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Infrastructure Capital and Private Sector Productivity: A Dynamic Analysis

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This paper examines the relationship between public capital and private sector productivity in the context of a dynamic framework that distinguishes long-run equilibrium relations from short-run disequilibrium values. Using annual data covering the 1948-1987 period we find that there is a stable long-run relationship among private sector productivity, private inputs of capital and labor, and core infrastructure capital. Public capital exerts a positive influence on private sector productivity along this path, although the effect is statistically significant only at low levels of confidence. On the other hand, there appears to be no discernible effect on productivity by core infrastructure capital in the short run. We also find that while public capital is weakly exogenous for the parameters of the long-run relation, it is not strongly exogenous, as it is Granger-caused by private sector productivity.

INTRODUCTION

In recent years, a sizable literature has emerged concerning the effect of public infrastructure capital on the productivity of labor or capital in the private sector. Two empirical approaches have been used to study this issue at the regional, national, and international levels. One approach uses a production function that includes the stock of public capital in addition to private inputs (Aschauer, 1989a, 1989b, 1989c; Eisner, 1991; Holtz-Eakin, 1994; Hulten and Schwab; 1991a, 1991b; Moomaw, Mullen, and Williams, 1995; Munnell, 1990a, 1990b; Tatom, 1991). The second approach uses a cost function dual to a production function that includes public capital as an argument (Berndt and Hansson, 1992; Lynde and Richmond, 1992, 1993; Nadiri and Mamuneas, 1991; Conrad and Seitz, 1994).

The findings concerning the productivity effect of public capital have been mixed. Whereas some find that public infrastructure capital contributes to private sector productivity or lowers production costs, others find no discernible effect attributable to public capital. Much of the time-series evidence on the productivity effect of public capital is based on models that suffer from a number of drawbacks.

One problem is that most studies assume unidirectional causality from public capital to private sector productivity. But one-way causation is only one of four possibilities.
Another possibility is reverse causation whereby higher levels of output in the private sector lead to the accumulation of public capital. As Eisner (1991, p. 49) puts it,

> [s]erious questions remain ... as to which is cause and which is effect. Does public capital contribute to more output? Or do states that have greater output and income, as a consequence of having more private capital and labor, tend to acquire more public capital ...?

A third possibility is a two-way causation or feedback between productivity and public capital. Finally, it is possible that there is no causal relationship between these two variables.

A second problem with much of the literature is the potential endogeneity of public capital. This is conceptually different from the issue of causality, as neither concept is a necessary nor a sufficient condition for the other. Almost all empirical studies of the productivity effect of public capital implicitly assume that public capital and private inputs are exogenous. Unless this assumption is satisfied, the single-equation approach that is used prominently in the literature will produce unreliable results.

A third problem is that most studies do not distinguish the short- and long-run effects of infrastructure capital on private sector productivity. In many instances the univariate properties of the data are not examined. Given that aggregate time-series data are typically nonstationary in the level or logarithmic form, not removing the unit root from the data can lead to spurious results. Recognizing this fact, some authors specify their models of productivity in the first-differenced form which means that they lose long-run information in the data. As Engle and Granger (1987) have shown, a first-differenced model of nonstationary variables is misspecified if the elements of the data vector are cointegrated, in which case an error-correction model is the proper specification.

This paper examines the relationship between public capital and private sector productivity while addressing the problems above in the context of the dynamic framework developed by Johansen (1988) and Johansen and Juselius (1990). A similar approach has been undertaken by Flores de Frutos and Pereira (1993).

Using annual data covering the 1948-1987 period, we find that there is a long-run relationship among private sector productivity, private inputs of capital and labor, and core infrastructure capital. Moreover, public capital exerts a positive influence on private sector productivity along this long-run path, although this effect is only statistically significant at low levels of confidence. On the other hand, the estimated short-run effect of public investment in core infrastructure on the growth rate of labor productivity is not statistically significant at conventional levels. We also find that while public capital is weakly exogenous for the parameters of the cointegrating relation, it is not strongly exogenous because it is Granger-caused by private sector productivity. Nor are the private inputs of labor and capital exogenous for the parameters of the long-run relation. This suggests that the proper approach to analyzing the productivity of public capital is a sys-

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1 Recognizing these limitations, Flores de Frutos and Pereira (1993) use an approach similar to that of the present study. Their model and findings are compared later in the current work.
tem of equations, such as that used in this paper, rather than the single-equation approach commonly employed in the literature.

**METHOD**

Much of the literature on the productivity effect of public capital uses a log-linear Cobb-Douglas production function that relates the level of output to inputs of private and public capital and private labor:

\[ y_t = \beta_0 + \beta_1 n_t + \beta_2 k_{pt} + \beta_3 k_{gt} \]

where:

- \( y_t \) = The logarithm of output;
- \( n_t \) = The log of labor input;
- \( k_{pt} \) = The log of private capital stock; and
- \( k_{gt} \) = The log of public capital stock.

A Cobb-Douglas production function can be transformed into a model of productivity by imposing a constant-returns-to-scale (CRS) restriction, in which case equation (1) can be written as:

\[ y_t - n_t = \beta_0 + \beta_2 (k_{pt} - n_t) + \beta_3 (k_{gt} - n_t) \]

where the left side is the log of the average product of labor. The CRS assumption is not the only way to specify a model of productivity, however. Instead one can start with a log-linear Cobb-Douglas functional form such as equation (1) and subtract the log of labor from both sides of the equation, thus making the left side the log of output per unit of labor:

\[ y_t - n_t = \beta_0 + (\beta_1 - 1)n_t + \beta_2 k_{pt} + \beta_3 k_{gt}. \]

In this case the coefficient on the right labor variable, \((\beta_1 - 1)\), represents the labor elasticity of output minus one. As a result, the sign of this coefficient is expected to be negative if labor is subject to diminishing marginal return. Let the four variables entering equation (3) be represented by the \(1 \times 4\) vector, \( X_t = (y_t - n_t, n_t, k_{pt}, k_{gt}) \). In this study, I am concerned with the following issues regarding the elements of this vector:

1. **Is there a long-run relationship among the elements of** \( X_t \)? This can be answered by testing for cointegration to determine whether the four elements of \( X_t \) share one or more common stochastic trends.

2. **If there is a cointegrating relation among the four series in** \( X_t \), **is public capital exogenous for the parameters of this relation?** If so, **is public capital weakly or strongly exogenous?** In the present context, weak exogeneity of public capital means that the conditional distribution of private sector productivity is entirely determined by the joint distribution of productivity and public capital. In this case, the marginal distribution of public capital adds...
nothing to the information set over which the conditional distribution of productivity is defined. The use of a single-equation approach, as in many empirical studies of the productivity effect of public capital, is valid only if public capital is weakly exogenous. If public capital is not weakly exogenous, then I must incorporate a separate equation for public capital when estimating the conditional mean of productivity.

3. In what direction does causality flow: from public capital to productivity or vice versa? While cointegration implies causality in at least one direction, cointegration tests cannot determine whether causality is unidirectional and in what direction. Unidirectional causality can be ascertained from Granger-noncausality tests that incorporate the cointegrating relation. Causality and exogeneity are two different concepts. Whereas Granger-noncausality would guarantee that we can forecast productivity conditional on future values of public capital, weak exogeneity of public capital would validate estimation of, and inference on, the parameters of a regression model of productivity. The two notions come together in the concept of strong exogeneity that requires both Granger-noncausality and weak exogeneity.

I study these issues in the maximum-likelihood framework of Johansen (1988) and Johansen and Juselius (1990). Suppose the elements of the vector $X_t = (y_t, n_t, k_{pt}, k_{gt})$ are integrated to order one, I(1); that is, they are nonstationary in the logarithmic form but achieve stationarity after they are differenced. Consider the following four equation vector error correction (VEC) model,

\begin{align}
\Delta(y_t - n_t) &= \mu_1 + \Sigma_1 \Delta(y_{t-1} - n_{t-1}) + \Sigma_2 \Delta n_{t-1} + \Sigma_3 \Delta k_{gt} + \Sigma_4 \Delta k_{pt} + \epsilon_{1t} \\
+ \Pi_{11}(y_{t-1} - n_{t-1}) + \Pi_{12} n_{t-1} + \Pi_{13} k_{gt} + \Pi_{14} k_{pt} + \epsilon_{1t} \\
\Delta n_t &= \mu_2 + \Sigma_2 \Delta(y_{t-1} - n_{t-1}) + \Sigma_3 \Delta n_{t-1} + \Sigma_4 \Delta k_{gt} + \Sigma_5 \Delta k_{pt} + \Pi_{21}(y_{t-1} - n_{t-1}) \\
+ \Pi_{22} n_{t-1} + \Pi_{23} k_{gt} + \Pi_{24} k_{pt} + \epsilon_{2t} \\
\Delta k_{gt} &= \mu_3 + \Sigma_3 \Delta(y_{t-1} - n_{t-1}) + \Sigma_4 \Delta n_{t-1} + \Sigma_5 \Delta k_{gt} + \Sigma_6 \Delta k_{pt} + \Pi_{31}(y_{t-1} - n_{t-1}) \\
+ \Pi_{32} n_{t-1} + \Pi_{33} k_{gt} + \Pi_{34} k_{pt} + \epsilon_{3t} \\
\Delta k_{pt} &= \mu_4 + \Sigma_4 \Delta(y_{t-1} - n_{t-1}) + \Sigma_5 \Delta n_{t-1} + \Sigma_6 \Delta k_{gt} + \Sigma_7 \Delta k_{pt} + \Pi_{41}(y_{t-1} - n_{t-1}) \\
+ \Pi_{42} n_{t-1} + \Pi_{43} k_{gt} + \Pi_{44} k_{pt} + \epsilon_{4t} \\
\end{align}

where:

\begin{align}
k &= \text{The lag length;} \\
i &= 1, 2, \ldots, k-1; \text{ and} \\
t &= 1, 2, \ldots, T \text{ is a time index.}
\end{align}

\footnote{Cointegration does not require all four elements of $X_t$ to be I(1) series. All that is needed is that at least two of them be I(1).}
In the above system of equations, the matrix $\Gamma$ represents the short-run dynamics of the relationship among the elements of the data vector and the parameter matrix, $\Pi$, captures the long-run information in the data.\footnote{With the exception of the error-correction term, $\Pi X_{t-1}$, equations (4) through (7) represent a typical vector autoregression (VAR) model in first-differenced form.} The rank of $\Pi$, denoted $r$, determines the number of cointegrating relationships among the members of $X_t$. When $0 < r < p$, two $p \times r$ matrices, $\alpha$ and $\beta$, can be found such that $\Pi = \alpha \beta^T$, where $\beta$ contains the cointegrating vectors and $\alpha$ consists of the corresponding weights or speeds of adjustment. Hypotheses concerning the rank of $\Pi$ can be tested using $-T \Sigma \ln \lambda_j$ as a test statistic, where $\lambda_j, j = r + 1, \ldots, p$, are estimates of the eigenvalues associated with $\Pi$.\footnote{This is known as the \textit{trace test}. Johansen also has developed another test known as the \textit{maximal eigenvalue test}. Unlike the conventional likelihood-ratio tests, the asymptotic distributions of these statistics do not follow the standard $\chi^2$ distribution. Johansen and Juselius (1990, Tables A1-A3) provide the proper critical values.} \footnote{The eigenvalues can be estimated as follows. Regress $\Delta X_i$ and $X_{t-k}$ on a constant and $\Delta X_{t-1}, \Delta X_{t-2}, \ldots, \Delta X_{t-k+1}$. Retrieve the residuals and denote them $R_q$ and $R_w$, respectively, where $k$ is the lag length. Use these to construct the product moment matrices,}

$$S_q = (1/T) \Sigma R_q R_q^T \quad \text{and} \quad S_w = (1/T) \Sigma R_w R_w^T$$

Using these matrices, find the eigenvalues, $\hat{\lambda}_1, \hat{\lambda}_2, \ldots, \hat{\lambda}_p$, and the associated eigenvectors, $V = (v_1, v_2, \ldots, v_p)$, by solving the following equation,

$$(\Delta S_{ak} - S_{ak} R_q R_q^T S_{ak}^{-1}) V = 0.$$

\footnote{It may be argued that cointegration analysis is best suited for high frequency data that typically increase the degrees of freedom. Using Monte Carlo simulations, Hakkio and Rush (1991) find that the power of cointegration tests is more a function of the span of the sample rather than the degrees of freedom.} \footnote{The original source for the data used in this study is the U.S. Department of Labor, Bureau of Labor Statistics. I retrieved the productivity and hours of work series from the DRI Basic Economics data tape, Chapter 7 (Capacity and Productivity), Section 2 (Productivity and Unit Costs), page 7-3. The code for output per hour is LBUOTU, and the code for hours of work is LBUNITU. The data on the stocks of private and public capital were supplied by Alicia Munnell, for which I am grateful to her.} \footnote{I also used total public capital and found results that were consistent with those reported below.}

**DATA AND PRE-TESTS**

I estimate the four-equation VEC model in equations (4) through (7) using annual data for the U.S. covering the years 1948 to 1987.\footnote{The original source for the data used in this study is the U.S. Department of Labor, Bureau of Labor Statistics. I retrieved the productivity and hours of work series from the DRI Basic Economics data tape, Chapter 7 (Capacity and Productivity), Section 2 (Productivity and Unit Costs), page 7-3. The code for output per hour is LBUOTU, and the code for hours of work is LBUNITU. The data on the stocks of private and public capital were supplied by Alicia Munnell, for which I am grateful to her.} For the input of labor, $n$, I use total hours of all persons in the private nonfarm business sector. I use total output also from this sector so that my measure of productivity, $y - n$, is output per hour in the private nonfarm business sector. For the stock of private capital, $k_p$, I use the stock of equipment and structures in the private nonfarm business sector. This excludes inventories, land, and residential structures. The measure of core infrastructure public capital, $k_p$, used in this paper includes highways, mass transit, airports, electrical and gas facilities, water supply facilities, and sewers.\footnote{I also used total public capital and found results that were consistent with those reported below.} Most authors who use the production function approach and time-series data also include the rate of capacity utilization in the manufacturing sector in their model to control for the cyclical movements of productivity. I will not include this variable in my analysis because cointegration is a long-run concept that allows one to determine whether there is a stable long-run relationship between the levels of productivity and other variables, whereas short-run changes in productivity represent deviations from the long-run equilibrium path. These short-run disequilibrium values are captured by $\Gamma$ in equations (4)
through (7); the inclusion of the rate of capacity utilization in the short-run dynamics of the model would be redundant.

I start my empirical analysis with a series of augmented Dickey-Fuller (1979) tests to determine whether the data contain unit roots. In performing this test I take the following steps. Given the well-known fact that the Dickey-Fuller test is a low power test, I first test the null hypothesis of two roots, (2), and then test the null of (1) only if the former hypothesis is rejected. When testing for (2), I include a constant term but not a deterministic trend in the test equation. I test for (1) once with a constant term and a deterministic trend and another time with a constant term but without a trend, however. Finally, I choose the lag length on the augmentation term based on whether the exclusion of lagged terms causes serial correlation in the test equation’s error term. The unit-root test results, which are reported in Table 1, indicate that all variables are I(1).

RESULTS

Based on tests of serial correlation, likelihood-ratio tests, and other model-selection criteria, a lag length of 4 is chosen for the four equation VEC system. The cointegration test results are reported in the top portion of Table 2. The mid-section of this table shows the estimated eigenvectors, β, while the bottom segment contains estimates of the corresponding weights, α. These results indicate that only the null hypothesis of no cointegration, r = 0, can be rejected at the 5 percent level, implying that there is a single cointegrating relationship among the four variables.

The fact that we find a single cointegrating vector has implications for the stability of the system represented by equations (4) through (7). Here stability is defined in the sense that departures from the underlying trend due to random shocks tend to revert to the trend. The more cointegrating relations there are, the more stable (stationary) paths there are to return to following a shock to one or more of its variables. As Dickey et al. (1991, p. 65) observe, “other things the same, it is desirable for an economic system to be stationary in as many directions as possible.” While with four variables in our VEC model, we could have up to three cointegrating relations, we find only one. This means that the long-run relationship among productivity, labor, and private and public capital stocks may be fragile.

Returning to Table 2, we observe that the largest eigenvalue in the top portion of this table (0.4926) corresponds to the equation in the first column in the mid-section of this table, which may be written as follows,

\[(y_t - n_t) = 4.20 - 0.88n_t + 0.62k_{ot} + 0.34k_{gt} \]

\[(10.28) \hspace{1cm} (4.13) \hspace{1cm} (2.74)\]

9 With k = 4, the Lagrange multiplier test for first-order autocorrelation yields \(\chi^2(16) = 18.70\), which is not significant (p-value = 0.28). Similarly, the Jarque-Bera test statistic for normality of the estimated residuals is \(\chi^2(4) = 10.13\), which is not statistically significant at conventional levels (p-value = 0.26).

10 The estimated long-run parameters in the top portion of Table 2 are reported as they would appear on the left side of the estimated equation. Thus, except for the first element of each vector, the other elements carry signs opposite what they would have as the coefficients of the corresponding right side variables.

11 If we were to reject the hypothesis that \(r \leq 1\), we would conclude that the long-run relationship between productivity and inputs is not unique. As discussed below, this would have implications for the stability of the system. Moreover, the multiplicity of the cointegrating vector would lead to identification problems, an issue that is a worthy extension of the present study.
Table 1—Augmented Dickey-Fuller Unit-Root Tests (lag length in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>I(2)*</th>
<th>I(1)*</th>
<th>I(1) With Trend*</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y - n )</td>
<td>-3.58(1)*</td>
<td>-2.38(2)</td>
<td>-0.67(2)</td>
</tr>
<tr>
<td>( n )</td>
<td>-5.94(1)**</td>
<td>0.09(1)</td>
<td>-2.61(1)</td>
</tr>
<tr>
<td>( k_p )</td>
<td>-3.71(1)**</td>
<td>-1.39(1)</td>
<td>-1.75(1)</td>
</tr>
<tr>
<td>( k_g )</td>
<td>-2.97(1)*</td>
<td>-1.55(3)</td>
<td>-0.90(3)</td>
</tr>
</tbody>
</table>

*The test equation is \( \Delta X_t = \alpha + (1 - \rho) \Delta X_{t-1} + \Sigma \Phi \Delta X_{t-

** Significant at the 5 percent level using MacKinnon critical values
*** Significant at the 10 percent level using MacKinnon critical values.

Glossary:
- \( y - n \) = Log of private nonfarm business-sector output per hour
- \( n \) = Log of labor input in the private nonfarm business sector
- \( k_p \) = Log of capital stock in the private nonfarm business sector
- \( k_g \) = Log of public core infrastructure capital stock

where the numbers in parentheses beneath the estimates are asymptotic \( \chi^2 \) statistics and *, **, *** indicate statistical significance at the 10 percent, 5 percent, and 1 percent levels, respectively. All three of the estimated coefficients have the expected signs. Moreover, the implied elasticity of labor equals 0.88 = 0.12 and that of private capital is 0.62. The sum of these two estimates equals 0.74 which is less than one. The estimated coefficient on public capital, 0.34, is also positive and statistically significant albeit at the 10 percent level. This last result is consistent with the findings by Aschauer (1989a) and Munnell (1990a), among others. Even though I did not impose the CRS restriction, the sum of the three estimated coefficients equals 1.08 which is slightly greater than 1.

My estimates of the point elasticities of output with respect to labor and private capital are not consistent with the general consensus in the profession that labor’s share of output is about 0.65 and that of private capital is 0.35. This discrepancy is symptomatic of many time-series studies of the issue using the production function approach, e.g., Munnell (1990a). With two exceptions, the results in equation (8) are similar to those found by Munnell for the case with no CRS constraint (Munnell, 1990a, Table 7, equation 1). My estimate of the elasticity of private capital is identical to hers, and my estimate of the elasticity of public capital is slightly smaller than her estimate of 0.37.

A major difference between our findings is that her estimated coefficient on labor is -1.06 which implies a negative marginal product for labor (-0.06) whereas mine implies a positive value (0.12). Another difference is that her estimates, including that of the coefficient on core infrastructure capital, are statistically significant at the 1 percent level, whereas my estimate of the coefficient on private capital stock is significant at the 5 percent level and that of the coefficient on public capital is barely significant at the 10 percent level (the critical value of \( \chi^2 \) with one degree of freedom equals 2.705). A possible

12 Under diminishing marginal returns, the estimated coefficient on the labor variables is expected to be negative.
13 My estimate of the constant term in equations (8) is also smaller than hers, which equals 4.37.
Table 2—Multivariate Johansen Cointegration Tests, Estimated Eigenvectors, and Weights* (y - n, n, k_p, and k_g)

<table>
<thead>
<tr>
<th>Cointegration Tests</th>
<th>Hypothesized Number of Cointegration Vectors</th>
<th>Eigenvalue</th>
<th>Likelihood Ratio</th>
<th>5% Critical Value</th>
<th>1% Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>r ≤ 3</td>
<td>0.0244</td>
<td>0.89</td>
<td>3.76</td>
<td>6.65</td>
</tr>
<tr>
<td></td>
<td>r ≤ 2</td>
<td>0.2070</td>
<td>9.24</td>
<td>15.41</td>
<td>20.04</td>
</tr>
<tr>
<td></td>
<td>r ≤ 1</td>
<td>0.3779</td>
<td>26.33</td>
<td>29.68</td>
<td>35.65</td>
</tr>
<tr>
<td></td>
<td>r = 0</td>
<td>0.4926</td>
<td>50.75</td>
<td>47.21</td>
<td>54.46</td>
</tr>
</tbody>
</table>

Normalized eigenvectors, β

<table>
<thead>
<tr>
<th>Row</th>
<th>y - n</th>
<th>n</th>
<th>k_p</th>
<th>k_g</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
<td>1.0000</td>
</tr>
<tr>
<td>2</td>
<td>0.8789</td>
<td>3.0367</td>
<td>-0.5134</td>
<td>-3.1551</td>
</tr>
<tr>
<td>3</td>
<td>-0.6238</td>
<td>-2.3557</td>
<td>0.4485</td>
<td>1.5465</td>
</tr>
<tr>
<td>4</td>
<td>-0.3367</td>
<td>0.7040</td>
<td>-1.0796</td>
<td>-1.3604</td>
</tr>
</tbody>
</table>

Weights, α

<table>
<thead>
<tr>
<th>Column</th>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>y - n</td>
<td>-0.300</td>
<td>-0.178</td>
<td>-0.291</td>
<td>0.017</td>
</tr>
<tr>
<td>n</td>
<td>1.538</td>
<td>0.023</td>
<td>-0.146</td>
<td>0.034</td>
</tr>
<tr>
<td>k_p</td>
<td>0.333</td>
<td>0.087</td>
<td>-0.107</td>
<td>0.013</td>
</tr>
<tr>
<td>k_g</td>
<td>-0.053</td>
<td>0.000</td>
<td>0.028</td>
<td>0.005</td>
</tr>
</tbody>
</table>

*All equations assume linear deterministic trend in the data and a lag of three years

Eigenvectors are normalized in terms of y - n

Glossary:

- y - n = Log of private nonfarm business-sector output per hour
- n = Log of labor input in the private nonfarm business sector
- k_p = Log of capital stock in the private nonfarm business sector
- k_g = Log of public core infrastructure capital stock

reason for this may be the fact that I use a system approach and maximum likelihood whereas Munnell uses a single-equation approach and OLS. As Engle and Granger (1987, p. 261) note, a model of I(1) variables expressed in level form "has been pejoratively called a 'spurious' regression by Granger and Newbold (1974) primarily because the standard errors are highly misleading."

I now turn to the issue of exogeneity of public capital. In order to test for weak exogeneity of infrastructure capital, we must test the statistical significance of the estimated weight associated with the stationary part of this variable. This is done using the likelihood-ratio test procedure described in Johansen and Juselius (1990). The results indicate that the estimated weight associated with public capital in the cointegrating equation, which equals 0.017, is not statistically significant [\(\chi^2(1) = 0.218\)] suggesting that public capital is weakly exogenous for the parameters of the cointegrating relation. On the other hand, the estimated weights associated with both private inputs are statistically significant at the 5 percent level [\(\chi^2(1) = 4.19\) for labor and 4.03 for private capital] suggesting that neither input is weakly exogenous for the parameters of the long-run relation. An implication of this finding is that the proper specification for estimating and drawing inference on the productivity of public capital is a system of equations such as that used here, rather than the single-equation approach used by others.

The next step is to test for Granger-noncausality between productivity and public capital. This is done in the context of the system summarized by equations (4) through (7)
Table 3—Maximum Likelihood Estimates of Vector Error Corrections Equations (asymptotic t-statistics in parentheses)

<table>
<thead>
<tr>
<th></th>
<th>$\Delta(y - n)_{t-1}$</th>
<th>$\Delta(y - n)_{t-2}$</th>
<th>$\Delta(y - n)_{t-3}$</th>
<th>$\Delta n_{t-1}$</th>
<th>$\Delta n_{t-2}$</th>
<th>$\Delta n_{t-3}$</th>
<th>$\Delta k_{gt-1}$</th>
<th>$\Delta k_{gt-2}$</th>
<th>$\Delta k_{gt-3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.281275</td>
<td>1.640248</td>
<td>0.374540</td>
<td>-0.21572</td>
<td>3.26362</td>
<td>2.52785</td>
<td>0.95103</td>
<td>-0.08529</td>
<td>-0.004037</td>
</tr>
<tr>
<td>$\Delta n_{t-1}$</td>
<td>-2.23013</td>
<td>2.32699</td>
<td>1.65343</td>
<td>-2.78593</td>
<td>2.69716</td>
<td>2.06787</td>
<td>-1.03012</td>
<td>0.014936</td>
<td>0.036883</td>
</tr>
<tr>
<td>$\Delta n_{t-2}$</td>
<td>-0.875872</td>
<td>1.842009</td>
<td>0.416336</td>
<td>-0.044247</td>
<td>0.144170</td>
<td>0.126894</td>
<td>0.08529</td>
<td>-0.004037</td>
<td>0.004037</td>
</tr>
<tr>
<td>$\Delta n_{t-3}$</td>
<td>-0.248066</td>
<td>1.470147</td>
<td>0.507956</td>
<td>-0.61533</td>
<td>0.283111</td>
<td>0.845262</td>
<td>0.404037</td>
<td>-0.004037</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-1}$</td>
<td>0.415935</td>
<td>0.356308</td>
<td>0.035808</td>
<td>-0.709706</td>
<td>0.738025</td>
<td>0.389364</td>
<td>-0.004037</td>
<td>0.004037</td>
<td>0.004037</td>
</tr>
<tr>
<td>$\Delta k_{gt-2}$</td>
<td>0.039439</td>
<td>0.320926</td>
<td>0.023236</td>
<td>-0.044066</td>
<td>1.475199</td>
<td>0.044212</td>
<td>0.086198</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-3}$</td>
<td>1.978686</td>
<td>0.153980</td>
<td>0.673620</td>
<td>-2.22909</td>
<td>0.079853</td>
<td>1.184999</td>
<td>0.020847</td>
<td>0.020847</td>
<td>0.004037</td>
</tr>
<tr>
<td>$\Delta k_{gt-4}$</td>
<td>0.384309</td>
<td>-2.794755</td>
<td>0.024689</td>
<td>1.219313</td>
<td>0.619498</td>
<td>0.550920</td>
<td>0.194999</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-5}$</td>
<td>-0.090424</td>
<td>-2.204970</td>
<td>0.667569</td>
<td>1.496040</td>
<td>-0.19892</td>
<td>0.604472</td>
<td>0.086198</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-6}$</td>
<td>-0.00986</td>
<td>-1.06188</td>
<td>0.109052</td>
<td>1.069922</td>
<td>1.324170</td>
<td>0.685189</td>
<td>0.194999</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-7}$</td>
<td>2.129131</td>
<td>-0.619498</td>
<td>0.550920</td>
<td>0.133389</td>
<td>-6.381777</td>
<td>-2.267360</td>
<td>0.279779</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-8}$</td>
<td>1.99315</td>
<td>-1.86850</td>
<td>-2.25183</td>
<td>2.47923</td>
<td>1.449343</td>
<td>1.681940</td>
<td>1.09341</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-9}$</td>
<td>-0.751560</td>
<td>1.449343</td>
<td>0.326520</td>
<td>-2.47923</td>
<td>2.200943</td>
<td>1.681940</td>
<td>-0.39291</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-10}$</td>
<td>-0.708363</td>
<td>1.387447</td>
<td>0.572120</td>
<td>-1.60359</td>
<td>1.449590</td>
<td>2.025233</td>
<td>-1.05500</td>
<td>0.020847</td>
<td>0.020847</td>
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<tr>
<td>$\Delta k_{gt-11}$</td>
<td>0.502018</td>
<td>-1.025253</td>
<td>-0.441454</td>
<td>1.53299</td>
<td>-1.44123</td>
<td>-2.105000</td>
<td>0.99415</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-12}$</td>
<td>0.267553</td>
<td>-0.389474</td>
<td>0.047730</td>
<td>1.075101</td>
<td>-0.720455</td>
<td>0.294999</td>
<td>-0.47599</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-13}$</td>
<td>0.146672</td>
<td>0.353476</td>
<td>0.581790</td>
<td>0.009675</td>
<td>0.021017</td>
<td>0.006196</td>
<td>0.001575</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
<tr>
<td>$\Delta k_{gt-14}$</td>
<td>0.972391</td>
<td>99.46923</td>
<td>143.4409</td>
<td>127.3971</td>
<td>99.46923</td>
<td>143.4409</td>
<td>192.7586</td>
<td>0.020847</td>
<td>0.020847</td>
</tr>
</tbody>
</table>

whose estimates are shown in Table 3.\textsuperscript{14} In order to test whether infrastructure investment Granger-causes private sector productivity, one must test the joint significance of the lagged log-differences of public capital, $\Delta k_{gt-1}$, $\Delta k_{gt-2}$, and $\Delta k_{gt-3}$ in the productivity equation (see the first column of Table 3). On the other hand, in order to test whether productivity Granger-causes infrastructure investment, we must test for the joint significance of the once, twice, and thrice lagged values of $\Delta(y - n)_{t-1}$ in the $\Delta k_{gt}$ equation (see the last column of Table 3). The results of Granger tests, reported in Table 4, indicate that the null hypothesis of Granger-noncausality of core infrastructure capital by private sector productivity can be rejected at the 5 percent level. Similarly, the null hypothesis of Granger-noncausality of productivity by infrastructure capital also can be rejected, although at the 1 percent level.

\textsuperscript{14} The estimates associated with the lagged logs of the variables, $(y - n)_{t-1}, n_{t-4}, k_{gt-4},$ and $k_{gt-7},$ constitute the long-run impact matrix, $\Pi$. This is the matrix that has been decomposed into the product $\Delta \Phi$ in Table 2.
Table 4—Pairwise Granger Causality Tests $\Delta(y - n), \Delta k_g$ (lag length = 3)

<table>
<thead>
<tr>
<th>Null Hypothesis</th>
<th>$\chi^2(3)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(y - n)$ does not Granger-cause $k_g$</td>
<td>8.10**</td>
</tr>
<tr>
<td>$k_g$ does not Granger-cause $(y - n)$</td>
<td>17.98***</td>
</tr>
</tbody>
</table>

** Significant at the 5 percent level  
* Significant at the 10 percent level

Glossary:
- $y - n = \log$ of private nonfarm business-sector output per hour
- $k_g = \log$ of public core infrastructure capital stock

Thus, there is feedback between these two variables—each causes and is caused by the other. An implication of the finding that private productivity Granger-causes infrastructure capital is that the latter is not strongly exogenous, even though earlier we found it to be weakly exogenous. This invalidates conditional forecasts of productivity given infrastructure capital.

Finally, consider the short-run effect of the growth of core infrastructure capital on the growth rate of labor productivity. (See the first estimated equation in Table 3.)

The estimated coefficient on the once lagged log-difference of public capital, while positive, is not statistically significant at conventional levels ($t = 1.496$). The estimated coefficients on the twice and thrice lagged values of this variables are negative and statistically insignificant. In the short run, public investment in core infrastructure has no discernible influence on the productivity growth rate. This may explain why those who difference the data to remove the unit root but do not incorporate the long-run (cointegrating) relationship in their analysis find that public capital has no discernible productivity effect (e.g., Tatom, 1991).

Before I conclude this exercise, I examine the sensitivity of the results to structural breaks in the output per hour of work in the nonfarm business sector. An anonymous referee identified two distinct regimes: the “good times” in the 1950s and 1960s when both productivity and public capital grew rapidly, and the “bad times” since. Graphical inspection of the productivity series used in this study reveals that there is a change of pattern starting in 1968 and continuing through the end of the sample period. A similar shift is evident for public capital starting at about the same time as that of productivity.

In order to control for this shift, I construct a dummy variable that assumes a value of zero from 1948 through 1967 and a value one afterward. I include this variable in the four equation model and perform all tests again. I find that the null of no cointegration still can be rejected, albeit at the 10 percent level. The estimated coefficient on the dummy variable is negative in both the productivity and public capital equations. While this estimate is only significant at the 10 percent level of a one tailed test in the productivity equation ($t = -1.45$), however, it is highly statistically significant in the public capital equation ($t = -3.48$). Together these results are consistent with the notion that public capital and productivity are positively correlated. In each of the two private inputs equa-

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15 Because all variables are in log-differenced form, they represent growth rates.
tions, the estimated coefficient associated with the dummy variable is positive but is only significant at the 10 percent level of a one tailed test (t = 1.33 in the labor equation).

The signs, significance, and magnitudes of the other point estimates are not much affected by the inclusion of the change-of-regime dummy variable. The only noticeable change is the estimated long-run coefficient on public capital, which is now 0.29 (which is smaller than the 0.35 estimate without the dummy variable).

**SUMMARY AND CONCLUDING REMARKS**

This paper studies the relationship between private sector productivity and public capital in a multivariate cointegration framework. Using annual data for the U.S. covering the 1948-1987 period, I find that there is a unique long-run relationship between private sector productivity, public capital, and private inputs of labor and capital. The estimated long-run relationship indicates that infrastructure capital exerts a positive effect on productivity, although the estimated effect is statistically significant only at low levels of confidence.

Using maximum likelihood, I estimate a four-equation vector error-correction system and decompose the resulting long-run impact matrix into the product of two matrices, one containing cointegrating vectors and the other consisting of the weights associated with the elements of the former. The weight associated with the stationary part of public capital is not statistically significant, implying that infrastructure capital is weakly exogenous for the parameters of the cointegrating vector. In contrast, the two private inputs are not weakly exogenous; their weights are statistically significant, a result that invalidates estimation and inference in a single equation framework.

Granger-noncausality tests indicate that there is a two way causation or feedback between productivity and infrastructure capital. The fact that productivity Granger-causes public capital means that public capital cannot be strongly exogenous.

Finally, I find no statistically significant short-run effect on productivity growth by government investment in core infrastructure. The relationship between public capital and productivity is essentially a long-run one. This makes sense, as the analytical framework used to estimate these results is a production function that relates to the supply-side of the economy.

Some of the issues examined in this paper (the presence of unit roots in the data, stochastic trends common to productivity and public capital, potential endogeneity of some of the variables, and the nature of causality between productivity and public capital) have been addressed in the literature, but often in different samples and using different econometric methodologies. In contrast, I examine these issues using a vector error-correction model that provides a unified framework that captures both the short- and long-run productivity effects of public capital.

A notable exception to much of the empirical literature in this area is the work of Flores de Frutos and Pereira (1993). Recognizing potential drawbacks of the single-equation approach, they study the dynamic relationship among private output, capital, labor, and public capital using a multivariate, multiple equation VARMA model and annual U.S. data from 1956 to 1989. The major difference between their findings and
those reported in this paper is that they find private output, capital, labor, and public capital are not cointegrated, a result that leads them to use a VAR(1) specification.\textsuperscript{16,17}

Another difference between the two sets of results is that Flores de Frutos and Pereira find public capital is not exogenous, whereas I find it to be weakly exogenous for the parameters of the cointegrating relation. In spite of this difference, there are a number of similarities between the findings of Flores de Frutos and Pereira and those of the present study: evidence of feedback between private productivity and public capital, lack of a discernible short-run effect by public capital, long-run elasticities that are consistent with a Cobb-Douglas production function that exhibits diminishing marginal returns to factors of productions while displaying constant returns to scale, and a large long-run effect on private output or productivity by public capital.

We reach this last result regarding the productivity effect of public capital in the long run even though we use different methods. Flores de Frutos and Pereira's (1993, p. 18) result is based on impulse response functions estimated over a 200 year period. On the other hand, in the present paper the long-run results are based on an estimated cointegrating relationship representing a stable equilibrium path in the sense that departures from this path due to random shocks tend to revert to the underlying stationary state.

In conclusion, even when a multivariate dynamic framework is used that corrects many of the econometric problems associated with the literature, the results indicate that public capital is productive, a result consistent with those reported by Aschauer (1989a, 1989b, 1989c) and Munnell (1990a, 1990b), among others. The only qualification is that this is a the long run effect.

REFERENCES


\textsuperscript{16} Flores de Frutos and Pereira study the effect of various inputs on private output, while I am concerned with the effect of the same inputs on the productivity (output per hour) of labor in the private sector.

\textsuperscript{17} This difference in the cointegration test result may be due, at least in part, to the fact that they use the Engle-Granger (1987) procedure whereas I use the Johansen approach which generally generates more robust results.


