Pre-Service Teachers’ Knowledge of Algebraic Thinking and the Characteristics of the Questions Posed for Students

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Pre-Service Teachers’ Knowledge of Algebraic Thinking and the Characteristics of the Questions Posed for Students

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**Abstract:** In this study, we explored the relationship between the strength of pre-service teachers’ algebraic thinking and the characteristics of the questions they posed during cognitive interviews that focused on probing the algebraic thinking of middle school students. We developed a performance rubric to evaluate the strength of pre-service teachers’ algebraic thinking.
across 130 algebra-based tasks. We used an existing coding scheme found in the literature to analyze the characteristics of the questions pre-service teachers posed during clinical interviews. We found that pre-service teachers with higher algebraic thinking abilities were able to pose probing questions that uncovered student thinking through the use of follow up questions. In comparison, pre-service teachers with lower algebraic thinking abilities asked factual questions; moving from one question to the next without posing follow up questions to probe student thinking.

The importance of mathematical discourse, that is classroom discussion, in the context of which students reveal their understanding of mathematical concepts, is widely acknowledged (National Council of Teachers of Mathematics, NCTM, 1989, 2000). Broadly speaking, mathematical discourse encompasses the ways in which ideas are exchanged in a mathematics classroom. While deeply rooted in mathematical ways of knowing, shaped by the nature of the tasks in which students are engaged, and fostered by the nature of the learning environment, discourse serves as a powerful facet through which students develop their understanding of mathematical ideas (Sfard, 2000a; 2000b). Mathematics teachers play a critical role in encouraging and facilitating mathematical discourse in a way that supports student thinking about mathematics. Often, asking and encouraging questions becomes the catalyst teachers use to initiate and sustain mathematical discourse with students. Questions shape the patterns of discourse by encouraging students to present ideas or concepts and to compare and clarify thinking. Questions also provide directions for the path of conversation. Posing and encouraging questions not only engages students in mathematical discourse by challenging their thinking about mathematics, but also serves as a window to students’ thinking about and understanding of mathematical ideas (Carpenter, Fennema, Franke, Levi, & Empson, 2000).

Eliciting Students’ Mathematical Thinking through Questioning

In recent years, mathematics educators have begun to examine the kinds of questions teachers pose with the goal of exploring how different types of questions support student learning and understanding of mathematics (Vaac, 1993; Kawanake & Stigler, 1999; Harrop & Swinson, 2003; Sahin & Kulm, 2008; Franke, Webb, Chan, Ing, Freund, & Battey, 2009). These studies typically identify the type of questions that stimulate cognitive processes, as
categorized in the taxonomy by Bloom, Englaehart, Furst, Hill, and Krathwohl (1956). For example, questions that encourage recollection of knowledge (i.e., focus on factual information) stimulate low-level cognitive processes in contrast to questions that encourage students to apply what they know, synthesize or evaluate. The latter foster higher levels of cognitive engagement and encourage deeper thinking and learning.

Over the course of a typical mathematics lesson, teachers pose a variety of questions. Kazemi and Stipek (2001) believe that teachers who know how to sustain mathematical dialogue effectively can enhance their students’ ability to communicate mathematical ideas, reason, justify, and construct valid mathematical arguments if they press students by asking high-level probing questions that demand explanations. Franke et al. (2009) explain that when teachers use probing questions they are providing support for students to construct a complete mathematical explanation. Sahin and Klum (2008) believe that teachers’ ability to ask high-level probing questions closely relates to their content knowledge. When investigating beginning and experienced teachers’ questioning in the context of mathematics lessons they reported that teachers with a limited content knowledge of mathematics predominantly generated low-level questions that focused on factual information. They struggled to ask high-level probing questions that had the potential to engage students in high levels of cognitive reasoning.

Formulating questions that focus on eliciting students’ mathematical thinking might be particularly difficult for pre-service teachers who typically have limited experience and underdeveloped content knowledge for teaching. Nicol (1999) analyzed the questions pre-service teachers posed to middle school students during their field experience. She reported that preservice teachers had difficulty understanding what questions to ask middle school students, and failed to understand the significance and purpose of asking questions. Moyer and Milewicz (2002) also used a field experience as a context to analyze pre-service teachers’ questioning. The pre-service teachers in their study were unable to ask elementary students questions that would promote students’ thinking about a problem. Instead, the pre-service teachers formulated questions that led the students toward an
answer to a problem. Both groups, Nicol and Moyer and Milewicz, observed that pre-service teachers predominantly posed questions to guide students to finish the mathematical task at hand rather than to engage students in revealing their thinking about the task and its solution. Asking good questions that create the opportunity to gain access to students’ thinking proves to be a difficult task for pre-service and veteran teachers alike (Buschman, 2001; Mewborn & Huberty, 1999).

The ability to pose questions to uncover student mathematical thinking develops as a form of a specialized knowledge for teaching that includes both mathematics content and pedagogy (Shulman, 1986; Ball & Bass, 2003; Ball, Thames, & Phelps, 2008). In teacher preparation programs, pre-service teachers need to strengthen their knowledge of mathematics while concurrently learning corresponding pedagogical knowledge (Capraro, Capraro, Parker, Kulm, and Raulerson, 2005). Mathematics content and methods courses need to create opportunities for pre-service elementary teachers to learn the mathematics underpinning the K-8 mathematics curriculum and at the same time to learn how to effectively engage students in discussions about the mathematics.

**Algebraic Thinking in the K-8 Mathematics Curriculum**

Current reform recommendations concerning K-8 mathematics (NCTM 2000) encourage the teaching and learning of algebra-based concepts at the K-8 level. The primary focus of teaching algebra-related concepts at the elementary and middle school level is to help students make a successful transition from the study of arithmetic to the study of algebra in the later grades. Calls for early algebra emphasize engaging students in activities that encourage mathematical ways of thinking that help bridge the divide between arithmetic and algebra. Many mathematics educators and policymakers believe that a productive way to link the concepts of arithmetic with the concepts of algebra is to focus K-8 mathematics instruction on algebraic thinking (Cuoco, Goldberg & Mark, 1999; Kieran, 1996, Swafford & Langrall, 2000). The core of the argument is that early algebra should not be equated with the early introduction of a traditional high school algebra course. For example, Kieran (1996)
highlighted the importance of introducing algebra concepts as a means to engage students in analyzing quantitative situations in a relational way. Silver (1997) advocated that early algebra instruction should give students access to algebraic ideas without putting an emphasis on symbolic manipulations, solving equations and simplifying expressions. Carpenter and Levi (2000) argued that the integration of algebra concepts in middle grades should involve algebraic reasoning with the goal of helping students develop new ways of mathematical thinking.

The term algebraic thinking closely relates to what Cuoco, Goldberg & Mark (1999) described as *useful* ways of thinking about mathematical content. Kieran and Chalouh (1993) interpreted algebraic thinking as the ability to build meaning for the symbols and operations of algebra in terms of arithmetic and further refined this perspective, defining algebraic thinking as the ability to analyze quantitative situations in a relational way (Kieran, 1996). Swafford and Langrall (2000) talked about algebraic thinking as the ability to think about unknown quantities as known. Driscoll (1999) provided more specificity for this term by identifying three useful kinds of algebraic thinking called mental habits of mind: (1) building rules to represent functions, (2) making generalizations by abstracting from computations, and (3) doing and undoing procedures and operations.

**Building rules to represent functions.** Our conception of algebraic thinking is consistent with Driscoll’s (1999, 2001), Swafford and Langrall’s (2000), and Kieran’s (1996). However, in this study, we limit our investigation of pre-service teachers’ algebraic thinking to the first of Driscoll’s (1999) algebraic habit of minds listed below, namely building rules to represent functions. We used Driscoll’s (1999) features of Building Rules to Represent Functions, listed in Figure 1, as our operational definition of the type of algebraic thinking that we investigated. That is, throughout this paper we narrow the meaning of the term algebraic thinking to connote ways of thinking essential to Building Rules to Represent Functions.
Features of Algebraic Thinking Underlying Building Rules to Represent Functions

- **Organizing information**: Thinking focused on organizing information in ways useful for uncovering patterns and rules that define them
- **Predicting a pattern**: Thinking focused on noticing a rule at work and trying to predict how it works
- **Chunking the information**: Thinking focused on searching for, and examining repeated chunks of information that reveal how a pattern works
- **Describing a rule**: Thinking focused on providing general descriptions for the steps of a rule
- **Different representations**: Thinking focused on exploring what different information about a situation or problem may be given by different representations
- **Describing change**: Thinking focused on examining and describing change in a process or relationship
- **Justifying a rule**: Thinking focused on seeking justifications for a general rule or a procedure in a general cases

Figure 1. Features of algebraic thinking examined in this study (adapted from Driscoll, 1999)

Focus on Teachers

A natural consequence of calls for early algebra is a heightened concern about the adequate preparation of K-8 mathematics teachers. Teachers’ knowledge has been identified as an important factor that influences teachers’ practice; one that closely relates to students’ achievement (Borko & Putman, 1996; Sowder & Schappelle, 1995; Hill, Rowan & Ball, 2005). In their recent reports, the U.S. Department of Education (2008) and the National Council of Teacher Quality (Geenberg & Walsh, 2008) shared a strong concern about the effective implementation of early algebra at the K-8 level. Both reports provided recommendations to strengthen the algebra-content knowledge and the pedagogical knowledge needed by K-8 mathematics teachers to implement algebra reform successfully. These recommendations are not surprising. Research shows that teachers’ knowledge of algebra is often dominated by a focus on symbols and symbolic manipulations, a focus that is in direct opposition to the philosophy of early algebra instruction (Ball 1990). Their perspectives on algebra and algebraic thinking are often strongly influenced by their own experiences with traditional, symbol oriented, school algebra. However, to teach algebra-based concepts in ways consistent with the philosophy of early algebra and accessible for K-8 students, teachers need to understand the ideas behind algebraic thinking. Without this
understanding they cannot effectively recognize and take advantage of opportunities to build on their students’ existing knowledge and to engage them in algebraic thinking.

**Goal**

The goal of this study was to address concerns related to pre-service teacher education and to provide insight into pre-service teachers’ readiness to meet the challenges of early algebra instruction. In particular, we sought to examine the relationship between pre-service teachers’ algebraic thinking proficiencies and their ability to engage students in algebraic thinking. Our primary goal was to provide an understanding of the relationship between pre-service teachers’ own algebraic thinking ability and their ability to ask questions to engage middle school students in algebraic thinking. To examine this relationship we analyzed (1) pre-service teachers’ own algebraic thinking as demonstrated in their solutions to algebra-based tasks and (2) the characteristics of questions that pre-service teachers posed when conducting interviews designed to elicit middle school students’ algebraic thinking.

**Method**

To seek an understanding of pre-service teachers’ algebraic thinking proficiency and its relationship to pre-service teachers’ questioning, we drew on the work of Clift and Brady (2005), Ebby (2000), and Sowder (2007). They emphasized that pre-service teachers’ should be provided with opportunities to connect what they learn in content and methods coursework with field experiences. Accordingly, we engaged pre-service teachers in interviewing and analyzing the algebraic thinking of a middle school student. Based on the theory of situated learning (Brown, Collins, & Duguid, 1989), we believed that this activity would enhance pre-service teachers’ knowledge of algebraic thinking (mathematics content knowledge) and their knowledge of questioning strategies that focus on examining the mathematical thinking of students (pedagogical content knowledge). Our goal was to create an opportunity for pre-service teachers to strengthen the content and pedagogical knowledge acquired during concurrent university-based mathematics content and methods classes.
by linking it with knowledge acquired during school-based field experiences.

**Context of the Study**

**Participants.** The study was conducted in the spring semester of 2009 in a large private university in the Midwest. The participants were eighteen elementary and middle school preservice teachers; sixteen females and two males. All were enrolled in a semester long mathematics content course integrated with field experience. The participants were junior or senior level students, all candidates for a 1-8 teaching license. The prior university mathematical experiences of the students were similar. The study took place in the last course of a 3-course sequence in mathematics, required in the elementary education program. All participants had completed the first two pre-requisite courses in the sequence.

**Mathematics content course.** The curriculum of the content course, which was taught in the mathematics department, addressed topics in elementary algebra and focused on helping preservice teachers develop an understanding of algebraic thinking. The goal of the content course was to engage the pre-service teachers in activities that would heighten their ability and awareness of different features of algebraic thinking and encourage them make connections among mathematics concepts fundamental to the K-8 curriculum. During the course, the preservice teachers worked individually or cooperatively on algebra-based tasks that led to multiple solutions and representations of mathematical ideas, and they engaged in discussions about them. Particularly, they were encouraged to share, explain, compare and make interpretations of various representations and reasoning. The content course helped pre-service teachers build an understanding of algebraic thinking while also introducing them to the pedagogical decisions made by middle school teachers when engaging students in algebraic thinking. The latter was introduced during activities in which the pre-service teachers analyzed middle school students’ written work for features of algebraic thinking and reflected on mathematical situations that might foster algebraic thinking in students. The term mathematical knowledge for teaching (Ball & Bass, 2003; Ball, Thames, & Phelps, 2008) best describes the focus of the mathematics
content class, during which pre-service teachers acquired knowledge of the subject matter, students, and curriculum.

Field component. The concurrent field component of the course was taught in the College of Education. Two weeks of university classroom instruction were followed by weekly observations of middle school mathematics instruction, during which time the pre-service teachers also engaged in one-on-one tutoring sessions with middle school students. The focus of the field component was to provide pre-service teachers with opportunities to directly link what they learned in their content course with practice (i.e., to engage the pre-service teachers’ in activities focused on probing the algebraic thinking of middle school students). The field component of the university classroom instruction was designed to engage the pre-service teachers in activities that assisted them in learning how to pose questions to probe students’ algebraic thinking. They practiced identifying and formulating different kinds of questions. The purpose was to help the pre-service teachers understand and differentiate among: (1) factual questions that often elicit a one-word response, (2) procedural questions that usually elicit descriptions of steps needed to solve a given problem, and (3) probing questions effective for eliciting responses related to student’s thinking about the problem.

Algebraic thinking clinical interviews. In the context of their weekly observations of mathematics classroom instruction, the pre-service teachers conducted two audio-taped problem-based algebraic thinking interviews with one middle school student. Each interview provided pre-service teachers with the opportunity to probe a middle school student’s algebraic thinking, in the context of an algebra-based task. First, the pre-service teachers were asked to select one of seven tasks suggested by the course instructors for the algebraic thinking interview. In this way, we restricted the pre-service teachers’ interview task selections to viable problems that encouraged the use of many features of algebraic thinking. The pre-service teachers were encouraged to ask questions to explore the middle school student’s algebraic thinking in the context of solving the selected tasks.
Data Analysis

Data sources for this study included (1) solutions to the algebra-based tasks pre-service teachers completed in the content class (130 total), (2) transcripts of the two algebraic thinking interviews each of the pre-service teachers conducted with a middle school student, including the written work generated by the middle school students during their interviews \( n=36 \).

Analyses of participants’ task solutions. To provide an understanding of the participants’ algebraic thinking ability we used our operational definition of algebraic thinking (Figure 1) to identify, in the context of each task, features of algebraic thinking that the preservice teachers demonstrated in their solutions. We assessed the pre-service teachers’ thinking with respect to each identified feature as (3) proficient, (2) emerging, or (1) not evident.

On a given task, we considered that the pre-service teacher was **proficient** (3) on an identified feature of algebraic thinking if the problem solution was correct and it exemplified characteristics of that feature (e.g., participant organized information in ways useful for uncovering patterns and linked this organization to the context of the problem).

On a given task, we considered that the pre-service teacher’s algebraic thinking concerning a specific feature was **emerging** (2) if the solution was correct but it exemplified characteristics of that feature without clear links to the context of the problem (e.g., participant organized information in useful ways for uncovering patterns, but did not link this organization to the context of the problem). We also considered the strength of the pre-service teacher’s algebraic thinking as **emerging** (2) if the answer to the problem was incorrect but the solution exemplified characteristics of that feature with clear connections to the context of the problem.

We assessed the pre-service teacher’s algebraic thinking concerning a specific feature as **not evident** (1) if the problem explicitly encouraged using a specific feature, but the evidence of this feature characteristics was absent from the written solution (e.g., the
problem explicitly asked to justify given answer but justification was not included in the solution.)

Finally, we quantified each participant’s strength of algebraic thinking by feature and overall. To do so, we averaged the participant’s assigned scores by feature across all analyzed tasks, and we also found the participant’s overall average score across all analyzed features and tasks.

Example analysis of participant task solution. In Figure 2 we present an example of a task that encouraged the solver to think about organizing information, predicting a pattern, describing a rule, and justifying a rule. We use this task and the solution (Figure 3) provided by one of the participants (PST #15) to provide further details about the task analysis process.

Assume that a sequence of circles in the figure below continues by adding one circle to each of the 5 “arms” of a figure in order to get the next figure in the sequence.

(a) Find a formula for the number of circles making up the Nth figure. Explain why your formula makes sense by relating it to the structure of the figures.
(b) Will there be a figure in the sequence that is made of 100 circles? If yes, which one? If no, why not? Determine the answer to these questions algebraically and in a way that a student in elementary school might be able to do.
(c) Will there be a figure in the sequence that is made of 206 circles? If yes, which one? If no, why not?

**Figure 2.** Sequence of Circles Task (adapted from Beckmann, 2007)
Organizing information. In her search for a formula that could be used to describe the total number of circles in any figure, and in response to part (a) of the problem, pre-service teacher #15 drew a sequence of figures that served to organize the information. She accurately labeled each figure with the information needed to derive a formula for the number of circles in any figure: the figure number, the number of circles that surround each center, and the total number of circles. It is evident in her answer to (a) that she explicitly linked the circle in the center of the figure, as well as the number of circles surrounding it, to the formula (rule) she derived. Her way of organizing the information about the circle pattern helped her to make sense of the problem’s regularity. Thus, we assessed this pre-service teachers’ ability to organize the information in this problem as proficient, and we assigned the score of (3).

Predicting a pattern. Her rule and explanation also served as evidence that the preservice teacher was able to think about observed regularities and make sense of how the pattern works. She not only correctly made sense of how the pattern works, but also ably predicted whether the pattern would generate figures made of one-hundred circles (part b) or 206 circles (part c). Despite our concern about her understanding of equality, as evidenced in part (b) by $20 \times 5 = 100 + 1 = 101$, her corresponding written explanations, e.g. in part (b):
The 20th figure would have 100 surrounding circles, but the middle circle would not be accounted for. [One] 1 more would need to be added in order to have the total number of circles, documented that this participant was proficient in predicting how the observed regularity works. In each part, we assessed this ability (predict a pattern) as proficient and assigned it a score of 3.

**Justifying a rule.** The pre-service teacher’s verbal description in part (a) acceptably justifies why her rule, $1 + F(5) = N$, predicts the total number of circles in any figure. Thus, in the context of substantiating the rule that she derived for this problem, we assessed the participant’s ability to justify as proficient (score 3)

The same could not be said about her ability to justify how the pattern works in parts (b) and (c). While it is true that she clearly makes reference to her rule in each part, her attempts to justify her answers to (b) and (c) are incomplete and somewhat immature. Specifically, in parts (b) and (c) she did not explicitly “undo” her rule to determine the relevant figure numbers (20 or 41), even though the problem statement encouraged algebraic and verbal explanations. In each part, the pre-service teacher “justified” by evaluating her rule (for $F = 20$ or 41), but she did not explain why she was using 20 or 41 in the first place. Accordingly, for these parts of the problem, we categorized her ability to justify as emerging (2).

**Analyses of algebraic thinking clinical interview transcripts.** Besides analyzing the tasks data, we also analyzed the transcripts of the algebraic thinking interviews that each preservice teacher conducted. We began our analysis of the transcripts by identifying the **questioning episodes** in each. We did so using the Franke et al. (2009) characterization of questioning episodes as segments of an interview transcript that start with the interviewer asking a question and continue through at least two conversational turns between the interviewer and the student. Franke described that the episode can end when (1) the interviewer moves on to explore a different mathematical issue related to the original question or (2) poses a new question that addresses the next question on the task.
We used Moyer and Milewicz’s (2002) work with pre-service teachers as a framework for analyzing the characteristics of the questions pre-service teachers posed. We categorized questions and questioning episodes as (1) checklisting, (2) instructing, or (3) probing. Then we categorized an episode as checklisting, instructing, or probing by considering all of the questions found in an episode. An episode might include more than one type of question. However, we categorized the episode based on the overall tenor of the questions within the episode.

**Checklisting.** We classified a questioning episode as checklisting if, within this episode, the pre-service teacher mainly asked questions without any attempt to probe a student’s response. As illustrated in the checklisting episode shown in Figure 4, the pre-service teacher does not follow up her interviewee’s response with a question that aims at that interviewee thinking. She asks “Do you see a pattern?” and follows up her student’s answer with another question “What does that tell you?” signaling to the student that she/he provided sufficient answer the preceeding question.

<table>
<thead>
<tr>
<th>Questioning Category Definition</th>
<th>Example of a Checklisting Questioning Episode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Checklisting:</td>
<td>1. PST: Read the next problem</td>
</tr>
<tr>
<td>Posing one question after another without probing the student’s response.</td>
<td>2. S: Can you build a letter V that uses 36 blocks and follows the pattern?</td>
</tr>
<tr>
<td></td>
<td>3. PST: Yeah. . . wait . . . 36?</td>
</tr>
<tr>
<td></td>
<td>4. S: Do you see a pattern?</td>
</tr>
<tr>
<td></td>
<td>5. PST: They’re odd.</td>
</tr>
<tr>
<td></td>
<td>6. S: What does that tell you?</td>
</tr>
<tr>
<td></td>
<td>7. PST: That you can’t.</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Okay, write that down. You’re right about that. Let’s do the last one.</td>
</tr>
</tbody>
</table>

**Figure 4.** Checklisting Questioning Episode Example

**Instructing.** We classified a questioning episode as instructing if within this episode the pre-service teacher mainly asked questions that directly led the student to an answer. Figure 5 includes an example of such an episode. Instructing question episodes often include leading questions that reveal part of the solution as a clue for the student. This situation is illustrated in the seventh conversational turn in Figure 5 when the pre-service teacher tries to focus the
student’s attention on odd and even numbers:

We’re just trying to say yes or no if there can be a figure made of 36. Look at what we have here. We have one, three, five, seven, so do you think the answer can be 36?

During an instructional questioning episode, sometimes the pre-service teacher stated the answer and asked the student if was correct. This is illustrated by the seventeenth conversational turn in Figure 5: “So you know from 36 since it’s even it won’t fit, right?” Instructing episodes aim at helping students get the solution to the problem, but they do not directly address their thinking about it.
## Questioning Episode

### Definition

**Instructing**
Posing leading questions to guide the student to an answer.

### Example of an Instructing Questioning Episode

| 1. PST: | So on this next one, can you build a letter V that follows the same pattern and uses 36 blocks? It means that you use 36 blocks in the end. So the answer would be 36. Would that work for this pattern? |
| 2. S: | I don’t get it. |
| 3. PST: | Can you build a figure that would have 36 of them in the V? |
| 4. S: | It’s going by two, 36. |
| 5. PST: | So could there be 36? |
| 7. PST: | We’re just trying to say yes or no if there can be a figure made of 36. Look at what we have here. We have one, three, five, seven. So do you think the answer can be 36? |
| 8. S: | No because you take off one. |
| 9. PST: | Good. What about 36? |
| 10. S: | You start with one. |
| 11. PST: | You take one off because you start off with one. But then, is there something about the number 36, that you know it won’t work? What? |
| 12. S: | It’s going by two. |
| 13. PST: | It’s going by two. So the two’s are what? Are they odd or even? Odd. Two is even and then one, three five, are odd. |
| 14. S: | And look at how this is going. . . |
| 15. PST: | Yeah, odd. |
| 16. S: | So you know from 36 since it’s even it won’t fit, right? |
| 17. PST: | Yeah, I was trying to say that. |
| 18. S: | Alright, so you got that. So do you understand that? |
| 19. PST: | Yeah because 36 is going by the pattern. |
| 20. S: | By the figure times two and then what do you do? |
| 21. PST: | Subtract one. |
| 22. S: | So 36 won’t work because? |
| 23. PST: | It’s even. |
| 24. S: | It’s even and you’d have to subtract one. Okay. Would any of the letter V’s in this pattern have an even number of blocks? |

### Figure 5. Instructing Questioning Episode Example

**Probing.** We categorized a questioning episode as probing, if during that episode the preservice teacher mainly asked questions that explored the thinking embedded in a student’s response. When posing probing questions the pre-service teachers demonstrated that they first listened to the student’s response to determine if the response was correct or incorrect, complete or incomplete, and then continued to ask questions until satisfied that the student’s thinking had been uncovered. Figure 6 illustrates this type of questioning situation. When pre-service teachers posed probing questions during an episode, they
asked questions that pushed the student to explain their thinking. For example, as illustrated in Figure 6 (conversational turns 4 and 5), the pre-service teacher followed the student’s response: “Because all of these numbers are odd numbers and 36 is an even number,” with the question “Why are they all odd numbers?” This questioning segment illustrates that the pre-service teacher was still not satisfied that the student’s response in turn 6 (“Because odd numbers are not divisible by 2”) revealed his thinking about the problem. So, she further probed the student’s thinking in the context of the problem (turn 7) asking: “But looking at the pattern, why can’t it ever be an even number?”

<table>
<thead>
<tr>
<th>Questioning Episode Definition</th>
<th>Examples of Probing Questioning Episode</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Probing</strong></td>
<td>Can you find the letter V that follows the pattern and uses 36 blocks?</td>
</tr>
<tr>
<td>Posing questions to probe and uncover the student’s thinking.</td>
<td>1. PST: No.</td>
</tr>
<tr>
<td></td>
<td>2. S: Why not?</td>
</tr>
<tr>
<td></td>
<td>3. PST: Because all of these numbers are odd numbers and 36 is an even number.</td>
</tr>
<tr>
<td></td>
<td>4. S: Why are they all odd numbers?</td>
</tr>
<tr>
<td></td>
<td>5. PST: Because odd numbers are not divisible by 2.</td>
</tr>
<tr>
<td></td>
<td>6. S: But looking at the pattern, why can’t it ever be an even number?</td>
</tr>
<tr>
<td></td>
<td>7. PST: Because if you start out with an odd number, or if you just keep adding, or an odd plus an even equals an odd and you are always adding an even number to the odd number.</td>
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<tr>
<td></td>
<td>8. S: Good job. Read the next question.</td>
</tr>
</tbody>
</table>

**Figure 6.** Probing Questioning Episode Example

**Analysis of questioning patterns of high and low algebraic thinkers.** Using the participants’ strength of algebraic thinking scores as our measure, we identified the group of preservice teachers with the lowest overall algebraic thinking scores (bottom 33%) and the group with the highest overall algebraic thinking scores (top 27%). Then we examined the questioning episodes that each group of pre-service teachers asked during their algebraic thinking interviews. In particular, we examined whether the proportions of checklisting, instructing, and probing episodes that were orchestrated by the high algebraic thinkers were different that the corresponding proportions orchestrated by the low algebraic thinkers.
In addition to this quantitative analysis, we selected one pre-service teacher from each group and conducted a qualitative comparison of the characteristics of the questions they posed. The two participants were selected because both used the same algebra task for one of their algebraic thinking interviews, and their overall algebraic thinking scores were further apart than any other pair of participants who used like tasks.

Results

We begin by presenting results pertaining to the identification of the pre-service teachers with high and low algebraic thinking scores. We then follow up with our analysis of the questioning episodes identified across analyzed transcripts overall, and within the high and low algebraic thinking groups. Finally, we present a detailed discussion of how the two selected participants: Lisa, representing the low algebraic thinking group, and Kelly, representing the high algebraic thinking group, posed questions to reveal middle students’ algebraic thinking in the context of the same algebra-based task.

Algebraic Thinking Proficiency

We divided the participants into three approximately equal sized groups (6, 7, 5) based on three clusters of participant algebraic thinking mean scores. As presented in Table 1, the mean algebraic thinking score of the low group (bottom 33%) was $\bar{M} = 2.18$ (max 3) with $SD = 0.15$. The mean score for the high group (top 27%) was $\bar{M} = 2.73$ (max 3) with $SD = 0.10$. By way of comparison, the overall mean algebraic thinking score, obtained by averaging the participants’ algebraic thinking tasks scores across all participants, tasks, and features, was $\bar{M} = 2.455$ (max 3) with $SD = 0.242$.

<table>
<thead>
<tr>
<th>AT Groups</th>
<th>$\bar{M}$</th>
<th>SD</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low (n = 6)</td>
<td>2.18</td>
<td>0.15</td>
<td>1.93 – 2.34</td>
</tr>
<tr>
<td>Average (n = 7)</td>
<td>2.49</td>
<td>0.03</td>
<td>2.45–2.54</td>
</tr>
<tr>
<td>High (n = 5)</td>
<td>2.73</td>
<td>0.10</td>
<td>2.58 – 2.82</td>
</tr>
</tbody>
</table>
Pre-service Teachers’ Questioning

Our analysis of the 36 algebraic thinking interview transcripts (2 transcripts/participant x 18 participants) revealed 236 questioning episodes. Therefore, on average, each pre-service teacher engaged the middle school student in 6.6 questioning episodes during the course of an algebraic thinking interview. When the questioning episodes were disaggregated by type, as illustrated in Table 2, the data revealed that the pre-service teachers overall predominantly engaged in checklisting episodes (48%) and instructing episodes (32%), with a significantly lower number of questioning episodes within which they asked probing questions (20%).

<table>
<thead>
<tr>
<th></th>
<th>Checklisting Episodes</th>
<th>Instructing Episodes</th>
<th>Probing Episodes</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low AT Group</td>
<td>42 (62%)</td>
<td>26 (38%)</td>
<td>0 (0%)</td>
<td>68</td>
</tr>
<tr>
<td>High AT Group</td>
<td>20 (29%)</td>
<td>18 (26%)</td>
<td>31 (45%)</td>
<td>69</td>
</tr>
<tr>
<td>All PSTs</td>
<td>113 (48%)</td>
<td>76 (32%)</td>
<td>47 (20%)</td>
<td>236</td>
</tr>
</tbody>
</table>

When the results were analyzed with respect to pre-service teachers’ algebraic thinking proficiency scores, however, the questioning pattern was very different. Table 2 illustrates that pre-service teachers in the high and low algebraic thinking groups engaged in approximately the same number of questioning episodes (68 in the low group, and 69 in the high group). However, 45% of all episodes identified in the high group transcripts were probing compared to 0% identified in the low group. Different questioning patterns were also noted for checklisting episodes; sixty two percent of the questioning episodes in the low group transcripts were checking compared to only 29% identified in the high group.

A z-test for two proportions revealed that, in the context of their algebraic thinking interviews, pre-service teachers in the high algebraic thinking group engaged their middle school students in...
significantly smaller proportion of checklisting questioning episodes compared to pre-service teachers in the low algebraic thinking group, \( z = 3.681, p < 0.05 \). On the other hand, pre-service teachers in the high algebraic thinking group engaged their middle school students in probing questioning episodes significantly more frequently than pre-service teachers in the low algebraic thinking group, \( z = 6.08, p < 0.05 \). These results may suggest that pre-service teachers’ own algebraic thinking competencies played an important role in their ability to pose questions to elicit algebraic thinking in students.

**Case Study Participants.** The two participants, Lisa (low algebraic thinking group) and Kelly (high algebraic thinking group), used the same task (see Figure 7) in their work with a middle school student. We use this task to discuss strength of algebraic thinking.

**Figure 7.** Letter V Task Used By Lisa and Kelly

Within the low group, Lisa exemplified the lowest overall algebraic thinking ability (\( \bar{M} = 1.93, SD = 0.151 \)). Of the seven features of algebraic thinking, her ability to justify was the weakest (\( \bar{M} = 1.48 \)) and her ability to organize information was the strongest (\( \bar{M} = 2.61 \)). In contrast Kelly’s performance score (\( \bar{M} = 2.81, SD = 0.108 \)) demonstrated the second highest score of pre-service teachers in the high algebraic thinking group. Kelly’s performance scores across the seven different features showed that she was near proficient (score 3) for predicting patterns and organizing information (\( \bar{M} = 2.91 \)). As in the case of Lisa, Kelly’s ability to justify was the weakest across all seven features of algebraic thinking; however Kelly’s mean justification score was much higher, \( \bar{M} = 2.70 \) when compared with Lisa’s, \( \bar{M} = 1.48 \).
We will present the results by comparing and contrasting the extent to which Lisa’s and Kelly’s questions revealed their student’s ability to use the following four features of algebraic thinking, as solicited by the V task: organizing information, predicting a pattern, describing a rule, and justifying a rule.

**Posing questions to probe student understanding of organizing information.** While interviewing her middle school student, Lisa posed checklisting questions that asked him how he organized information. As illustrated in the following questioning episode, Lisa directly asked the student if he could make a table. She followed the student’s affirmative response by saying “Okay,” thus implying that the student should make a table. The student proceeded to describe how he constructed a table that listed the figure number on the left side of and the number of yellow blocks on the right. The questioning episode concluded as Lisa complimented the student on his work, and asked him to read the next question.

1. S: Number two. How many blocks will be in the 15th figure in the sequence? How did you figure out your answer? Well, I haven’t got to the fifteenth figure yet, so I have to count up two, I’d say twelve more times or thirteen more times. So I start out with 9 blocks and I count up.
2. L: Can you make a table or something?
3. S: Yes.
4. L: Okay.
5. S: So now I’m going to make a table. So for my first blocks I got 9, so that’s the fifth one [figure]. So the sixth goes up two which is eleven. Then go up another two which is thirteen. The eighth one would be fifteen. The ninth, seventeen. Then the tenth, nineteen. The eleventh, twenty-one. Then the twelfth, twenty-three. Then the thirteenth will be twenty five. In the fourteenth there will be seven. And the last one, that hits up fifteen, is twenty-nine. So the answer is twenty-nine.
6. L: Okay, good job, now number three.

Lisa simply affirmed the student’s responses and moved on, rather than follow up on the responses the student provided. For instance, when the student estimated that he needed to count up twelve or thirteen more times to find the 15th figure (first conversational turn) Lisa could have asked the student why he might use this strategy. Rather than probe the student’s thinking, Lisa simply asked if the student could make a table (turn 2). At the end of the
questioning episode, when the student provided the answer (turn 5), Lisa affirmed the answer and moved on to the next question (turn 6). She could have asked the student how the table he created helped to organize the information in the problem to find a pattern. The questioning episode demonstrates the characteristics of checklisting because Lisa moved from one question to the next without probing the student’s responses, which described how he organized the information in the problem, but not how he used it.

Using the same task Kelly also focused on organizing information. However, as the questioning episode demonstrates, when the student produced an organizational scheme Kelly followed up with questions that explored the student’s thinking about organizing problem information.

1. K: While you’re working I’m going to stop you and ask questions. But also while your working if you could vocalize what you are thinking and what you’re doing and why you’re doing it. That will help me understand your mathematical thinking process.

2. S: I say 4 because for this one this is the vertex and then they add one on each side and then they add two on each side and then they add three on each side. So I say 4.

3. K: Can you explain what you mean?

4. S: It says if the pattern continues how many yellow blocks will be in the next Letter V. So that question means how many in all?

5. K: Yes.

6. S: So how many in all that would be . . . because this is 3+3=6 and that’s 7 and so then 4+4=8 and 9, so 9.

7. K: Is there a way you can organize the information to represent what you’re writing?

8. S: I could draw another V.

9. K: What do you mean?

10. S: So this would be the vertex of the boxes and then that would be, four. And this is four. And then what I mean by that is there are four boxes on each side and then in all there would be 8, I mean there would be 9 boxes because 4+4 is 8 plus one is nine.


12. S: How many blocks would be in the 15th figure in the sequence (reading the next question).

First, Kelly demonstrated her awareness of the purpose of her interaction with the student as she began the questioning episode by telling the student that she needs to explain her thinking so Kelly could understand the student’s mathematical processes (turn 1). Although
not a question per se, the statement set the expectation that Kelly would ask questions to explore the student’s thinking. When the student provided what appeared to be an incorrect response (turn 2), rather than correct the student, Kelly pressed the student by asking for clarifications. This provided the student the opportunity to self-correct her understanding of the problem (turn 4). Kelly’s persistence in asking “what do you mean” (turn 9) lead the student to reveal her thinking about the problem, as illustrated in turn 10. In contrast to Lisa, Kelly consistently posed follow up questions to examine the student’s thinking after a response was provided.

**Posing Questions to Probe Student Understanding of Predicting a Pattern.** The checklisting segment below illustrates how Lisa asked a series of questions to stimulate the student’s response related to predicting a pattern: “OK so what is the pattern here? (turn 2), “So what will be in the fifth one” (turn 4), “Try to notice a pattern that you see, okay?” (turn 6). Once the student produced an answer, Lisa praised the response and moved on to the next question. She neglected to probe the student’s understanding of how and why the pattern works. Lisa’s questions proceeded from one to the next without any attempt to uncover the student’s thinking about the pattern.

1. S: If the pattern continues how many yellow blocks will be contained in the next letter v?
2. L: Okay, so what is the pattern here?
3. S: Uhm...
4. L: So what will be in the fifth one?
5. S: Uhm...
6. L: Try to notice a pattern that you see okay?
7. S: I know that it goes up by two. Then it increases by another two, so the fourth one is seven. Count up two more and that would be nine, so nine.
8. L: Okay, well good job, that’s right.

The questioning segment below illustrates Kelly’s interactions with her student. Kelly’s first question “What are you confused about?” (turn 2) aimed to uncover the students’ understanding of the problem, as expressed in conversational turn 1. Once the student verbalized her thinking, Kelly posed a follow up question that built on the student’s response. She pushed the student to further consider how to use what
she already knew to solve the problem “So, do you notice a pattern?” (turn 6), and “How does that help you?” (turn 8).

1. S: How many blocks would be in the 15th figure in the sequence? How could you figure out your answer? I’m confused.
2. K: What are you confused about?
3. S: Do you know how that in the fourth figure there are three boxes on each side?
5. S: And in the third one there are two boxes on each side and in the second one there is one box.
6. K: So do you notice a pattern?
7. S: Yeah, there’s a pattern. So in each pattern they add a box to the side.
8. K: Okay. Well look at figure number one. Before you told me that for the fifth figure, there’s only 4 on each side. How does that help you?
9. S: Well, they are actually subtracting one box. So I figured for the 15th one there would be 14 on each side, so in all there would be 29 boxes.

Kelly’s questions probed what the student already knew and understood about the pattern, encouraged the student to think deeper, and uncovered her algebraic thinking ability. In contrast, Lisa did not attempt to use questions as a mean to uncover student’s thinking and understanding of the problem.

Posing questions to probe student understanding of describing a rule. The questioning segment below illustrates how Lisa used an instructing type questioning episode to lead her student to describe a rule. Her questioning led the student to construct a formula that showed how the rule worked for any sized letter V in the letter V sequence “Ok but can you think of a formula?” (turn 4), “Did you notice anything that you could do to figure it out for any one? (turn 6), “Okay do you see anything, what can we do about it, maybe doubling it? The number two, what would that be doubled? Three doubled what would this be?” (turn 8).

1. S: How could you figure out the number of blocks in any letter V in this pattern? Figure out the numbers for any one?
2.L: Yes for any one. If I told you to figure out the hundredth one or thirtieth?
3.S: Go up two every v pattern.
4.L: Okay but can you think of a formula?
5.S: I believe I did one.
6.L: Well what you said is go up two from the previous figure. But did you notice anything that you could do to figure it out for any one?
7.S: Just start up and keep on going by two until you reach your destination.
8.L: Okay do you see anything, what can we do about it, maybe doubling it? the number two, what would that be doubled? Three doubled what would this be?
9.S: Six. So we double it so that would be two plus two. Keep adding two.
10.L: So if you’re making a formula, we know we would have two somewhere in the formula because you already know we are going up by two. So do you think we could do anything? Let me use the pencil. This is number one, two, three, four, five. If we took times the number?
11.S: Oh, two times the number, so two.
12.L: Do you see what we could do with that?
13.S: Multiply?
14.L: Good. Because look, what is two times one?
15.S: Two.
16.L: Right, and then minus what’s the number?
17.S: One.
18.L: Equals one. So does that work? Try this.
19.S: So it would be two again.
20.L: Times?
21.S: Two times two is four. And then you do two minus two equals...
22.L: Minus one. So you have a formula with the number two times the figure number. Try that. So try to plug in six for n. So you always have six in your formula.
23.S: So you put six right here so 2 x 6 – 1.
24.L: Okay, so what is our formula? We’re taking two each time... times the number, minus one. And that will give you your answer for any shape. See how that works?. Okay, ready for number four?

Lisa’s series of questions clearly intended to lead the student to construct a formula for the pattern. She used questions to provide explicit clues for the student. Unsatisfied with the student’s responses, Lisa engaged in instructing (turn 10) and began to demonstrate to the student how to generate a formula under the guise of questioning to prompt student’ thinking about the problem. In her interactions with the student Lisa never probed the student’s understanding, but
instead, explained her own thinking in the form of asking instructing leading questions.

Kelly’s interactions with her student are illustrated in the questioning episode below. Initially, to develop a rule that allows predicting the number of blocks needed for any figure, the student focused on both sides of the letter V, counting the number of blocks on both sides (turn 1). The student’s response in turn 1, however, does not reveal a complete understanding of how the pattern works. Kelly’s follow up question “So, is there a way you can represent that pattern?” (turn 2) aimed to uncover the thinking that led to the student’s initial response. Kelly’s persistence to uncover the student’s thinking about how the pattern works was evident in her next two questions, which pushed the student to further explain: “So for any V is there a way you can show how many blocks there are?”, and “What were you saying about the bottom one?” (turns 4 and 5). Kelly continued asking questions that would allow her to understand the student’s thinking about the rule. In particular, the student’s explanation in turn 7 is somewhat unclear and does not provide unambiguous insight into the student’s thinking. In fact, it could be construed from turn 7 that the student thinks that doubling the figure number and subtracting one (rather than adding 1) will yield the total number of blocks needed to build the given figure. So Kelly followed up with a series of questions (turn 8, 12, 14, 16, 18) to uncover the student’s reasoning about the rule.

1.S: How could you figure out the number of blocks in any letter V in this pattern? That you subtract one from what there should be. So if it’s the 16th figure, there would be 15 boxes on each side instead of 16 boxes Because I figured out the pattern of the V’s.
2.K: So is there a way you can represent that pattern?
3.S: I believe there is a way but I don’t know if I can figure out a way.
4.K: So for any V is there a way you can show how many blocks there are?
5.S: Yeah. For any V you just subtract the number. You subtract one box from the number of boxes that it should be, or from the figure number.
6.K: What were you saying about the bottom one?
7.S: The bottom one is what I call the vertex of the v because it’s where it starts. If you use the number 48, instead of there being 48 boxes on each side you subtract one from each side so there’s 47 boxes on each side because that’s the pattern.
You subtract one from how many there is supposed to be on each side... from the figure number.

8.K: So it’s the figure number?
10.K: And then what are you doing with the figure number?
11.S: You’re subtracting one from the figure number.
12.K: Okay, and what does that give you?
13.S: If you know the figure number all you have to do is subtract one.
14.K: Can you use your formula and explain it for me one more time?
15.S: This is the figure number one, so one minus one is zero. This is figure number two. Two minus one is one.
16.K: So what is one?
17.S: One is the number you subtract from. What I mean is that because of the pattern, you subtract one from whatever the figure number is.
18.K: So how are you figuring out how many blocks are in the whole V?
19.S: If you use the figure number three, three minus one is two so there would be two boxes on each side. Once you know how many boxes on each side you add that one in. So two plus two is four, and then you add that vertex box and there will be five in all.

In contrast to Lisa who used an instructing questioning episode to lead her student to the description of a rule, Kelly systematically used her series of questions to probe student’s thinking about the rule.

**Posing questions to probe student understanding of justifying a rule.** We use the checklisting questioning episode below to illustrate Lisa’s interactions to engage her student in considering and justifying whether there exists a V-figure constructed of 36 blocks. Lisa’s questions “Would they?” and “Right, because you start with one which is? (turns 2 and 4) do not invite the student to provide any explanations. In fact, once her student provided a short response (turn three), Lisa simply accepted it and moved on to the next question without probing the student’s thinking that led to that response.

1.S: Would any of the letter V in this pattern have an even number of blocks? Why or why not?
2.L: Would they?
4.L: Right, because you start with 1 which is?
5.S: An even . . . no an odd number.

Lisa missed an important opportunity to check the student’s thinking at the end of the episode (turn 5) when the student responded “even” and then corrected himself, stating “odd.”. This change in the student’s response did not prompt Lisa to use the opportunity to ask a followup question that could elucidate her student’s thinking. Even though the problem statement clearly encouraged justification, Lisa did not use it to formulate questions that would probe the student’s thinking about the answers he provided. For Lisa the purpose of asking questions was to get the student to produce an answer.

We contrast Lisa’s questioning episode above with Kelly’s below, which also addressed justification. In contrast to Lisa, Kelly followed up each of her student’s responses with a question to probe the student’s thinking. Kelly frequently asked “why?”, encouraging the student to justify and to reveal her thinking about the problem. For example, after the student shared “But. . . I don’t see how it can be eighteen” (turn 5), Lisa pressed the student to explain by asking “why?” (turn 6), and followed up with “Can you show me why that makes sense if it is no?” (turn 8) to further inquire about the student’s thinking and understanding.

1.S: [reads problem statement] Can you build a letter v that follows the pattern and uses 36 boxes? Yes, if you just follow the pattern, which is subtract one from the figure.
3.S: Thirty-six minus two is four. I’m trying to think if there would be 18 boxes. No, there wouldn’t be. What about, because eighteen plus eighteen equals 36, so thirty and subtract 18 from 2, wait don’t add the vertex box because 36 in all. Sixteen plus sixteen is thirty-two so you couldn’t do that. Seventeen plus seventeen is thirty-four.
4.K: So you’re saying it can’t be sixteen, it can’t be seventeen?
5.S: But... I don’t see how it can be eighteen.
6.K: Why?
7.S: Because of there is eighteen boxes on each side, that’s already thirty-six and plus one would be thirty-seven so that would be no.
8.K: Can you show me why that makes sense if it is no?
9.S: Because we already tried it for seventeen, and seventeen would be seventeen plus seventeen equals thirty-four, plus that vertex
box would equal thirty-five so you still need one more. If you go any less than that, the total number of boxes would go down. And if you go more than that, the number of boxes would go up.

11. S: You would not be able to do it for thirty-six. So there might be other numbers that you can’t do it for, but there must be a pattern of the numbers you can’t do it for.
12. K: So might there be some numbers you can’t have a total of?
13. S: A square number?
14. K: Is there a way you can show that? On paper?
15. S: If you are trying to get nine boxes in all you can put three boxes on each side, which is this, and that’s six and that’s seven. And if you add four boxes on each side, which is here, that’s eight. Wait! You can get it with nine boxes. Eight plus eight that’s one, that’s nine. I mean four plus four is eight and with that nine, and so that wouldn’t work with square numbers.

16. K: Is it square numbers?
17. S: Well, the total is one, three, five . . . oh!
18. K: What do you notice?
19. S: It goes by odd numbers. You can only do it with odd numbers.
20. K: Why?
21. S: Because there wouldn’t be an even number of boxes on each side. What I did in my head was pictured 15 boxes on each side. So sixteen plus sixteen is thirty-two plus the vertex box is thirty-three which I pictured as two boxes on each side, which is not right. So, I’m trying to do it with even numbers and I already tried two because putting two on each side is four plus the vertex box because you always have to include the vertex box so that would be five. And then sixteen one each side, that would be thirty-three. So I’m going to say you can’t do it with any even number.

22. K: Okay, could you read number five?

When the student expressed an idea that the pattern might involve square numbers (turn 15), rather than correct this response, Kelly encouraged further explanations (turn 16, 18). Her questions, unlike those of Lisa’s, were formulated to examine her student’s understanding and to engage her student in thinking about justifying a rule.

Discussion and Conclusion

Our study sheds light on the relationship between pre-service teachers’ algebraic thinking ability and their ability to elicit algebraic thinking in students during problem-based clinical interviews. In
particular, we examined characteristics of patterns of questioning of pre-service teachers with high and low algebraic thinking skills.

Overall, while conducting clinical interviews to engage students in algebraic thinking, our pre-service teachers predominantly used questioning as a means to prompt students for an answer. We identified that in almost half (48%) of all questioning episodes across all interview transcripts, pre-service teachers’ used questions only to prompt students for responses (checklisting). In addition, about 32% of all the participants’ questioning episodes were identified as ones in which the pre-service teachers used questions as a guide to instruct students. Neither type of interaction (checklisting and instructing) provided opportunities for the preservice teachers to gain access to students’ thinking about problems they posed. In fact, the preservice teachers used questions as a mean to elicit students’ thinking about a problem in only about 20% of all the questioning episodes overall.

Our results indicate that pre-service teachers’ questioning ability might relate to their own algebraic thinking proficiency. While the high and low algebraic thinking groups did not significantly differ with respect to the number of instructing episodes, our data uncovered statistically significant differences in the proportion of checklisting, and probing episodes for the two groups. The high algebraic thinking group of pre-service teachers engaged students in probing episodes significantly more frequently compared to the low algebraic thinking group, who in fact did not use this form of questioning at all. The high algebraic thinking group of preservice teachers engaged students in checklisting questioning episodes significantly less frequently than the low algebraic thinking group, who used this form of questioning interactions with a great frequency. Our analysis confirms Nicol’s (1999) and Moyer and Milewicz’s (2002) finding that asking questions to elicit students’ thinking appears to be a difficult task for preservice teachers.

The analysis of the questions asked in the interviews conducted by the two case study participants supported the group results. When Lisa (low algebraic thinking) posed questions, she consistently used a checklisting approach that moved from one question to the next without following up on the student’s responses. This pattern might
suggest that for Lisa the goal of asking questions was to help the student to succeed by providing instruction leading to an answer. It might be that Lisa’s own limited algebraic thinking ability prevented her from understanding the task well enough to ask questions that probed the student’s responses.

In her interactions with her middle school student Kelly (high algebraic thinking), consistently included probing questions to explore the student’s reasoning. Kelly’s questioning episodes consisted of series of questions through which she attempted to gain an understanding of, and to further explore, thinking processes that gave rise to the responses of her student.

Our results suggest an important connection between pre-service teachers’ own algebraic thinking and the characteristics of the questions they posed for students. They also suggest that pre-service teachers’ own algebraic thinking abilities might shape questions pre-service teachers pose as they attempt to engage students in algebraic thinking. The results add to the growing body of evidence that a connection exists between content and pedagogical knowledge (Kazemi and Stipek, 2001; Sahin and Kulm, 2008; Franke et al., 2009; Nicole 1999; Moyer and Milewicz, 2002).

Learning how to elicit and gain access to students’ mathematical thinking through questioning is an important skill that pre-service teachers need to develop during their teacher preparation program. Strengthening the content and pedagogical knowledge that pre-service teachers need in order to pose questions to elicit students’ thinking is an important goal for teacher preparation programs. This study draws further attention to the concerns related to the adequate preparation of pre-service teachers by providing insights about the importance of recognizing that strong content knowledge is needed to establish pedagogical knowledge.
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