

DYNAMIC MODELING OF A BELT DRIVEN ELECTROMECHANICAL XY PLOTTER CUTTER

Joseph V. Prisco, B.S.

Marquette University, 2013

Current industrial XY plotter cutters that use a belt driven gantry for the X motion and media feed for the Y motion sometimes have performance issues in cutting out high quality shapes. Mathematical models for these plotter cutters are not publicly available and thus the parameters critical to cut quality are not well understood. This thesis develops a dynamic, electromechanical model for the gantry arm and media feed using first principles and a non-linear friction model. These models are independently simulated and experimentally verified. In order to verify the effectiveness of the individual models, they are combined with a control system and trajectory generation algorithm. A rectangle, star and oval are simulated with the combined system using both a detuned and tuned controller and compared to experimental results. The effectiveness of the model is demonstrated with good agreement between theoretical and experimental results for both controllers. The resulting model can be used to improve and optimize the performance of XY plotter cutters.
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Joseph V. Prisco, B.S.

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DEDICATION

I would like to dedicate this thesis to my family; for their patience and support.

Thank You
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CHAPTER 1

Introduction

1.1 Background and Motivation

XY motion control is used in industry for many applications. One specific application is XY plotter cutters for both the industrial labeling and arts and crafts industries. These devices are programmable machines that allow the user to define a specific shape to be cut out of flat media (e.g., paper and vinyl). These devices can be used to generate precision shapes and labels more quickly than traditional die cutting operations.

1.1.1 XY Plotter Cutter System Description

Based on the concept of a milling machine, an XY plotter cutter consists of a cutting blade and media which are moved relative to each other. In one particular system (Fig. 1.1), the Y motion is controlled by moving the media and the X motion is controlled by a gantry. The carriage on the gantry is driven by a timing belt and a geared DC servomotor system with a freely rotating cutting blade (rotation about Z-axis) on the end. The media is driven back and forth by a geared DC servomotor system and two pinch rollers. One roller is connected through gearing to the motor shaft. The other roller is an idler roller that is clamped down to the drive roller in order to pinch the media to the drive roller. The system uses a position controller with coordinated control of the axes in order to cut out desired shapes.

1.1.2 Poor System Optimization

Many industrial XY plotter cutters are not optimized with respect to cost and performance. While numerous XY plotters are able to cut out high quality shapes at high speeds, these machines are often over-designed due to a poor understanding of the system dynamics. As a result, these machines are very costly. In other situations, XY plotter cutters are designed for low cost and the system performance is low. These systems are often use open loop stepper motors which
result in overall poor quality shapes, rounded corners and open contours. As shown in Figs. 1.2 and 1.3, the cutters cannot repeatedly cut out ovals and stars; the resulting shapes are severely deformed and frequently are not closed shapes. The fundamental causes for poor shape cutting is not fully understood. As a result, the design of either the high performing, high cost or low performing, low cost device follows historical precedent set by industry which includes the building of multiple prototypes and extensive testing, a “design-build-test” approach.

1.1.3 Improving Design Techniques

The goal of this thesis is not only to optimize XY plotter cutters, but also to improve the design process for them and other similar devices. It is important to break out of the design, build, test mode of design. This can be done by creating a mathematical model of a specific XY plotter cutter. In order to create a model, a gray box approach will be used. That is, the model will be developed using a combination of first principles and experimental data. Experimental data is needed because some phenomena are too difficult to model from a first principles standpoint. Unfortunately, mathematical models of XY plotter cutters are not publicly available.

Creation of a model will allow a better understanding of the fundamentals of XY plotter cutting and reduce the number of engineering builds of the product. The model will be used to address issues with cutting closed shapes. The model will also be used for control design. In addition the model will aid in the identification of critical design parameters. A thorough understanding of the system will allow designers to create an optimized machine instead of an overdesigned or underperforming machine. Finally and most importantly, the model will be able to be used to predict what parameters most influence certain errors. For example, it will tell if friction is too difficult to overcome, if the actuator is sized correctly and whether or not the control algorithm is adequate. Without this, designers are forced into a guess and check mode of design.

The goal of the project is to develop a minimum fidelity mathematical model of an XY plotter that can be be used for system optimization for cost and performance. The model will aid in
understanding why the errors with starting and stopping and shape quality are seen in XY plotter cutters. In addition, it will help to predict the parameters that are critical to cut performance. The model will be developed for the configuration described in Section 1.1.1.

Figure 1.1: Industrial XY Plotter Cutter

Figure 1.2: Shows the performance issue of some XY plotter cutters on circular cutouts. The inability to start and stop in the same position can be observed.

1.2 Previous Work

The following section explains the previous work done in XY motion control in addition to work done on control of machinery with respect to frictional effects.
1.2.1 Review of XY Motion Control Models

Significant work has been done in the area of XY motion control but none were found to be readily available for XY plotter cutters as stated above specific to this application: X motion is controlled by a gantry arm on a linear slide and Y motion is controlled by pinch rollers driving media back and forth. Because of this, none of them answer the questions of (1) what are the fundamental parameters governing the motion of a commercial XY plotter cutter? and (2) how do these parameters affect the quality of cut? The goal of this project is to understand the dynamics understand why issues with shape quality are seen.

Park et al. [1] discuss the dynamics of a dual-drive servo mechanism and develops an XY gantry model consisting of two motors for Y control with another motor sliding the gantry in the X direction. This design uses two parallel rails for Y-motion with a bar spanning across the two rails which holds the end effector of the system. The Y rail is driven by the two servomotors. This thesis focuses on the bar connecting the two rails with a flexible coupling. Park et al. looks to improve the positioning of this system by understanding the structural dynamics of the system. They model the joints of the connecting bar as torsional springs which improves the accuracy of the positioning model. As will be shown in this thesis, in order to accurately model XY plotter cutters, it is not necessary to model structural dynamics for the system. Also, the physical configuration of this system is significantly different than that of the XY plotter cutter studied here.

Lin et al. [2] discuss ultra-precision machining which focuses on a dual servo controlled stage but does not focus on a low fidelity control model of XY motion
control. Lin et al. develop a high fidelity model for the X and Y motion that is insightful, but is overly complicated. Their model is used to investigate the hysteresis introduced by a ball screw. However, the X and Y control for a XY plotter cutter as in this thesis is typically done with a linear slide and pinch rollers for media feed rather than a ball screw. Hidenori and Hishizume [3] also focus on a similar application for micromachining applications. This particular application involves two linear slides which are supported by aerostatic bearings. There are also electrorheological fluid dampers for precise position control. While in principle this is similar to XY plotter cutters, the component configuration being analyzed is significantly different and therefore the dynamic analysis is significantly different.

Hong et al. [4] discuss the dynamics of XY gantry systems specific to the configuration of dual drive servo mechanism as done by Park et al. [1]. This application focuses on path optimization with the use of a dynamic model. While path planning is important, extensive path planning is not the focus of this project. A very simplified approach will be used. In addition, Hong et al. does not do path planning for the system configuration.

Babaie and Khaznadi [5] develop a model for an XY positioning table. They were able to develop a mathematical model of the XY positioning table and use a Neuro-Fuzzy model that predicts the friction of the table. This helps to improve position control of the system. While the form of the mathematical is similar for the XY positioning system, the configuration is different.

Lim et al. [6] discuss the position control for an XY table with a non-rigid ballscrew. The focus is on a torsional displacement estimation feedback method in order to reduce positioning error. The configuration is not the same as a XY plotter cutter. In addition, the torsional displacement method is not applicable to XY plotter cutters because motion is not achieved through a ball screw on the XY plotter cutter.

Weikert et al. [7] develop a model for an H-Bot XY motion controller. This thesis focuses mainly on the kinematics of the H-Bot. This again is similar to the XY plotter cutter but is not the correct configuration. A more in-depth analysis is also needed. Sollmann et al. [8] develop another model for an H-Bot. The kinematics of this system are the same as the H-Bot developed by Weikert et al. [7]; however,
Sollmann et al. [8] develop a much more complex and complete dynamic model. The equations of motion are developed for the system which are applicable to an XY plotter cutter. The system is also controlled by a current controlled servo-amp and a PWM signal from the control system, which is again applicable to an XY plotter cutter. The model for this system also takes into account belt compliance and non-linear friction. According to Sollman et al., understanding of non-linear friction is very important to controlling these systems in a precise manner. Sollman et al. also claim that belt compliance is very important in understanding precision control of coordinated motion machines. These claims will be investigated further in this thesis due to the fact that the H-Bot is similar to the XY plotter cutter.

The phenomena of the effect of belt compliance on XY motion control systems can be investigated further in work by Hace et al. [9]. This work explains that modeling of belt compliance is important when there is significantly large loads and high speed movements. When this occurs, there can be significant uncertainty in the position of one joint relative to another (the position of a pulley relative to another). There can also be serious vibrations due to belt stretch oscillations. For the XY plotter cutter system, this effect likely does not need to be accounted for. The mass of the end effector is quite small and the speeds of operation are relatively slow. Based off work by Sollman et al. [8] and Hace et al. [9], further investigation needs to be done in friction modeling and the control of it in dynamic systems.

1.2.2 Friction Modeling and the Control of it in Dynamic Systems

As mentioned in Sollmann et al. [8], it is very important to understand the friction in XY motion systems. A good place to start in understanding friction with respect to motion in control is in work done by Armstrong-Helouvry et al. [10].

Armstrong et al. describe many detailed models for friction and the mechanisms behind it. A very helpful table is included in their work which explains the advantages and disadvantages of different friction models. This table shows seven different friction models which can be used. The models are viscous, Coulomb, Coulomb + viscous, Stribeck, rising static friction, frictional memory and presliding displacement. Each of these models is good for different things however the Coulomb + viscous model is advantageous because it can predict stick-slip motion along with
hunting. Hunting is oscillation around the position set point. The Stribeck model is also very useful, however the Stribeck effect is often very hard to physically realize and leads to very complex models and should only be used if needed.

1.2.3 Issues Caused by Friction in Dynamic Machines

Armstrong-Helouvry et al. [10] also discuss many different techniques for friction compensation in control. Another table is developed in which the different control tasks are outlined with significant friction in the system. In regulator or position control of systems with friction, there are issues with steady-state error and hunting. The main phenomena that can cause this problem is stick-slip friction. In tracking with velocity reversal control, there can be problems with stand still or lost motion during a direction change. For example, a system may pause at zero velocity until enough force is applied to exceed the static friction force due to the direction change.

In tracking at low velocities, systems with non-linear friction can sometimes fail to track a position command at low velocities. This is explained by the Stribeck friction model. This phenomenon occurs when the operating point velocity, defined as $V_0$, lies on the negatively sloped portion of the steady-state-friction-velocity curve, which is the Stribeck portion of the friction curve.

In tracking control at high velocities, there can be issues tracking a velocity setpoint. This problem is defined for the region where viscous friction effects dominate. Since the slope is predictable for this region, stability is not the issue, but rather tracking error is. The tracking error can become even worse when the machine has to run at high and low speeds. A gain scheduling technique is often implemented for this kind of situation.

Additional info of the control techniques for systems with friction can be found in Appendix A. Since the XY plotter cutter is a precisely controlled machine that has significant friction in the mechanisms, the above phenomena need to be understood when controlling the machine.
1.3 Literature Review Conclusion

Overall, the literature review is broken into two sections: XY motion control and frictional effects in machinery. Significant work has been done in XY motion control, however none is specific to the problem of XY plotter cutters. In addition, it is important to understand the effects of friction in machinery as friction plays a significant role in the performance of the XY plotter cutter.

1.4 Outline of Thesis

Chapter 2 will discuss the development and verification of the mathematical model for the gantry. Chapter 3 will discuss the development and verification of the mathematical model for the media feed. Chapter 4 will discuss the development of the model of the blade for the XY plotter cutter. In addition, the trajectory generation for shape cutouts for the system is discussed. The model of the XY plotter cutter (gantry, media feed and blade) will be verified using a detuned and tuned control system. Chapter 5 will discuss the uses of the model in improving the design and performance of the cutter. Chapter 6 will conclude the thesis and introduce future work.
CHAPTER 2

Model of the Gantry

The XY plotter cutter will be analyzed by independently modeling the gantry, media feed and blade. Once these systems have been modeled, they are combined to form the model of the XY plotter cutter. This chapter will describe the process of mathematically modeling the gantry and experimentally verifying the model.

2.1 Description of Gantry

The following section will describe the physical system being analyzed, the gantry, along with the process of taking the physical system and expressing it in terms of the physical model.

2.1.1 Gantry Physical System

The gantry portion of the XY plotter cutter controls the X-motion of the system as shown in Fig. 2.1. The system contains a rigid, linear slide which functions as a track for the linear sliding system. The cutter carriage fits around the linear slide. The cutter carriage holds the knife for cutting. The system is actuated by a DC servomotor and timing belt system. The timing belt is driven by a DC servomotor through a gear train. The belt rotates around two pulleys as shown in Fig. 2.1.

2.1.2 Gantry Physical Model

Based on the physical system described above, a physical model is created. In order to represent the physical system as a physical model, certain simplifying assumptions are made. The assumptions for the system are as follows:

1. The timing belt is inextensible and massless. This assumption is made because the mass of the timing belt (about 4 g) relative to the mass of the rest
of the system is insignificant. Belt compliance needs to be taken into account in systems that have large loads and high speeds [9]. This system has small loads with low speeds. Adding these effects to the model adds unnecessary complexity to the model.

2. **The motor can be modeled by a torque constant and applied current.** This is made because a current driver is used to drive the motor. This will be explained in more detail under the Section 2.4.2.

3. **A Coulomb + viscous friction model will be used to represent friction in the system.** The frictional model used is called a Coulomb + viscous friction model. This is used because prior research shows that most frictional effects are captured by this type of model. It is sometimes advantageous to add in the Stribeck effect as discussed in Chapter 1; however, it is very hard to model and measure. In addition, the XY plotter cutter does not exhibit characteristics that Stribeck friction models show such as tracking errors at low velocities [10]. Therefore, a Coulomb + viscous friction model is likely adequate for this system investigation. In addition, a Coulomb + viscous friction model is much more practical mathematically and physically in that the friction parameters are reasonably realized.
4. **Friction will be treated as a lumped term measured at the motor.**
   This is done because friction parameters are very difficult to measure on a component level. A test to measure Coulomb and viscous friction is developed in Section 2.3.2 and will be explained in detail in this section.

5. **There is no backlash in the gear train.** Backlash is neglected because the effects of it are so small compared to other effects seen in the system. Typical backlash models are geometric models that take into account the gap in mating gears. In work done by Shing [11], it is shown that backlash effects are very small compared to that of friction and inertia.

6. **The linear slide is rigid, straight and aligned.** The linear slide is a steel beam that has very little flexure and is precision manufactured for straightness.

7. **The friction of the blade interacting with the media is ignored.** This will be considered when the model of the gantry and media feed are coupled in Chapter 5.

8. **The geometry of the blade is ignored.** The blade rotates on a caster like design. This has an impact on the direction of friction and the geometry of cut. However, the geometry of the blade is outside the scope of this project.

   Based on the assumptions made, the gantry system is represented as a physical model as shown in Fig. 2.2. The idler pulley is modeled as a rigid, rotating mass (pulley 1). The second gear/pulley is also modeled as a rigid, rotating mass (pulley/gear 2). The cutter carriage is modeled as a rigid, sliding mass. The motor is modeled as a torque source with a rotating mass attached to the shaft which will be explained in more detail in Section 2.4.2. The slide is treated as perfectly rigid.

**2.2 Equations of Motion of Gantry**

The equations of motion are derived based on the physical model shown in Fig. 2.2. The equation is derived using Lagrange’s approach based off the
Figure 2.2: Shows the physical model of the gantry developed from the physical system.

formulation presented by Ginsberg [12]. Specifically,

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_j} \right) - \frac{\partial L}{\partial q_j} = Q_j, \ j = 1, 2, ..., N \quad (2.1)$$

where the Lagrangian is defined as:

$$L = T - V \quad (2.2)$$

where

- $T = \text{System Kinetic Energy}$
- $V = \text{System Potential Energy}$

and where

- $Q_j = \text{Generalized Force}$
- $q_j = \text{Generalized Coordinate}$
- $\dot{q}_j = \text{Generalized Velocity}$

For the gantry system, the generalized coordinate (Eqn. 2.3) and generalized velocity (Eqn. 2.4) are written as:

$$q_1 = \theta_{3,g} \quad (2.3)$$

$$\dot{q}_1 = \dot{\theta}_{3,g} \quad (2.4)$$

These are chosen because $\theta_3$ is the drive coordinate of the gantry. In order to
determine the Lagrangian of the system, the velocities of each joint are defined in
terms of the generalized velocity from simple kinematics:

\[
\dot{\theta}_{1,g} = \frac{r_{3,g}}{r_{2,g}} \dot{\theta}_{3,g} \\
\dot{\theta}_{2,g} = \frac{r_{3,g}}{r_{2,g}} \dot{\theta}_{3,g} \\
\dot{x} = \frac{\dot{\theta}_{1,g} r_{3,g}}{r_{2,g}}
\]  

(2.5)

(2.6)

(2.7)

For the gantry system, the Lagrangian is written as:

\[
L = \frac{I_{3,g}}{2} \dot{\theta}_{3,g}^2 + \frac{I_{1,g} r_{3,g}^2}{2 r_{2,g}^2} \dot{\theta}_{3,g}^2 + \frac{I_{2,g} r_{3,g}^2 \theta_{3,g}^2}{2 r_{2,g}^2} + \frac{m_4 r_{1,g}^2 r_{3,g}^2 \theta_{3,g}^2}{2 r_{2,g}^2}
\]  

(2.8)

In order to determine the equation of motion, the Lagrangian (Eqn. 2.8) is used in
Lagrange’s equation (Eqn. 2.1). The equation of motion for the gantry is:

\[
I_{3,g} \ddot{\theta}_{3,g} + \frac{I_{1,g} r_{3,g}^2}{r_{2,g}^2} \ddot{\theta}_{3,g} + \frac{I_{2,g} r_{3,g}^2}{r_{2,g}^2} \ddot{\theta}_{3,g} + \frac{m_4 r_{1,g}^2 r_{3,g}^2}{r_{2,g}^2} \dot{\theta}_{3,g} + B_{pantry} \ddot{\theta}_{3,g} + T_{friction} = \tau_1
\]  

(2.9)

where

\[
T_{friction} = \begin{cases} 
T_{f_s} = \text{Static Coulomb friction torque for impending motion} & (\dot{\theta}_{3,g} = 0) \\
T_{f_d} = \text{Dynamic Coulomb friction torque for motion} & (\dot{\theta}_{3,g} \neq 0) \\
-T_{f_s} < T_{friction} < T_{f_s} & \text{for static equilibrium} (\dot{\theta}_{3,g} = 0)
\end{cases}
\]

The torque at the motor can be found using a classical DC servomotor model:

\[
\tau_1 = k_t i_a
\]  

(2.10)

where

\[ k_t = \text{torque constant of motor} \]

\[ i_a = \text{current applied to motor} \]

Based on this equation, an equivalent inertia term can be formed (Eqn. 2.11).

\[
I_{\text{equivalent}} = I_{3,g} + \frac{I_{2,g} r_{3,g}^2}{r_{2,g}^2} + \frac{I_{1,g} r_{3,g}^2}{r_{2,g}^2} + \frac{m_4 r_{1,g}^2 r_{3,g}^2}{r_{2,g}^2}
\]  

(2.11)

The equivalent damping for the entire gantry is defined as:
$B_{\text{gantry}} = \text{viscous damping of gantry system}$ \hspace{1cm} (2.12)

2.3 Parameter Identification for the Gantry

The system parameters for the mathematical model of the gantry are identified. The techniques to determine mass, inertia and friction parameters are described in this section.

2.3.1 Identification of Mass and Inertia Parameters for Gantry

<table>
<thead>
<tr>
<th>Body 1</th>
<th>Material</th>
<th>Mass (kg)</th>
<th>Inertia (kg-m^2)</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>Idler Pulley 1</td>
<td>Aluminum</td>
<td>6.00E-08</td>
<td></td>
<td>$I_{1,g}$</td>
</tr>
<tr>
<td>Shaft 1</td>
<td>Steel</td>
<td>5.02E-09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Body 2</td>
<td>Pulley Gear 2 (Spur Gear)</td>
<td>Aluminum or 303 Stainless</td>
<td>9.00E-08</td>
<td>$I_{2,g}$</td>
</tr>
<tr>
<td>Timing Pulley</td>
<td>Aluminum</td>
<td>8.00E-08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shaft 2</td>
<td>Steel</td>
<td>9.33E-09</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Body 3</td>
<td>Motor Gear</td>
<td>Steel</td>
<td>3.80E-07</td>
<td>$I_{3,g}$</td>
</tr>
<tr>
<td>Motor</td>
<td>N/A</td>
<td>9.90E-07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Body 4</td>
<td>Carriage</td>
<td>Plastic/Bronze</td>
<td>0.03</td>
<td>$m_4$</td>
</tr>
</tbody>
</table>

Table 2.1: Mass, Mass Moments of Inertia and Radii Parameters for the Gantry.

The mass and mass moment of inertia parameters are found first due to the relative simplicity of the task. For the gantry, the only mass needed is the carriage, $m_4$. This cutter carriage is removed from the slide and weighed. The next task is to obtain the mass moments of inertia of the remaining bodies, $I_{1,g}$, $I_{2,g}$, and $I_{3,g}$ which are made up of multiple components. For this system, the task is performed using a CAD package and assigning material properties to each component. The CAD models are obtained from the manufacturer of the component. When models are not available from the manufacturer, models can be approximated by using key dimensions of the parts. The mass moments of inertia are approximated by the
CAD software and converted to the desired units and coordinate system. For the case of the inertia of the motor rotor, this data is obtained from the manufacturer. The parameters obtained for the system are shown in Table 2.1.

### 2.3.2 Identification of Friction Parameters for Gantry

Friction is a substantially more difficult phenomenon to characterize. A naive approach to modeling friction is to identify friction parameters for each individual component on the gantry. Friction comes from many different sources and many different material interactions in this system. There is friction present on the linear slide, the bearing for the gear shafts and inherent in the motor. It becomes very hard to physically realize the friction parameters from each component. Because of this, a system level approach is developed to find the effective friction as seen by the motor. This method will not give any insight to the specific causes of friction, but will provide the proper data needed to analyze and model the friction from a system dynamics perspective. In other words, the goal of the friction model is to understand the overall effects the total friction has on the system output. The procedure for measuring these friction parameters is adapted from Dr. Kevin Craig [13].

**Procedure for Measuring Static Friction Parameter**

The procedure for measuring the friction parameters for a DC servomotor is developed by using the model of a DC servomotor with a series of simple experiments. Once this procedure is developed for a DC servomotor, it can be used in a general approach for a DC servomotor and the load attached to it. The first parameter to measure is static Coulomb friction torque because it is the simplest to obtain. For the measurement of all friction parameters, the general equation for a DC servomotor found in Eqn. 2.13 is used.

\[
I_{rotor} \ddot{\theta} + B_{motor} \dot{\theta} + T_{friction} = k_i i_a
\]  
(2.13)
where

\[ B_{\text{motor}} = \text{viscous damping of motor} \]
\[ T_{\text{friction}} = \text{friction torque of system} \]
\[ k_t = \text{motor torque constant} \]
\[ i_a = \text{applied current to motor} \]

For the static case, Eqn. 2.13 simplifies to Eqn. 2.14 because the angular velocity and acceleration is zero.

\[ T_{fs} = k_t i_a \]  \hspace{1cm} (2.14)

In order to measure this constant, the motor is configured for current control using a variable power supply. The current is monitored by the power-supply and is slowly increased until the rotor slightly turns. Fig. 2.3 shows the the current slowly increasing over time until the rotor moves; then the current drops off to a constant value due to it being in a dynamic, or moving state (the friction torque is now in the dynamic region). The current is recorded at the point circled in red which is the max current applied before motion occurs.

Once this current is found, the calculation for static Coulomb friction torque is performed. The max recorded current is multiplied by the torque constant obtained from the motor manufacturer which gives the value for static Coulomb friction torque (Eqn. 2.14).

\[ \text{Procedure for Measuring Dynamic and Viscous Friction Parameters} \]

Dynamic Coulomb friction torque and viscous friction can be found in a similar manner. Again, the general equation (Eqn. 2.13) is utilized. This equation

![Figure 2.3: Current vs. Time for Static Friction Test.](image-url)
simplifies down to Eqn. 2.15 in the steady-state case (i.e., constant velocity).

\[ B_{\text{motor}+\text{load}}\dot{\theta} + T_{fd} = k_i i_a \]  

(2.15)

and

\[ B_{\text{motor}+\text{load}} = \text{Viscous Friction of Motor and Load} \]

In this case, \( k_i \) is a known parameter and \( i_a \) and \( \dot{\theta} \) are measured. The current, \( i_a \), is applied to the motor and recorded. The corresponding rotational velocity at steady-state, \( \dot{\theta} \), is measured using the encoder on the motor. This gives a mathematical expression with one equation and two unknowns, where \( B_{\text{motor}} \) and \( T_{fd} \) are unknowns. Therefore, the \( i_a \) and \( \dot{\theta} \) parameters are measured at an additional operating point at steady-state. Then, the system of equations (2 equations 2 unknowns) is solved for \( B_{\text{motor}+\text{load}} \) and \( T_{fd} \). This gives an approximate value of dynamic Coulomb friction torque and viscous friction of the motor along with the system attached to the motor. It is important to note that this approximation is only good if the motor behaves in a linear fashion, meaning that speed increases linearly with voltage and torque increases linearly with current which is true of a DC servomotor.

### 2.3.3 Results of Gantry Friction Parameters

When applying the friction testing procedure to the gantry, the motor experiences greater loads as the belt tension increases. This suggests that it is likely that the frictional torques are a function of belt tension. As a result, the tests for Coulomb and viscous friction are performed for a range of different belt tensions that the gantry experiences. The belt tension is measured using an ultrasonic belt tension meter and the Coulomb and viscous friction values are obtained for different belt tensions and shown in Figs. 2.4, 2.5 and 2.6. In the case of dynamic Coulomb friction torque, an increase in belt tension from 1 lbf to 3 lbf causes the dynamic friction to increase by about 28 percent. For static friction, an increase in belt tension from 1 lbf to 3 lbf causes the static Coulomb friction torque to increase by about 30 percent. For viscous friction, an increase in belt tension from 1 lbf to 3 lbf causes the viscous friction to increase by about 67 percent. The exact relationships
between static Coulomb, dynamic Coulomb and viscous friction are not clear; however, there is a clear trend that friction increases with belt tension. This is important because the gantry behaves significantly differently for varying belt tensions. This is important to understand from a design standpoint and the model must be able to account for this. The tabulated friction parameters for this system are shown in Table 2.2.

![Static Friction vs. Belt Tension](image)

**Figure 2.4:** Static Friction vs. Belt Tension for the gantry.

### 2.3.4 Friction Parameter Conclusions

In this section, it is seen that a method of characterizing friction parameters for the DC servomotor and the load attached is needed. More importantly, the outlined method of friction measurement is applicable to any system that exhibits stick slip characteristics and is run by a DC servomotor. Since this system is run by two independent DC servomotors, this method is used to characterize the friction in both the gantry and media feed systems.

### 2.4 Gantry System Control and Measurement

A control system and measurement system for the gantry is created. It is necessary to set up a real-time control system with data acquisition in order to be
able verify the mathematical model which was previously developed and test different control schemes to verify that the cut issues can be predicted and resolved.

**Figure 2.5:** Dynamic Friction vs. Belt Tension for the gantry.

**Figure 2.6:** Viscous Friction vs. Belt Tension for the gantry.
### Friction Data for Gantry

<table>
<thead>
<tr>
<th>Sample</th>
<th>Tension (lbf)</th>
<th>RPM (rad/s)</th>
<th>Current (stick) Amps, $i_s$</th>
<th>Current (Dynamic) Amps, $i_d$</th>
<th>Torque Constant (N-m/A), $K_t$</th>
<th>Viscous Friction (N-m), $F_{\text{motor+load}}$</th>
<th>Dynamic Coulomb Friction (N-m), $T_{fd}$</th>
<th>Static Coulomb Friction (N-m), $T_{fs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.95-1.00</td>
<td>53.77</td>
<td>0.22-0.25</td>
<td>0.270</td>
<td>0.0137</td>
<td>4.260E-06</td>
<td>3.470E-03</td>
<td>3.014E-3 - 3.425E-3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>118.07</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.50-1.53</td>
<td>55.01</td>
<td>0.22-0.26</td>
<td>0.275</td>
<td>0.0137</td>
<td>5.184E-06</td>
<td>3.482E-03</td>
<td>3.014E-3 - 3.562E-3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>121.08</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>3</td>
<td>1.95-2.02</td>
<td>84.74</td>
<td>0.25-0.30</td>
<td>0.325</td>
<td>0.0137</td>
<td>5.168E-06</td>
<td>4.015E-03</td>
<td>3.425E-3 - 4.11E-3</td>
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<td></td>
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<td></td>
<td></td>
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<td>4</td>
<td>2.45-2.55</td>
<td>50.61</td>
<td>0.26-0.29</td>
<td>0.305</td>
<td>0.0137</td>
<td>6.550E-06</td>
<td>3.847E-03</td>
<td>3.562E-3 - 3.973E-3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>102.89</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3.00-3.05</td>
<td>77.18</td>
<td>0.30-0.34</td>
<td>0.375</td>
<td>0.0137</td>
<td>7.090E-06</td>
<td>4.590E-03</td>
<td>4.11E-3 - 4.658E-3</td>
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<tr>
<td></td>
<td></td>
<td>125.49</td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>6</td>
<td>3.52-3.58</td>
<td>56.63</td>
<td>0.32-0.34</td>
<td>0.330</td>
<td>0.0137</td>
<td>1.181E-05</td>
<td>3.899E-03</td>
<td>4.384E-3 - 4.658E-3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>99.02</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 2.2:** Friction vs. Belt Tension Data.

#### 2.4.1 System Configuration

Fig. 2.7 shows the block diagram for the system measurement and control configuration. For this system, a dSPACE DS1103 RTI board is utilized along with the dSPACE control desk software to implement the real time control system and acquire real time data. The dSPACE software is also convenient because it interfaces seamlessly with Simulink, which is already being used to simulate the equations of motion for the gantry and is able to automatically generate the code to be used on the dSPACE board.

A motor driver connects to the dSPACE board. For this system, a servo driver (Advanced Motion Controls, model AZ20A8DDC) operating in current mode is used. It takes the voltage output from the dSPACE controller, and converts it to a current which drives the DC servomotor. The DC servomotor is connected to the gear train as shown in Fig. 2.1. The system is made closed loop by the feedback of...
the optical encoder installed on the shaft of the DC servomotor. This signal is fed back into the dSPACE control system. Finally, the motor driver and motor are powered by a DC power supply.

![Block Diagram of System Configuration for Measurement and Control.](image)

**Figure 2.7:** Block Diagram of System Configuration for Measurement and Control.

### 2.4.2 Simulink Model of System

The gantry model that is implemented in Simulink is separated into two parts. The first part is the control system and motor driver. The second part is the mechanical system of the gantry.

**Simulink Model of Control System and Motor Driver**

In order to verify the mathematical model of the gantry, the entire system described in the above section must be modeled and simulated. In this case, it is coded using Simulink. The first component to be modeled is the dSPACE system. In this case, the dSPACE outputs the control signal in the form of a 5V PWM signal. The PWM block in Simulink takes a signal out of the controller on a scale from 0-1 and generates a PWM with a duty cycle proportional to the value between 0 and 1 input into the block. The control signal is scaled by a factor of five volts. Because the dynamics of the dSPACE are so fast compared to the dynamics of the XY plotter cutter, this conversion can be approximated as an instantaneous response. Fig. 2.8 shows the dSPACE board being approximated using a gain block.

The motor driver is modeled easily using a similar approach to that of the dSPACE board. The main function of the driver board is to take the voltage from the control system and convert it to a current. This conversion factor is specified by the manufacturer as 6.4 $\frac{A}{V}$. Again, due to the high quality of the driver, the
dynamics of it are so fast relative to the dynamics of the XY plotter cutter, this can be approximated as an instantaneous response. Fig. 2.8 shows the motor driver being approximated using a gain block.

Finally, Fig. 2.8 shows the PID block used along with the position input to the system.

![Figure 2.8: Block Diagram of PID Controller, dSPACE Board and Motor Driver.](image)

**Simulink Model of Gantry**

Now that the dSPACE board and motor driver are modeled in Simulink, the equations of motion of the gantry are coded. Because a current driver is used for the motor, the model for the motor can be greatly simplified. The use of the motor driver allows the motor circuit to be ignored since a current is being directly applied to the motor. According to Eqn. 2.10, the torque output of the motor is represented by the applied current, $i_a$, which is output from the motor driver, multiplied by the torque constant, $k_t$. In Simulink, a gain block is used which is shown in Fig. 2.9. The equivalent inertias (Eqn. 2.11) are modeled as shown in Fig. 2.9 with a gain block. The viscous damping is also fed back using a gain block which is shown in Fig. 2.9. It is apparent that this system follows the general form of a second order differential equation (Eqn. 2.9).

The Coulomb friction model is added. A flow diagram of the function of the model is shown in Fig. 2.10 and the Simulink model is shown in Fig. 2.11. The model is separated into three areas. The static friction section of the diagram is
boxed in blue in Fig. 2.9. The value for the static friction torque is implemented as previously discussed in Section 2.3.2. The total applied torque to the system is fed in through the Resultant Force input. The absolute value of this force is compared to the value for static friction torque. If the Resultant Force value is larger than the static friction torque value, then the value for static friction torque is used. Sign 1 also measures the direction of the torque and applies this to the output friction torque.

The dynamic friction model is explained which is boxed in green in Fig. 2.11. The value for dynamic friction torque or Slip Friction is determined from the procedure described in Section 2.3.2 and implemented in the model. The direction of the velocity is measured by the sign block which determines the direction of the friction force.

The section outlined in red in Fig. 2.11 determines which state of friction is used. The Hit Crossing detects when the velocity changes directions. When the system is at rest or changing directions in velocity, a value of 1 is output. When the system is in motion, a value of 0 is output. The Switch Threshold then outputs the dynamic friction torque when a value of 0 from the Hit Crossing is output. It outputs the static friction torque when a value from 1 from the Hit Crossing is output.
### 2.4.3 Gantry Model Verification

In this section, the model of the gantry is verified. For the model verification a step input is used. The output of a step input is verified using a P controller, which controls the position of the motor. A command of 10 radians is used as it is an aggressive move. Significant overshoot and oscillations are seen on this command which make it good for model verification. The system response start time for the
experimental data was difficult to determine. There was no way to synchronize the
timing between the simulation and experimental data. Therefore, it was assumed
that the start time for the response of the simulated and experimental data was the
same. This also applies for the verification of the media feed system in Section 3.2.3.

As explained in Section 2.3.3, the friction of the system varies with belt
tension. For the gantry, the model must be verified for all the belt tension operating
points tested. Fig. 2.12 shows strong correlation between simulated data and
experimental data for a belt tension of 3 lbf.

There is slight mismatch in the phase and steady-state error; however for an
aggressive input this correlation is quite strong. From this test, it is determined
that model for the gantry is adequate as any cut commands will be significantly
slower and more closely controlled. There is also a small mismatch in the phase of
the response. The number of oscillations for the experimental vs. simulated data is
the same. The rest of the verification data is shown in Appendix B. The additional
testing for varying belt tensions shows that there is minor dependance on system
response as belt tension varies. Fig B.1 shows a max position value of about 18
radians while Fig 2.12 shows a max position value of around 17 radians. The
oscillations and phase of the system responses varying with belt tension are also
very similar. This shows that the system output is not very sensitive to belt tension
in this application. However, in other systems where the range of possible belt
tensions is larger, this relationship may need to be considered in manufacturing and
assembly.
Figure 2.12: Experimental Data vs. Simulated Data for 3.52-3.58 lbf tension.
CHAPTER 3
Model of the Media Feed

As explained in Chapter 2, the XY plotter cutter will be analyzed by independently modeling the gantry, media feed and blade. This chapter will describe the process of mathematically modeling the media feed and experimentally verifying the model.

3.1 Description of Media Feed

The following section will describe the physical system being analyzed (the media feed of the XY plotter cutter) along with the process of a taking a physical system and expressing it in terms of a physical model.

3.1.1 Media Feed Physical System

The media feed of the XY plotter cutter controls the Y-motion of the system. This system is shown in Figs. 3.1, 3.2 and 3.3. The system is driven by a DC servo motor which has a gear attached to it. The bottom roller is the drive roller which has a gear mounted to it. The drive roller is mounted to the frame of the cutter by two ball bearings. The upper roller is the idler roller that pinches the media to the drive roller.

3.1.2 Media Feed Physical Model

Based on the physical system of the media feed, a physical model is created. In order to represent the physical system as a physical model, simplifying assumptions are made. The assumptions for the system are as follows:

1. **There is no slip between the media and drive roller.** It is reasonable to assume that there is no slippage in the media relative to the rollers due to the knurled surface of the drive roller.

2. **The motor can be represented by a torque constant and applied**
A Coulomb + viscous friction model will be used to represent friction in the system. This assumption is made for the same reason as stated in Section 2.1.2.

3. A Coulomb + viscous friction model will be used to represent friction in the system. This assumption is made for the same reason as stated in Section 2.1.2.

4. Friction will be treated as a lumped term measured at the motor. This assumption is made for the same reason as stated in Section 2.1.2.

5. There is no backlash in the gear train. This assumption is made for the same reason as stated in Section 2.1.2.
6. **The mass of the media is negligible.** The mass of the media is small relative to the other system parameters and therefore can be ignored. The equivalent inertia of the mass of the media is about $5.8 \times 10^2 \text{ kg} - \text{m}^2$. Table 3.1 shows that this is much smaller than the other inertia parameters.

7. **The friction of the blade interacting with the media is ignored.** This assumption is made for the same reason as stated in Section 2.1.2.

   Based on the assumptions made, the media feed system can also be represented as a physical model. The physical model for this system is shown in Fig. 3.4. The drive gear and motor are modeled as body 1 and are represented as a rigid, rotating mass and torque source. The media drive roller and roller gear are also modeled as body 2 and are represented as rigid, rotating masses. The idler roller is modeled as body 3 and represented as a rigid, rotating mass.

   The physical model of this system is shown in Fig. 3.5 with the gears removed from the system for simplicity. This model shows the media pinched between two rollers and being driven.

### 3.1.3 Equations of Motion of Media Feed

The equations of motion are derived based on the physical model shown in Fig. 3.4. The equation is again derived using Lagrange’s approach based off the Ginsberg [12] formulation (Eqn. 2.1). For the media feed system, the generalized coordinate (Eqn. 3.1) and generalized velocity (Eqn. 3.2) are written as:
These are chosen because $\theta_1$ is the drive coordinate of the media feed. In order to determine the Lagrangian of the system, the velocities of each joint are defined in
terms of the generalized velocity:

\[
\dot{\theta}_{2,m} = -\dot{\theta}_{1,m} \frac{r_{1,m}}{r_{3,m}} \\
\dot{\theta}_{3,m} = \dot{\theta}_{1,m} \frac{r_{1,m} r_{2,m}}{r_{3,m} r_{4,m}}
\] (3.3) (3.4)

For the media feed system, the Lagrangian is written as:

\[
L = \frac{I_{1,m} \dot{\theta}_{1,m}^2}{2} + \frac{I_{2,m} r_{1,m}^2 \dot{\theta}_{1,m}^2}{2 r_{3,m}^2} + \frac{I_{3,m} r_{1,m}^2 r_{2,m}^2 \dot{\theta}_{1,m}^2}{2 r_{3,m}^2 r_{4,m}^2}
\] (3.5)

In order to determine the equation of motion, the Lagrangian (Eqn. 2.8) is used in Lagrange’s equation (Eqn. 2.1). The equation of motion for the media feed is:

\[
I_{1,m} \ddot{\theta}_{1,m} + \frac{I_{2,m} r_{1,m}^2 \ddot{\theta}_{1,m}}{r_{3,m}^2} + \frac{I_{3,m} r_{1,m}^2 r_{2,m}^2 \dot{\theta}_{1,m}}{r_{3,m}^2 r_{4,m}^2} + B_{\text{media feed}} \dot{\theta}_{1} + T_{\text{friction}} = \tau_{2}
\] (3.6)

The \( T_{\text{friction}} \) term is defined in the same way it is defined for the gantry and can be found under Eqn. 2.9. In addition, the torque term, \( \tau_{2} \) is defined as:

\[
\tau_{2} = k_{t} i_{a}
\] (3.7)

where

\[
k_{t} = \text{torque constant of motor}\\n\]

\[
i_{a} = \text{current applied to motor}
\]

Based off this equation, an equivalent inertia term can be formed (Eqn. 3.8)

\[
I_{\text{equivalent}} = I_{1,m} + \frac{I_{2,m} r_{1,m}^2}{r_{3,m}^2} + \frac{I_{3,m} r_{1,m}^2 r_{2,m}^2}{r_{3,m}^2 r_{4,m}^2}
\] (3.8)

The equivalent damping for the entire media feed is defined as:

\[
B_{\text{media feed}} = \text{viscous damping of media feed system}
\] (3.9)

3.2 Parameter Identification for the Media Feed

In the following section, the system parameters for the media feed are identified using a similar process to that of the gantry and explained in detail in Chapter 2.
### Media Feed System Parameters

<table>
<thead>
<tr>
<th>Variable</th>
<th>Material</th>
<th>Inertia (kg-m²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Body 1</td>
<td>Motor Rotor N/A</td>
<td>9.90e-07</td>
</tr>
<tr>
<td>Motor Gear 303 Stainless Steel</td>
<td>3.00E-07</td>
<td></td>
</tr>
<tr>
<td>Body 2</td>
<td>Drive Gear 303 Stainless Steel</td>
<td>7.90E-07</td>
</tr>
<tr>
<td>Driver Roller Steel</td>
<td>1.88E-06</td>
<td></td>
</tr>
<tr>
<td>Insert Brass</td>
<td>1.50E-07</td>
<td></td>
</tr>
<tr>
<td>Body 3</td>
<td>Idler Roller Steel</td>
<td>1.88E-06</td>
</tr>
</tbody>
</table>

**Table 3.1:** Inertia Parameters for Media Feed.

### 3.2.1 Identification of Mass and Inertia Parameters for the Media Feed

The same procedure that is applied to the gantry is applied to media feed for the parameter identification. For the roller, the inertias of all three bodies are calculated. Each component is again modeled using a CAD software package, material properties are assigned and the inertias of each component are calculated. The inertias are provided in Table 3.1. Body 1 consists of the motor rotor and the spur gear attached to the rotor. Body 2 consists of the drive roller, drive gear and a brass insert on the roller. Body 3 consists of the idler roller.

### 3.2.2 Identification of Friction Parameters for the Media Feed

The same approach that was taken for friction modeling and parameter identification that was taken on the gantry is used on the media feed. Parameters for static Coulomb, dynamic Coulomb and viscous friction are measured for the media feed system using the process from Section 2.3.2. As shown in Table 3.2, this system has a significantly higher static friction term than dynamic term. The static friction value is nearly twice the value of the dynamic friction. It takes significantly more effort from the motor to start the system into motion than it does to maintain the system in motion because of the difference in static and dynamic friction torque. This result suggests a system that is very difficult to control from a starting and stopping perspective.
3.2.3 Media Feed System Measurement

The control and measurement system for the media feed is set up in the same way as the gantry system was set up. The same configuration as shown in Fig. 2.7 applies to this system.

Simulink Model of the Media Feed

The Simulink model of the media feed is very similar to that of the gantry. The only difference lies in the mechanical system, which is now the media feed system rather than the gantry. The model is shown in Fig. 3.6. It is apparent that this model is nearly identical to the model for the gantry which is seen in Fig. 2.9. Fig. 3.6 shows three blocks boxed in blue: the equivalent inertia, damping constant and friction model. These are the three terms that are different. The equivalent inertia terms change based on the inertia of Eqn. 3.8 and given in Table 3.1. The damping and friction models changed based on the experimental data from Table 3.2.

It is interesting to note that even though the two systems of the gantry and media feed are physically very different being that one is a sliding mass and the other is two rollers pinched together, they can still be modeled in a similar fashion.

Media Feed Model Verification

The media feed model is verified with a similar process to the gantry. For the media feed, three different step inputs are used. The magnitudes of the step input are 10, 20 and 30 radians respectively. The additional testing of the 20 radian and 30 radian position input is done to ensure the model works well for inputs with harsh direction changes. The larger step inputs caused additional oscillations around

<table>
<thead>
<tr>
<th>RPM (rad/s)</th>
<th>Current (stick) Amps, $i_a$</th>
<th>Current (Dynamic) Amps, $i_b$</th>
<th>Torque Constant (N-m/A), $K_t$</th>
<th>Viscous Friction (N-m-s), $B_{motor+load}$</th>
<th>Dynamic Coulomb Friction (N-m), $T_{fd}$</th>
<th>Static Coulomb Friction (N-m), $T_{fs}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>194.74</td>
<td>1.07</td>
<td>0.740</td>
<td>0.0137</td>
<td>7.736E-06</td>
<td>8.700E-03</td>
<td>1.4659E-02</td>
</tr>
<tr>
<td>433.76</td>
<td>0.880</td>
<td>0.880</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 3.2: Friction Parameters for Media Feed.
Figure 3.6: Simulink Model for Roller System.
the position set point as there is greater overshoot as the step input increases.

As seen in Fig. 3.7, the position of input of 10 radians does not cause multiple direction changes. To ensure the model is tested for this, the 20 and 30 radian inputs are tested. A P controller is again used with a slightly smaller control gain of 0.08 rather than 0.1. The three different step inputs are used to investigate how the model will respond to aggressive moves with overshoot. However, the XY plotter cutter will never have motion inputs that are this aggressive. All motion inputs to the XY cutter are well controlled and smoothly shaped. Figs. 3.7, 3.8 and 3.9 show that even for aggressive position inputs, the model tracks quite well. There is some mismatch on the number of oscillations, the steady-state error and the percent overshoot, however it still matches well.

The 10 radian step response, Fig. 3.7, has the correct number of oscillations and a close match on the overshoot, however significant difference on the steady-state error. The 20 radian step response, Fig. 3.8, over predicts the amount of oscillations and overshoot and has a significant difference on the steady-state error. The 30 radian step response, Fig. 3.9, over predicts the amount of oscillations and overshoot with the difference in steady-state error not being as great as the previous two. As there are more oscillations in the step responses, the model tends to deviate more. The model appears to be under predicting the amount of friction in the system. This is of little concern for predicting shape cutouts because harsh direction changes will never occur in an XY plotter cutter. The rest of the mismatch is again likely due to the difficulty of characterizing friction along with the difficulty of measuring the parameters for the model. In addition, it is important to note that media feed system response is very sensitive to friction parameters. A small change in friction parameters mean a large change in system response.
Figure 3.7: Experimental vs. Simulated Data for 10 Rad Step Input Using the Media Feed.
Figure 3.8: Experimental vs. Simulated Data for 20 Rad Step Input Using the Media Feed.
Figure 3.9: Experimental vs. Simulated Data for 30 Rad Step Input Using the Media Feed.
CHAPTER 4

Model of the XY Plotter Cutter

Using the models of the gantry, media feed and blade, the model of the XY plotter cutter is developed. In addition to the gantry and media feed, a model is developed for the resistance force of the cutting blade interacting with the media for cutting. The effectiveness of the model is tested by simulating and experimentally verifying a rectangle, star and oval.

4.1 Combined Model of Gantry and Media Feed

The independent, verified models of the gantry and media feed are combined and controlled together to form the full XY plotter cutter. The XY plotter cutter is designed so that each axis is decoupled from the other. In other words, each motor has its own, independent control system. The X and Y position commands are generated by various methods and independently sent to each motor to cut out the desired shapes. The Simulink model for the XY plotter cutter is shown in Fig. 4.1. This model reflects the description of independent commands being sent to each motor to cut desired shapes.

4.1.1 Cutting Resistance for Blade

Although the controllers are decoupled from each other, this does not mean that X and Y motion is completely decoupled from one another. There is some interaction between the X and Y motion due to the the cut resistance force of the cutting blade. Experimentation performed for this project has shown that the cut resistance force of the blade can be modeled as a constant force. A simplified model is shown in Fig. 4.2. This diagram is shown in the XZ plane.

Determination of the Magnitude of Cut Resistance Force

The blade can modeled simply as shown in Fig 4.2. This model shows the swivel of the blade about the z direction. It also shown that the cut resistance force
Position Input to Gantry Motor

Position Input to Media Feed Motor

Model of Gantry

Model of Rollers

Position Input for Theta_1

Position Input for Theta_2

Position Output_1

Position Output_2

Conversion to radians

Conversion to radians

x_out

x_out

Figure 4.1: Simulink Model for XY Cutter.

opposes the direction of the force applied to drive the blade. Cut resistance force is much more difficult to model for the cutting blade as it is not just two surface rubbing together.

In order to measure the cut resistance force, a pull test is used. The knife blade is mounted in a fixture as shown in Fig. 4.3. The velocity is set on the pull tester to a relatively low velocity (200mm/min, max velocity of pull tester). It was determined that for low velocities the blade friction is speed independent. The fixture is pulled at a constant velocity without the knife blade mounted in it, and the force to pull over a set length is recorded. Then, the knife blade is added to the fixture and the same test is repeated with the cutting blade engaged in the media. The values from each test are averaged and the test with just the fixture is subtracted from the test with the fixture and the knife blade to find determine the force from the knife blade. Each test showed consistent results for the force from the
It is important to note that the amount of cut resistance force generated is dependent on many factors. Some of these factors include knife geometry, depth of blade engagement in media, media type and amount of downward force the blade exerts on the media. For this case, only one scenario is tested. The knife geometry is held constant, along with the blade engagement, media type and amount of downward force. Further testing could be done to characterize this situation; however, the detailed testing is outside the scope of this project.

**Determination of the Direction of Cut Resistance Force**

For this scenario shown in Fig. 4.2, the cut resistance force is acting completely in the X direction. However, as demonstrated by Fig. 4.2, the cutting blade rotates very similar to a caster. Therefore, the cut resistance force opposes
the direction of cut in almost all situations. Since the XY plotter cutter cuts in the
X and Y directions, the total cut resistance force force is broken up into components
in the X and Y directions.

The direction of the blade with respect to the XY coordinate system can be
explained mathematically. Eqn. 4.1 describes the total velocity of the system which
is given by the square root of the sum of the squares of the X and Y velocities.

\[
\text{Magnitude of Velocity} = v_{\text{blade}} = \sqrt{x^2 + y^2}
\]  
(4.1)

The ratio of X velocity to total velocity is given by Eqn. 4.2.

\[
\text{X Velocity Ratio} = l_{\text{blade},x} = \frac{\dot{x}}{v_{\text{blade}}}
\]  
(4.2)

The ratio of Y velocity to total velocity is given by Eqn. 4.3.

\[
\text{Y Velocity Ratio} = l_{\text{blade},y} = \frac{\dot{y}}{v_{\text{blade}}}
\]  
(4.3)

The amount of cut resistance force in the X direction is proportional to the X
Velocity Ratio and is given by Eqn. 4.4.

\[
F_{\text{cutting resistance},x} = F_{\text{cutting resistance}} \ast l_{\text{blade},x}
\]  
(4.4)

The amount of cut resistance force in the Y direction is proportional to the Y
Velocity Ratio and is given by Eqn. 4.5.

\[
F_{\text{cutting resistance},y} = F_{\text{cutting resistance}} \ast l_{\text{blade},y}
\]  
(4.5)

### 4.1.2 Implementation of Blade Cut Resistance Force in Simulink

The blade cut resistance force model is implemented in Simulink using the
concept described in Section 4.1.1. The X and Y velocities of the blade are fed back
to a block to perform the calculations shown by Eqns. 4.1, 2.1 and 4.3 and is shown
in Fig. 4.4. It is important to note that in Fig. 4.4 that \( \frac{\dot{x}}{\sqrt{x^2+y^2}} = \sqrt{\frac{x^2}{x^2+y^2}} \). This was
done in Simulink to make the diagram simple. The velocity ratios are found using
Eqns. 4.4 and 4.5. The direction is also found in Fig. 4.4 by using the sign block to
tell the direction of each the X and Y velocities.
Since the total cut resistance force can be measured using the test described in Section 4.1.1, the cut resistance force is modeled using a constant block denoted as *Cut Resistance Force* and is shown in Fig. 4.5. This block is multiplied by the gear ratio of the system. In addition the direction signal and relative velocity signal is fed in. Cutting resistance force, the direction signal and the velocity ratio are multiplied together to get the frictional force for each direction.

**Figure 4.5:** Magnitude and Direction of Friction Force for Cutting Blade.

### 4.2 Shape and Trajectory Generation

In order to do shape cutout testing, a method for to generate the cut commands to the XY plotter cutter is developed. The following section describes a
procedure to construct a shape using the Cartesian coordinates of the XY plotter cutter and develop the needed trajectory based off the angular position of the motor. Trajectory planning techniques are used to provide smooth commands to the motor. In order to demonstrate the algorithm that has been generated, a rectangle is explained as an example in the following section.

### 4.2.1 Development of Cartesian Coordinates and Joint Coordinates for Shapes

The rectangular shape is shown in Fig. 4.6. The corresponding Cartesian points are defined in Fig. 4.7. In this case, the Cartesian points defining the rectangle were simply defined by inspection. For a more complex shape such as a oval, one would need to use the equation of a oval to generate a series of points on the curve to define the oval. After the Cartesian points are defined, they are converted to the joint space using the inverse kinematics. The inverse kinematic equations are defined by Eqns. 4.6 and 4.7. The joint coordinates are shown in Fig. 4.7.

\[
y = \frac{r_{1,m}r_{3,m}}{r_{2,m}} \theta_{1,m}
\]

\[
x = \frac{r_{1,g}r_{3,g}}{r_{2,g}} \theta_{3,g}
\]

### 4.2.2 Trajectory Generation for Shapes

Using the joint space points for the rectangle, smooth, controlled commands are used to cut in a straight line path (in Cartesian space) from point to point. A technique from robotics can be used to achieve this called trajectory planning. A trajectory is defined as the path the joint, or motor follows as a function of time \[14\]. Significant work has been done in this field and many different approaches are available.

Some of the options discussed by Niku \[14\] include third-order polynomial trajectories, fifth-order polynomial trajectories and linear segments with parabolic blends. All of these trajectories provide the motors with smooth position inputs.
Third-order trajectories allow the user to specify boundary conditions for the initial and ending velocities and positions. Fifth-order trajectories allow the user to specify boundary conditions for the starting and endpoint velocities and accelerations. Linear segments with parabolic blends give the user even more control by allowing the user to specify the max angular velocity and max angular acceleration. For this work, minimal control over the boundary conditions and max velocities and accelerations are needed, therefore a third-order trajectory is used. The main purpose of the trajectory is to provide a smooth motion profile.

**Figure 4.6:** Rectangle in Cartesian Space.

**Figure 4.7:** Conversion from Cartesian Space to Joint Space.

### Third-Order Polynomial Trajectory Planning

In order to specify the input command to the motor from a known start position to end position, a third-order trajectory is used. In addition to the start and end positions, the start and end velocities are also specified in a third-order...
trajectory. The form of the equation is shown in Eqn. 4.8.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$  \hspace{1cm} (4.8)

The derivative of the third-order trajectory is given by Eqn. 4.9

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$  \hspace{1cm} (4.9)

The initial and final positions and velocities are:

$$\theta(t_i) = \theta_i$$  \hspace{1cm} (4.10)
$$\theta(t_f) = \theta_f$$
$$\dot{\theta}(t_i) = 0$$
$$\dot{\theta}(t_f) = 0$$

The coefficients for the trajectory equation are solved using the matrix defined as Eqn. 4.11

$$\begin{bmatrix} \theta(t_i) \\ \theta(t_i) \\ \theta(t_f) \\ \dot{\theta}(t_f) \end{bmatrix} = \begin{bmatrix} 1 & t_i & t_i^2 & t_i^3 \\ 0 & 1 & 2t_i & 3t_i^2 \\ 1 & t_f & t_f^2 & t_f^3 \\ 0 & 1 & 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} c_0 \\ c_1 \\ c_2 \\ c_3 \end{bmatrix}$$  \hspace{1cm} (4.11)

The unknowns in this systems of equations (Eqn. 4.11) are found. In order to solve the system of equations, $\theta_i$ and $\theta_f$ are defined by the joint coordinates solved for using the inverse kinematics (Fig. 4.7). As an example, the first set of $\theta_i$ and $\theta_f$ for the $\theta_3$ joint are defined as:

$$\theta_i = 0rad$$
$$\theta_f = 59rad$$

An algorithm is developed to determine the values of $t_i$ and $t_f$ (initial and final times, respectively). Fig. 4.8 shows the flow chart that describes the algorithm used to determine these values. The magnitude of the cut segment is determined to decipher whether the X or Y cut command is larger. The larger value is passed through to the next step and the approximate desired speed is set by dividing the length of the vector by the desired average speed in order to determine the elapsed
time of the trajectory. This determines the value for $t_f$ while $t_i$ is already known. This process is repeated based on the number of trajectories needed to cut out a desired shape.

![Diagram of cutting time step determination](image)

**Figure 4.8: Algorithm For Cutting Time Step Determination.**

The coefficients determined from Eqn 4.11 are used in Eqn. 4.10 to generate the position trajectory from one point to another. This concept can be extended to generating trajectories for a series of points. The Cartesian points constructing shapes are determined and the series of trajectories are generated using the above concepts to cut the desired shape. Figs. 4.9 and 4.10 show the X and Y trajectories for the whole rectangular shape.
4.3 Verification of Model

In order to verify the model’s effectiveness, some simple tests are run. Three shapes are tested: a rectangle, star and oval. The shapes are tested with a detuned control system and then with a tuned control system. The average cut speed is set at $2 \frac{\text{in}}{\text{s}}$ due to this being an industry standard.
4.3.1 Selection of Shape Cutouts

A rectangle, star and oval are tested which consist of varying degrees of complexity and are shown in Figs. 4.6, 4.11 and 4.12.

![Star Cutout Input](image1.png)

**Figure 4.11:** Star Cutout Input.

![Oval Cutout Input](image2.png)

**Figure 4.12:** Oval Cutout Input.

**Rectangle Cutout**

A rectangle is chosen because of its simplicity and is shown in Fig. 4.6. It is made up of four different segments determined by inspection in which the X and Y
motion are decoupled from each other. The generated trajectories are shown in Figs. 4.9 and 4.10. One can observe that the X and Y trajectories are relatively simple for this shape.

**Star Cutout**

A star is chosen because it is significantly more complex than the rectangle, but still relatively simple. The star is made up of ten segments determined by inspection which require coupled X and Y motion to form straight line paths. This shape will help to test the interaction between the X and Y motion. The generated trajectories are shown in Figs. 4.13 and 4.14. One can observe the these trajectories are significantly more complex than the rectangle trajectories.

![X Trajectory for Star Cutout](image)

**Figure 4.13:** X Trajectory for Star Cutout.

**Oval Cutout**

An oval is chosen because it is the most complex shape to be cut. The methodology to cut a oval is shown in Fig. 4.15. The oval is too complex to determine the segments by inspection as is done in the rectangle and star. Therefore, the equation of the desired oval must be defined and points that lie on the path of the oval are plotted. These points are then connected with straight lines in order to make up a oval. It is important to ensure that the oval is broken up into
enough points so that the series of straight line segments appears smooth. This shape is the most difficult because it is made up of 480 very small cut vectors in order to construct the oval. These cut vectors also require coordinated X and Y motion. The X and Y trajectories are seen in Figs. 4.16 and 4.17. Upon first glance, these trajectories look simpler than the star trajectories. In fact, they appear to be a sinusoidal function. However, this is on a macroscopic level. On a more detailed level, the trajectories are made up of very fine third-order trajectories. This is seen in Fig. 4.18, which show a the detail of X trajectory. For this reason, the oval is the most difficult shape tested.

4.3.2 Detuned Control Testing

The rectangle, star and oval are used to test the effectiveness of the model. The shapes are simulated using the XY plotter cutter model (gantry, media feed and blade) and experimentally verified using the data from the motor encoder. First, a detuned control system is used. That is, control values are assigned which cause excessive overshoot and ringing. For this experiment, each joint (gantry and media feed) use a P controller with a gain of 0.1. This control design causes significant overshoot, steady-state error and oscillations which make it useful for testing the effectiveness of the model. The goal of this testing is not to see how well
the input trajectories match the output trajectories (although this is ultimately desired), but to determined how well the predicted output trajectory matches the measured output trajectory and if the model correctly predicts when errors occur. In addition, the ultimate goal is to understand how closely the predicted output shape matches the experimental output shape.
Figure 4.17: Y Trajectory Input for Oval Shape.

Figure 4.18: X Trajectory Detail View to Show Third-Order Trajectories.

Detuned Rectangle

The rectangle is tested and the reference inputs for the rectangle are shown in Figs. 4.9 and 4.10. Figs. 4.19 and 4.20 show detuned experimental vs. simulated data for the X and Y trajectories for the rectangle. Fig 4.19 shows that the experimental and predicted data match very well for the X direction. Both the simulated and experimental data show that the X trajectory curve is not as smooth as desired, especially at about 3.7 s.
Fig. 4.20 shows that the experimental and predicted data do not match as well for the Y direction; however, many of the key features are predicted. There is a time delay in the experimental data at the first movement at about 3 seconds which the simulation shows in general. There is a large difference (about 1 radian) in the flat part of the graph between the experimental and simulated data between approximately 3 and 7 seconds. More importantly, the model correctly predicts that the desired value of close to 10 radians will not be achieved. Also, there is discrepancy between the steady-state values at the end (greater than 7s). However, both the model and experimental data show that a Y position of 0 will not be achieved for the ending position. Even though the errors were not predicted completely, the model predicted the locations that errors are likely to occur and the type of error that occurs (tracking error, time delay, “stair stepping”, etc.).

Fig. 4.21 shows the shape output produced by the X and Y trajectories and confirms what was shown for the X and Y trajectories (Figs. 4.19 and 4.20) In order to plot Fig. 4.21, the Y trajectory joint values are plotted vs. the X trajectory joint values. The joint values are multiplied by the kinematics (Eqns. 4.6 and 4.7) to convert the joint angles to Cartesian space. One can see that the model correctly predicts the main issues with the shape cutout. There is a mismatch in position on the top line of the rectangle and a difference in the start stop point.

**Figure 4.19:** Experimental vs. Simulated vs. Command Data for Detuned X Trajectory for Rectangle.
Figure 4.20: Experimental vs. Simulated vs. Command Data for Detuned Y Trajectory for Rectangle.

Figure 4.21: Experimental vs. Simulated vs. Command Data for Detuned Rectangle.

Detuned Star

The star is tested and will be more difficult to accurately predict due to the increased amount of segments along with its coupled X and Y motion. The X trajectory again has very good results. As shown in Fig. 4.22 there is very little discrepancy in the experimental vs. predicted trajectory. There are certain areas where the rounded corners become flat and this is successfully predicted by the
model and is shown in Fig. 4.23. The model continues to show that it is able to predict when and where errors are happening.

![Figure 4.22: Experimental vs. Simulated vs. Command Data for Detuned X Trajectory for Star.](image)

For the Y motion shown in Fig. 4.24, the simulated and experimental data do not match match as well as the X motion. However, the model does an adequate job of predicting the general trends of the Y trajectory. The model shows that there is a time delay in starting the first cut command at around 1 s and that there is significant difficulty in tracking the smoothness of the input curves. It also shows that there are significant issues hitting steady-state values (i.e., between 2.25 and 3.5 s). It shows a general phase lag in the trajectory and a general “stair-stepping” trend which is shown in the experimental data. While the experimental vs. simulated curves don’t match exactly, it is shown that the model correctly predicts the type of errors occurring and approximately where they are occurring.

Fig. 4.25 is plotted in the same manner as the rectangle (Fig. 4.21) and shows the star shape has significant quality issues which is consistent with the observations from the X and Y trajectories. The model shows difficulty in hitting steady-state values which is confirmed by the experimental data. The model also shows that the cutter cannot make “clean” corners and becomes extremely distorted on the lower half of the star. Finally, the model shows that there is discrepancy between starting and stopping points which is consistent with experimental data.
Figure 4.23: Experimental vs. Simulated vs. Command Data for Detuned X Trajectory Star, Detailed View.

The star shape again shows that the model is able to predict cutting errors.

Figure 4.24: Experimental vs. Simulated vs. Command Data for Detuned Y Trajectory for Star.

Detuned Oval

The oval is tested and as mentioned before is the most difficult shape to cut and therefore is the most difficult test for the model. As shown in Fig. 4.26, the X trajectory has a strong match in experimental vs. simulated data which is
consistent with previous shapes. The model shows that there will be some tracking error along with an flat area on a curve which is shown in Fig. 4.27. These predictions are consistent with the experimental data shown.

The Y trajectory, shown in Fig. 4.28, has difficulty tracking the Y command and matching the experimental and simulated values. The model shows that there is a significant phase lag in tracking the position signal, which is reflected in the
experimental data. The model also shows the “stair-stepping” effect which is also shown in the experimental data. Subsequently, the model predicts that the smoothness of the curve is not able to be tracked and is confirmed by the experimental data. Finally, there is steady-state error at the final value which both the model and experimental data show. This is again consistent with previous shapes in that the “magnitude” of the errors are not completely predicted by the model but the types of errors and where they occur are correctly predicted.

As expected, the model shows a shape that deviates significantly from the commanded oval. It shows the “stair-stepping” effect which was seen in the Y Trajectory (Fig. 4.28). It also shows discrepancy in the starting and stopping positions which is further shown in Fig. 4.30. While the model and experimentation do not match completely, the general predictions hold. For example, the “stair-stepping” is less prominent in the experimental data as opposed to the model data but the same general trend is there. In addition, the discrepancy is starting and stopping is not exactly the same in the model as it is in experimentation; however the error is still shown.
Figure 4.28: Experimental vs. Simulated vs. Command Data for Detuned Y Trajectory for Oval.

Figure 4.29: Experimental vs. Simulated vs. Command Data for Detuned Oval.

Conclusions

Overall, the gantry, or X model, is able to very accurately and precisely predict issues with the shape cutouts for the set control gains. It is apparent that the gantry is both easier to model and control. The media feed or Y model is able to predict issues with shape cutouts, however not as precisely as the gantry. Subsequently, the media feed is more difficult to control and model. Further
discussion of why this is the case will be discussed later.

4.3.3 Tuned Control Testing

Now that the effectiveness of the model has been verified in the previous section, a tuning procedure is done using Simulink in order to improve the quality of the shape cutouts. The goal is to show that the model can be used as an instrument to eliminate issues in cutting and understand where they are coming from. The improved quality is observed in simulation and verified with experimentation.

Control Tuning

In order to tune the control system, an adaptation of the Ziegler-Nichols approach as described by Ogata is used [15]. The procedure is traditionally done experimentally on the machine being controlled. In this case, because the model is verified for the gantry and media feed, it is easier to perform the procedure using the Simulink model. To perform this tuning method, the controller is reduced to P controller. The control gain, $K_p$ is increased slowly until it reaches an ultimate gain, denoted as $K_u$ where the output of the control oscillates at a constant amplitude. The period of this oscillation is denoted as $P_u$. The control parameters are then solved for using the formulas shown in Table 4.1.
The form of the PID controller is:

\[ G(s) = K_p + \frac{K_i}{s} + K_d(s) \]  \hspace{1cm} (4.12)

The Ziegler-Nichols results are shown in Table 4.2.

<table>
<thead>
<tr>
<th>Tuned Controller Parameters</th>
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</thead>
<tbody>
<tr>
<td>System</td>
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<tr>
<td>Gantry</td>
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<tr>
<td>Media Feed</td>
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</table>

Table 4.2: Tuned Control Parameters.

The Ziegler-Nichols values are further refined for the gantry to improve the cut quality. This is done using further experimentation. The final values used for the Tuned Control Testing are shown in Table 4.3.

<table>
<thead>
<tr>
<th>Tuned Controller Parameters</th>
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</thead>
<tbody>
<tr>
<td>System</td>
</tr>
<tr>
<td>Gantry</td>
</tr>
<tr>
<td>Media Feed</td>
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</tbody>
</table>

Table 4.3: Tuned Control Parameters.

Tuned Rectangle

The system is retested using the tuned PID control system. As shown in Fig. 4.31, the match between the experimental data and model prediction is even stronger for the rectangle for the X trajectory. The model indicates that the tracking of the commanded curve would be improved with the tuned control system. There is virtually no discrepancy between the simulated data, experimental data and commanded data.

The Y trajectory simulated data shown in Fig. 4.32, indicates that the tracking error and time delay issues that are shown in Fig. 4.19 have been rectified.
Figure 4.31: Experimental vs. Simulated vs. Command Data for X Trajectory for Tuned Rectangle.

It also indicates that the “stair-stepping” issue also shown in Fig. 4.19 has been rectified. There is virtually no discrepancy between the simulated data, experimental data and commanded data. As expected, since the simulation and experimentation confirms that most issues shown for the detuned square have been eliminated, the cut quality of the resulting rectangle is nearly flawless and is shown in Fig. 4.33. However, there are still minor issues in the cut quality as shown in
Fig. 4.34. The simulated data shows there is steady-state error and is confirmed by experimental data. The model predicted that the main issues would be eliminated and was also able to predict that there were still some minor quality issues.

Figure 4.33: Experimental vs. Simulated vs. Command Data for Tuned Rectangle.

Figure 4.34: Experimental vs. Simulated vs. Command Data for Tuned Rectangle Detail View.
**Tuned Star**

The correlation between the model data and experimental data for the star shape X trajectory is strong (Fig. 4.35). There are still no major issues that the model predicts. Some of the minor tracking error is eliminated in the model as confirmed by experimental data.

**Figure 4.35:** Experimental vs. Simulated vs. Command Data for X Trajectory for Tuned Star.

The model for the Y trajectory (Fig. 4.36) predicts that many of the issues shown in Fig. 4.24 have been rectified. For example, issues such as tracking errors, time delays, “stair-stepping” and steady-state error have been eliminated with the tuned control system.

The total star shape is plotted in Fig. 4.37. As expected, the major issues that were seen in Fig. 4.25 were eliminated. However, not all issues were rectified and there is still room for improvement in shape quality. Fig. 4.38 shows a detailed view on the bottom right corner of the star which shows that there is some issue with the quality of the star corner. Most importantly, the model was able to predict that there is a problem with the star corner.
Figure 4.36: Experimental vs. Simulated vs. Command Data for Y Trajectory for Tuned Star.

Figure 4.37: Experimental vs. Simulated vs. Command Data for Tuned Star.

Tuned Oval

The correlation between the model data and experimental data for the oval X trajectory is again strong and is shown in Fig. 4.39. Just like in the detuned case (Fig. 4.26), there were no major issues. The model data predicts that it will track the commanded curve well and is confirmed by the experimental data.

The model predicts that the main issues seen in Fig. 4.28 have been
corrected for the Y trajectory. Issues such as time delay, “stair-stepping”, steady-state error and tracking error are all eliminated by the tuned control system according to the model and is confirmed by experimental data shown in Fig. 4.40. The model still predicts that there will be minor issues such as “waviness” to the cuts and this is confirmed by the experimental data.

The total oval shape is plotted in Fig. 4.41. As expected, the model predicts that the overall shape quality is significantly improved and the experimental data confirms this. The model and experimental data shown in Fig. 4.41 are virtually indistinguishable and match the commanded input closely.
Fig. 4.40: Experimental vs. Simulated vs. Command Data for Y Trajectory for Tuned Oval.

Fig. 4.41: Experimental vs. Simulated vs. Command Data for Tuned Oval.

Fig. 4.42 shows shape quality is not perfect. There is some “waviness” to the curved line predicted by the model and is confirmed by the experimental data. Some of this is due to the “waviness” of the input trajectory; but, the quality can still be improved. Further work can be done to improve the quality of cut, however the model accurately predicts issues with the cutouts.
Conclusions

Overall, the quality of the shape cutouts were significantly improved using tuned the control system. This shows that model was able to effectively predict an improvement in cutout quality. However, the quality of the shapes still has room for improvement as shown by model data and confirmed by experimental data. The model can be used to further guide a designer how to continue to improve the quality of shapes and will be discussed in more detail in the next chapter.
CHAPTER 5

Using the Model in Design

It is important to understand how this model can be used for design now that it has been verified and well understood. The chapter will identify what factors are important in the mechanics of the system and how other factors such as the control system and trajectory generation methodology affect the cut output.

5.1 Dominant Factors in System Design

In order to investigate what the driving factors for the system design are, it is advantageous to classify and quantify the major factors in the system. In this case, there are two systems to be investigated: the gantry and media feed. The blade also has effects on system performance but they are small comparatively. Since each system is modeled as a second order system with non-linear friction, each system is broken up into three main parameters: inertia, viscous friction (damping), and Coulomb friction (static and dynamic). Each parameter can be discussed in general in addition to comparing parameters between the gantry and media feed systems.

5.1.1 System Coulomb Friction

Table 5.1 shows the breakdown of static friction torque and dynamic friction torque for the gantry and media feed systems. In the gantry system (X-direction), the static and dynamic friction torque values are very similar in magnitude. This is advantageous to avoid any control problems due to stick-slip friction [10]. For the media feed (Y-direction), the static friction torque is significantly higher than the dynamic friction torque as found in Section 3.2.2.

Using the model, one can investigate the effects of reducing static friction in the system. In Fig. 5.2, the oval shape cutout is simulated and plotted with the original measured friction parameters of the XY plotter cutter and labeled “High Static Friction Value”. The same, detuned control gains from Section 4.3.2 are used. It is desired to understand the effects of reducing the stick-slip phenomena of the
media feed. The oval shape cutout is also plotted with a reduced static friction value for the media feed where the static friction value is reduced to the same value as that of the dynamic friction value. The output shape for this is plotted and labeled “Low Static Friction Value”. One can see that there are some differences between the two curves. The stair stepping effect seen in the “High Static Friction Value” simulated is reduced in the “Low Static Friction Value” simulated. The stair stepping is still present, however it is less severe. Overall, the “Low Static Friction Value” oval is smoother. This makes the system much easier to control once the control system is tuned. In Fig. 5.3, the same “High Static Friction Value” vs. “Low Static Friction Value” is plotted for the tuned control system from Section 4.3.3. The results on this are much less drastic as the circle cutout quality is already good. Therefore, the designer can see for the tuned control system that the effect of lower friction has minor impact on system performance. However, the affect of this may be more drastic on other shapes such as a rectangle or star.

Based on results from modeling the gantry and media feed, it is apparent that the system is very sensitive to friction. This means that a small change in friction causes a significant change in system performance. This relationship is very important to understand when designing a machine. Friction must be well controlled (i.e., consistent) from machine to machine to achieve consistent results. Other techniques may be needed to mitigate its effects.

In addition, Table 5.1 shows that the friction in the media feed is much higher than that of the gantry. For static friction torque, the media feed’s friction torque is greater by a factor of nearly 5 and nearly 2.5 for the dynamic friction torque. This should alert a designer that significant work can be done in this area to improve the performance of the machine. Understanding and optimizing friction allows the designer to improve the accuracy, efficiency, life and overall quality of the machine.

5.1.2 Cutting Blade Resistance

Significant work was not done on the cutting blade resistance. As discussed previously, cutting blade resistance is dependent on media type and the force applied to the blade perpendicular to the media. For the vinyl material, the cutting
### System Friction Parameters

<table>
<thead>
<tr>
<th></th>
<th>Gantry</th>
<th>Media Feed</th>
<th>Ratio of Media Feed Friction to Gantry Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Static Friction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torque (N·m)</td>
<td>3.000E-03</td>
<td>1.466E-02</td>
<td>4.9</td>
</tr>
<tr>
<td><strong>Dynamic Friction</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Torque (N·m)</td>
<td>3.500E-03</td>
<td>8.700E-03</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>Ratio of Static to Dynamic Friction</strong></td>
<td>0.86</td>
<td>1.7</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.1:** System Friction Parameter Comparison.

**Figure 5.2:** Comparison between the oval cutouts for high and low levels of static friction with detuned controller.

Blade resistance was not a significant factor. However, different material types may prove to be larger factors in the overall XY cutting model.

### 5.1.3 System Inertia

Inertia is another significant factor in the system. Inertia is an inherent resistance to change in rotary motion and most directly affects the acceleration of a system which can cause some different phenomena in control systems. It is also important to understand inertia characteristics of a system for torque requirements for motor sizing. In general, the lower the inertia of a system, the easier it is to
control and the less torque the system needs to run. Table 5.1 shows the media feed system has more inertia than the gantry system by a factor of about 1.8. As expected, the media feed system requires more torque to run the gantry system.

Table 5.1: System Friction Inertia Comparison.

Another important phenomena in motion control systems is inertia ratios. The inertia ratio is defined as the amount of load inertia relative to amount of inertia in the rotor of the motor. Past studies have shown that the closer the inertia ratio is to 1:1 the better the performance in the system is [16]. Systems with a large inertia ratio can have increased overshoot, longer oscillation periods and long settling times. Large inertia ratios also increase settling time and cause mechanical resonance to be more likely. This occurs when their is compliance in the motor shaft.
couplings. Experimentation shows that a good rule of thumb is that the inertia ratio should be less than 10:1. This, however, is application dependent and is only a guideline. As shown in Table 5.2, the inertia ratios of the gantry and media feed systems are relatively small (0.7 and 1.9 respectively).

<table>
<thead>
<tr>
<th>Inertia Ratio</th>
<th>Gantry</th>
<th>Media Feed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inertia Ratio</td>
<td>0.7</td>
<td>1.9</td>
</tr>
</tbody>
</table>

Table 5.2: Inertia Ratios.

5.1.4 System Viscous Friction

Viscous friction is another important factor in the system. Table 5.3 shows the parameters for the gantry and media feed systems. Viscous friction is a parameter that has increasing effects as velocity increases. For this system, viscous friction does not have a significant impact on the system because shape cutouts are made at relatively low velocities. Also, between systems, the viscous friction is relatively similar.

<table>
<thead>
<tr>
<th>Viscous Friction Parameters</th>
<th>Gantry</th>
<th>Media Feed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Viscous Friction(N\text{-}m\text{-}s)</td>
<td>5.18E-06</td>
<td>7.74E-06</td>
</tr>
</tbody>
</table>

Table 5.3: Viscous Friction Parameters.

5.2 Trajectory Planning for XY Cutting

Another significant factor in the quality of cut is the trajectory planning of the system. In Section 4.2.2, a third-order polynomial trajectory was used. However, significant work can be done in this area to improve cut quality which can subsequently be tested on the model. According to Niku [14], there are many other trajectories that can be used. In addition, a different approach used to cutting altogether could be used. As explained in Section 4.3.1, the oval shape is broken up
into many small, straight, segments making it the hardest shape to cutout. This tends to cause agitated motion in the cutting for any curved cutout and causes the most issues in overall shape quality.

5.3 Control Design

A verified mathematical model provides a multitude of techniques for control design. Since this is a non-linear system, classical control techniques such as the root-locus method do not apply. However, one can use Simulink to linearize the model around an operating point and tune the model. In addition, instead of performing the Ziegler-Nichols tuning procedure on a physical system, this procedure can now be done using the mathematical model is Simulink. One can also use Simulink optimization routines to tune PID gains for non-linear plants which Simulink provides. The model can be used for other model predictive control techniques as mentioned in Appendix B. For this system, the most effective tuning method was the Ziegler-Nichols method using the mathematical model.

5.4 System Interaction

Some of the common performance parameters that are important to improve in XY cutting are cut speed and cut accuracy. Manufacturers want to increase cut speed without sacrificing any quality. As discussed in Sections 5.1, 5.2 and 5.3, XY cutting is broken into three categories: system mechanics, trajectory planning and the control system. Ultimately, a designer can use this model in order to find out which parameters to optimize in order to be able to improve cut speed without sacrificing cut quality. There are some key takeways on these three sections to consider in a XY plotter cutter design.

5.4.1 System Mechanics

In system mechanics, it is first and foremost important to eliminate stick-slip friction from the system (keep static and dynamic friction values that are similar) and keep overall friction values low. It is also important to keep the system inertia low and keep the system inertia ratio low. Viscous friction is not as large of a factor; however, the lower this value is kept, the better.
In addition, the model helps the designer to understand the tradeoffs between each of the design parameters. For example, the XY plotter cutter from this thesis has relatively high friction and low inertia. A designer may do a study to understand the affect of adding more inertia to the system to mitigate frictional effects. However, more system inertia means larger actuators. The model can be used to find this optimum point.

5.4.2 Trajectory Planning

Overall, the smoother one can get a shape trajectory, the better. Mechanical and control issues are accentuated by the the unrefined techniques used for shape trajectory planning.

5.4.3 Control System

As the system mechanics and trajectory planning become less optimized, it is more difficult to develop a robust control algorithm. As the system mechanics and trajectory planning become less optimized, the control system becomes a “cover-up” for the system mechanics and trajectory planning.

5.4.4 Conclusion

A small list of techniques and points for consideration for use with a verified model were mentioned in this section. Once a mathematical model is verified, it can be used for many different design optimization techniques and be instrumental in the design process of an XY plotter cutter. This section is meant to give some examples of what to consider in design for a plotter cutter, but is just the start. One can use the identified design factors and parameters to optimize a machine for cost, speed, accuracy and many other specifications.
CHAPTER 6

Conclusion and Future Work

In the following chapter, conclusions about the XY plotter cutter are drawn and future work is discussed.

6.1 Conclusion

A mathematical model was developed for the gantry and media feed of the XY plotter cutter. Each model was independently verified using the dSPACE board as a control system. After each model was verified, a model was developed for the cutting blade which coupled the gantry and media feed together. In order to verify the total model (gantry, media feed and cutting blade), a detuned control system was used to cut a rectangle, star and oval. The shapes were tested in simulation and then experimentally verified using the XY plotter cutter. A Ziegler-Nichols tuning approach was used and the shapes were retested in simulation and again experimentally verified using the XY plotter cutter. The combination of these two experiments proved that the model could accurately predict how shapes would be cut out.

Once the model was proven to accurately predict shape cutouts, it was shown that the model could be used for design and system optimization. Now that a fundamental understanding of XY plotter cutters is achieved, this model can be used a starting point for any future designs. This investigation also showed that the key system characteristics can be obtained with a relatively low fidelity model. The gantry and media feed were each represented using a 2nd order system with a stick-slip friction model. Therefore, this model is easily adaptable to other XY plotter cutter designs.

Overall, the model can assist in increasing product performance, decreasing the cost and reducing the development time of an XY plotter cutter. It also enables designers to test different hardware before making an engineering build. For
example, if a designer wanted to know if the system performance could be achieved using a open loop stepper motor instead of a closed loop servo motor, this model could be used as the basis for an improved model using stepper motors. The model can also help a designer understand the cost to benefit ratio in an XY plotter cutter system. For example, one could understand whether the performance gained out of a lower friction system is worth the extra cost. In addition, this can be used to break out of the “design, build, test” mode.

6.2 Future Work

While the XY plotter cutter was effectively modeled as a second order non-linear system, there still is work that needs to be done regarding the optimization of the system.

6.3 Mechanical Optimization

As discussed throughout this thesis, both the gantry and media feed are dominated by frictional forces. Due to the approach taken, it is not understood what the contributing mechanical components are to the friction in the system. Work needs to be done in order to understand what components cause friction, especially the stick-slip friction in the media feed and how these effects can be reduced or eliminated using more effective mechanical components. It is apparent that the high friction values are coming from the gantry and media feed systems directly even though there is some friction inherent in the motor. This is shown in Appendix D. One area that could be causing high stick-slip friction on the media feed is the frame. The frame is made of sheet metal and has significant flex. Also, the sheet metal design does not guarantee good alignment for the drive roller to mount in. Guaranteeing good frame alignment with the use of ball bearings is likely conducive to low stick-slip friction values.

In addition, work needs to be done to ensure inertia parameters are optimized for performance and cost. Factors such as the inertia ratio and total inertia of the system can be analyzed. The total inertia can be changed by the gear ratios of each subsystem. More analysis needs to be performed to ensure optimum gear ratios are used.
6.4 Trajectory Planning Optimization

Additional work needs to be done using trajectory planning for shape cutting development. Limited investigation was done in this project for shape development. Work needs to be done to determine what trajectories work best for certain shapes, including investigation on boundary condition types. Also, work in continuous path planning for robotic applications could be applied to XY cutting, such as work done by Angeles et al. [17] to improve the smoothness of shape cutting.

In addition, the boundary conditions of Eqn. 4.10 can be modified for certain shapes. One example of this would be to change $\dot{\theta}_i$ and $\dot{\theta}_f$ to a non-zero number so that there is not a complete start and stop between vectors on a oval. This would allow for smoother cutting. Experimentation could also be done on fifth-order polynomials and trapezoidal profiles to determine the effect on cut quality.

6.5 Control Optimization

The focus of this work was not control tuning techniques for the XY plotter cutter. A crude, Ziegler-Nichols approach was used. Section 5.3 explains some of the tuning procedures that can be done for PID control. However, additional model predictive control techniques can be developed for high friction systems and system with stick-slip friction. Some of these techniques include a friction compensation control model [10] and a feed forward friction compensation control model [18] to eliminate errors induced by friction.

6.6 Open Loop Control Optimization

This system was developed to run as a closed loop system. However, it is advantageous to be able to run an XY plotter cutter as an open loop system using stepper motors. This significantly reduces the cost of the system as the added cost of an encoder is no longer needed. However, a mechanically optimized system and strong understanding of system dynamics is needed for such an approach. The model that was developed in this thesis can be extended to a stepping motor design. Once the stepping motor model is developed, performance characteristics of the machine could be developed before building a machine.
REFERENCES


APPENDIX A

Design for Control in Machinery

Managing Frictional Effects in Machinery

Armstrong-Helouvry et al. [10] explain some of the common techniques for dealing with frictional effects in machinery. The first section discussed is on designing for control. Significant work has been done in this area and can be summarized in a statement. The amplitude or effects of stick-slip can be reduced by decreasing the mass, increasing the damping or increasing the stiffness of the mechanical system [19] [20] [21].

Managing Friction in Control

There are a few ways to manage friction through control techniques as summarized by Armstrong-Helouvry et al. [10]. One way to increase the damping and stiffness of a machine is to increase the PD gains. PD controllers can help to stabilize systems with significant stick-slip friction. Another method of control is to implement integral control. Integral control is used to eliminate steady-state error. Problems with limit cycling, or hunting, (oscillation around a steady-state value) can arise when tracking at low or zero velocities. There are a couple of methods of mitigating this issue. First, a deadband can be used at the integrator block to prevent oscillations. A deadband is an period of neutral signal to prevent oscillation cycles around a set point [22]. This, however, can cause additional steady-state error so care needs to be taken when using this method. Integral control can also cause issues when passing through the zero position point [23]. The integral control can cause issues “breaking-away” from the static friction force causing significant tracking errors. Systems with this issue typically use a reset on the integrator block when changing directions.

There are methods of compensation available based on mathematical models of friction. The concept is that if the friction force is known based off the model used, then a control compensation equal and opposite to the frictional force is used.
This of course depends on how accurate the friction model is. It also depends on how the “state” of the system is observed, being that friction is a non-linear term.

One such example of this is done by Camudas de Wit et al. [24]. A feed forward method of friction compensation is done in this paper. Based on the frictional model, 80-90 percent of the frictional effects can be captured. Thus, the error in estimation results in an undershoot or overshoot for friction compensation. In this paper, it was shown that significant improvements in tracking error were made based on friction estimation and including this in the control algorithm.

Another example of this is done by Khayati et al. [18]. In this paper, three different friction models are used and implemented in a feedforward control scheme. The three friction models used are a four-stage-GMS model, monostage-GMS model and a LuGre model which again aim to cancel out the frictional effects in control. These compensation techniques significantly reduced tracking errors with the largest tracking error being less than 2 percent. Most simulations showed that tracking errors were 0.
APPENDIX B

Gantry Friction vs. Belt Tension Graphs

Figs. B.1, B.2, B.3, B.4 and B.5 show the experimental data vs. simulated data for the rest of the gantry step responses with a detuned controller. The data correlation between experimental and simulated data is strong for all belt tensions.

Figure B.1: Experimental Data vs. Simulated Data for 0.95-1.00 lbf tension.
Figure B.2: Experimental Data vs. Simulated Data for 1.50-1.53 lbf tension.
Figure B.3: Experimental Data vs. Simulated Data for 1.95-2.02 lbf tension.
Figure B.4: Experimental Data vs. Simulated Data for 2.45-2.55 lbf tension.
Figure B.5: Experimental Data vs. Simulated Data for 3.00-3.05 lbf tension.
APPENDIX C

Experimental Shape Cutouts

This appendix shows the actual shape cutouts that were made from the detuned and tuned shape tests.

Figure C.1: Cutout of detuned rectangle shape.
Figure C.2: Cutout of detuned star shape.
Figure C.3: Cutout of detuned oval shape.
Figure C.4: Cutout of tuned rectangle shape.
Figure C.5: Cutout of tuned star shape.
Figure C.6: Cutout of tuned oval shape.
## APPENDIX D

**Friction Comparison**

<table>
<thead>
<tr>
<th>System</th>
<th>Static Coulomb Friction Torque (N-m), $T_{fs}$</th>
<th>Dynamic Coulomb Friction Torque (N-m), $T_{fd}$</th>
<th>Viscous Friction (N-m-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Motor</td>
<td>1.37E-03</td>
<td>1.12E-03</td>
<td>7.00E-07</td>
</tr>
<tr>
<td>Gantry</td>
<td>3.43E-03</td>
<td>3.47E-03</td>
<td>4.26E-06</td>
</tr>
<tr>
<td>Media Feed</td>
<td>1.47E-02</td>
<td>8.70E-03</td>
<td>7.74E-06</td>
</tr>
</tbody>
</table>

| Ratio Between Gantry Friction to Motor Friction | 2.5 | 3.1 | 6.1 |
|-----------------------------------------------------------------------------------------------|
| Ratio Between Media Feed Friction to Motor Friction | 10.7 | 7.8 | 11.1 |

**Figure D.1:** Comparison of Friction From System Load Relative to Friction of Motor.